WikiJournal of Science, 2018, 1(1):6
doi: 10.15347/wjs/2018.005
Research Article

A card game for Bell's theorem and its loopholes

Guy Vandegrift[1]*, Joshua Stomel[1]

Abstract

In 1964 John Stewart Bell made an observation about the behavior of particles separated by macroscopic distances that had puzzled physicists for at least 29 years, when Einstein, Podolsky and Rosen put forth the famous EPR paradox. Bell made certain assumptions leading to an inequality that entangled particles are routinely observed to violate in what are now called Bell test experiments. As an alternative to showing students a "proof" of Bell's inequality, we introduce a card game that is impossible to win. The solitaire version is so simple it can be used to introduce binomial statistics without mentioning physics or Bell's theorem. Things get interesting in the partners' version of the game because Alice and Bob can win, but only if they cheat. We have identified three cheats, and each corresponds to a Bell's theorem "loophole". This gives the instructor an excuse to discuss detector error, causality, and why there is a maximum speed at which information can travel.

The conundrum

Although this can be called a theorem, it might be better viewed as something "spooky" that has been routinely observed, and is consistent with quantum mechanics. But this puzzling behaviour violates what might be called common notions about what is and is not possible.[1][2] Students typically encounter a mathematical theorem as an incomprehensible statement that cannot be digested until it is first proven and then applied in practice. It is not uncommon for novices to refer to some version of Bell's *inequality* as Bell's *theorem* because the inequality can be mathematically "proven".[3] The problem is that what is proven turns out to be untrue.

David Mermin described an imaginary device not unlike that shown in Fig. 1, and refers to the fact that such a device actually exists as a *conundrum*, then pointed out that many physicists deny that it is a conundrum.[4]

A simple Bell's theorem experiment

It is customary to name the particles[5] in a Bell's theorem experiment "Alice" and "Bob", an anthropomorphism that serves to emphasize the fact that a pair of humans cannot win the card game ... unless they cheat. To some experts, a "loophole" is a constraint on any theory that might replace quantum mechanics.[6] It is also possible to view a loophole as a physical mechanism by which the outcome of a Bell's theorem experiment might seem less "spooky". In this paper, we associate loopholes with ways to cheat at the partners' version of the card game. It should be noted that the three loophole mechanisms introduced in this paper raise questions that are even spookier than quantum mechanics: *Are the photons "communicating" with each other? Do they "know" the future? Do they "persuade" the measuring devices to fail when the "cards are unfavorable"?*[7]

Since entanglement is so successfully modeled by quantum mechanics, one can argue that there is no need for a mechanism that "explains" it. Nevertheless, there are reasons for investigating loopholes. At the most fundamental level, history shows that a successful physical theory can be later shown to be an approximation to a deeper theory, and the need for this new theory is typically associated with a failure of the old paradigm. It is plausible that a breakdown of quantum mechanics might be discovered using a Bell's theorem experiment designed to investigate a loophole. But the vast majority of us (including most working physicists) need other reasons to care about loopholes: Many find it interesting that we seem to live in a universe governed by fundamental laws, and Bell's theorem yields insights into the bizarre nature of those laws. Also, those who teach can use these card games to motivate introductory discussions about statistical inference, polarization, and modern physics.

[1] Wright State University Lake Campus, Celina, Ohio, 45822
***Author correspondence**: guy.vandegrift@wright.edu
Supplementary material:
en.wikiversity.org/wiki/WikiJournal_of_Science/A_card_game_for_Bell's_theorem_and_its_loopholes

Received 02-02-2018; accepted 31-05-2018

WikiJournal of Science, 2018, 1(1):6
doi: 10.15347/wjs/2018.005
Research Article

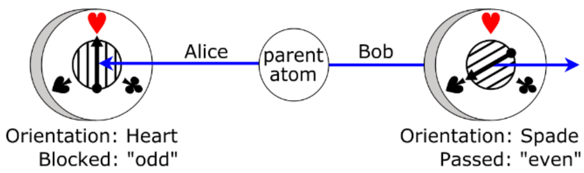

Figure 1 | The outside casing of each device remains stationary while the circle with parallel lines rotates with the center arrow pointing in one of three directions (♥, ♣, ♠.) If Jacks are used to represent these directions, Alice will see J♥ as her question card. She will respond with an "odd"-numbered answer card (3♥) to indicate that she is blocked by the filter. If Bob passes through a filter with the "spade" orientation, he sees J♠ as the question card, and answers with the "even" numbered 2♠. This wins one point for the team because they gave different answers to different questions.

Figure 1 shows a hypothetical and idealized experiment involving two entangled photons simultaneously emitted by a single (parent) atom. After the photons have been separated by some distance, each is exposed to a measurement that determines whether the photon would pass or be blocked by the polarizing filter.[8] To ensure that the results seem "spooky" it should be possible to rotate the filter while the photons are en route so that the filter's angle of orientation is not "known" to either photon until it encounters the filter. If the filters are rotated between only three polarization angles, we may use card suits (hearts ♥, clubs ♣, spades ♠) to represent these angles. These three polarization angles are associated with "question" cards, because the measurement essentially asks the photon a question:

> *"Will you pass through a filter oriented at this angle?"*

For simplicity we restrict our discussion to symmetric angles (0°, 120°, 240°.) The filter's axis of polarization is shown in the figure as parallel lines, with the center line pointing to the heart, club, or spade. Any face card can be used to "ask" the question, and the four face cards (jack, queen, king, ace) are equivalent. If the detectors are flawless, each measurement is binary: The photon either passes or is blocked by the filter (subsequent measurements on a photon would yield nothing interesting.) The measurement's outcome is represented by an even or odd numbered "answer" card (of the same suit). The numerical value of an answer card is not important: all even numbers (2,4,6,8) are equivalent and represent a photon passing through the filter, while the odd cards (3,5,7,9) represent a photon being blocked.

