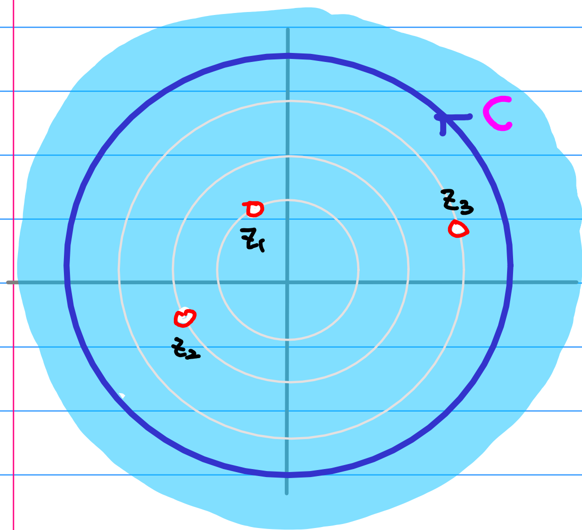
z-Transform 3.Principles

20170714

Series Expansion at $z = 0$



$$f(z) = \sum_{n=n_1}^{\infty} a_n^{\{m\}} z^n$$

$$a_n^{\{m\}} = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z^{n+1}} \, dz$$
$$= \sum_k \mathrm{Res}\left(\frac{f(z)}{z^{n+1}}, z_k\right)$$

Poles z_k

$n \geq 0$	$z_1, z_2, z_3, 0$
$n < 0$	z_1, z_2, z_3

* General Series Expansion at $z=0$

$$f(z) = \sum_{n=n_1}^{\infty} a_n z^n$$

$$a_n = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z^{n+1}}\, dz$$

$$= \sum_k \operatorname{Res}\left(\frac{f(z)}{z^{n+1}}, z_k\right)$$

* z-transform

$$X(z) = \sum_{k=0}^{\infty} x_k z^{-k}$$

$$x_n = \frac{1}{2\pi i} \oint_C X(z) z^{n-1}\, dz$$

$$= \sum_k \operatorname{Res}\left(X(z) z^{n-1}, z_k\right)$$

z - Transform	$X(z)$
Laurent Series	$f(z)$

z-Transform $X(z) \longleftrightarrow x_n$

Laurent Series $f(z) \longleftrightarrow a_n$

$$\boxed{X(z) = f(z^{-1}) \longleftrightarrow x_n = a_n}$$

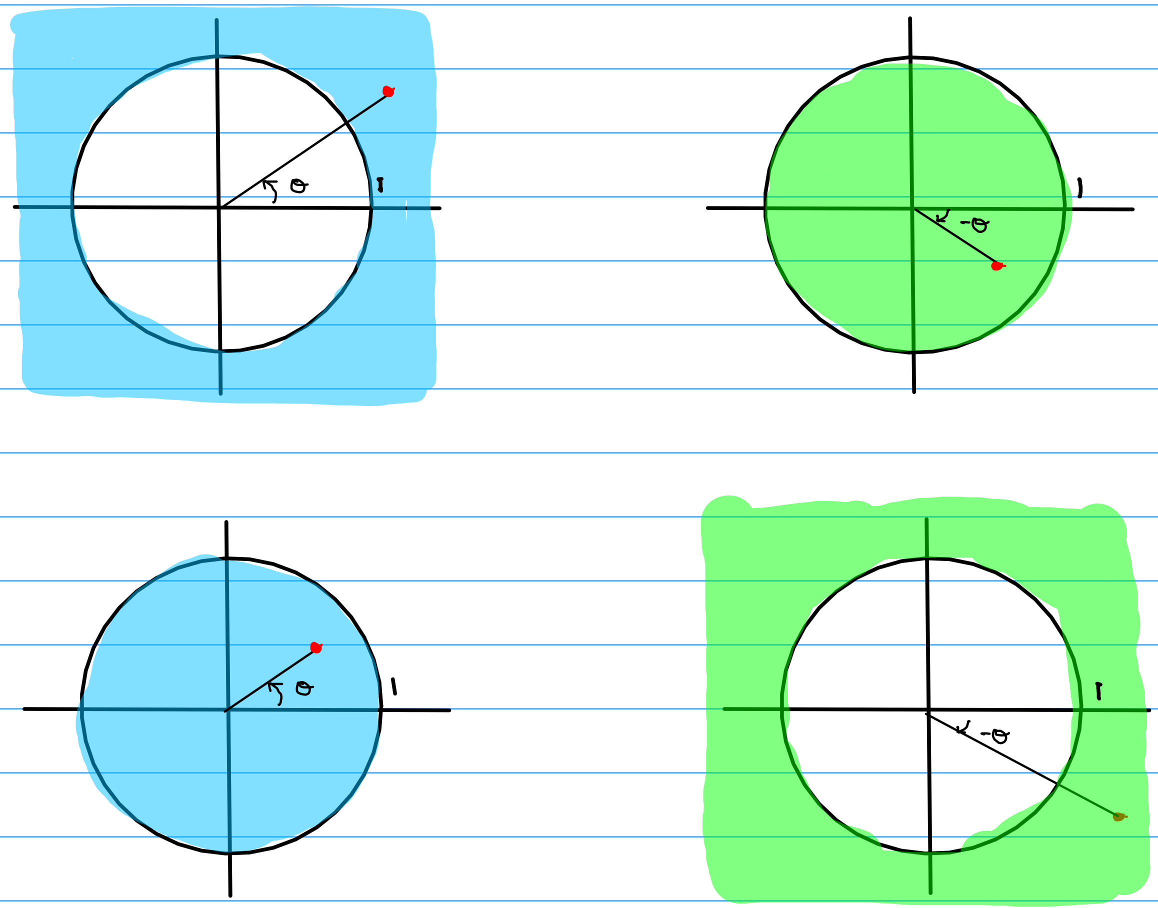
z-Transform $X(z) \longleftrightarrow x_n$

Laurent Series $f(z) \longleftrightarrow a_n$

$$\boxed{X(z) = f(z) \longleftrightarrow x_n = a_{-n}}$$

Mapping $w = \frac{1}{z}$

$z = \rho e^{j\theta}$ $\qquad$ $z^{-1} = \frac{1}{\rho} e^{-j\theta}$



- inverse magnitude
- negative phase

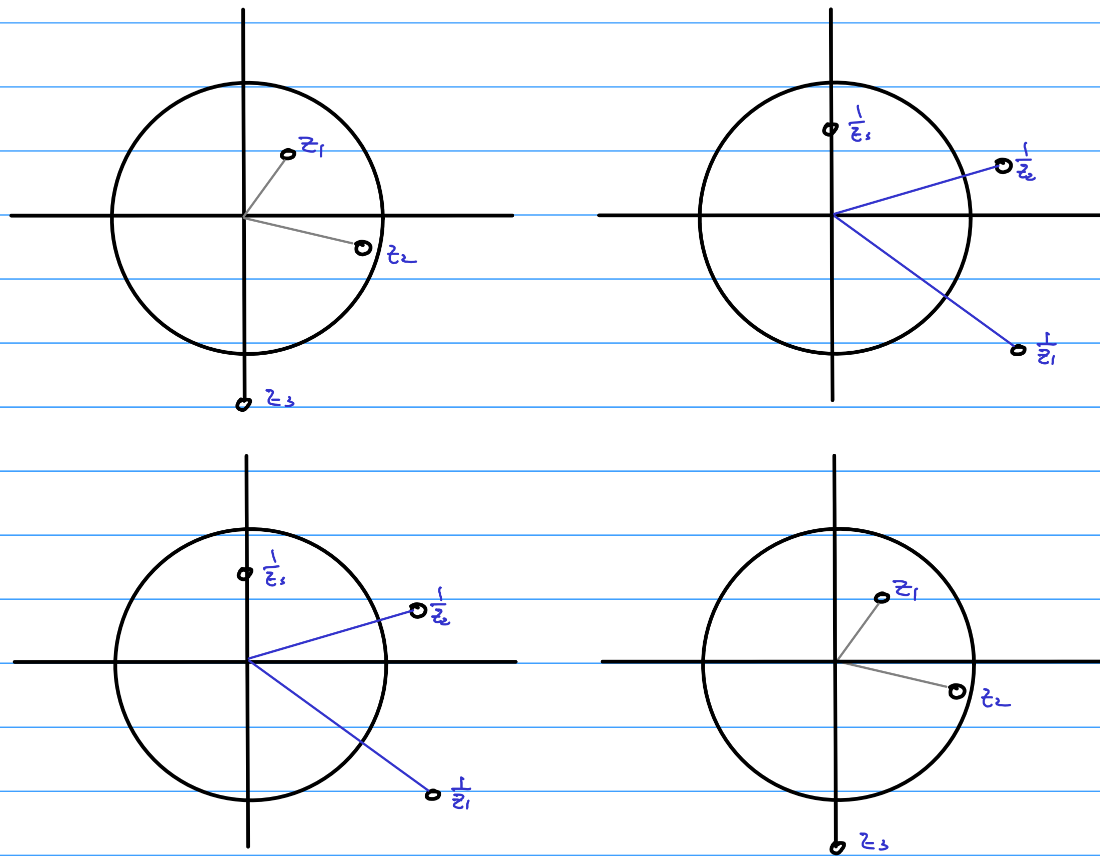
$$f(z) = \frac{(z - z_1)(z - z_2)}{(z - p_1)(z - p_2)(z - p_3)}$$

