Applications of Array Pointers (1A)

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Multi-dimensional Array Pointers

(*n-1*)-d array pointer to a *n*-d array

```
int a[4] ;
int (*p) ;
```
1-d array
0-d array pointer (p = a)

```
int b[4] [2];
int (*q) [2];
```
2-d array
1-d array pointer (q = b)

```
int c[4] [2][3];
int (*r) [2][3];
```
3-d array
2-d array pointer (r = c)

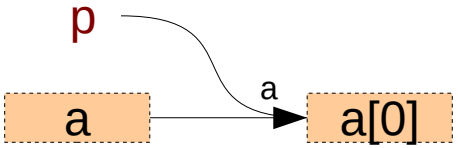
```
int d[4] [2][3][4];
int (*s) [2][3][4];
```
4-d array
3-d array pointer (s = d)

the 1st dimension can be accessed by incrementing (n-1)-d array pointer

n-d array name and (*n-1*)-d array pointer

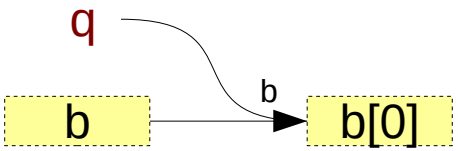
```
int a[4] ;
int (*p) ;
```

```
p = &a[0];
p = a;
```



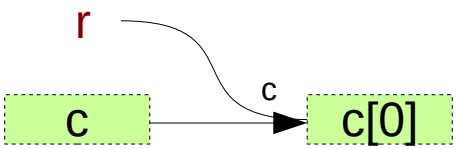

```
int b[4] [2];
int (*q) [2];
```

```
q = &b[0];
q = b;
```



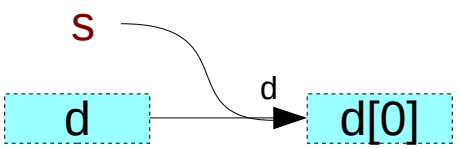

```
int c[4] [2][3];
int (*r) [2][3];
```

```
r = &c[0];
r = c;
```




```
int d[4] [2][3][4];
int (*s) [2][3][4];
```

```
s = &d[0];
s = d;
```



the 1st dimension can be accessed by incrementing (n-1)-d array pointer

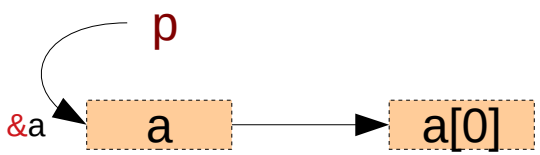
***n*-d** array pointer to a ***n*-d** array

```
int  a  [4] ;          1-d array
int (*p) [4];          1-d array pointer      (p = &a)

int  b  [4][2];        2-d array
int (*q) [4][2];       2-d array pointer      (q = &b)

int  c  [4][2][3];     3-d array
int (*r) [4][2][3];    3-d array pointer      (r = &c)

int  d  [4][2][3][4];  4-d array
int (*s) [4][2][3][4]; 4-d array pointer      (s = &d)
```

***n*-d** array name and ***n*-d** array pointer

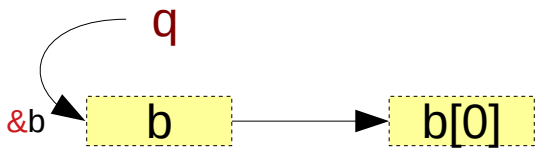
```
int  a  [4] ;
int (*p) [4];
```

p = &a;



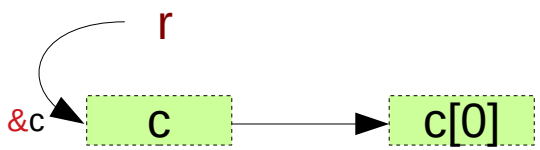

```
int  b  [4][2];
int (*q) [4][2];
```

q = &b;



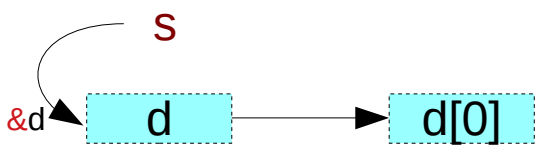

```
int  c  [4][2][3];
int (*r) [4][2][3];
```

r = &c;

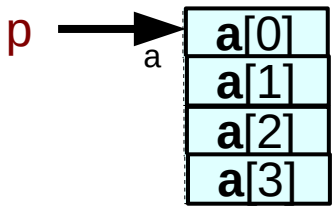


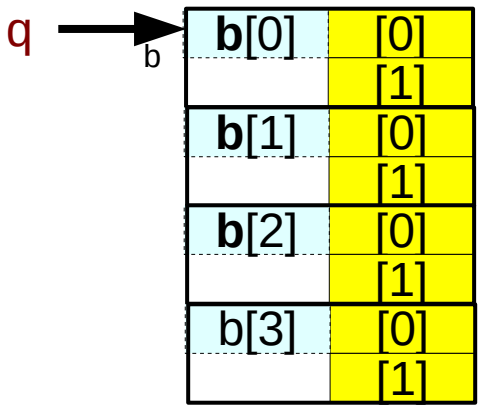

```
int  d  [4][2][3][4];
int (*s) [4][2][3][4];
```

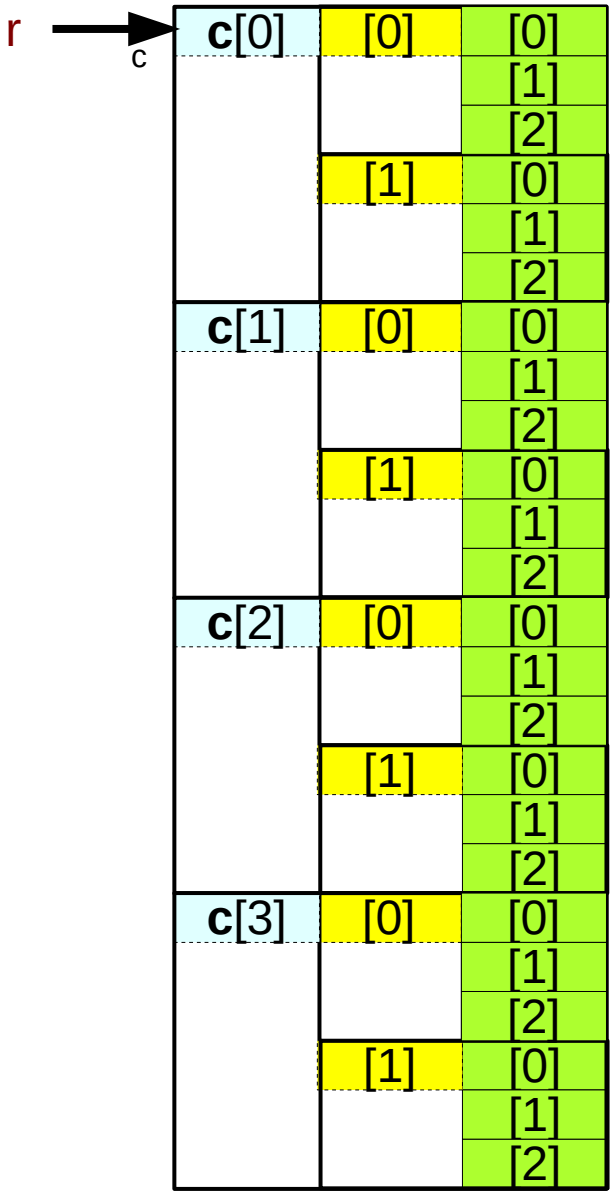
s = &d;



multi-dimensional array pointers

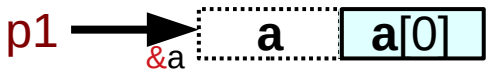




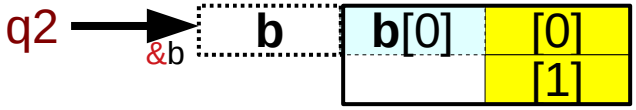



```
int a[4] ;          1-d array
int (*p) ;          0-d array pointer

int b[4]   [2];     2-d array
int (*q)   [2];     1-d array pointer

int c[4]   [2][3];  3-d array
int (*r)   [2][3];  2-d array pointer

int d[4]   [2][3][4];  4-d array
int (*s)   [2][3][4];  3-d array pointer
```

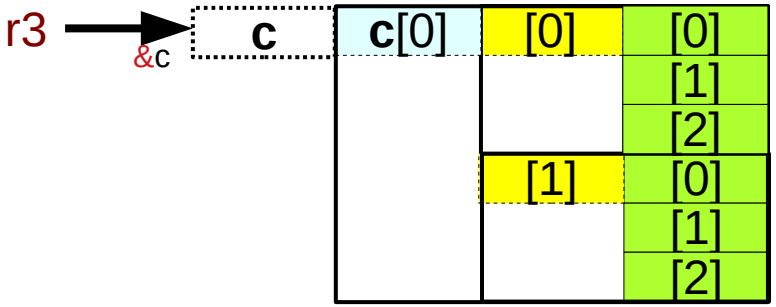
Initializing ***n-d*** array pointers



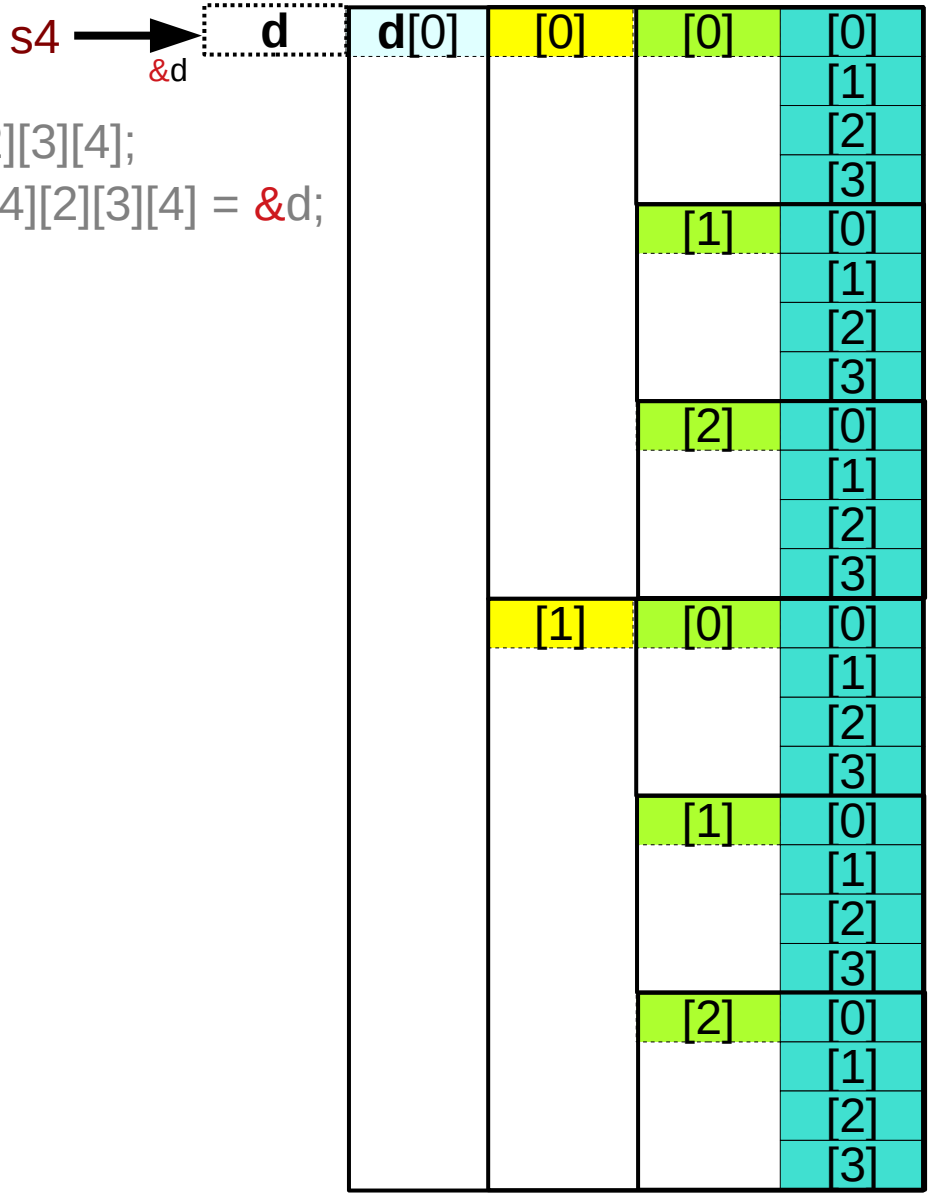

```
int a[4] ;
int (*p1)[4] = &a ;
```




```
int b[4][2];
int (*q2)[4][2] = &b;
```

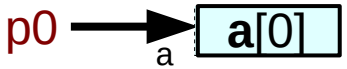



```
int c[4][2][3];
int (*r3)[4][2][3] = &c;
```

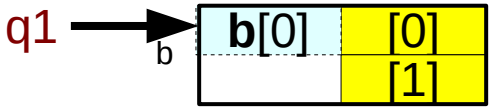



```
int d[4][2][3][4];
int (*s4)[4][2][3][4] = &d;
```