Although Bell's inequality is easy to prove[9], we avoid it here because the card game reverses roles regarding probability: Instead of the investigators attempting to ascertain the photons' so-called hidden variables, the players are acting as particles attempting to win the game by guessing the measurement angles. Another complication is that the original form of Bell's inequality does not adequately model the partners' version of the game because humans have the freedom to exhibit a behavior not observed by entangled particles (under ideal experimental conditions). This behavior involves a 100% correlation (or anti-correlation) whenever the polarization measurement angles are either parallel or perpendicular to each other.[10] In the partners' version of the card game, this behavior must be enforced by deducting a penalty from the partners' score whenever they are caught using a forbidden strategy (which we shall later call the β-strategy). The minimum required penalty is calculated in **Supplementary file:The car and the goats**. Fortunately students need not master this calculation because the actual penalty should often be whatever it takes to encourage a strategy that mimics this aspect of entanglement (which we shall call the α-strategy.)

A theoretical understanding of how one can model entanglement using the Schrödinger equation can be found in **Supplementary file:Tube entanglement**.

The solitaire card game

Figure 2 shows the three possible outcomes associated with one hand of the solitaire version of the game. The solitaire version requires nine cards. The figure uses a set with three "jacks" (♥♣♠) for the questions, and (2,3) for the six (even/odd) answer cards. To play one round of the game, the player first shuffles the three question cards and places them face down so their identity is not known. Next, for each of the three suits, the player selects an even or odd answer card. The figure shows the player choosing the heart and club to be even, while the spade is odd: 2♥ 2♣ 3♠. This is the only viable strategy, since the alternative is to always lose by selecting three answers that are all even or all odd. In the partner's version we shall introduce a second, β-strategy, which is not possible in the solitaire game.

After three answer cards are selected and turned face up, two of the three question cards are randomly selected and also turned face up. Figure 2 depicts all three equally probable outcomes, or ways to select two out of three cards (3 choose 2.)[11] The round is scored by adding or subtracting points, as shown in Table 1: First the suit of each of the two upturned question cards is matched to the corresponding answer card. In case 1 (shown in the figure), the player wins one point because answers are different: ♥ is an even number, while ♠ is

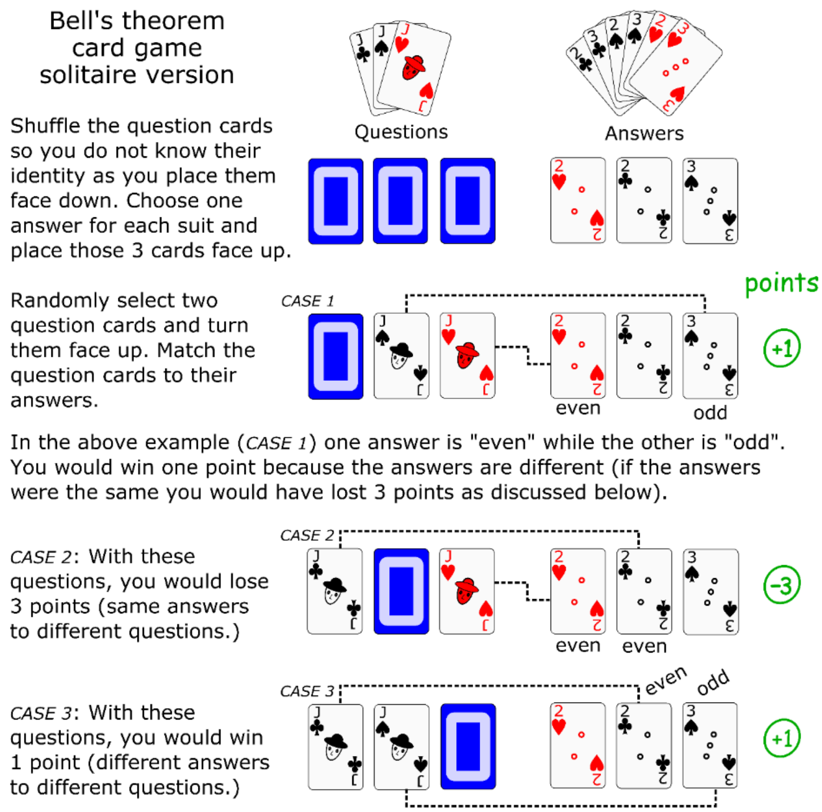

Figure 2 | Solitaire version of game. Cases 1, 2, and 3 represent three possible outcomes if the player chooses the best strategy (later called the "α-strategy": One answer (here, "odd" for ♠) differs from that given for the other two questions (here, "even" for ♥ & ♣).

odd. The player loses three points in case 2 because the ♥ and ♣ are the same (even). Case 3 wins one point for the player because the answers are different. It is evident that the player has a 2/3 probability winning a round. The conundrum of Bell's theorem is that entangled particles in an actual experiment manage to win with a probability of 3/4. Table 1 shows that this scoring system causes humans to average a loss of at least 1/3 of a point per round, while entangled particles maintain an average score of zero.[12] How do particles succeed where humans fail?