$$f(z^{-1}) = \frac{(\frac{1}{z} - z_1)(\frac{1}{z} - z_2)}{(\frac{1}{z} - p_1)(\frac{1}{z} - p_2)(\frac{1}{z} - p_3)}$$

$$= \frac{(1 - z_1 z)(1 - z_2 z)}{(1 - p_1 z)(1 - p_2 z)(1 - p_3 z)}$$

$\frac{1}{z_1}, \frac{1}{z_2}$

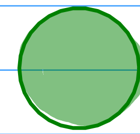
$\frac{1}{p_1}, \frac{1}{p_2}, \frac{1}{p_3}$



$g(z)$ with a simple pole

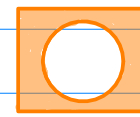
$b > 0$ assumed

$$g(z) = \frac{1}{1-bz} = \boxed{\frac{b^{-1}}{b^{-1}-z}} \qquad |bz| < 1$$



$|z| < \frac{1}{b}$

$$h(z) = \frac{1}{1-\frac{b}{z}} = \boxed{\frac{z}{z-b}} \qquad \left|\frac{b}{z}\right| < 1$$



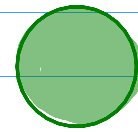
$|z| > b$

$$g(z^{-1}) = \frac{b^{-1}}{b^{-1}-z^{-1}} = \frac{z}{z-b} = h(z)$$

$$h(z^{-1}) = \frac{z^{-1}}{z^{-1}-b} = \frac{b^{-1}}{b^{-1}-z} = g(z)$$

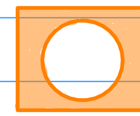
$$\boxed{\begin{aligned} g(z^{-1}) &= h(z) \\ h(z^{-1}) &= g(z) \end{aligned}}$$

$$g(z) = \boxed{\frac{b^{-1}}{b^{-1}-z}} = \frac{\bigcirc}{\bigcirc - z}$$



$$|z| < \frac{1}{b}$$

$$h(z) = \boxed{\frac{z}{z-b}} = \frac{z}{z - \square}$$



$$|z| > b$$

$$g(z^{-1}) = \frac{b^{-1}}{b^{-1}-z^{-1}} = \frac{z}{z-b} = h(z)$$

$$\frac{\bigcirc}{\bigcirc - z^{-1}} = \frac{z}{z - \square}$$

$$h(z^{-1}) = \frac{z^{-1}}{z^{-1}-b} = \frac{b^{-1}}{b^{-1}-z} = g(z)$$

$$\frac{z^{-1}}{z^{-1} - \square} = \frac{\bigcirc}{\bigcirc - z}$$

$$\bigcirc = b^{-1}$$
$$\square = b$$

$$\bigcirc^{-1} = \square$$
$$\square^{-1} = \bigcirc$$

Infinite Sum of G. P.

Simple pole $\Rightarrow$

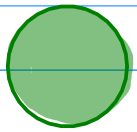
$$\frac{☁}{z-\square} \Rightarrow \frac{z}{z-\square} \Rightarrow \frac{1}{1-\frac{\square}{z}}$$ infinite sum of G.P

$$\frac{☁}{\triangle - z} \Rightarrow \frac{\bigcirc}{\bigcirc - z} \Rightarrow \frac{1}{1-\frac{z}{\bigcirc}}$$ infinite sum of G.P

Convergence Condition

$$\frac{b^{-1}}{b^{-1}-z} \Rightarrow$$ think this way $b^{-1}-z > 0$ $\quad \boxed{b^{-1} > |z|}$ $\quad$ pole b^{-1}

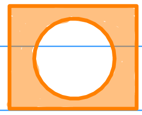
$$\boxed{\frac{b^{-1}}{b^{-1}-z}} = \frac{\bigcirc}{\bigcirc - z}$$



$|z| < \frac{1}{b}$ $\quad |z| < p$

$$\frac{z}{z-b} \Rightarrow$$ think this way $z - b > 0$ $\quad \boxed{|z| > b}$ $\quad$ pole b

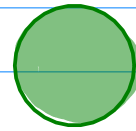
$$\boxed{\frac{z}{z-b}} = \frac{z}{z-\square}$$



$|z| > b$ $\quad |z| > p$

Two Sequences are involved (causal, anti-causal)

$\boxed{\dfrac{b^{-1}}{b^{-1}-z}} = \dfrac{\bigcirc}{\bigcirc - z}$



$|z| < \frac{1}{b}$ $\quad$ $|z| < p$

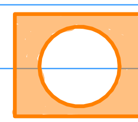
positive seq

① $(n \ge 0)$ $\quad (bz)^0 + (bz)^1 + (bz)^2 + \cdots = \sum_{n=0}^{\infty} b^n z^n$ $\quad$ L.S.

negative seq

② $(n \le 0)$ $\quad (b^{-1}z^{-1})^0 + (b^{-1}z^{-1})^{-1} + (b^{-1}z^{-1})^{-2} + \cdots = \sum_{n=0}^{-\infty} b^{-n} z^{-n}$ $\quad$ Z.T.

$\boxed{\dfrac{z}{z-b}} = \dfrac{z}{z - \square}$



$|z| > b$ $\quad$ $|z| > p$

positive seq

① $(n \ge 0)$ $\quad (bz^{-1})^0 + (bz^{-1})^1 + (bz^{-1})^2 + \cdots = \sum_{n=0}^{\infty} b^n z^{-n}$ $\quad$ Z.T.

negative seq

② $(n \le 0)$ $\quad (b^{-1}z)^0 + (b^{-1}z)^{-1} + (b^{-1}z)^{-2} + \cdots = \sum_{n=0}^{-\infty} b^{-n} z^{n}$ $\quad$ L.S.

$\boxed{n \geq 0}$ $\boxed{n \leq 0}$ $\boxed{\text{L.S.}}$ $\boxed{\text{Z.T.}}$

$$(\quad)^0 + (\quad)^1 + (\quad)^2 + \cdots \longrightarrow (n \geq 0)$$

$$(\quad)^0 + (\quad)^{-1} + (\quad)^{-2} + \cdots \longrightarrow (n \leq 0)$$

$$\sum \boxed{z^n} \longrightarrow \text{L.S.}$$

$$\sum \boxed{z^{-n}} \longrightarrow \text{Z.T.}$$

$$(b\,z) = (b^{-1}z^{-1})^{-1}$$

$(n \geq 0)$		$(n \leq 0)$	
$(b\,z)^{n}$		$(b^{-1}z^{-1})^{n}$	
$b^{n}z^{n}$	L.S.	$b^{-n}z^{-n}$	Z.T.
$n=1,2,\dots$		$n=-1,-2,\dots$	

$$(b\,z^{-1}) = (b^{-1}z)^{-1}$$

$(n \geq 0)$		$(n \leq 0)$	
$(b\,z^{-1})^{n}$		$(b^{-1}z)^{n}$	
$b^{n}z^{-n}$	Z.T.	$b^{-n}z^{n}$	L.S.
$n=1,2,\dots$		$n=-1,-2,\dots$	

$$(b\,z) = (b^{-1} z^{-1})^{-1}$$

	$(n \geq 0)$		$(n \leq 0)$	
L.S.	$\sum\limits_{n=1,2,\cdots} b^{n} z^{n}$	$=$	$\sum\limits_{n=-1,-2,\cdots} b^{-n} z^{-n}$	Z.T.
	$\lvert b\,z \rvert \leqslant 1$	$=$	$\lvert b^{-1} z^{-1} \rvert^{-1} \leqslant 1$	

$$(b\,z^{-1}) = (b^{-1} z)^{-1}$$

	$(n \geq 0)$		$(n \leq 0)$	
Z.T.	$\sum\limits_{n=1,2,\cdots} b^{n} z^{-n}$	$=$	$\sum\limits_{n=-1,-2,\cdots} b^{-n} z^{n}$	L.S.
	$\lvert b\,z^{-1} \rvert \leqslant 1$	$=$	$\lvert b^{-1} z \rvert^{-1} \leqslant 1$	