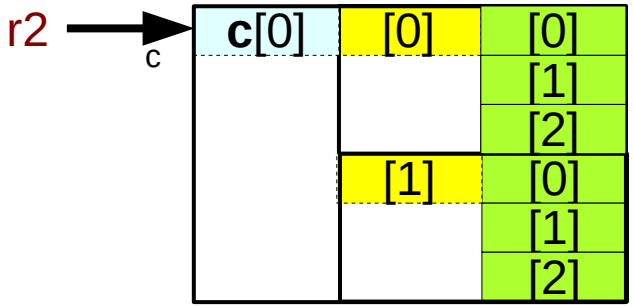
Initializing *(n-1)-d* array pointers



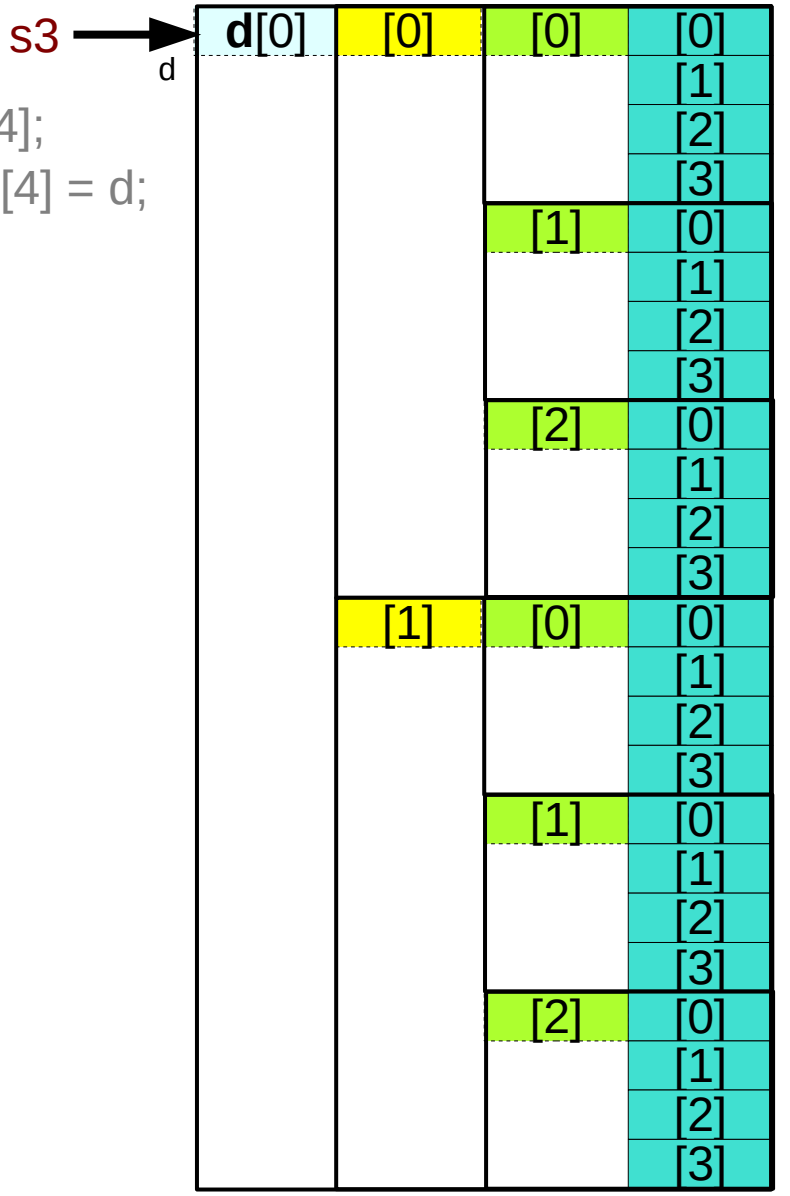

```
int a[4] ;
int (*p0) = a ;
```




```
int b[4][2];
int (*q1)[2] = b;
```




```
int c[4][2][3];
int (*r2)[2][3] = c;
```




```
int d[4][2][3][4];
int (*s3)[2][3][4] = d;
```

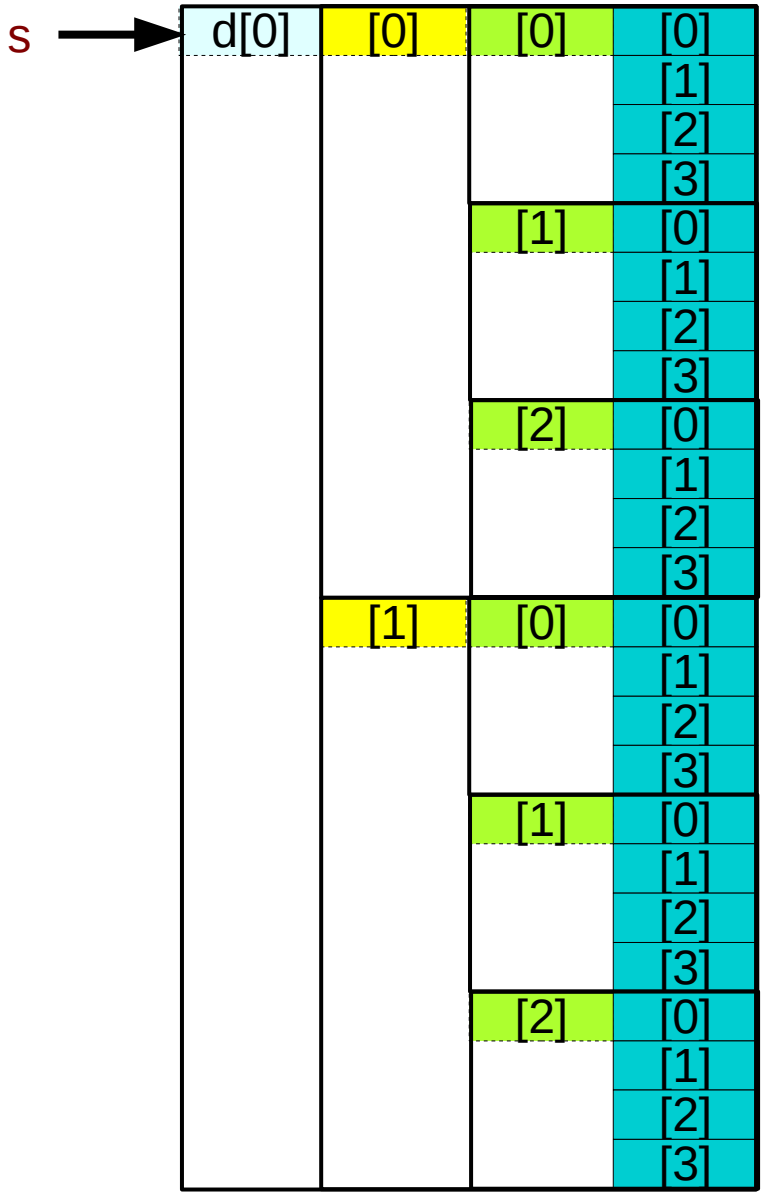
array pointers to multi-dimensional subarrays

```
int d[4] [2][3][4];
int (*s) [2][3][4];
```

d	4-d array name	d[4][2][3][4]
	3-d array pointer	(*p)[2][3][4]
d[i]	3-d array name	d[i][2][3][4]
	2-d array pointer	(*q)[3][4]
d[i][j]	2-d array name	d[i][j][3][4]
	1-d array pointer	(*r)[4]
d[i][j][k]	1-d array name	d[i][j][k][4]
	0-d array pointer	(*s)

i,j,k are specific index values

i =[0..3], j = [0..1], k= [0..2]



Initializing array pointers to multi-dimensional subarrays

```
int d[4] [2][3][4];
int (*s) [2][3][4];
```

d	4-d array name	d[4][2][3][4]	p[i][j][k][l]
	3-d array pointer	(*p)[2][3][4]	int (*p)[2][3][4] = d;
d[i]	3-d array name	d[i][2][3][4]	q[j][k][l]
	2-d array pointer	(*q)[3][4]	int (*q)[3][4] = d[i];
d[i][j]	2-d array name	d[i][j][3][4]	r[k][l]
	1-d array pointer	(*r)[4]	int (*r)[4] = d[i][j];
d[i][j][k]	1-d array name	d[i][j][k][4]	s[l]
	0-d array pointer	(*s)	int (*s) = d[i][j][k];

i =[0..3], j = [0..1], k= [0..2]

Passing multidimensional array names

```
int a[4] ;
int (*p) ;
```

call

```
funa(a, …);
```

prototype

```
void funa(int (*p), …);
```

```
int b[4] [2];
int (*q) [2];
```

call

```
funb(b, …);
```

prototype

```
void funb(int (*q)[2], …);
```

```
int c[4] [2][3];
int (*r) [2][3];
```

call

```
func(c, …);
```

prototype

```
void func(int (*r)[2][3], …);
```

```
int d[4] [2][3][4];
int (*s) [2][3][4];
```

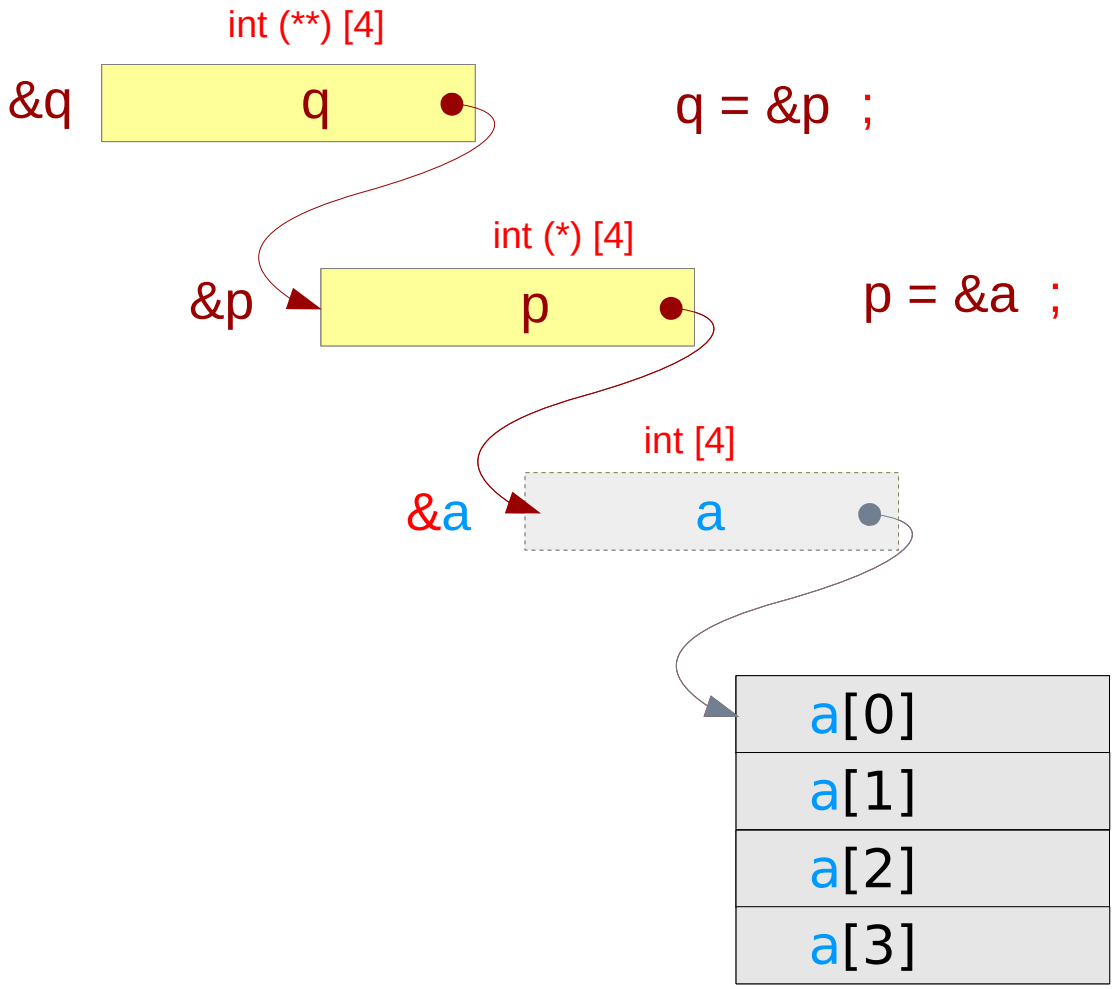
call

```
fund(d, …);
```

prototype

```
void fund(int (*s)[2][3][4], …);
```

Double pointer to a **1-d** array – a variable view (p, q)