Points	Answers are:	Example[13]
+1	different	2♥ and 3♠
+1	different	2♥ and 3♣
−3	same	2♥ and 2♣

Table 1 | Solitaire scoring

The game for entangled partners

In the partners' version, Alice and Bob each play one (even/odd) answer card in response to the suit of a question card. Every round is played in two distinctly different phases. Alice and Bob are allowed to discuss strategy during phase 1 because it simulates the fact that the particles are (effectively) "inside" the parent atom before it emits photons. Then, all communication between the partners must cease during phase 2, which simulates the arrival of the photons at the detectors for measurement under conditions where communication is impossible. In this phase each player silently plays an (even/odd) answer that matches the question's suit. The player cannot know the other's question or answer during phase 2.

In the solitaire version, the player held a deck of six numbered cards and pre-selected (even/odd) answers for each of the three (question) suits. This simulated the parent atom "deciding" the responses that each photon will give to all possible polarization measurements.[14] In an "ideal" Bell's theorem experiment, the two photons'

WikiJournal of Science, 2018, 1(1):6
doi: 10.15347/wjs/2018.005
Research Article

responses to identical polarization measurement angles are either perfectly correlated or perfectly anticorrelated.[8][15] This freedom to independently choose different answers when Alice and Bob are faced with the same question creates a dilemma for the designers of the partners' version of the card game. Adherence to any rule forbidding different answers to the same question cannot always be verified. To enforce this rule, we deduct Q points whenever they give different answers to the same question. No points are awarded for giving the same answer to the same question. Note how this complexity is relevant to actual experiments because detectors can register false events. The minimum penalty that should be imposed depends on how often the partners are given question cards of the same suit, and is derived at **Supplementary file:The car and the goats**

$$Q \geq \frac{4}{3}\left(\frac{1-P_S}{P_S}\right), \qquad (1)$$

where P_S is the probability that Alice and Bob are asked the same question. The equality holds if $P_S = 1/4$ and $Q = 4$, which can be accomplished by randomly selecting two question cards from nine (K♠, K♥, K♣, Q♠, Q♥, Q♣,J♠, J♥, J♣), as shown in Fig. 3. If the equality in (1) holds, the partners are "neutral" with respect to the selection of two different strategies, one of which risks the 4 point penalty. Both strategies lose, but the loss rate is reduced to −1/4 points per round, because the referee must dilute the number of times different questions are asked

A sample round begins in the top part of Fig. 3 as phase 1 where the pipe smoking referee has selected different