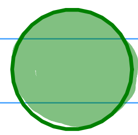
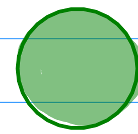
$(n \geq 0)$ $\qquad\qquad$ $(n \leq 0)$

$k \geq 0$

$$b\,z \Leftarrow \left(b^{-1} z^{-1}\right)^{-1}$$

$$b^{n} z^{n} \qquad\qquad b^{-n} z^{-n} \qquad n \leq 0$$

$$\left|\frac{z}{b^{-1}}\right| < 1 \qquad\qquad \left|\frac{z}{b^{-1}}\right| < 1$$

$$\boxed{\frac{b^{-1}}{b^{-1} - z}}$$  $$\boxed{\frac{b^{-1}}{b^{-1} - z}}$$ 

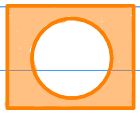
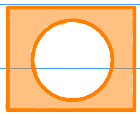
$(n \geq 0)$ $\qquad\qquad$ $(n \leq 0)$

$k \geq 0$

$$b\,z^{-1} \Leftarrow \left(b^{-1} z\right)^{-1}$$

$$b^{n} z^{-n} \qquad\qquad b^{-n} z^{n} \qquad n \leq 0$$

$$\left|\frac{b}{z}\right| < 1 \qquad\qquad \left|\frac{b}{z}\right| < 1$$

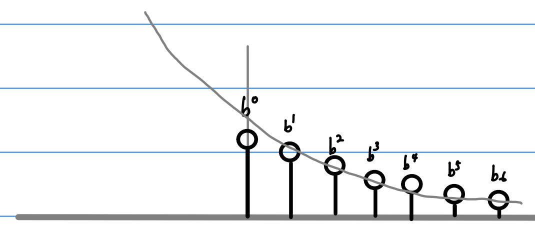
$$\boxed{\frac{z}{z - b}}$$  $$\boxed{\frac{z}{z - b}}$$ 

	$\dfrac{\bigcirc}{\bigcirc - z}$	$\dfrac{z}{z - \square}$
	pole $p = \bigcirc$	pole $p = \square$
	c.r $\left(\dfrac{z}{\bigcirc}\right)$	c.r $\left(\dfrac{\square}{z}\right)$
	r.o.c $\lvert z \rvert < \bigcirc$	r.o.c $\lvert z \rvert > \square$
$(n \geq 0)$	$\sum_{n=0}^{\infty} \left(\dfrac{z}{\bigcirc}\right)^{n} = \sum_{n=0}^{\infty} \left(\dfrac{1}{\bigcirc}\right)^{n} z^{n}$	$\sum_{n=0}^{\infty} \left(\dfrac{\square}{z}\right)^{n} = \sum_{n=0}^{\infty} \square^{n} z^{-n}$
$(n \leqslant 0)$	$\sum_{n=0}^{-\infty} \left(\dfrac{z}{\bigcirc}\right)^{-n} = \sum_{n=0}^{-\infty} \bigcirc^{n} z^{-n}$	$\sum_{n=0}^{-\infty} \left(\dfrac{\square}{z}\right)^{-n} = \sum_{n=0}^{-\infty} \left(\dfrac{1}{\square}\right)^{n} z^{n}$
	L.S: $b^{n} z^{n}$ $(n \geqslant 0)$	Z.T: $b^{n} z^{-n}$ $(n \geqslant 0)$
	Z.T: $b^{-n} z^{-n}$ $(n \leqslant 0)$	L.S: $b^{-n} z^{n}$ $(n \leqslant 0)$
	$p = \bigcirc = b^{-1}$	$p = \square = b$

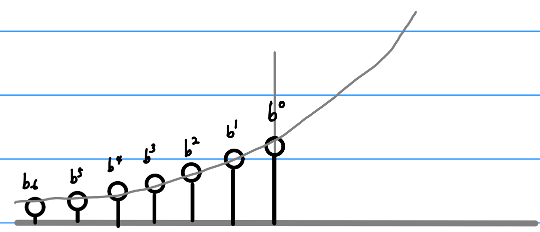
$\boxed{n \geq 0}$ $\boxed{b^n}$ & $\boxed{n \leq 0}$ $\boxed{b^{-n}}$ assumed

	$\dfrac{\bigcirc}{\bigcirc - z}$	$\dfrac{z}{z - \square}$
$(n \geq 0)$	L.S. $\left(\dfrac{z}{\bigcirc}\right)^n \Leftrightarrow b^n z^n$ $(n \geq 0)$	Z.T $\left(\dfrac{\square}{z}\right)^n \Leftrightarrow b^n z^{-n}$ $(n \geq 0)$
$(n \leq 0)$	Z.T $\left(\dfrac{z}{\bigcirc}\right)^{-n} \Leftrightarrow b^{-n} z^{-n}$ $(n < 0)$	L.S. $\left(\dfrac{\square}{z}\right)^{-n} \Leftrightarrow b^{-n} z^{n}$ $(n < 0)$
	$p = \bigcirc = b^{-1}$	$p = \square = b$

$(n \geq 0)$ n $\boxed{b^n}$



$(n \leq 0)$ $-n$ $\boxed{b^{-n}}$



$$\left(\frac{z}{\bigcirc}\right)^{n},\quad \left(\frac{z}{\bigcirc}\right)^{-n},\quad \left(\frac{\square}{z}\right)^{n},\quad \left(\frac{\square}{z}\right)^{-n} \qquad \boxed{p}$$

$\dfrac{z^{+}}{\bigcirc^{-}}$ $\quad \dfrac{z}{p} \longrightarrow$ L.S. $(n \geq 0)\quad p^{-n} z^{n}$ L.S $(n \geq 0) \longrightarrow \boxed{b^{n}}\, z^{n}$ L.S. $p = \bigcirc = b^{-1}$	$\dfrac{\square^{+}}{z^{-}}$ $\quad \dfrac{p}{z} \longrightarrow$ Z.T. $(n \geq 0)\quad p^{n} z^{-n}$ Z.T. $(n \geq 0) \longrightarrow \boxed{b^{n}}\, z^{-n}$ Z.T. $p = \square = b$
$\dfrac{\bigcirc^{+}}{z^{-}}$ $\quad \dfrac{p}{z} \longrightarrow$ Z.T. $(n \leq 0)\quad p^{n} z^{-n}$ Z.T. $(n \leq 0) \longrightarrow \boxed{b^{-n}}\, z^{-n}$ Z.T. $p = \bigcirc = b^{-1}$	$\dfrac{z^{+}}{\square^{-}}$ $\quad \dfrac{z}{p} \longrightarrow$ L.S. $(n \leq 0)\quad p^{-n} z^{n}$ L.S. $(n \leq 0) \longrightarrow \boxed{b^{-n}}\, z^{n}$ L.S. $p = \square = b$

$\dfrac{z^{+}}{p^{-}}$	$\dfrac{p^{+}}{z^{-}}$
L.S.	Z.T.

$$p = \bigcirc = \square^{-1}$$

$$p = \square = \bigcirc^{-1}$$

	L.S. $\qquad p^{-n}z^{n}$	Z.T. $\qquad p^{n}z^{-n}$
$(n \geq 0)$	$\left(\frac{z}{\bigcirc}\right)^{n} \qquad \left(\frac{1}{\bigcirc}\right)^{n}\boxed{z^{n}}$ $\boxed{\frac{\bigcirc}{\bigcirc - z}} \qquad p^{-n}z^{n}$ $z^{0}, z^{1}, z^{2}, \dots$	$\left(\frac{\square}{z}\right)^{n} \qquad \square^{n}\boxed{z^{-n}}$ $\boxed{\frac{z}{z - \square}} \qquad p^{n}z^{-n}$ $z^{0}, z^{-1}, z^{-2}, \dots$
$(n \leq 0)$	$\left(\frac{\square}{z}\right)^{-n} \qquad \left(\frac{1}{\square}\right)^{n}\boxed{z^{n}}$ $\boxed{\frac{z}{z - \square}} \qquad p^{-n}z^{n}$ $z^{0}, z^{-1}, z^{-2}, \dots$	$\left(\frac{z}{\bigcirc}\right)^{-n} \qquad \bigcirc^{n}\boxed{z^{-n}}$ $\boxed{\frac{\bigcirc}{\bigcirc - z}} \qquad p^{n}z^{-n}$ $z^{0}, z^{1}, z^{2}, \dots$
	$(n \geq 0) \quad p = \bigcirc = b^{-1}$ $(n \leq 0) \quad p = \square = b$	$(n \geq 0) \quad p = \square = b$ $(n \leq 0) \quad p = \bigcirc = b^{-1}$

L.S.	Z.T.
L.S.	Z.T.