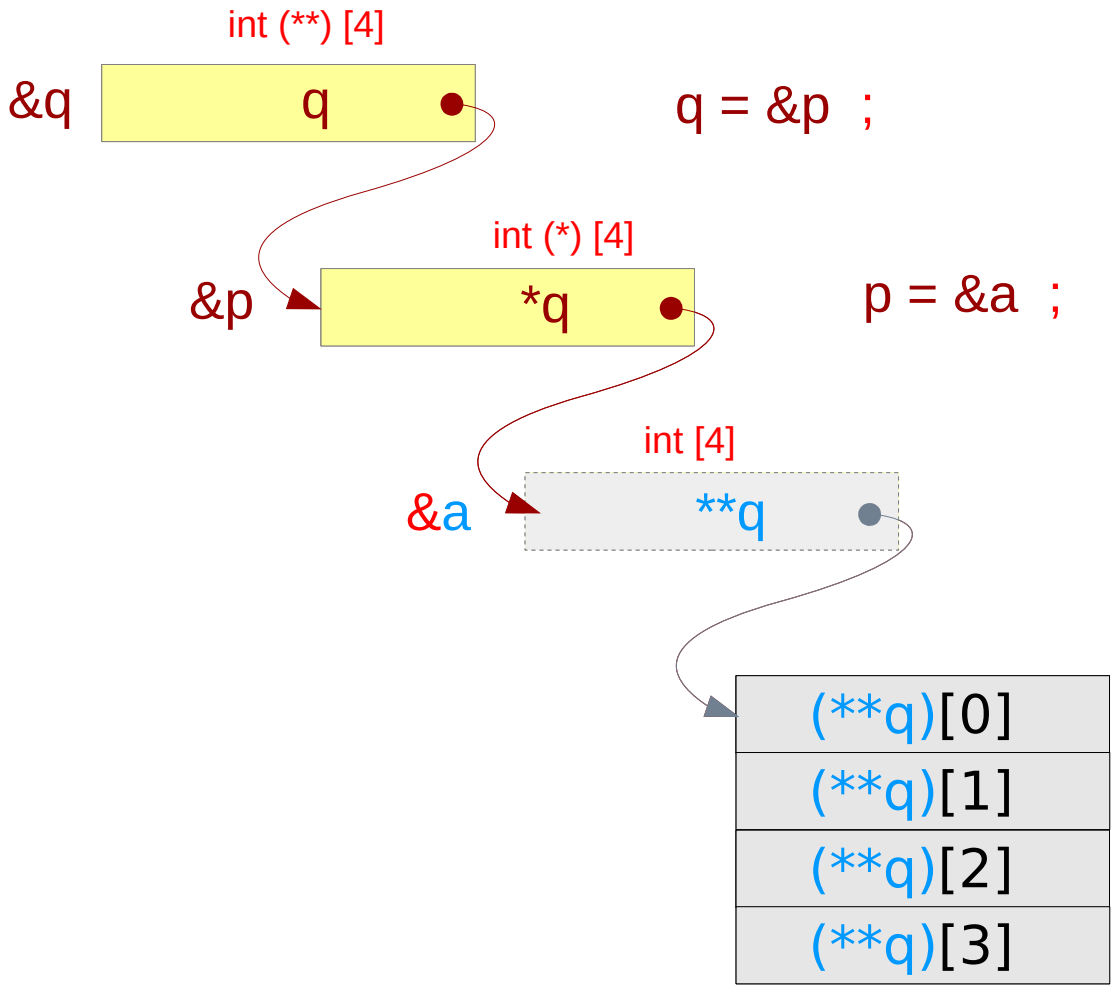



```
int  a[4]  ;
int  (*p) [4] = &a ;
int  (**q) [4] = &p ;
```

➡ p = &a ;

➡ q = &p ;

Double pointer to a **1-d** array – a variable view (q)

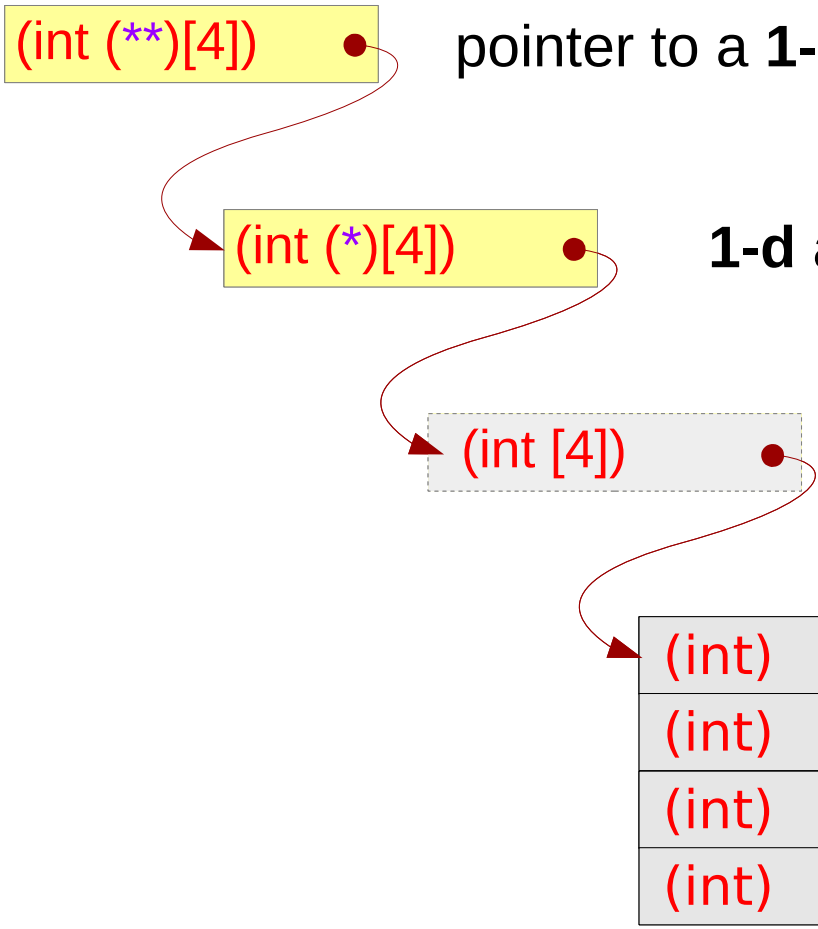



```
int  a[4]  ;
int  (*p) [4] = &a ;
int  (**q) [4] = &p ;
```

➡ p = &a ;

➡ q = &p ;

Double pointer to a **1-d** array – a type view



pointer to a **1-d** array pointer

1-d array pointer

(int *) a pointer to an int

```
int  a[4]  ;
int  (*p) [4] = &a ;
int  (**q) [4] = &p ;
```

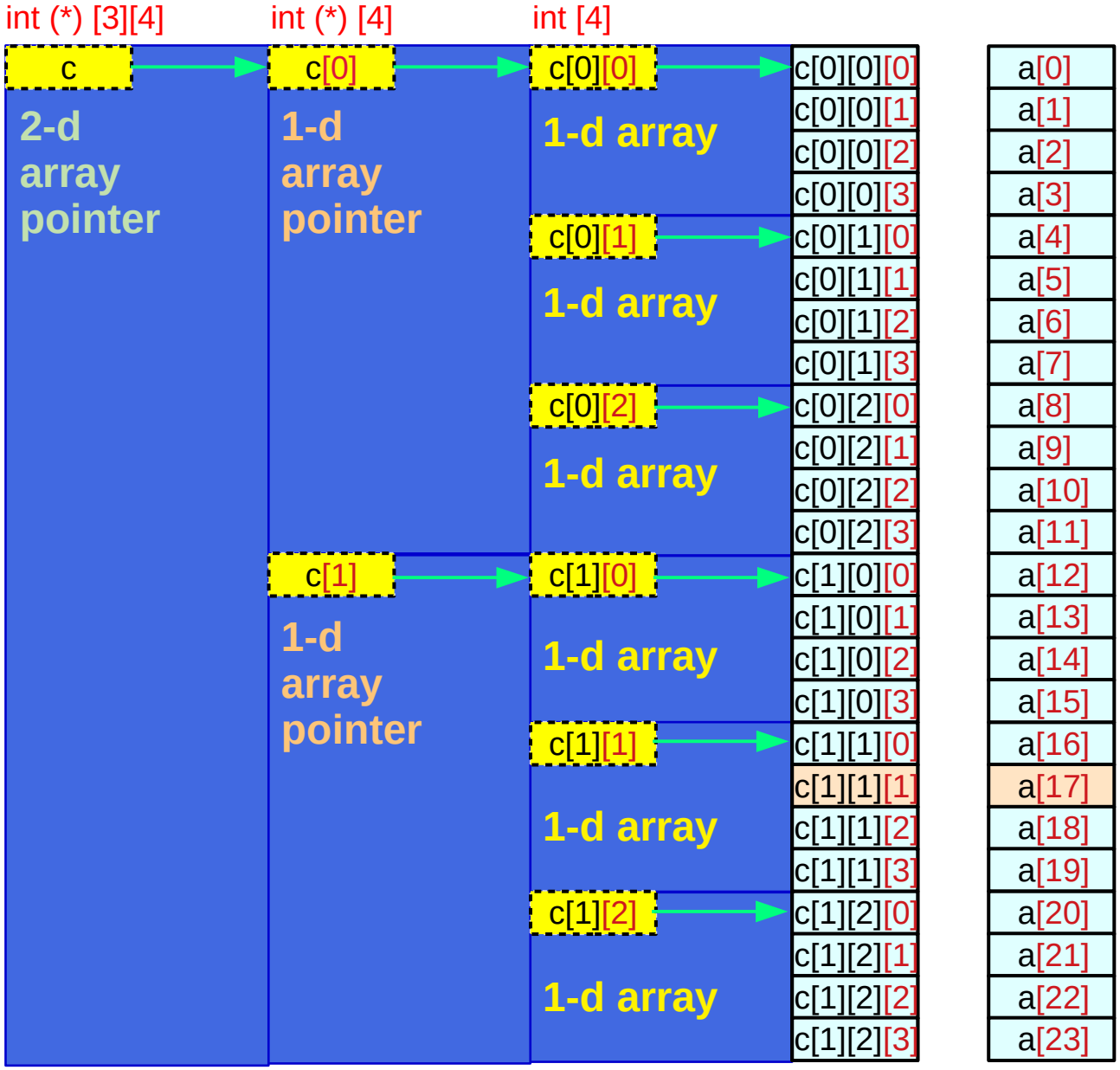
➡ p = &a ;

➡ q = &p ;

Virtual Array Pointers in Multi-dimensional Arrays

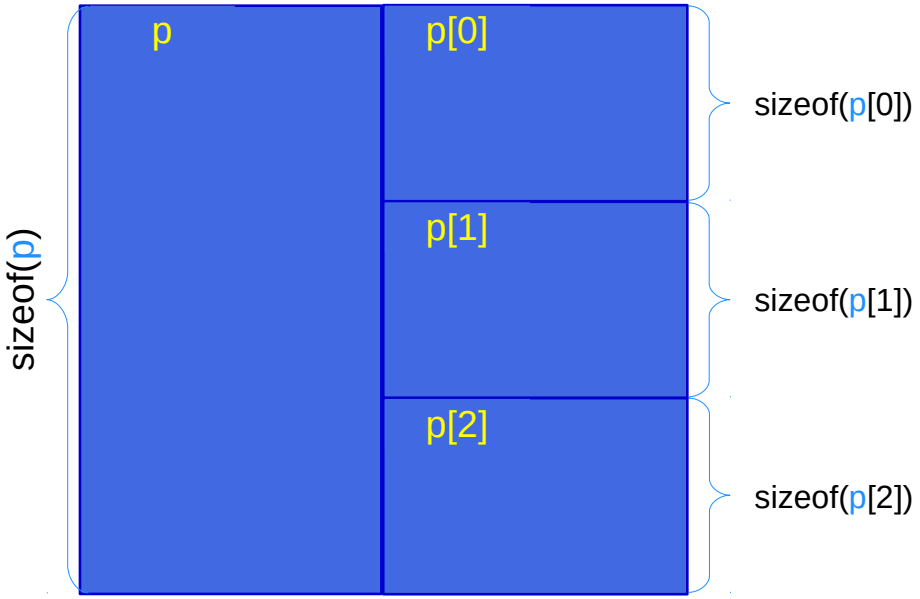
3-d array structure

- Hierarchical
- Nested Structure
- Virtual Array Pointers over
  - Contiguous
  - Linear Layout