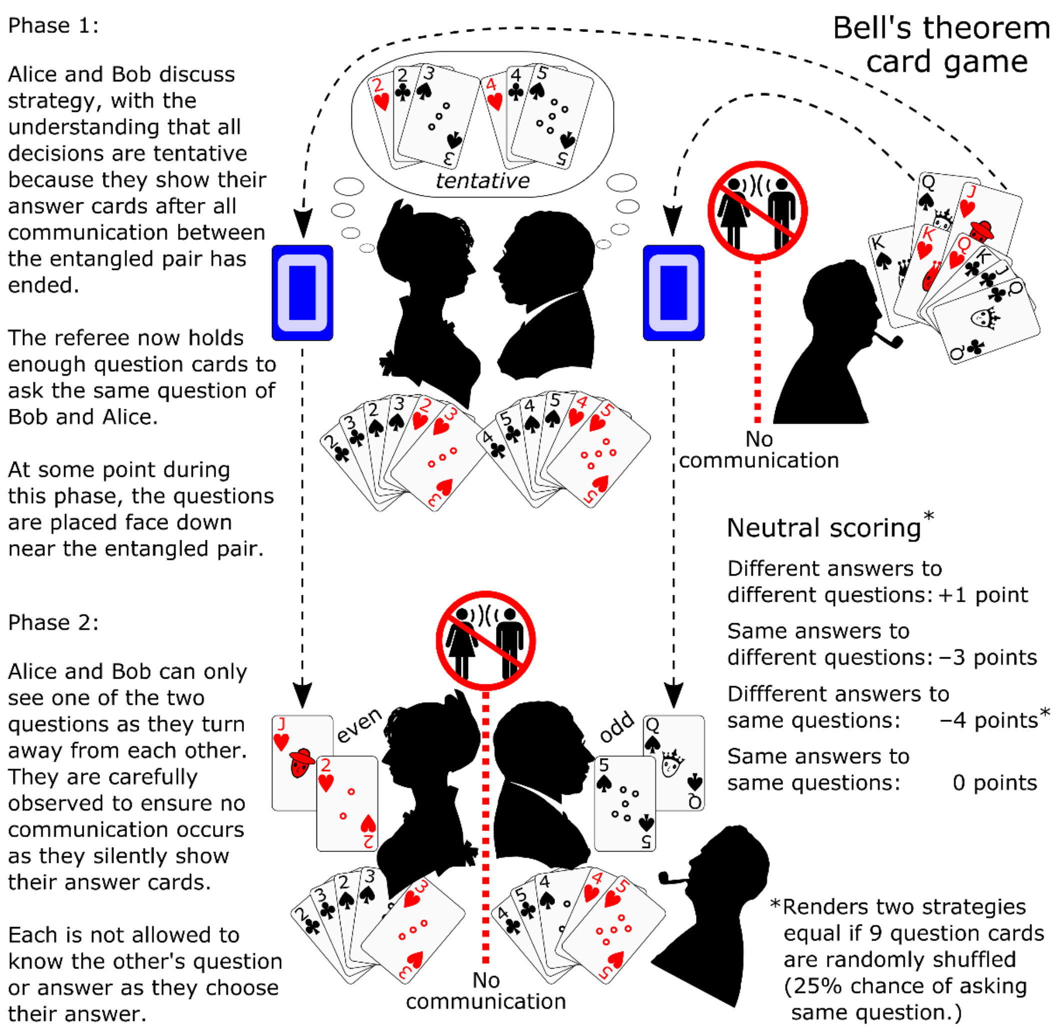

Figure 3 | One round of the partners' version with Alice and Bob employing the same strategy (α) introduced in the solitaire game. Here, a version of "neutral" scoring is used in which the referee randomly selects from the nine question cards, with a penalty of 4 points assessed if different answers are given to the same question. Instructors might wish to override this "neutral" scoring by asking the same question more often than called for in the random selection.

questions (hearts and spades). In a classroom setting, consider allowing Alice and Bob to side-by-side, facing slightly away from each other during phase 2. Arrange for the audience to sit close enough to listen and watch for evidence of surreptitious communication between Alice and Bob. The prospect of cheating not only makes the game more fun, but also allows us to introduce "loopholes". The "thought-bubbles" above the partners show a tentative agreement by the partners to play the same α-strategy introduced in the solitaire version (both say "even" to ♥♣, and "odd" to ♠.) It is important to allow both players to hold all the answer cards in phase 2 so that each can change his or her mind upon seeing the actual question. The figure shows them following their original plan and winning because the referee selected a heart for Alice and a spade for Bob.

But the partners have another strategy that might win: Suppose Alice agrees to answer "even" to any question, while Bob answer is always "odd". This wins (+1) if different questions are asked, and loses (−Q) if the same question is asked. This is called the β-strategy. The Supplementary file:The car and the goats establishes that no other strategy is superior to the α and/or β strategies:

> **α-strategy:** Alice and Bob select their answers in advance, in such a way that both give the same answer if asked the same question. For example, they might both agree that ♥♣ are even, while ♠ is odd. This strategy was ensured in the solitaire version because only three cards are played: If the heart is chosen to be "even", the solitaire version models a situation where both Alice and Bob would answer "even" to "heart". This α-strategy requires that one answer differs from the other two (i.e., all "even" or all "odd" is never a good strategy). The expected loss is 1/3 for each round whenever different questions are asked.

> **β-strategy:** One partner always answers "even" while the other always answers "odd". This strategy gains one point if different questions are asked, and loses Q points if the same question is asked.

For pedagogical reasons, the instructor may wish to discourage the β-strategy. If Alice and Bob are not asked the same question often, they might choose to risk large losses for the possibility winning just a few rounds using the β-strategy, perhaps terminating the game prematurely with a claim that they lost "quantum entanglement". To counter this, the referee can raise the penalty to six points and randomly shuffle only six question cards that result from the merging of two solitaire decks. We refer to any scoring that favors the players' use of the α-strategy as "biased scoring". To further inhibit use of the β-strategy, the referee should routinely override the shuffle and deliberately select question cards of the same suit. The distinction between biased and neutral scoring lies in whether the equality or the inequality holds in (1). Table 2 shows examples of each scoring system. Both were selected to match an integer value for Q. The shuffle of 9 face cards exactly matches the equality in (1) if $Q = 4$, while the more convenient collection of 6 face cards will bias the players towards the the α-strategy if $Q = 6$[16]

Cheating at cards and Bell's theorem "loopholes"

In the card game, Alice and Bob could either win by surreptitiously communicating after they see their question cards, or by colluding with the referee to learn the questions in advance. Which seems more plausible, information travelling faster than light, or atoms acting as if they "know" the future? A small poll of undergraduate math and science college students suggests that they

Neutral scoring: Points if Bob and Alice give...		Shuffle 9 face cards to ask the same question exactly 25% of the time.
+1	different answers to different questions	"even" to hearts "odd" to spades
−3	the same answer to different questions	"even" to clubs "even" to hearts
−4	Different answers to the same question	"even" to clubs "odd" to clubs
0	the same answer to the same question	"even" to clubs (for both players)

Biased scoring: Points if Bob and Alice give...		Shuffle 6 face cards and/or ask the same question with a probability higher than 2/11.
+1	different answers to different questions	"even" to hearts "odd" to spades
−3	the same answer to different questions	"even" to clubs "even" to hearts
−4	Different answers to the same question	"even" to clubs "odd" to clubs
0	the same answer to the same question	"even" to clubs (for both players)

Table 2 | Examples of neutral and biased scoring are inclined to favor faster-than-light communication as being more plausible. We shall use a space-time diagram to illustrate how faster-than-light communication violates causality by allowing people to send signals to their own past. And, we shall argue that one can make the case that decisions made today by humans regarding how and where to perform a Bell's theorem experiment next week, might be mysteriously connected to the behavior of an obscure atom in a distant galaxy billions of years ago.[17]

The third loophole was a surprise for us. In an early trial of the partners' game, a student[18] stopped playing and attempted to construct a modified version of the α-strategy that uses the new information a player gains upon seeing his or her question card. After convincing ourselves that no superior strategy exists, we realized that a player could cheat by terminating the game after seeing his or her own question card, but before playing the answer card. This is related to an important detector efficiency loophole.[19] The student's discovery also alerted us to the fact that our original calculation of (1) was just a lucky guess based on flawed logic.

