Probability Rules (3A)

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Independent Events

If two events, A and B are independent then the joint probability is

$$P(A \text{ and } B) = P(A \cap B) = P(A)P(B),$$

for example, if two coins are flipped the chance of both being heads is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.[

https://en.wikipedia.org/wiki/Probability

Mutually Exclusive Events

If either event *A* or event *B* occurs on a single performance of an experiment this is called the union of the events *A* and *B* denoted as $P(A \cup B)$. If two events are mutually exclusive then the probability of either occurring is

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B).$$

For example, the chance of rolling a 1 or 2 on a six-sided die is
$P(1 \text{ or } 2) = P(1) + P(2) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}.$

https://en.wikipedia.org/wiki/Probability

Not Mutually Exclusive Events

If the events are not mutually exclusive then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B).$$

For example, when drawing a single card at random from a regular deck of cards, the chance of getting a heart or a face card (J,Q,K) (or one that is both) is $\frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{11}{26}$, because of the 52 cards of a deck 13 are hearts, 12 are face cards, and 3 are both: here the possibilities included in the "3 that are both" are included in each of the "13 hearts" and the "12 face cards" but should only be counted once.

https://en.wikipedia.org/wiki/Probability

Conditional Proabability

Conditional probability is the probability of some event *A*, given the occurrence of some other event *B*. Conditional probability is written $P(A \mid B)$, and is read "the probability of *A*, given *B*". It is defined by[31]

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

If $P(B) = 0$ then $P(A \mid B)$ is formally undefined by this expression. However, it is possible to define a conditional probability for some zero-probability events using a σ-algebra of such events (such as those arising from a continuous random variable).[citation needed]

https://en.wikipedia.org/wiki/Probability

Conditional Probability Examples

For example, in a bag of 2 red balls and 2 blue balls (4 balls in total), the probability of taking a red ball is $1/2$; however, when taking a second ball, the probability of it being either a red ball or a blue ball depends on the ball previously taken, such as, if a red ball was taken, the probability of picking a red ball again would be $1/3$ since only 1 red and 2 blue balls would have been remaining.

https://en.wikipedia.org/wiki/Probability

Inverse Proabability

In probability theory and applications, **Bayes' rule** relates the odds of event A_1 to event A_2, before (prior to) and after (posterior to) conditioning on another event B. The odds on A_1 to event A_2 is simply the ratio of the probabilities of the two events. When arbitrarily many events A are of interest, not just two, the rule can be rephrased as **posterior is proportional to prior times likelihood**, $P(A|B) \propto P(A)P(B|A)$ where the proportionality symbol means that the left hand side is proportional to (i.e., equals a constant times) the right hand side as A varies, for fixed or given B (Lee, 2012; Bertsch McGrayne, 2012). In this form it goes back to Laplace (1774) and to Cournot (1843); see Fienberg (2005). See Inverse probability and Bayes' rule.

https://en.wikipedia.org/wiki/Probability

Probability Rules Summary

Event	Probability
A	$P(A) \in [0,1]$
not A	$P(A^{\complement}) = 1 - P(A)$
A or B	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \cup B) = P(A) + P(B) \quad \text{if A and B are mutually exclusive}$
A and B	$P(A \cap B) = P(A \vert B)P(B) = P(B \vert A)P(A)$ $P(A \cap B) = P(A)P(B) \quad \text{if A and B are independent}$
A given B	$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B \vert A)P(A)}{P(B)}$

Counting tails only

References

[1] http://en.wikipedia.org/
[2] https://en.wikiversity.org/wiki/Discrete_Mathematics_in_plain_view