$(n \geq 0)$ $(n \geq 0)$
$(n \leq 0)$ $(n \leq 0)$

p^{-n}	p^{n}
p^{-n}	p^{n}

b^{n} b^{n}
b^{-n} b^{-n}

$p = \bigcirc$	$p = \square$
$p = \square$	$p = \bigcirc$

$\bigcirc = b^{-1}$	$\square = b$
$\square = b$	$\bigcirc = b^{-1}$

$\square^{n}$	$\square^{n}$
$\square^{-n}$	$\square^{-n}$

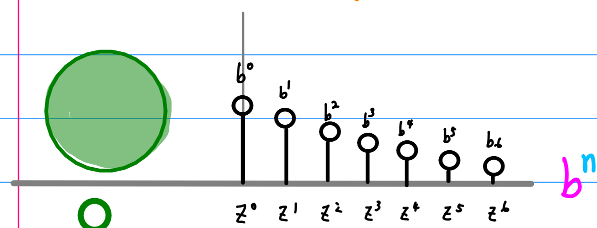
$\square^{n}$ $\square^{n}$
$\square^{-n}$ $\square^{-n}$

$\frac{\bigcirc}{\bigcirc - z}$	$\frac{z}{z - \square}$
$\frac{z}{z - \square}$	$\frac{\bigcirc}{\bigcirc - z}$

$\frac{\square^{-1}}{\square^{-1} - z}$	$\frac{z}{z - \square}$
$\frac{z}{z - \square}$	$\frac{\square^{-1}}{\square^{-1} - z}$

L.S.: $a_n z^n$ Z.T.: $x_n z^{-n}$

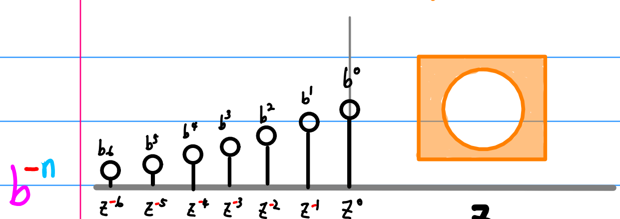
$p^{-n} z^n$



$$\frac{\bigcirc}{\bigcirc - z}$$

$$\frac{b^{-1}}{b^{-1} - z}$$

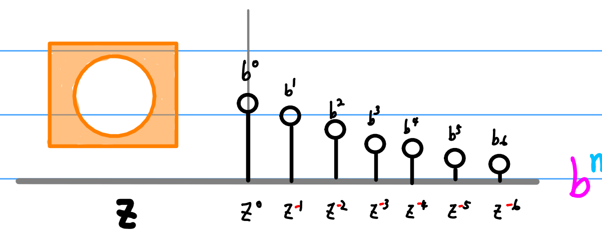
$p^{-n} z^n$



$$\frac{z}{z - \square}$$

$$\frac{z}{z - b}$$

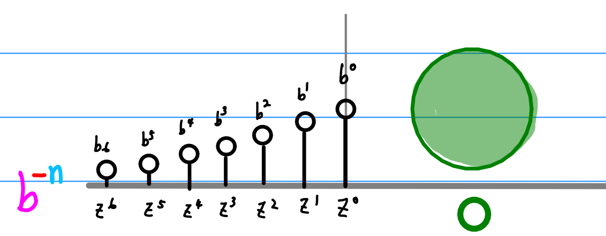
$p^n z^{-n}$



$$\frac{z}{z - \square}$$

$$\frac{z}{z - b}$$

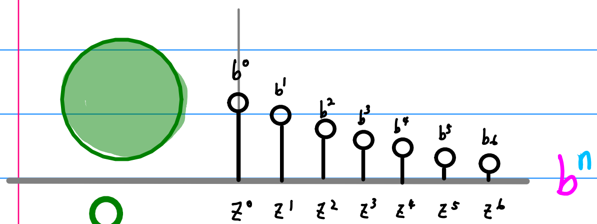
$p^n z^{-n}$



$$\frac{\bigcirc}{\bigcirc - z}$$

$$\frac{b^{-1}}{b^{-1} - z}$$

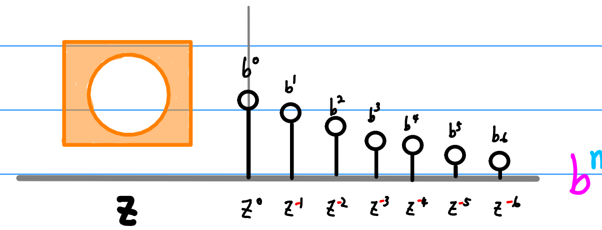
$b^n z^n$



$$\frac{\bigcirc}{\bigcirc - z}$$

$$\frac{b^{-1}}{b^{-1} - z}$$

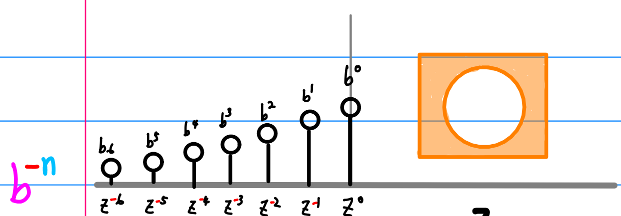
$b^n z^{-n}$



$$\frac{z}{z - \square}$$

$$\frac{z}{z - b}$$

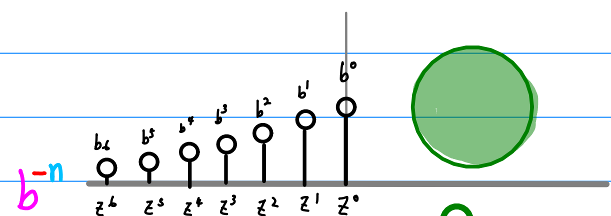
$b^{-n} z^n$



$$\frac{z}{z - \square}$$

$$\frac{z}{z - b}$$

$b^{-n} z^{-n}$



$$\frac{\bigcirc}{\bigcirc - z}$$

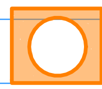
$$\frac{b^{-1}}{b^{-1} - z}$$

Z. T.

$p z^{-1} = b z^{-1}$ $(n \geq 0)$

$$\sum_k (b z^{-1})^k = \frac{z}{z-b}$$

$(k = n \geq 0)$

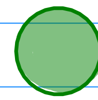
$|b z^{-1}| < 1$ 

$p = b$

$p z^{-1} = b^{-1} z^{-1}$ $(n < 0)$

$$\sum_k (b z)^k = \frac{b^{-1}}{b^{-1} - z}$$

$(k = -n \geq 0)$

$|b z| < 1$ 

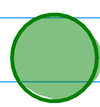
$p = b^{-1}$

L. S.

$p^{-1} z = b z$ $(n \geq 0)$

$$\sum_k (b z)^k = \frac{b^{-1}}{b^{-1} - z}$$

$(k = n \geq 0)$

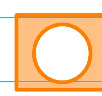
$|b z| < 1$ 

$p = b^{-1}$

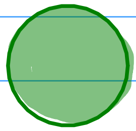
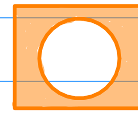
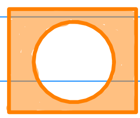
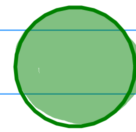
$p^{-1} z = b^{-1} z$ $(n < 0)$