Referee collusion: Determinism loophole

Alice and Bob could win every round of the partners' version if they cheat by communicating with each other after seeing their question cards in phase 2. In an actual experiment, this loophole is closed by making the measurements far apart in space and nearly simultaneous, which in effect requires that these communications travel faster than the speed of light.[20] While any faster-than-light communication is inconsistent with special relativity, we shall limit our discussion to information that travels at nearly infinite speed.[21]

Figure 4 shows Alice and Bob slightly more than one light-year apart. The dotted world lines for each is vertical, indicating that they remain at rest for over a year. The slopes of world lines of the train's front and rear are roughly 3 years per light-year, corresponding to about 1/3 the speed of light. Both train images are a bit confusing because it is difficult to represent a moving train on a space-time diagram: A moving train can be defined by the location of each end at any given instant in time. This requires the concept of simultaneity, which is perceived differently in another reference frame. The horizontal image of the train at the bottom represents to location of each car on the train on the first day of January, as time and simultaneity are perceived by Alice and Bob. To complicate matters, the horizontal train image is not what they would actually see due to the finite transit time required for light to reach their eyes. It helps to imagine a distant observer situated on a perpendicular to some point on the train. The transit time for light to reach this distant observer will be nearly the same for every car on the train. Many years later, this distant observer will see the horizontal train as depicted at the bottom of the figure. It will be instructive to return to the perspective of this distant observer after the paradox has been constructed.

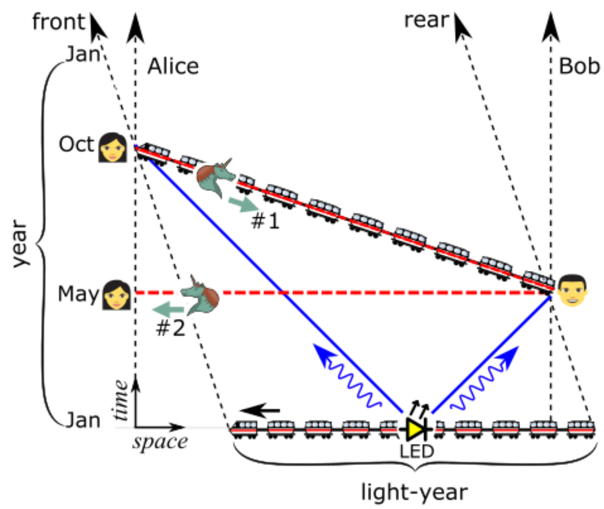

Figure 4 | "Magic phone#1" is situated on a moving train and can be used by Alice to send a message to Bob's past, which Bob relays back to Alice's past using the land-based "Magic phone #2". These magic phones transmit information with near infinite speed.

The slanted image of the train depicts the location of each car on the day that the (moving) passengers perceive the front to be adjacent to Alice, at the same time that the train's rear is perceived to be adjacent to Bob. It should be noted that Alice and Bob do not perceive these two events as simultaneous. The figure shows that the rear passes Bob several months before the front passes Alice (in the partners' reference frame.)

Now we establish that the passengers perceive the front of the train to reach Alice at the same time that the rear reaches Bob. The light-emitting-diode (LED) shown at the bottom of Fig. 4 emits two pulses from the center of the train in January. It is irrelevant whether the LED is stationary or moving because all observers will see the pulses travelling in opposite directions at the speed of light (±1 ly/yr.) Note how the backward moving pulses reaches the rear of the train in May, five months before the other pulse reaches the train's front in October. But, the passengers see two light pulses created at the center of the train, directed at each end of the train, and will therefore perceive the two pulses as striking simultaneously.

To create the causality paradox, we require two "magic-phones" capable of sending messages with nearly infinite speed. Unicorn icons use arrows to depict the information's direction of travel: magic phone #1 transmits from Alice to Bob, while #2 transmits from Bob to Alice. Magic phone #1 is situated on the moving train. When Alice shows her message through the front window as the train passes her in October, a passenger inside relays the message via magic phone #1 to the train's rear, where Bob can see it through a window. Bob immediately relays the message back to Alice via the land-based magic phone #2 in May, five months before she sent it.

Our distant observer will likely take a skeptical view of all this. The slope of the slanted train's image indicates that the distant observer will see magic phone #1 sending information from Bob to Alice, opposite to what the passengers perceive. The distant observer will first see the message inside the rear of the train (when it was adjacent to Bob in May). That message will immediately begin to travel towards of Alice, faster than the speed of light, but slow enough so that Alice will not receive it until October. Meanwhile, Bob sends the same message via land-based phone #2 to Alice, who receives it in May. Alice waits for almost five months, until she prepares to send the same message, showing it through the front window just before the message also arrives at the front via the train-based magic phone #1. It would appear to the distant observer that the events depicted in Fig. 4 had been artificially staged.

This communications loophole in an actual Bell test experiment was closed by arranging for the measurements to coincide so that any successful effort to communicate would suggest that humans could change their own past using this ability to send information faster than light.