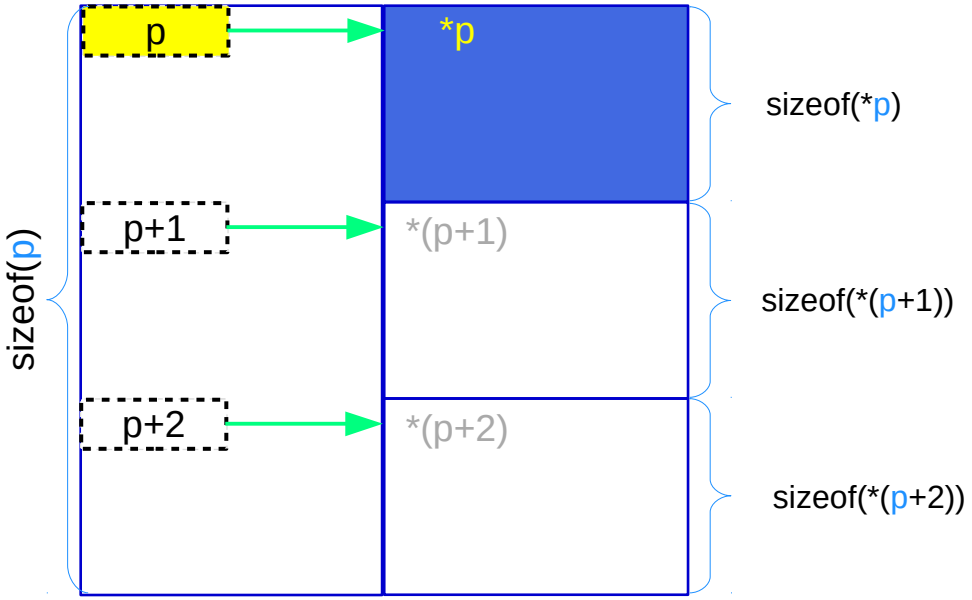


Array **p** and virtual array pointer **p**

Abstract data (array) p



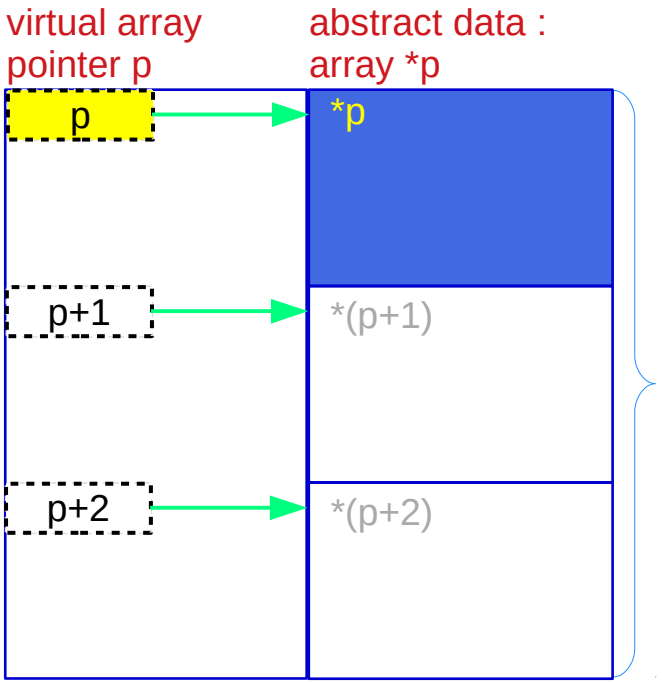
Virtual array pointer p

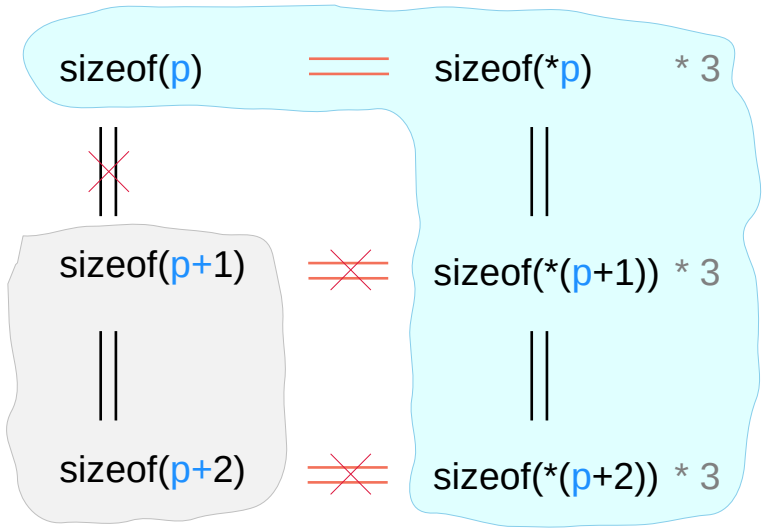


p is the name of an array and
has a array pointer type but
has a size of the array

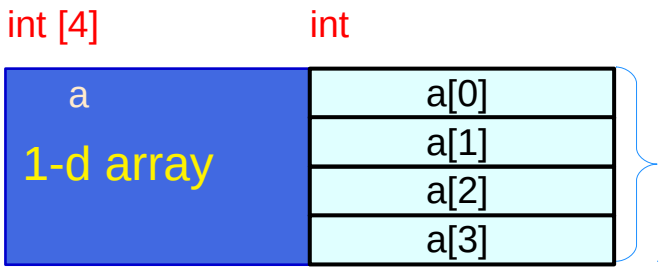
p is a virtual array pointer

Virtual array pointer to abstract data



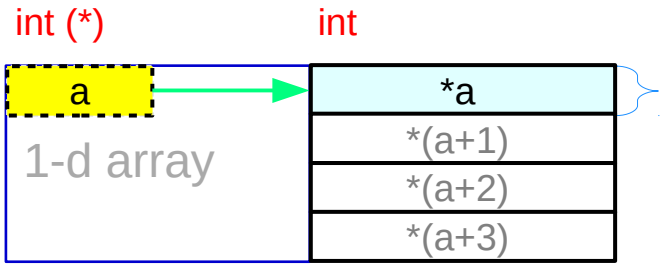


Array **a** and pointer **a**



1-d array a specific array type

sizeof(a)



pointer a general pointer type

sizeof(a) = sizeof(*a) * 4

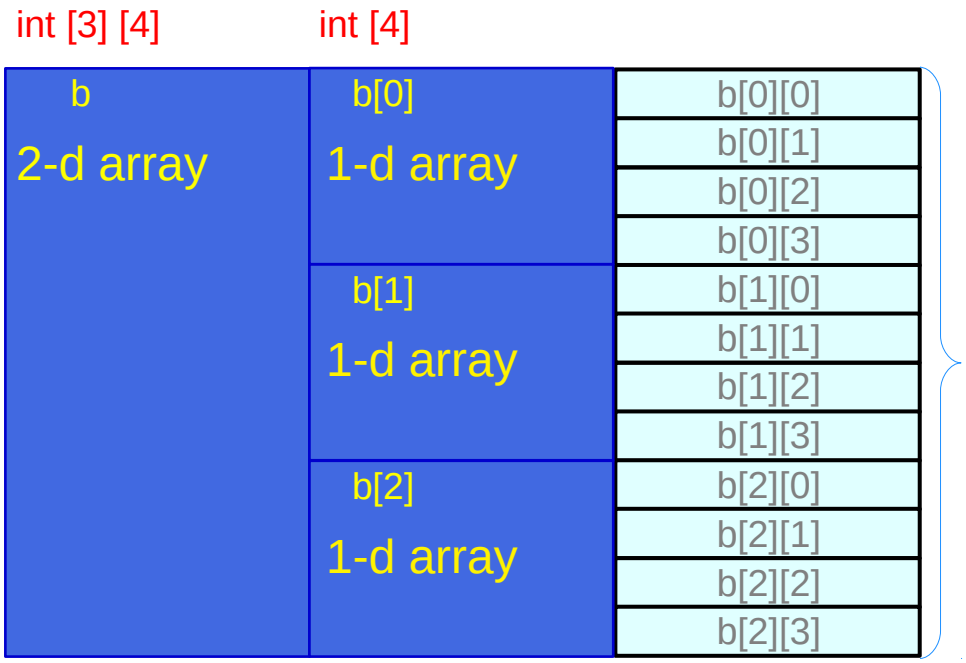
a is the name of a 1-d array and
has a pointer type but
has a size of the array

a is a virtual array pointer

Array **b** and pointer **b**

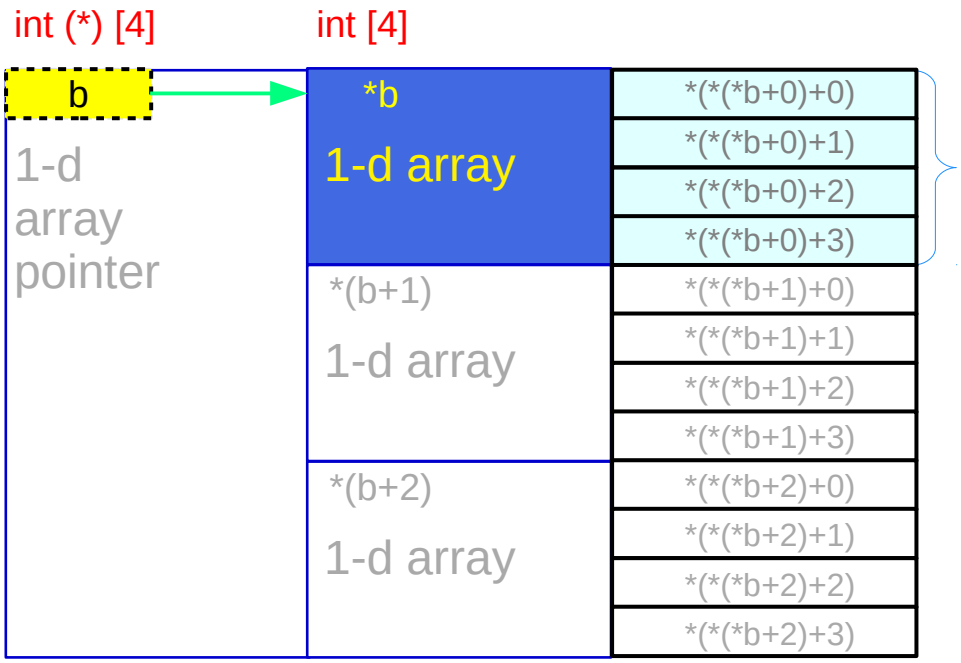
2-d array b specific array type

sizeof(b)



1-d array pointer b general pointer type

sizeof(b) = sizeof(*b) * 3



b is the name of a 2-d array and has a 1-d array pointer type but has a size of the array

b is a virtual array pointer

Array **c**

3-d array c

specific array type

sizeof(c)

c is the name of a 3-d array and has a 2-d array pointer type but has a size of the array

c is a virtual array pointer

Pointer **c**

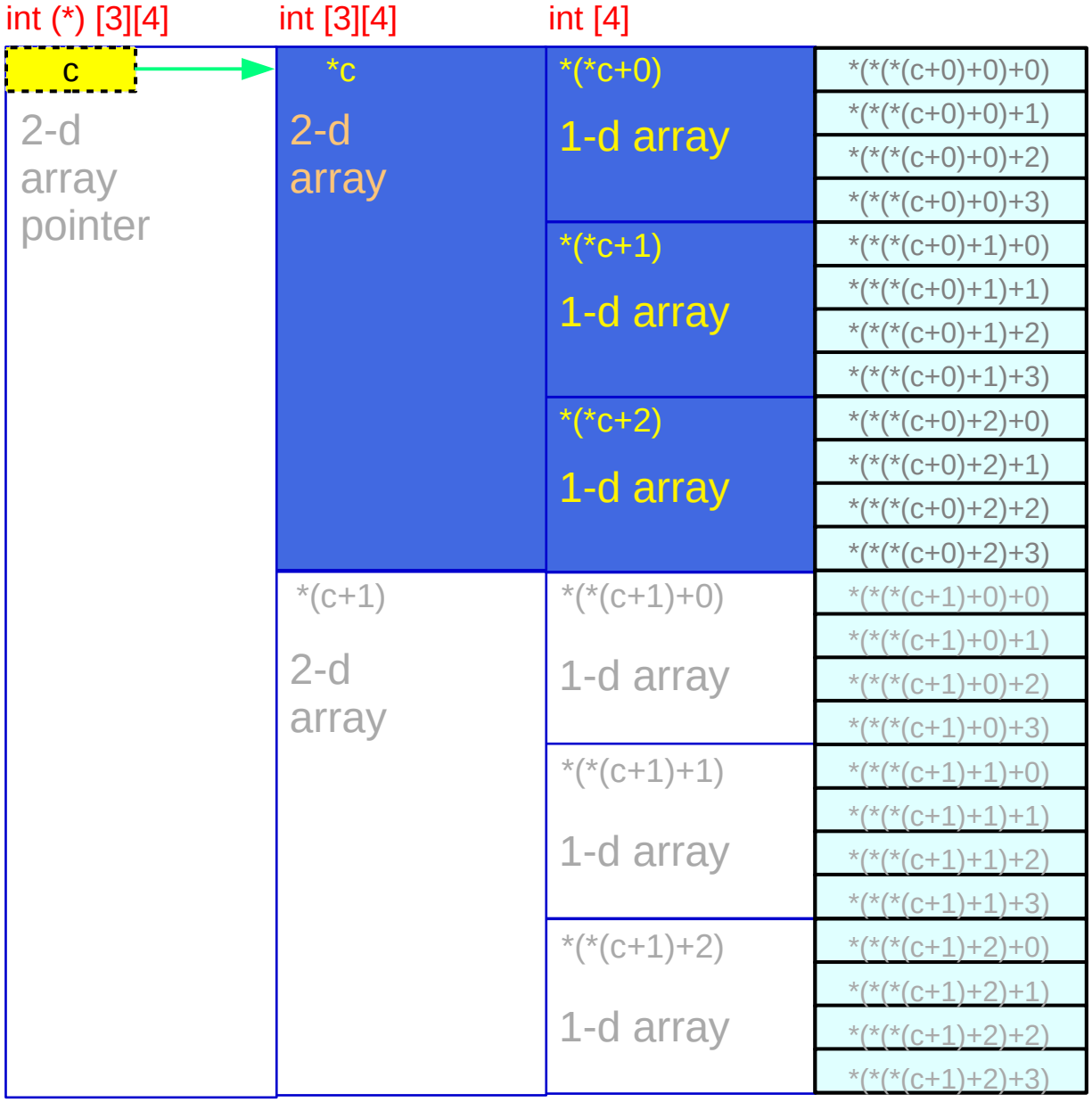
2-d array pointer c

general pointer type

sizeof(c) = sizeof(*c) * 2

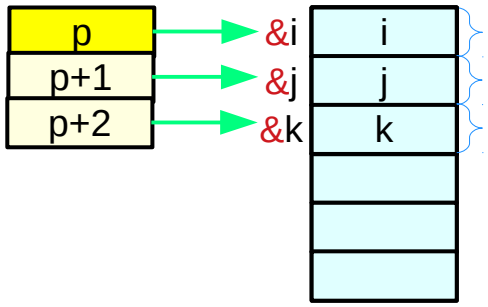
c is the name of a 3-d array and has a 2-d array pointer type but has a size of the array