$$\sum_k (b z^{-1})^k = \frac{z}{z-b}$$

$(k = -n \geq 0)$

$|b z^{-1}| < 1$ 

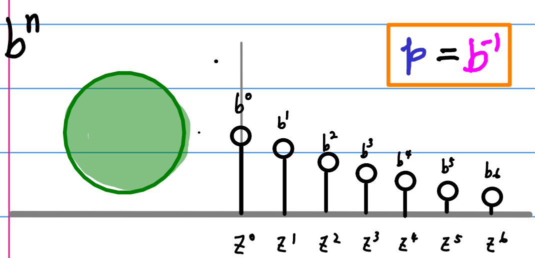
$p = b$

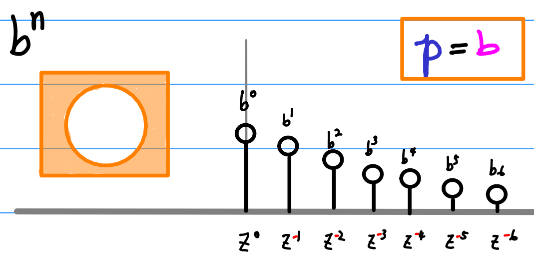
	L.S.	Z.T.
$n \geq 0$	 $\left\|\frac{z}{p}\right\| < 1$	 $\left\|\frac{p}{z}\right\| < 1$
	$\frac{p}{p-z}$ $\frac{\bigcirc}{\bigcirc - z}$ $\frac{b^{-1}}{b^{-1}-z}$	$\frac{z}{z-p}$ $\frac{z}{z-\square}$ $\frac{z}{z-b}$
	$a_n = p^{-n} = b^n$	$x_n = p^n = b^n$
$n < 0$	 $\left\|\frac{p}{z}\right\| < 1$	 $\left\|\frac{z}{p}\right\| < 1$
	$\frac{z}{z-p}$ $\frac{z}{z-\square}$ $\frac{z}{z-b}$	$\frac{p}{p-z}$ $\frac{\bigcirc}{\bigcirc - z}$ $\frac{b^{-1}}{b^{-1}-z}$
	$a_n = p^{-n} = b^{-n}$	$x_n = p^n = b^{-n}$

$(n \geq 0)$	$a_n = x_n = b^n$
$(n < 0)$	$a_n = x_n = b^{-n}$

Laurent Series	$a_n = p^{-n}$	$(n \geq 0, \ n < 0)$
z-Transform	$x_n = p^n$	$(n \geq 0, \ n < 0)$

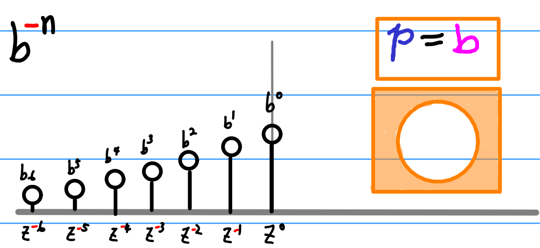
L.S. | Z.T.

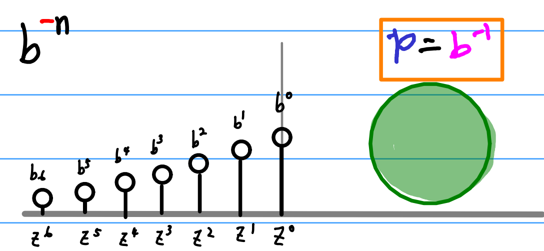




$$\boxed{a_n = p^{-n} \quad (n \geq 0)}$$

$$\boxed{x_n = p^n \quad (n \geq 0)}$$



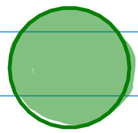
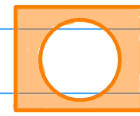
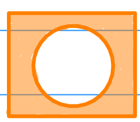
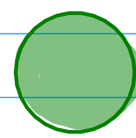


$$\boxed{a_n = p^{-n} \quad (n \leq 0)}$$

$$\boxed{x_n = p^n \quad (n \leq 0)}$$

$(n \geq 0)$	$a_n = x_n = b^n$
$(n < 0)$	$a_n = x_n = b^{-n}$

Laurent Series	$a_n = p^{-n}$	$(n \geq 0,\ n < 0)$
z-Transform	$x_n = p^n$	$(n \geq 0,\ n < 0)$

	Laurent Series	Z - Transform
$n \geq 0$	 $\lvert z \rvert < p$ $\left\lvert \frac{z}{p} \right\rvert < 1$ ① $z^1, z^2, z^3, \cdots$ ② $\frac{1}{1-\frac{z}{p}} = \frac{p}{p-z}$ ③ $a_n = p^{-n} = b^n \quad (p = b^{-1})$	 $\lvert z \rvert > p$ $\left\lvert \frac{p}{z} \right\rvert < 1$ ① $z^{-1}, z^{-2}, z^{-3}, \cdots$ ② $\frac{1}{1-\frac{p}{z}} = \frac{z}{z-p}$ ③ $x_n = p^n = b^n \quad (p = b)$
$n \leq 0$	 $\lvert z \rvert > p$ $\left\lvert \frac{p}{z} \right\rvert < 1$ ① $z^{-1}, z^{-2}, z^{-3}, \cdots$ ② $\frac{1}{1-\frac{p}{z}} = \frac{z}{z-p}$ ③ $a_n = p^{-n} = b^{-n} \quad (p = b)$	 $\lvert z \rvert < p$ $\left\lvert \frac{z}{p} \right\rvert < 1$ ① $z^1, z^2, z^3, \cdots$ anti-causal ② $\frac{1}{1-\frac{z}{p}} = \frac{p}{p-z}$ ③ $x_n = p^n = b^{-n} \quad (p = b^{-1})$

$$a_n = \left(\frac{1}{2}\right)^n \quad (n \geqslant 0)$$
$$= p^{-n} \quad (n \geqslant 0) \quad p = 2$$
$$f(z) = \frac{2}{2-z}$$

$$x_n = \left(\frac{1}{2}\right)^n \quad (n \geqslant 0)$$
$$= p^{n} \quad (n \geqslant 0) \quad p = \frac{1}{2}$$
$$X(z) = \frac{z}{z-0.5}$$

$$a_n = \left(\frac{1}{2}\right)^{-n} \quad (n \leqslant 0)$$
$$= p^{-n} \quad (n \leqslant 0) \quad p = \frac{1}{2}$$
$$f(z) = \frac{z}{z-0.5}$$

$$x_n = \left(\frac{1}{2}\right)^{-n} \quad (n \leqslant 0)$$
$$= p^{n} \quad (n \leqslant 0) \quad p = 2$$
$$X(z) = \frac{2}{2-z}$$

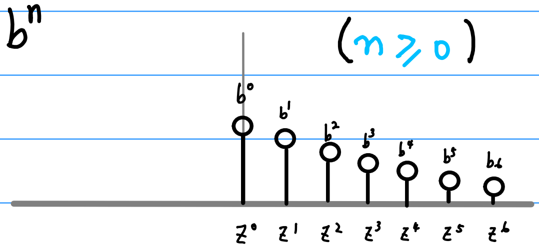
$$a_n = b^n \quad (n \geqslant 0)$$
$$= p^{-n} \quad (n \geqslant 0) \quad p = b^{-1}$$
$$f(z) = \frac{b^{-1}}{b^{-1}-z}$$

$$x_n = b^n \quad (n \geqslant 0)$$
$$= p^{n} \quad (n \geqslant 0) \quad p = b$$
$$X(z) = \frac{z}{z-b}$$

$$a_n = b^{-n} \quad (n \leqslant 0)$$
$$= p^{-n} \quad (n \leqslant 0) \quad p = b$$
$$f(z) = \frac{z}{z-b}$$

$$x_n = b^{-n} \quad (n \leqslant 0)$$
$$= p^{n} \quad (n \leqslant 0) \quad p = b^{-1}$$
$$X(z) = \frac{b^{-1}}{b^{-1}-z}$$