Referee collusion: Determinism loophole

This "determinism", or "freedom-of-choice" loophole involves the ability of the quantum system to predict the future. Curiously, the strategy would not be called "cheating" in the card game if Alice or Bob relied on intuition to guess which cards the referee will play in the upcoming round. But what makes this loophole bizarre when applied to a Bell test experiment is that it would have been necessary to predict the circumstances under which the experiment was designed and constructed by human beings who evolved on a plant that was formed almost five billion years ago. On the other hand, viewing the parent atom, the two photons, and the detectors as one integrated quantum entity is consistent with the proper modeling of a quantum-mechanical system. The paradoxical violation of Bell's inequality arises from the need to model two remote particles as one system, so it is not unreasonable to assume that the conundrum can be resolved by including the devices that make the measurements into that model.

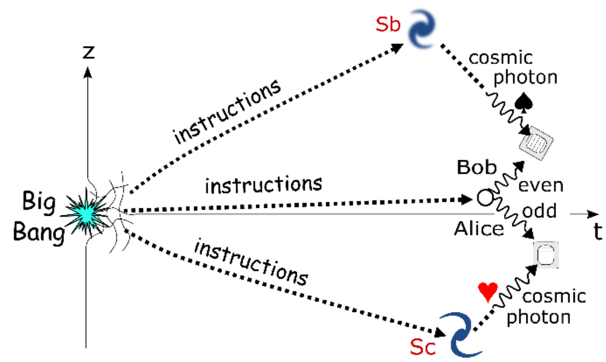

Figure 5 | Cosmic photons from two distant spiral galaxies arrive on Earth with properties that trigger the filters to ask the ♥ & ♠ questions of photons just prior to their arrival with a winning combination of (even/odd) answers.

Figure 5 is inspired by a comment made by Bell during a 1985 radio interview that mentioned something he called "superdeterminism".[22][23] It is a timeline that depicts the big bang, beginning at a time when space and time were too confusing for us to graph. At this beginning, "instructions" were established that would dictate the entire future of the universe, from every action taken by every human being, to the energy, path, and polarization of every photon that will ever exist. Long ago, obscure atoms in two distant galaxies (Sb and Sc) were instructed to each emit what will become "cosmic photons" that strike Earth. Meanwhile, "instructions" will call for humans to evolve on Earth and create a Bell's theorem experiment that uses the frequency and/or polarization of cosmic photons to set the polarization measurement angles while the entangled photons Alice and Bob are still en route to the detectors. Alice and Bob will arrive at their destinations already "knowing" how to respond because the cosmic photons were "instructed" to have properties that cause the questions to be "heart" and "spade".

Viewed this way, the events depicted in Fig. 5 are just the way things happen to turn out. Efforts to enact the scenario with an actual experiment using cosmic photons in this way are being carried out. The most recent experiment looks back at photons created 600 years

WikiJournal of Science, 2018, 1(1):6
doi: 10.15347/wjs/2018.005
Research Article

ago.[24][25] Note also how this experiment does not "close" the loophole, but instead greatly expands the scale of any "collusion" between the parent atom and detectors.

It is claimed that the results of Bell test experiments do not contradict special relativity, despite what may appear to some as faster-than-light "communication" between Alice and Bob.[26] Figure 5 can help us visualize this if the "instructions" represent the time evolution of an exotic version of Schrödinger's equation for the entire universe. If this wave equation is deterministic, future evolution of all probability amplitudes is predetermined. One flaw in this argument is that it relies on an equation that governs the entire universe, and for that reason is not likely to be solved or written down. Perhaps this is why the paradox seems to have no satisfactory resolution.

The Rimstock cheat: Detector error loophole

The following variation of the α-strategy allows the team to match the performance of entangled particles by achieving an average score of zero: Alice preselects three answers and informs Bob of her decision. Bob will either answer in the same fashion, or he might abruptly stop the hand upon seeing his question card, perhaps requesting that the team take a brief break while another pair of students play the role of Alice and Bob. In a card game, this request to stop and replay a hand would require the cooperation of a gullible scorekeeper. But no detector in an actual Bell's theorem experiment is 100% efficient, and this complicates the analysis of a Bell's theorem experiment in a way that requires both careful calibration of the detector's efficiency, as well as detailed mathematical analysis.

♥ (desired)				♠ (undesired)				♣ (undesired)			
Don't stop		Don't stop		Don't stop		Do stop		Don't stop		Do stop	
+1	+1	+1	+1	+1	−3	0	0	+1	−3	0	0
win	win	win	win	win	lose	win	lose	win	lose	win	lose

Figure 6 | The Rimstock cheat: Bob flips a coin to determine whether to play the cheat on this round. Alice will play "even" to hearts and "odd" to spades or clubs.

Since this strategy never calls for Alice and Bob to give different (even/odd) answers to the same question, we may consider only rounds where the players get different questions. To understand why Bob might refuse to play a card, suppose Alice plans to answer "even" to hearts and "odd" to clubs and spades. As indicated at the top of Fig. 6, Bob the heart is the "desired" suit because he knows they will win if he sees that question. But their chances of winning are reduced to only 50% if Bob sees the "undesired" club or spade. To avoid raising suspicion, Bob does not stop the game each time he sees an unfavorable question. Instead, he stops with a 50% probability upon seeing an unfavorable card. To calculate the average score, we construct a probability space consisting of equally probable outcomes, beginning with the three possible suits that Bob might see. We quadruple the size of this probability space (from 3 to 12) by treating the following two pairs of events as independent, and occurring with equal probability:

1. Bob will either stop the hand, or play round (*Do stop* or *Don't stop.)*
2. After seeing his question, Bob knows that Alice might receive one of only two possible questions (ignoring rounds with the same question asked of both.)