c is a virtual array pointer



Pointers to primitive data

int *p; **int i, j, k;**



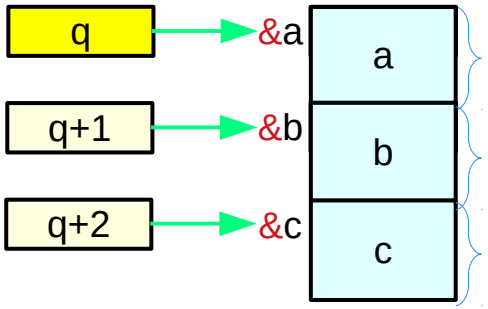
size = sizeof(i) = sizeof(*p) ≠ sizeof(p)

size = sizeof(j) = sizeof(*(p+1)) ≠ sizeof(p+1)

size = sizeof(k) = sizeof(*(p+2)) ≠ sizeof(p+2)

pointer size
4 or 8 bytes

double *q; **double a, b, c;**



size = sizeof(a) = sizeof(*q) ≠ sizeof(q)

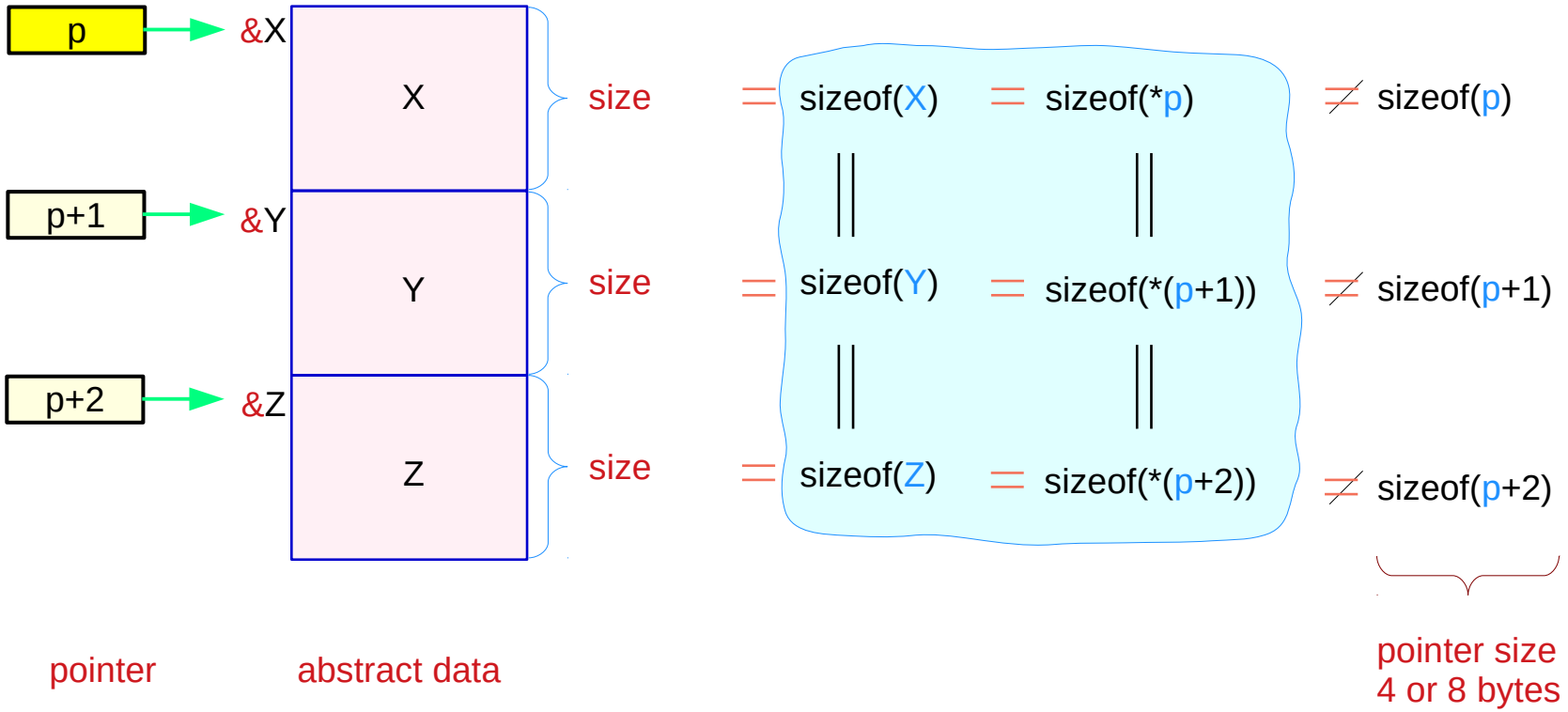
size = sizeof(b) = sizeof(*(q+1)) ≠ sizeof(q+1)

size = sizeof(c) = sizeof(*(q+2)) ≠ sizeof(q+2)

pointer size
4 or 8 bytes

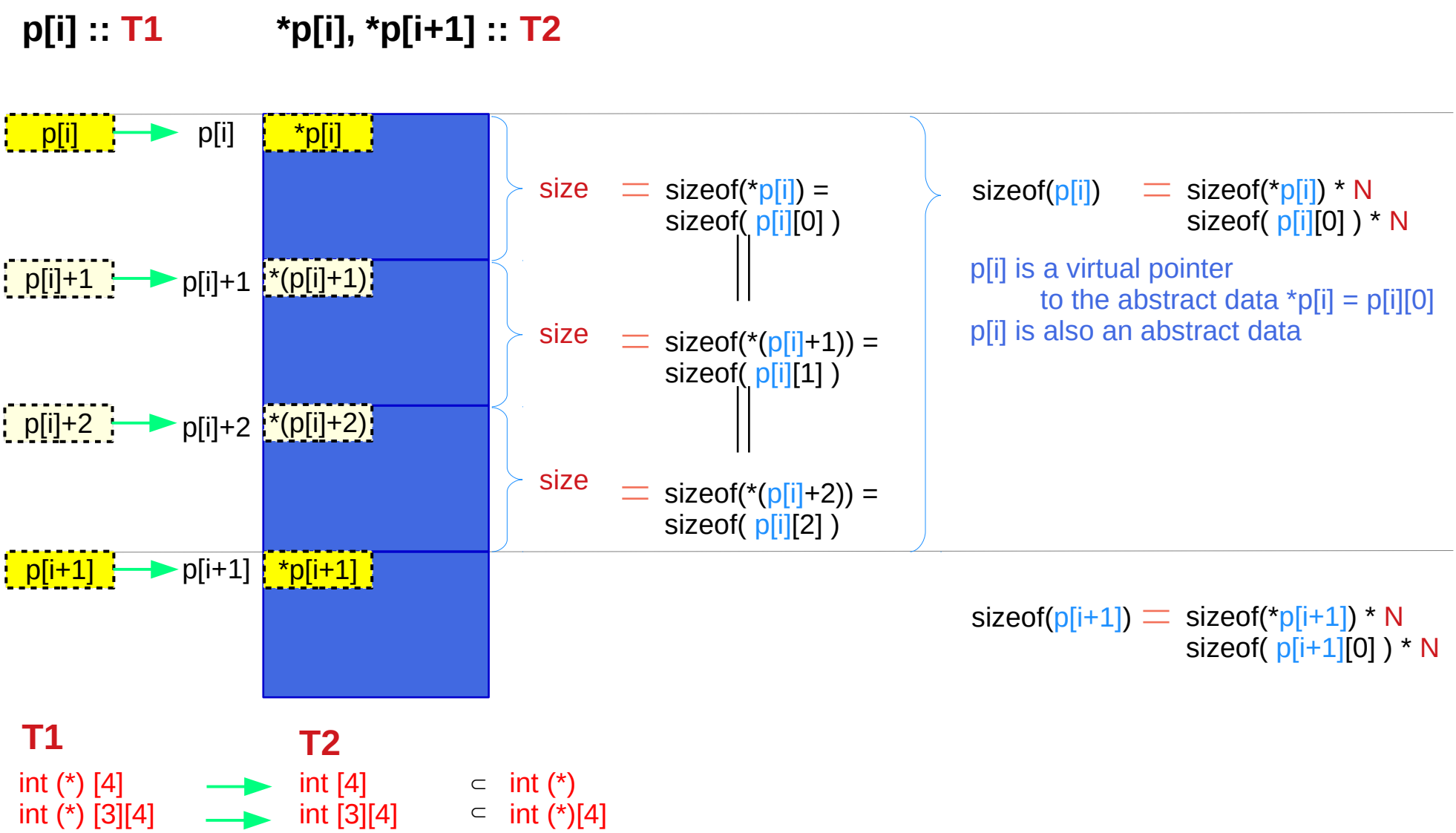
Pointers to abstract data

T *p; **T X, Y, Z;**

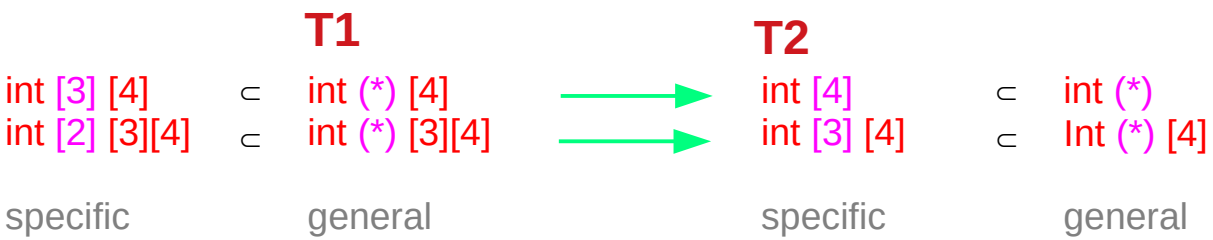


type -------- array
value -------- start address
increment size -------- size

Virtual pointers in a multi-dimensional array



Virtual pointers in a multi-dimensional array




```
typedef int (*T1) [4];          typedef int T2[4];
typedef int (*T1) [3][4];       typedef int T2[3][4];

  T1    a;                        T2    b;
```