b^n $(n \geq 0)$

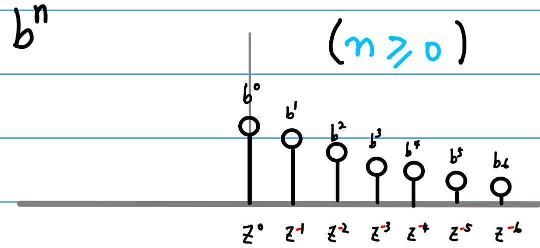


$$X(z^{-1}) = \frac{z^{-1}}{z^{-1} - 0.5} \qquad |z| < 2$$

$$f(z) = \frac{2}{2-z} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^n$$

$$a_n = \left(\frac{1}{2}\right)^n = p^{-n} \qquad \boxed{p = 2}$$

b^n $(n \geq 0)$

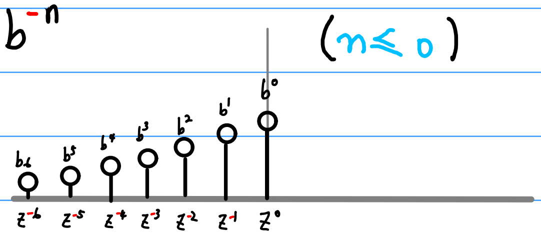


$$|z| > \frac{1}{2}$$

$$X(z) = \frac{z}{z - 0.5} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$x_n = \left(\frac{1}{2}\right)^n = p^n \qquad \boxed{p = \frac{1}{2}}$$

b^{-n} $(n \leq 0)$

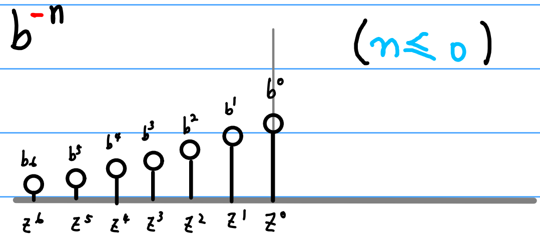


$$X(z^{-1}) = \frac{2}{2 - z^{-1}} \qquad |z| > \frac{1}{2}$$

$$f(z) = \frac{z}{z - 0.5} = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-n} z^n = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$$

$$a_n = \left(\frac{1}{2}\right)^{-n} = p^{-n} \qquad \boxed{p = \frac{1}{2}}$$

b^{-n} $(n \leq 0)$

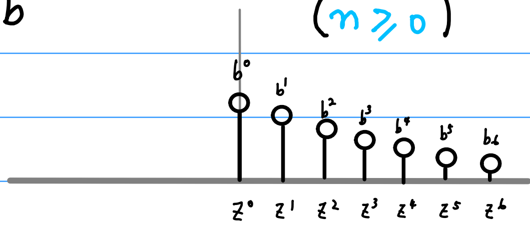


$$|z| < 2$$

$$X(z) = \frac{2}{2 - z} = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-n} z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^n$$

$$x_n = \left(\frac{1}{2}\right)^{-n} = p^n \qquad \boxed{p = 2}$$

b^n $(n \geq 0)$



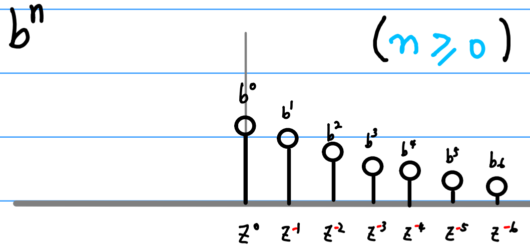
$$X(z^{-1}) = \frac{z^{-1}}{z^{-1} - b} \qquad |z| < b^{-1}$$

$$f(z) = \frac{b^{-1}}{b^{-1} - z} = \sum_{n=0}^{\infty} b^n z^n$$

$$a_n = b^n$$

$$= p^{-n} \qquad \boxed{p = b^{-1}}$$

b^n $(n \geq 0)$



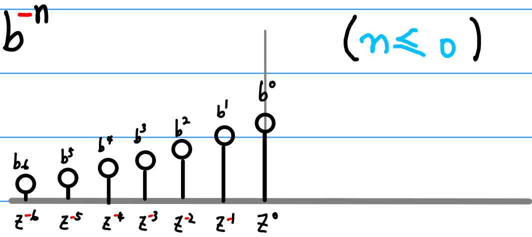
$$|z| > b$$

$$X(z) = \frac{z}{z - b} = \sum_{n=0}^{\infty} b^n z^{-n}$$

$$x_n = b^n$$

$$= p^n \qquad \boxed{p = b}$$

b^{-n} $(n \leq 0)$



$$X(z^{-1}) = \frac{b^{-1}}{b^{-1} - z^{-1}} \qquad |z| > b$$

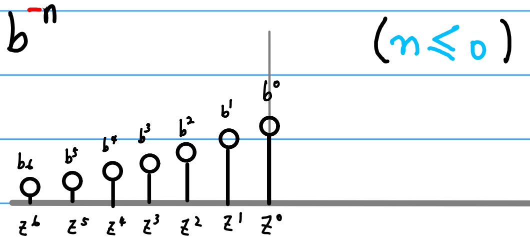
$$f(z) = \frac{z}{z - b} = \sum_{n=-\infty}^{0} b^{-n} z^n$$

$$= \sum_{n=0}^{\infty} b^n z^{-n}$$

$$a_n = b^{-n}$$

$$= p^{-n} \qquad \boxed{p = b}$$

b^{-n} $(n \leq 0)$



$$|z| < b^{-1}$$

$$X(z) = \frac{b^{-1}}{b^{-1} - z} = \sum_{n=-\infty}^{0} b^{-n} z^{-n}$$

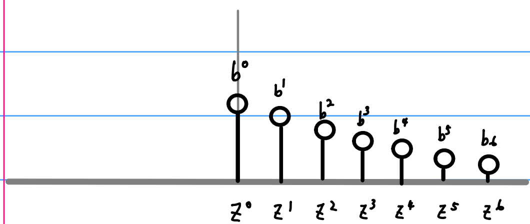
$$= \sum_{n=0}^{\infty} b^n z^n$$

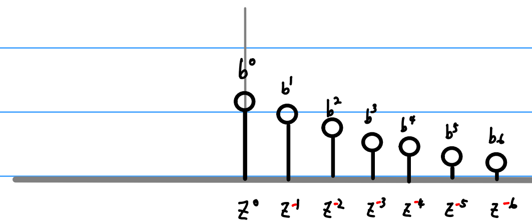
$$x_n = b^{-n}$$

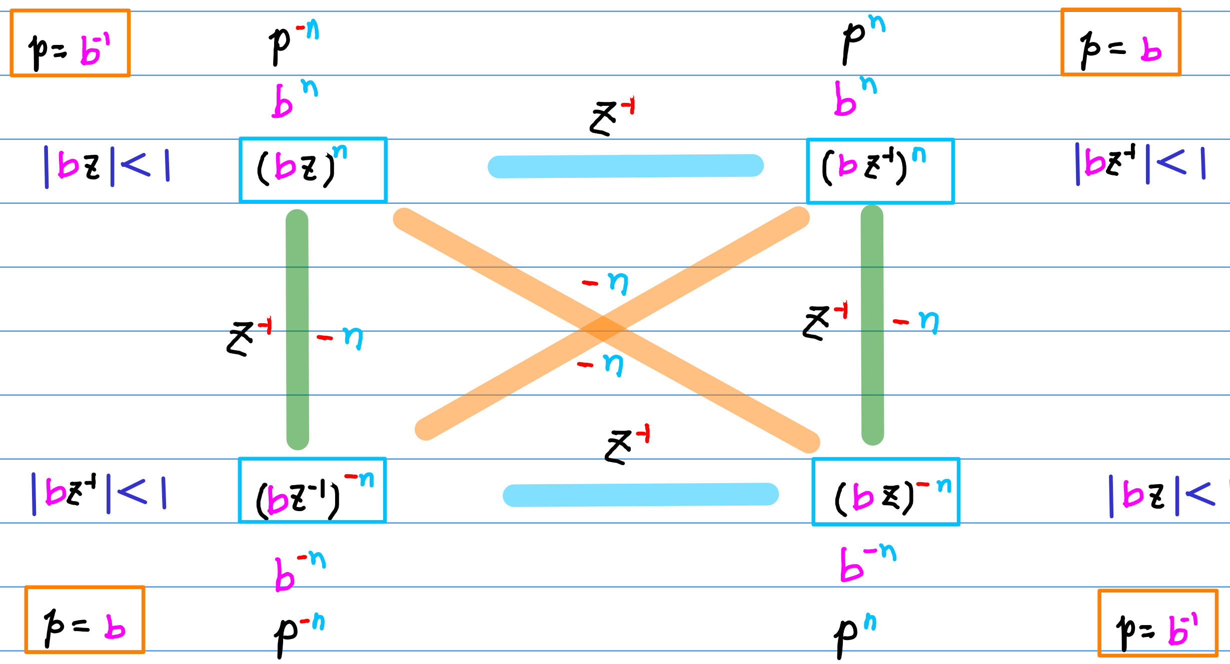
$$= p^n \qquad \boxed{p = b^{-1}}$$

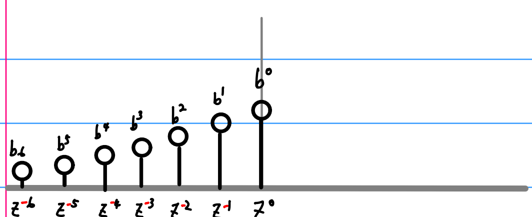
$$\frac{1}{1-(bz)} = \frac{b^{-1}}{b^{-1}-z}$$