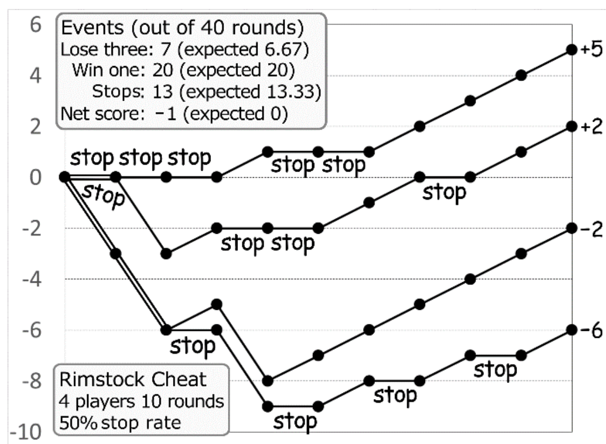

Figure 7 | Four teams of players engaging in the detector error cheat. Each connected dot represents a hand in which different questions were asked, and the horizontal dots simulate a detector error that coincided with a player receiving an unfavorable question.

Figure 6 can be used to show that Bob will stop the game with a probability of 1/3.[27] But if Bob and Alice randomly share this role of stopping the game, each player will stop a given round with only 1/6, yielding an apparent detector efficiency of 5/6 = 83.3%.[19] Typical results for a team playing this ruse are illustrated in Fig. 7. Ten rounds are played on four different occasions. The vertical axis represents in the team's net score, with upward steps corresponding to winning one point, and downward corresponding to losing three points. The horizontal lines showing no change in score indicate occasions where Bob or Alice refused to play an answer card (it was never necessary to ask both partners the same question in this simulation.)

Figure 6 was generated using an Excel spreadsheet using the rand() function, which caused the graphs to change at every ctrl+= keystroke. It took several strokes to get a graph where the lines did not cross, and all the event counts were this close to expected values. As discussed in a supplement, an Excel verification lab is an appropriate activity in a wide variety of STEM courses.

Pedagogical issues

To make sixteen solitaire decks, purchase three identical standard 52 card decks. Remove only one suit (hearts, clubs, spades) from each deck to create four solitaire sets. Each group should contain 3-5 people, and two solitaire decks (for "biased" scoring with Q=6.) To avoid confusion of an ace (question card) with an (even/odd) answer card, reserve the ace for groups with large even/odd number cards. For example, one group might have solitaire sets with (ace,8,9) and (king, 6,7). In a small classroom, the entire audience can observe or even give advice to one pair playing the partners' version at the front of the room. Placing the question cards adjacent to the players at the start will permit the instructor and entire class to join the partners' discussion regarding strategy during phase 1. For "neutral" scoring with Q=4, the instructor can either borrow question cards from the class, or convert unused "10" cards into questions. Since cheating will come so naturally, this game is not suitable for gambling (even for pennies).

Bell's theorem can lead to topics ranging from baseless pseudoscience to legitimate (but pedagogically unnecessary) speculation regarding alternatives to the theory of quantum mechanics. While few physicists are experts in such topics, all teachers will eventually face such issues in the classroom. The authors of this paper claim no expertise in any of this, and the intent is to illustrate the "spookiness" of Bell's theorem, show how one can use simple logic to prove that faster-than-light communication violates special relativity,[21] and introduce students to the concept of a "deterministic" theory or model.[26]

Supplementary material

Subpages

Supplementary file 1 | The car and the goats

(Download) A rigorous proof of the penalty that yields "neutral scoring".

Supplementary file 2 | Impossible correlations

(Download) Extends Bell's inequality to non-symmetric cases and also proves the CHSH inequality without using calculus.

Supplementary file 3 | Tube entanglement

(Download) Describes a simple analog to entanglement with polarized photons. It relies on Maluss' Law, and also introduces Dirac notation as a shorthand representation for the wavefunction of two non-interacting massive particles confined to a narrow tube.

Acknowledgements

Versions of this manuscript have received five referee reports (it was first submitted to the *American Journal of Physics*.) It is obvious that each referee was highly qualified, and that each exerted a considerable effort to improve the quality of this paper.

Competing interests

Guy Vandegrift is a member of the *WikiJournal of Science* editorial board.