T1 references T2
T2 is a dereference of T1

T1 is a pointer type
T2 is an array type
T1 has one more dimension than T2

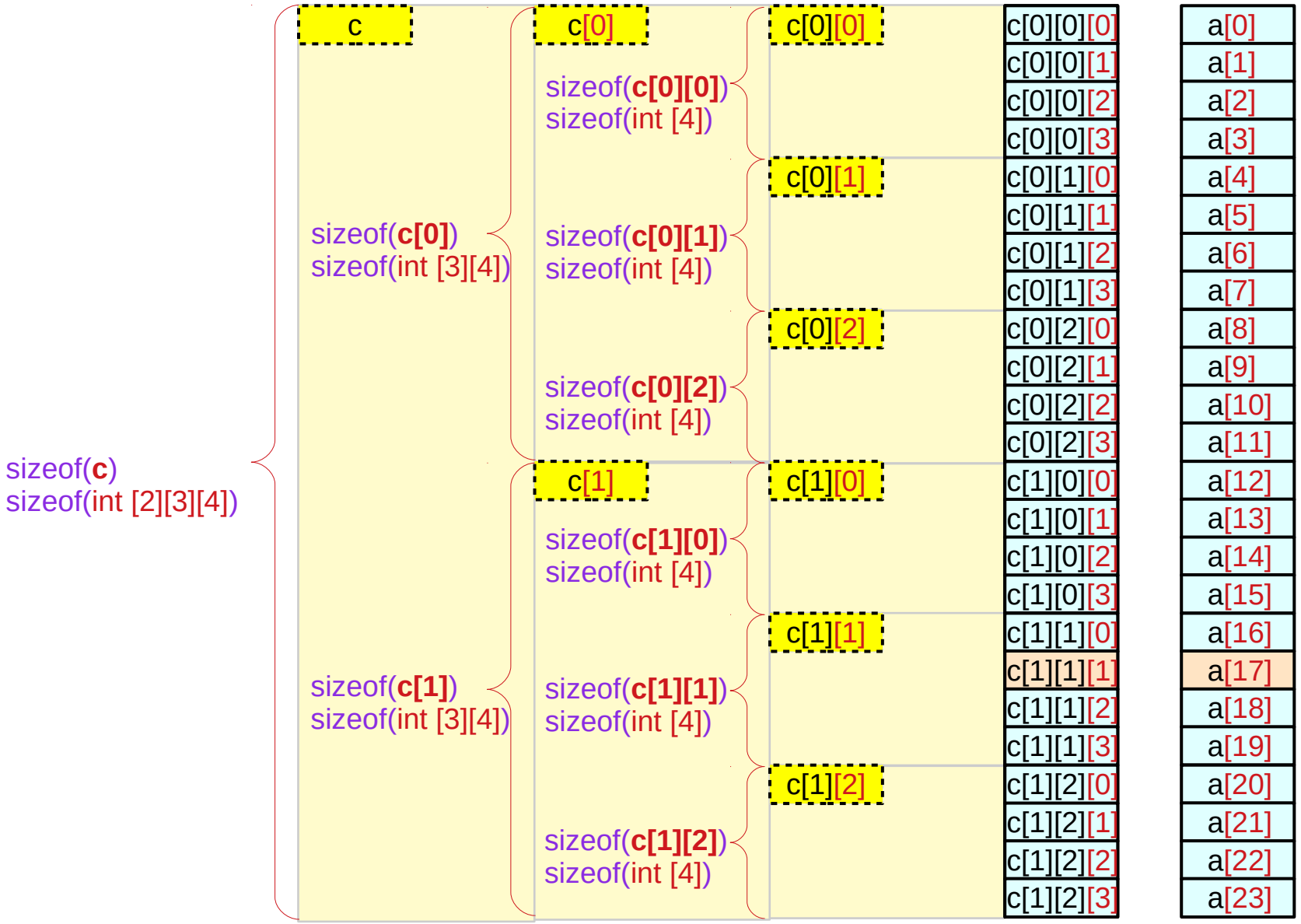
Virtual array pointers – types, sizes, and values

int c[2][3][4];	c[i][j]	c[i][j][0]	
type	int [4] int (*)	int int	• abstract data type • array pointer type
size	sizeof(c[i][j]) =	sizeof(c[i][j][0]) * 4	= sizeof(int) * 4
value (address)	c[i][j] =	&c[i][j][0]	

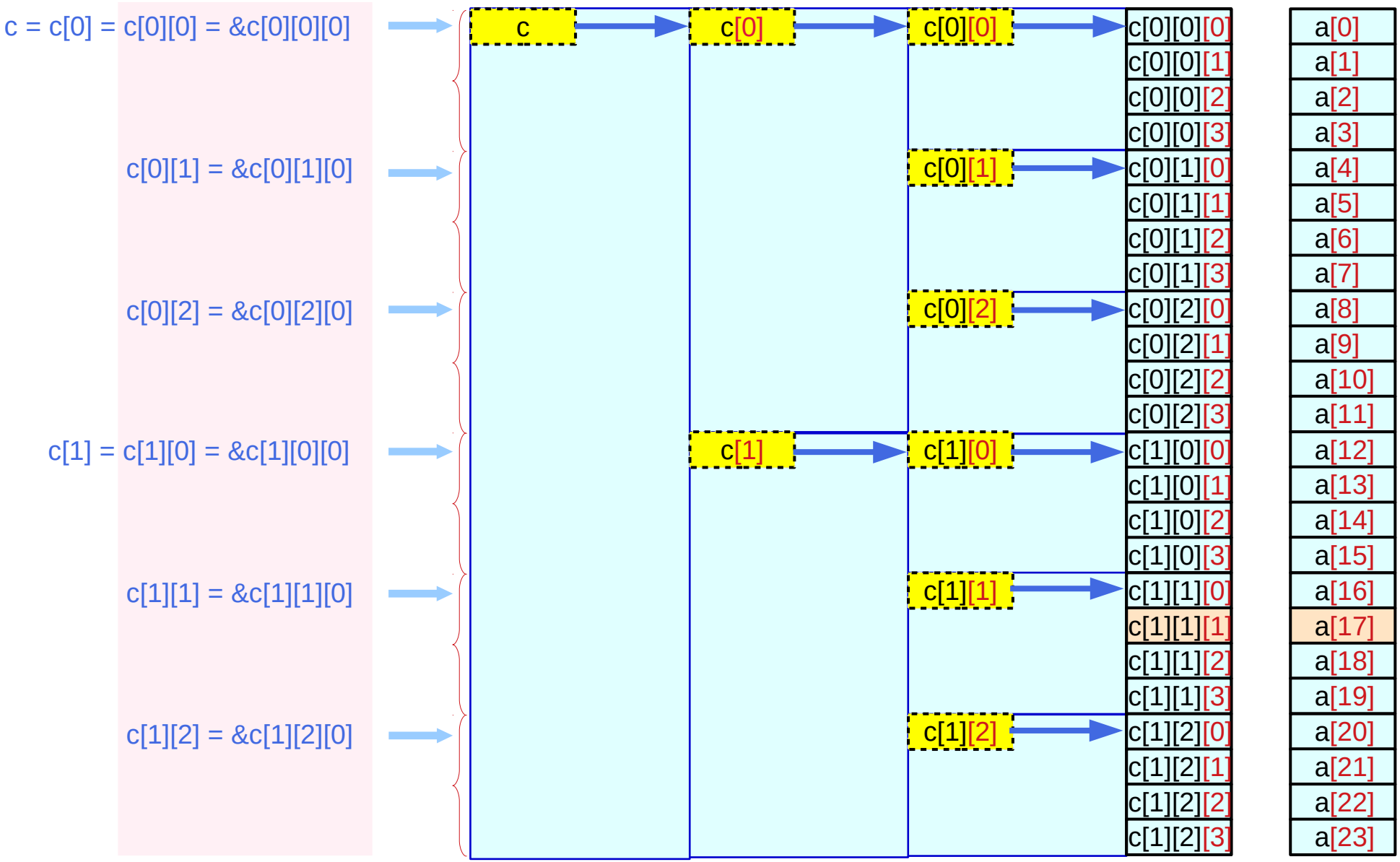
int c[2][3][4];	c[i]	c[i][0]	
type	int [3][4] int (*)[4]	int [4] int (*)	• abstract data type • array pointer type
size	sizeof(c[i]) =	sizeof(c[i][0]) * 3	= sizeof(int) * 4 * 3
value (address)	c[i] =	&c[i][0]	

int c[2][3][4];	c	c[0]	
type	int [2][3][4] int (*)[3][4]	int [3][4] int (*)[4]	• abstract data type • array pointer type
size	sizeof(c) =	sizeof(c[0]) * 2	= sizeof(int) * 4 * 3 * 2
value (address)	c =	&c[0]	

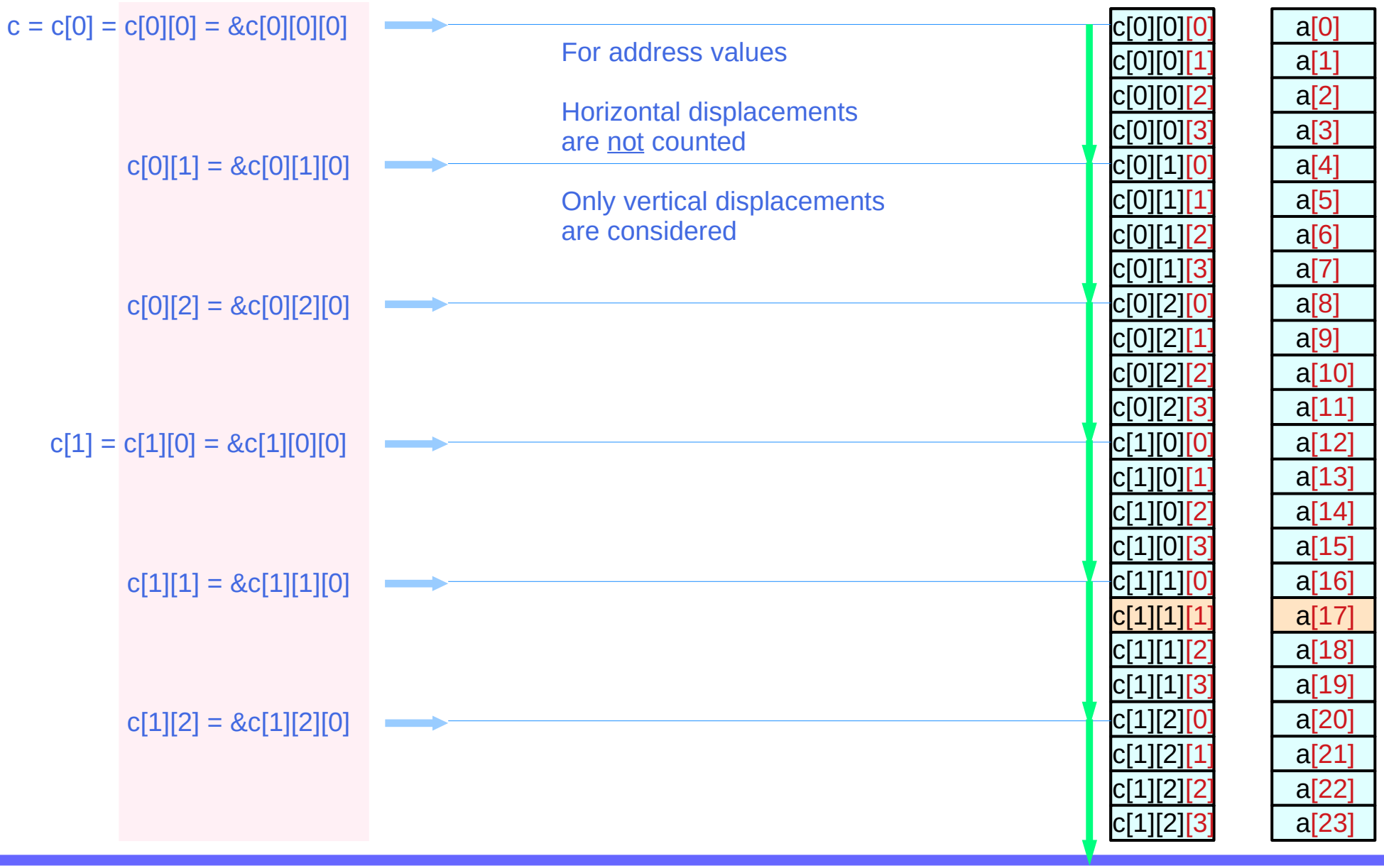
virtual array pointers c, c[i], c[i][j] – sizes



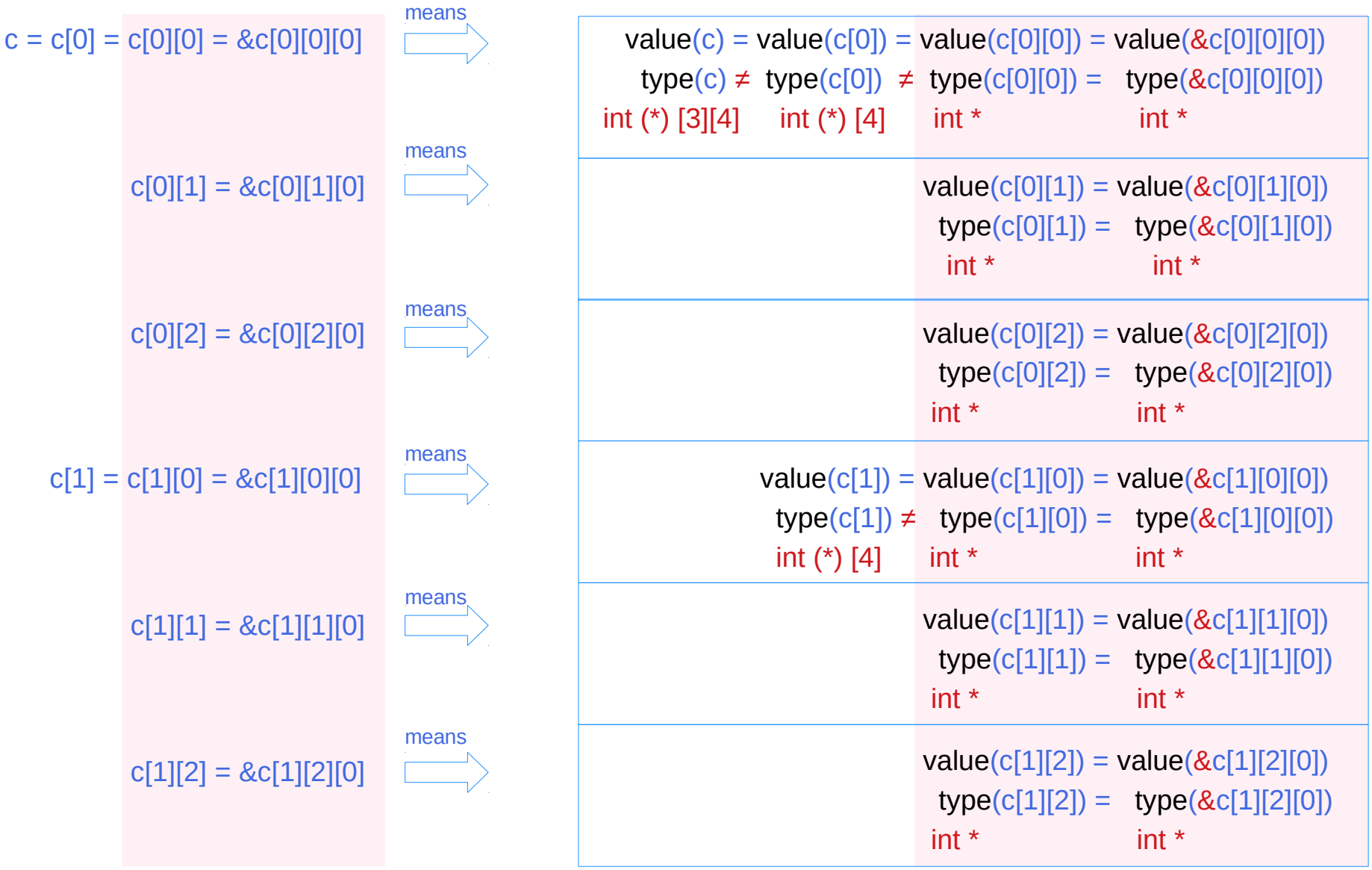
Virtual array pointer c, c[i], c[i][j] – values (addresses)