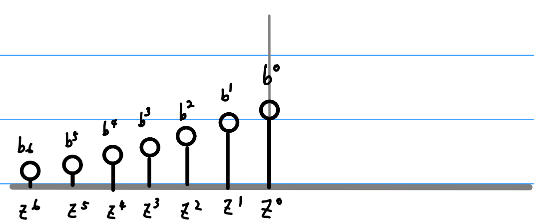
$$\frac{1}{1-(bz^{-1})} = \frac{z}{z-b}$$





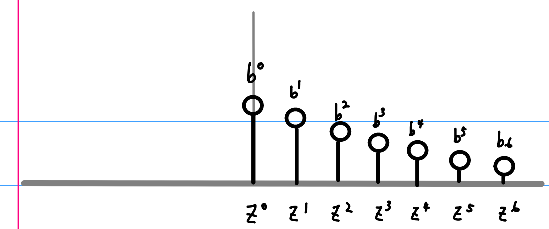


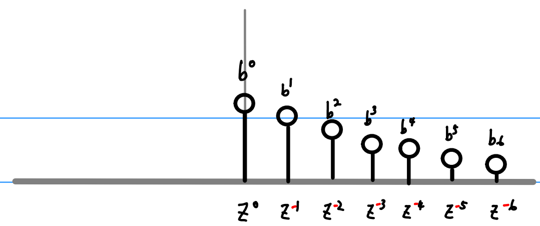




$$\frac{1}{1-(bz^{-1})} = \frac{z}{z-b}$$

$$\frac{1}{1-(bz)} = \frac{b^{-1}}{b^{-1}-z}$$



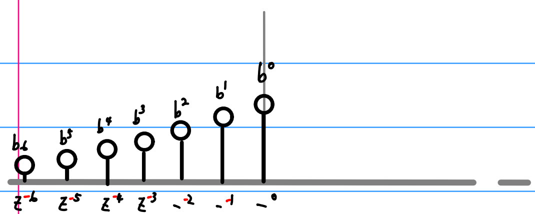


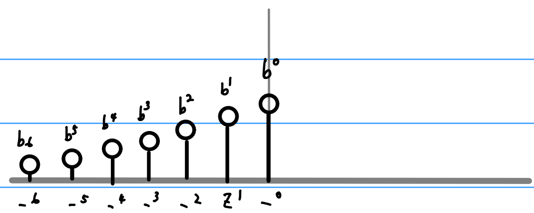
$$\boxed{\frac{1}{1-(bz)} = \frac{b^{-1}}{b^{-1}-z}}$$

$$\boxed{\frac{1}{1-(bz^{-1})} = \frac{z}{z-b}}$$

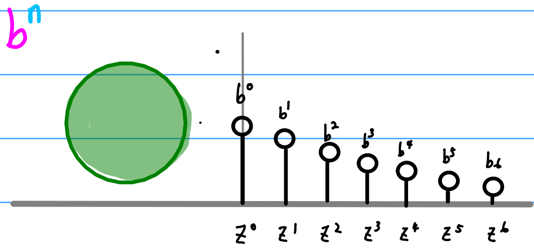
$$\boxed{\frac{1}{1-(bz^{-1})} = \frac{z}{z-b}}$$

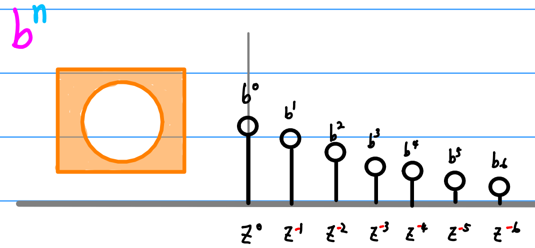
$$\boxed{\frac{1}{1-(bz)} = \frac{b^{-1}}{b^{-1}-z}}$$





$$\boxed{x_n = a_{-n}} \qquad \boxed{X(z) = f(z)}$$





$$f(z) = \frac{1}{1-(bz)} \qquad |z| < b^{-1}$$

$$X(z) = \frac{1}{1-(b/z)} \qquad |z| > b$$

$$\boxed{a_n = b^n \quad (n \geqslant 0)}$$
$$= p^{-n} \quad (p = b^{-1})$$

$$\boxed{x_n = b^n \quad (n \geqslant 0)}$$
$$= p^n \quad (p = b)$$

$-n$

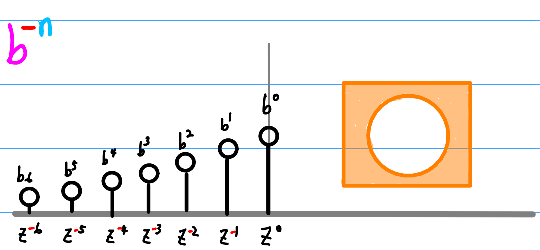
$-n$

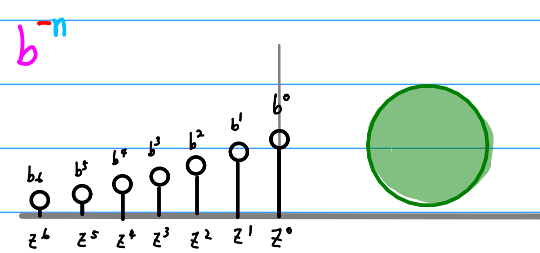
$$f(z) = \frac{1}{1-(b/z)} \qquad |z| > b$$

$$X(z) = \frac{1}{1-(bz)} \qquad |z| < b^{-1}$$

$$\boxed{a_n = b^{-n} \quad (n \leqslant 0)}$$
$$= p^{-n} \quad (p = b)$$

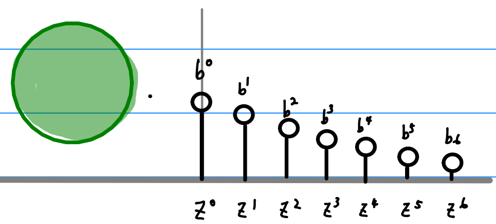
$$\boxed{x_n = b^{-n} \quad (n \leqslant 0)}$$
$$= p^n \quad (p = b^{-1})$$



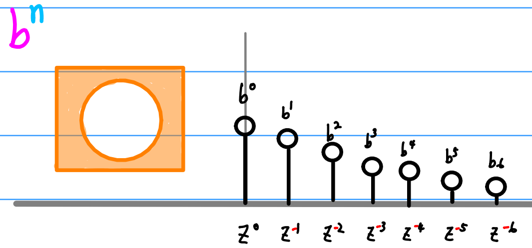


$$\boxed{x_n = a_n} \qquad \boxed{X(z) = f(z^{-1})}$$

b^n



b^n



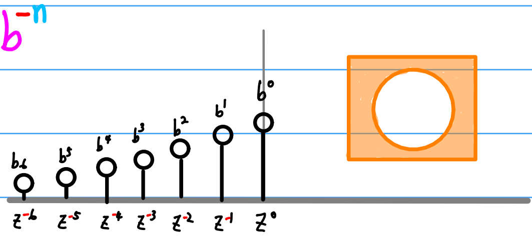
$$\boxed{f(z) = \frac{1}{1-(bz)} \quad |z| < b^{-1}} \xleftrightarrow{Z^{-1}} \boxed{X(z) = \frac{1}{1-(b/z)} \quad |z| > b}$$