References

1. ↑ Bell, John S. (1964). "On the Einstein Podolsky Rosen Paradox". *Physics* **1** (3): 195–200. doi:10.1103/physicsphysiquefizika.1.195. https://wikiversity.miraheze.org/wiki/File:Bell_1964_on_EPR_paradox.pdf.
2. ↑ Vandegrift, Guy (1995). "Bell's theorem and psychic phenomena". *The Philosophical Quarterly* **45** (181): 471–476. doi:10.2307/2220310. http://www.wright.edu/~guy.vandegrift/shortCV/Papers/bell.pdf.
3. ↑ See for example this discussion on the Wikipedia article's talk page, or Wikipedia's effort to clarify this with w:No-go theorem
4. ↑ Mermin, N. David (1981). "Bringing home the atomic world: Quantum mysteries for anybody". *American Journal of Physics* **49** (10): 940–943. doi:10.1119/1.12594. https://wikiversity.miraheze.org/wiki/File:Mermin_AJP_mysteries_for_everybody_1981.pdf. "(referring to those who do not consider this a conundrum) In one sense they are obviously right. Compare Tovey's remark that (Beethoven's) *Waldstein Sonata has no more business than sunsets and sunrises to be paradoxical.*"
5. ↑ or detectors
6. ↑ These (hypothetical) theories are called "hidden variable" theories Larsson, Jan-Åke (2014). "Loopholes in Bell inequality tests of local realism". *Journal of Physics A: Mathematical and Theoretical* **47** (42): 424003. doi:10.1088/1751-8113/47/42/424003.
7. ↑ In w:special:permalink/829073588 these questions are associated with the "communication (locality)", the "free choice" and a "fair sampling" loophole, respectively.
8. ↑ [8.0] [8.1] In most experiments electro-optical modulators are used instead of polarizing filters, and often it is necessary to rotate one set of orientations by 90°. Giustina, Marissa; Versteegh, Marijn A. M.; Wengerowsky, Sören; Handsteiner, Johannes; Hochrainer, Armin; Phelan, Kevin; Steinlechner, Fabian; Kofler, Johannes *et al*. (2015). "Significant-Loophole-Free Test of

Bell's Theorem with Entangled Photons". *Physical Review Letters* **115** (25): 250401. doi:10.1103/physrevlett.115.250401.

9. ↑ Maccone, Lorenzo (2013). "A simple proof of Bell's inequality". *American Journal of Physics* **81** (11): 854–859. doi:10.1119/1.4823600.
10. ↑ See equation 29 in Aspect, Alain (2002). "Bell's theorem: the naive view of an experimentalist". In Bertlmann, Reinhold A.; Zeilinger, Anton. *Quantum [un] speakables* (PDF). Berlin: Springer. p. 119-153. doi:10.1007/978-3-662-05032-3_9.
11. ↑ $\binom{n}{k}$ or "*n* choose *k*" is defined in w:Binomial coefficient
12. ↑ The player can lose more than 1/3 of a point per round by adopting the obviously bad strategy of making all three answers the same (all even or all odd.) This is closely related to the fact that Bell's *"inequality"* is not Bell's *"equation"*.
13. ↑ Since *3-choose-2* equals 6, three other cases exist; all involve 3♥.
14. ↑ Keep in mind that it seems artificial for the parent atom to "know" that these photons are part of an experiment involving just three possible polarization measurements. This need to somehow orchestrate all possible fates for each emitted photon created the EPR conundrum long before Bell's inequality was discovered. See w:EPR paradox.
15. ↑ It is best not to assume that this correlations implies that the "decision" regarding polarization was actually made as the two photons are created by the parent atom. In physics, mathematical models should be judged by whether they yield predictions that can be verified by experiment, not whether these models make any sense.
16. ↑ Equation (1) shows that the case is neutral at
17. ↑ One can also make the case that it is not the role of physics (or science) to speculate in such matters.
18. ↑ User:Rimstock
19. ↑ [19.0] [19.1] Garg, Anupam; Mermin, N. David (1987). "Detector inefficiencies in the Einstein-Podolsky-Rosen experiment". *Physical Review D* **35** (12): 3831–3835. doi:10.1103/physrevd.35.3831.
20. ↑ Aspect, Alain; Dalibard, Jean; Roger, Gérard (1982). "Experimental Test of Bell's Inequalities Using Time- Varying Analyzers". *Physical Review Letters* **49** (25): 1804–1807. doi:10.1103/physrevlett.49.1804.
21. ↑ [21.0] [21.1] Liberati, Stefano; Sonego, Sebastiano; Visser, Matt (2002). "Faster-than-c Signals, Special Relativity, and Causality". *Annals of Physics* **298** (1): 167–185. doi:10.1006/aphy.2002.6233.
22. ↑ Bell, John S. (2004). "Introduction to hidden-variable question". *Speakable and unspeakable in quantum mechanics: Collected papers on quantum philosophy*. Cambridge University Press. pp. 29–39. doi:10.1017/cbo9780511815676.006.
23. ↑ Kleppe, A. (2011). "Fundamental Nonlocality: What Comes Beyond the Standard Models". *Bled Workshops in Physics*. **12**. pp. 103–111. In that interview, Bell was apparently speculating about a deterministic "hidden variable theory where all outcomes are highly dependent on initial conditions.
24. ↑ Gallicchio, Jason; Friedman, Andrew S.; Kaiser, David I. (2014). "Testing Bell's Inequality with Cosmic Photons: Closing the Setting-Independence Loophole". *Physical Review Letters* **112** (11): 110405. doi:10.1103/physrevlett.112.110405.
25. ↑ Handsteiner, Johannes; Friedman, Andrew S.; Rauch, Dominik; Gallicchio, Jason; Liu, Bo; Hosp, Hannes; Kofler, Johannes; Bricher, David *et al*. (2017). "Cosmic Bell Test: Measurement Settings from Milky Way Stars". *Physical Review Letters* **118** (6): 060401. doi:10.1103/PhysRevLett.118.060401.
26. ↑ [26.0] [26.1] See also Ballentine, Leslie E.; Jarrett, Jon P. (1987). "Bell's theorem: Does quantum mechanics contradict relativity?". *American Journal of Physics* **55** (8): 696–701. doi:10.1119/1.15059.
27. ↑ 2/3 is the product of the probability of receiving an unfavorable card, and 1/2 is the probability of stopping; hence (2/3)(1/2)=1/3