Virtual array pointer c, c[i], c[i][j] – vertical displacement

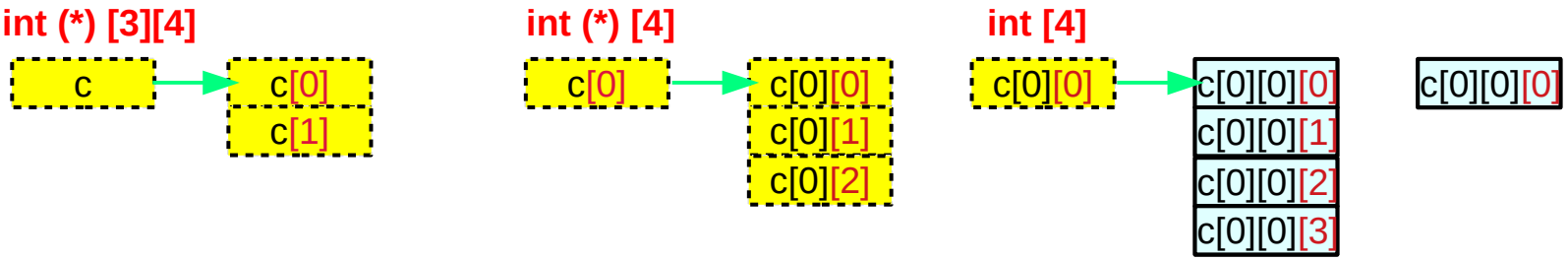


Virtual array pointer c, c[i], c[i][j] – values and types



Virtual array pointer c, c[0], c[0][0] – types and sizes

Types – array pointers



Sizes – abstract data

sizeof(**c**)
sizeof(**c[0]**) * 2
sizeof(**c[0][0]**) * 2 * 3
sizeof(**c[0][0][0]**) * 2 * 3 * 4

sizeof(**int [2][3][4]**)

sizeof(**int [2][3][4]**) = 96
sizeof(**int (*)[3][4]**) = 4 / 8

sizeof(**c[0]**)
sizeof(**c[0][0]**) * 3
sizeof(**c[0][0][0]**) * 3 * 4

sizeof(**int [3][4]**)

sizeof(**int [3][4]**) = 48
sizeof(**int (*)[4]**) = 4 / 8

sizeof(**c[0][0]**)
sizeof(**c[0][0][0]**) * 4

sizeof(**int [4]**)

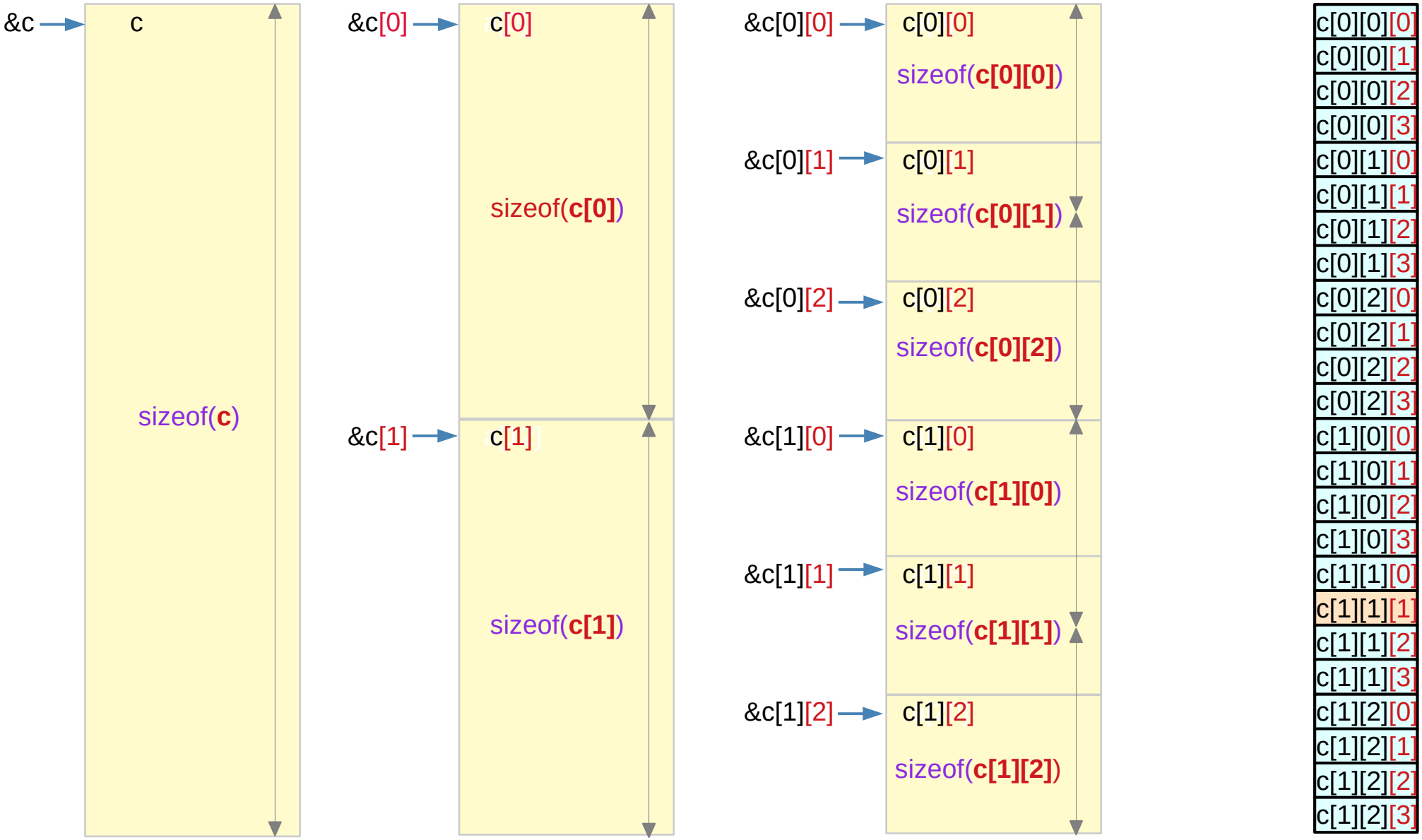
sizeof(**int [4]**) = 16
sizeof(**int (*)**) = 4 / 8

sizeof(**c[0][0][0]**)

sizeof(**int**)

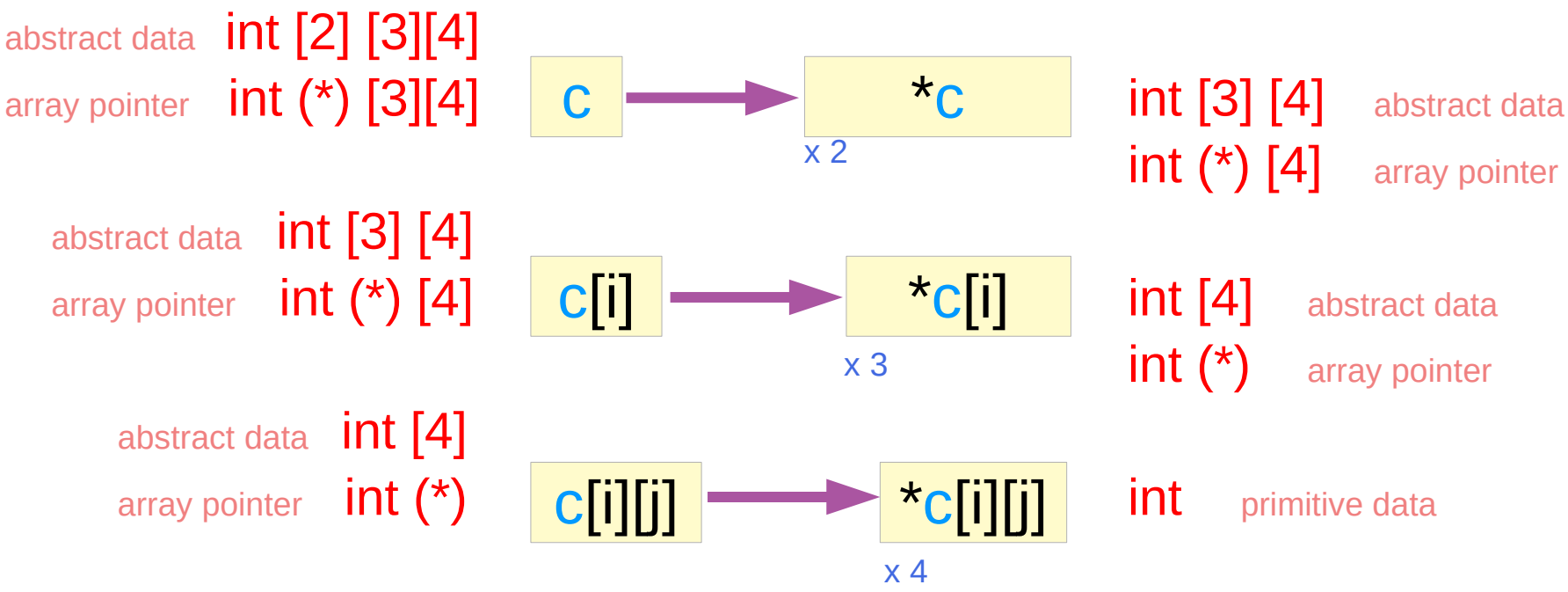
sizeof(**int**) = 4

Abstract Data c, c[i], c[i][j] – start addresses and sizes



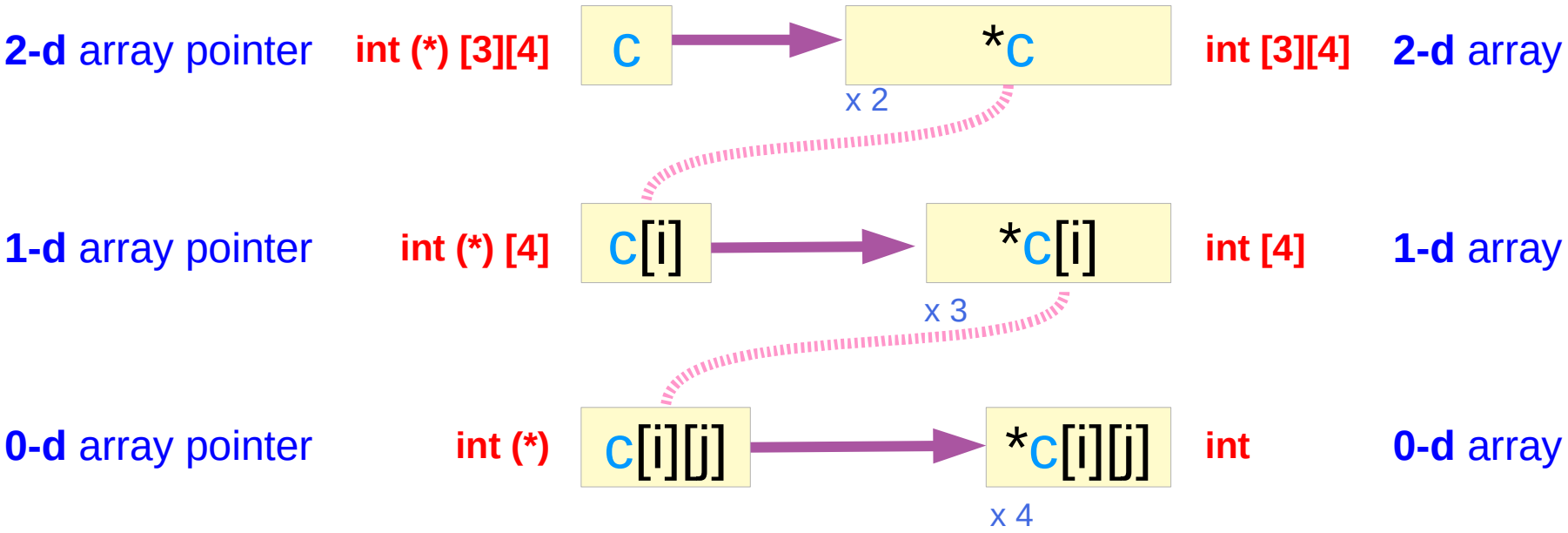
Types in a multi-dimensional array

```
int     c [2][3][4];
```