$a_n = b^n \quad (n \geqslant 0)$ $\qquad$ $x_n = b^n \quad (n \geqslant 0)$

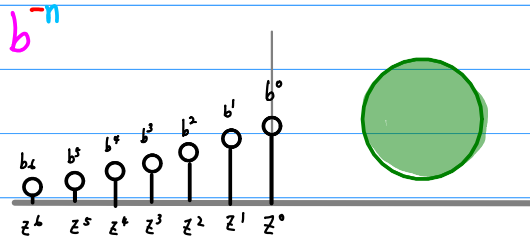
$$\boxed{f(z) = \frac{1}{1-(b/z)} \quad |z| > b} \xleftrightarrow{Z^{-1}} \boxed{X(z) = \frac{1}{1-(bz)} \quad |z| < b^{-1}}$$

$a_n = b^{-n} \quad (n \leqslant 0)$ $\qquad$ $x_n = b^{-n} \quad (n \leqslant 0)$

b^{-n}

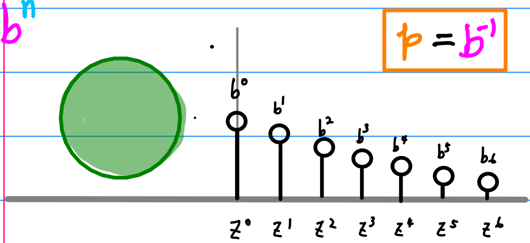


b^{-n}



$$\boxed{a_n = p^{-n}}$$

b^n



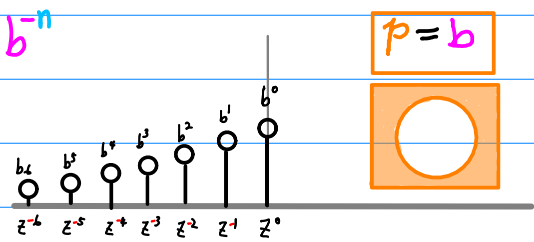
$$f(z) = \frac{1}{1-(bz)} \qquad |z| < b^{-1}$$

$$\boxed{a_n = p^{-n} \quad (n \geq 0)}$$

$$f(z) = \frac{1}{1-(b/z)} \qquad |z| > b$$

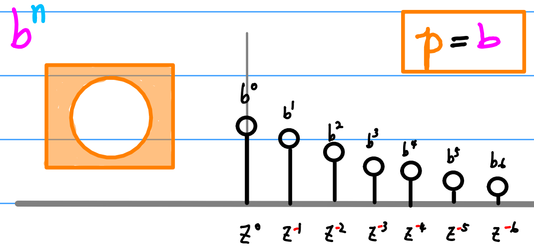
$$\boxed{a_n = p^{-n} \quad (n \leq 0)}$$

b^{-n}



$$\boxed{x_n = p^{n}}$$

b^n



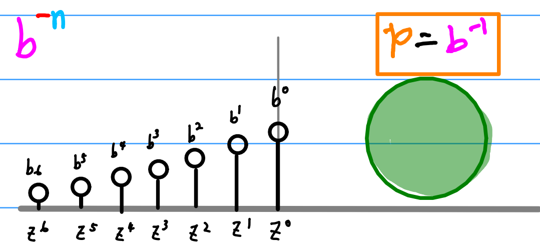
$$X(z) = \frac{1}{1-(b/z)} \qquad |z| > b$$

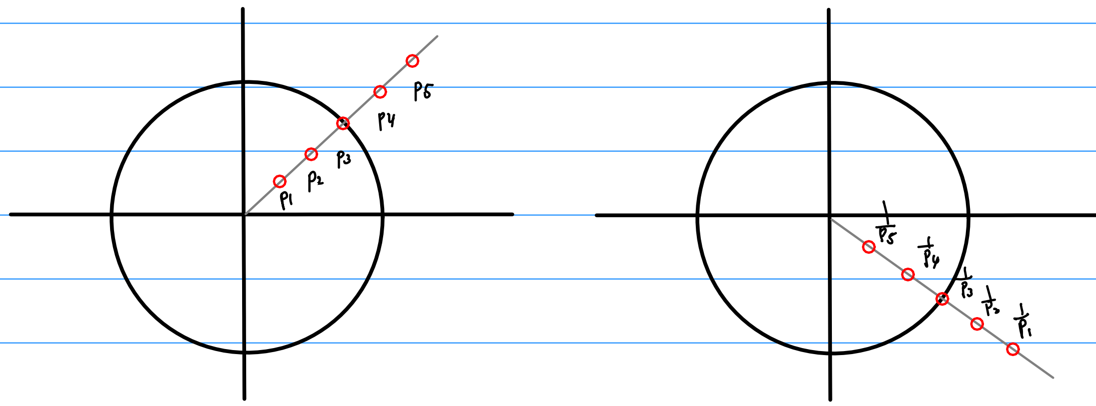
$$\boxed{x_n = p^{n} \quad (n \geq 0)}$$

$$X(z) = \frac{1}{1-(bz)} \qquad |z| < b^{-1}$$

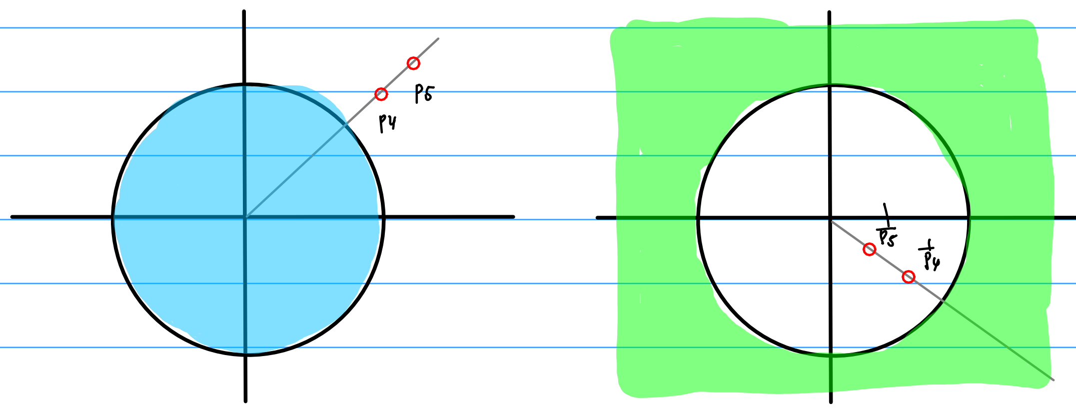
$$\boxed{x_n = p^{n} \quad (n \leq 0)}$$

b^{-n}



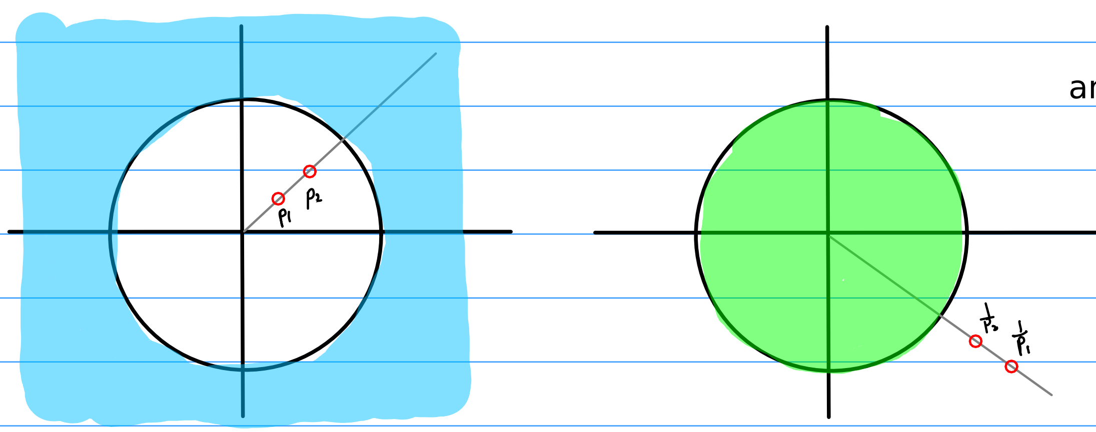


causal



$n \geq 0$

anticausal



$n < 0$

two-sided