Virtual array pointers and abstract data

```
int     c [2][3][4];
```

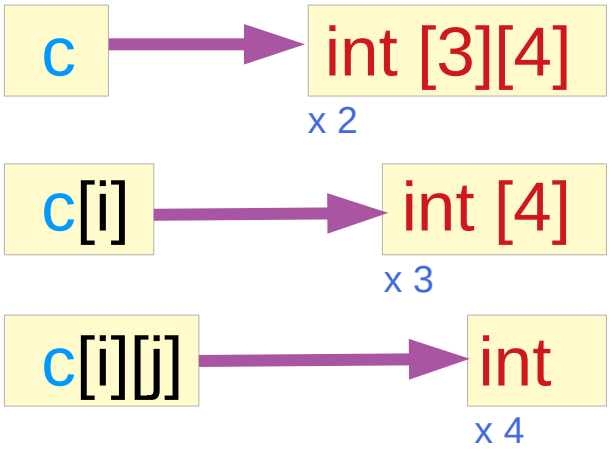


all these pointers are virtual, and take no actual memory locations

exploiting the **contiguity** of allocated memory locations

Abstract Data Sizes

```
int     c [2][3][4];
```



c, c[i], c[i][j] are virtual array pointers
and they are also abstract data (arrays)

when sizes are considered,
view them as abstract data (arrays)

the size of a pointer type is fixed
Here, the sizes of virtual pointers are shown
i.e, the sizes of different abstract data types

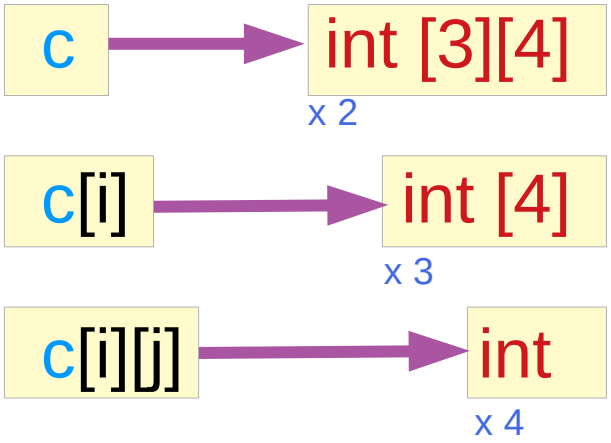
```
sizeof( c)          = sizeof(int [2][3][4])
sizeof(*c)          = sizeof(int [3][4])

sizeof( c[i])       = sizeof(int [3][4])
sizeof(*c[i])       = sizeof(int [4])

sizeof( c[i][j])    = sizeof(int [4])
sizeof(*c[i][j])    = sizeof(int)
```

all are sizes of arrays

Virtual array pointer sizes and abstract data sizes

```
int     c [2][3][4];
```

size of a virtual array pointer = size of the pointed abstract data type * the number of such types



sizeof(c) = sizeof(*c) * **2**

sizeof(c[i]) = sizeof(*c[i]) * **3**

sizeof(c[i][j]) = sizeof(*c[i][j]) * **4**

Sizes of array pointer types

```
int     c [2][3][4];
```

c → int [3][4]

c[i] → int [4]

c[i][j] → int

not real array pointers
virtual array pointers

c int (*)[3][4]
sizeof(int (*) [3][4]) = pointer size ≠ sizeof(c)

c[i] int (*) [4]
sizeof(int (*) [4]) = pointer size ≠ sizeof(c[i])

c[i][j] int [4]
sizeof(int [4]) = pointer size ≠ sizeof(c[i][j])

4 bytes for 32-bit machines
8 bytes for 64-bit machines

Hierarchical nested array pointers

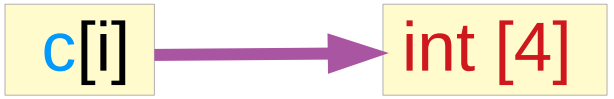
```
int     c [2][3][4];
```

c points to a **2-d** array
increment size: sizeof(int)***2****3*4

int (*) [3][4]

c[i] points to an **1-d** array
increment size: sizeof(int)***3****4

int (*) [4]



c[i][j] points to an integer
increment size: sizeof(int)***4***

int (*)

Sub-array properties in multi-dimensional arrays

int c [2][3][4];  3-d access c **[i][j][k]**

2-d array pointer	c	int (*) [3][4]
1-d array pointers	c[i]	int (*) [4]
0-d array pointers	c[i][j]	int (*)

Hierarchical Sub-arrays in a 3-d array

```
int     c [L][M][N];
```

```
        c [i][j][k]
```

left-to-right associativity

```
c    [i]  [j][k]
c[i] [j]  [k]
c[i][j]   [k]
```

Array Names and Types

Pointers to hierarchical sub-arrays

c	3-d array names	int (*) [M][N]	2-d array pointer
c[i]	2-d array names	int (*) [N]	1-d array pointer
c[i][j]	1-d array names	int (*)	0-d array pointer

General requirements for accessing **c[i][j][k]**

c [i][j][k]

```
&c[i][j][k] = c[i][j]+k
&c[i][j]    = c[i]+j
&c[i]       = c+i
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]       = c
```

```
c[i][j][k] = *(c[i][j]+k)
c[i][j]    = *(c[i]+j)
c[i]       = *(c+i)
```

```
c[i][j][0] = *(c[i][j])
c[i][0]    = *(c[i])
c[0]       = *(c)
```

3-d access pattern **c[i][j][k]**

General requirements

```
c [i][j][k]
```

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

Pointer array approach

```
int** c[2];
int*  b[2*3];
int   c[2*3*4];
```

```
c[i][j][k]    :: int
c[i][j]       :: int *
c[i]          :: int **
```

```
c[i]  ⬅  &b[i*3]
b[j]  ⬅  &a[j*4]
```

Explicit
Arrays of pointers with
Multiple Indirection

N-dim Array approach

```
int c[2][3][4];
```

```
c[i][j][k]    :: int
c[i][j]       :: int [4]
c[i]          :: int (*) [4]
```

```
c[i][j]  ⬅  &c[i][j][0]
c[i]     ⬅  &c[i][0][0]
c        ⬅  &c[0][0][0]
```

Implicit
Nested
Virtual Array Pointers

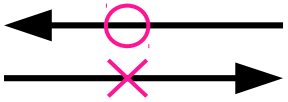
3-d access pattern **c[i][j][k]** – array pointer approach

General requirements

c [i][j][k]

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```



N-dim array approach

```
int c[2][3][4];
```

```
c[i][j][k]   ::    int
c[i][j]   ::   int [4]
c[i]   ::  int (*) [4]
c   :: int (*) [3][4]
```

```
c[i][j]  ⟸ &c[i][j][0]
c[i]     ⟸ &c[i][0][0]
c        ⟸ &c[0][0][0]
```

Implicit
Nested
Virtual Array Pointers

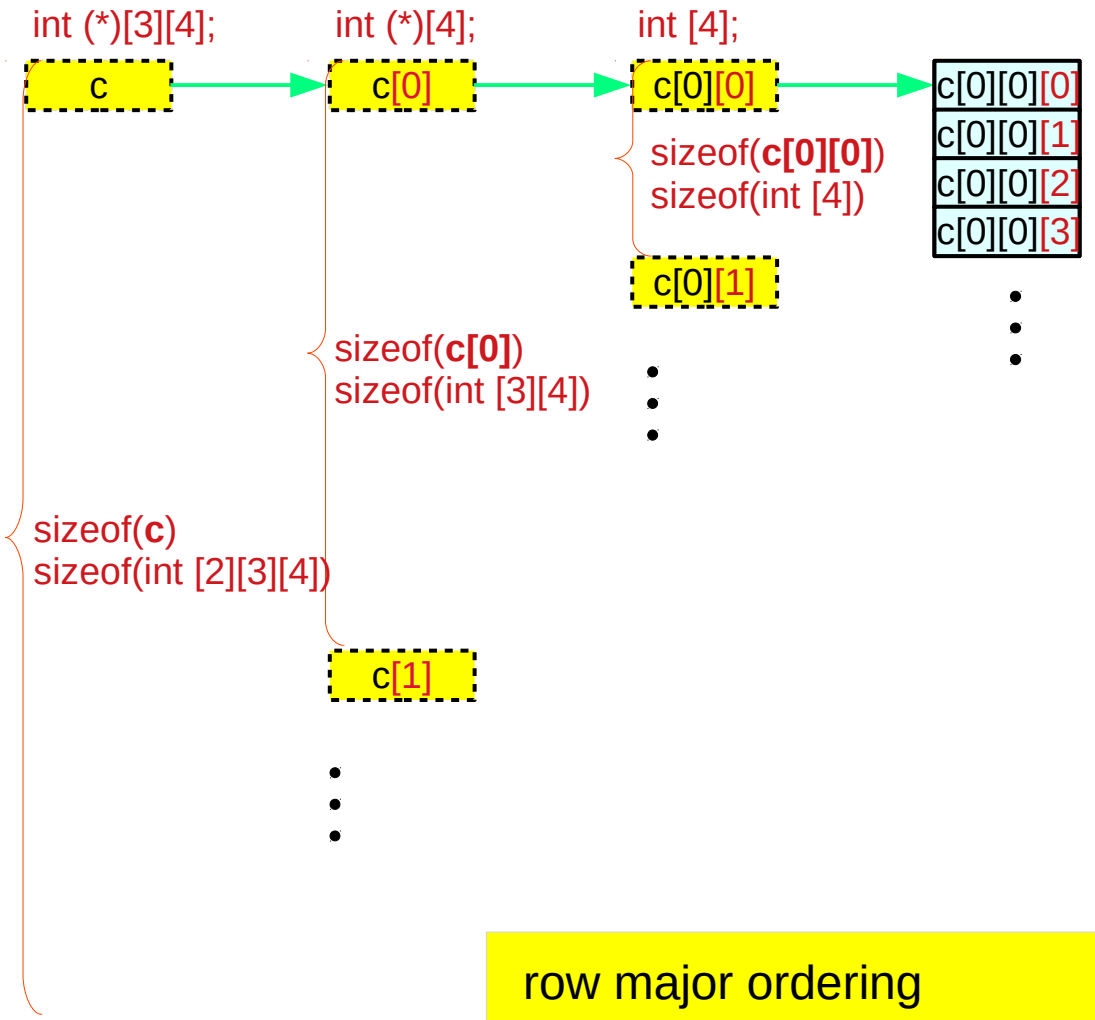
Using N-dimensional arrays

```
int     c [2][3][4];
```

```
        c [i][j][k];
```

constraints

```
c       ⟸ &c[0][0][0]
c[i]    ⟸ &c[i][0][0]
c[i][j] ⟸ &c[i][j][0]
```



row major ordering
contiguous linear layout

Types of **c[i]** and **c[i][j]**

```
c [i][j][k];
```

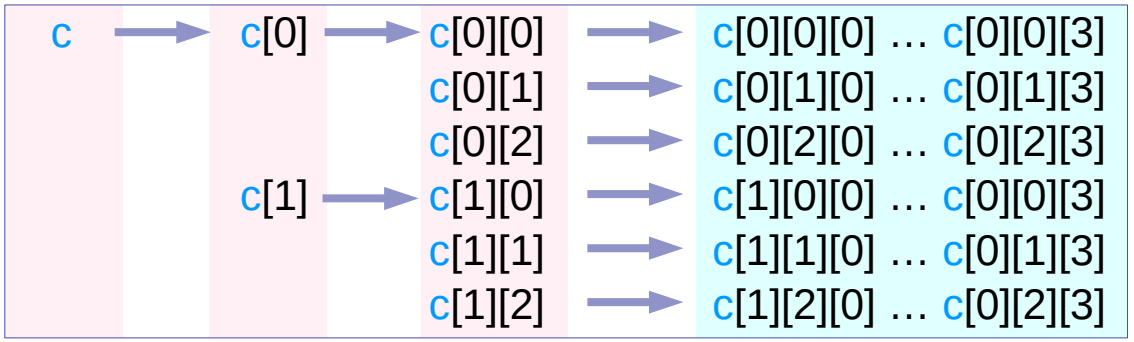
```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
int     c [2][3][4];
```

c[i] virtual array pointer of the type int (*) [4]
c[i][j] : the name of 1-d array with 4 integers int [4]

c[i][j] (virtual array) pointer of the type int (*)
c[i][j][k] : an element of a 4-integer array int



int [2] [3][4]	int [3] [4]	int [4]	int … int
int (*) [3][4]	int (*) [4]	int (*)	int … int
pointers to a 2-d array	pointers to a 1-d array	1-d array names	leading element of 4-integer array

Values of **c[i]** and **c[i][j]**

```
c [i][j][k];
```

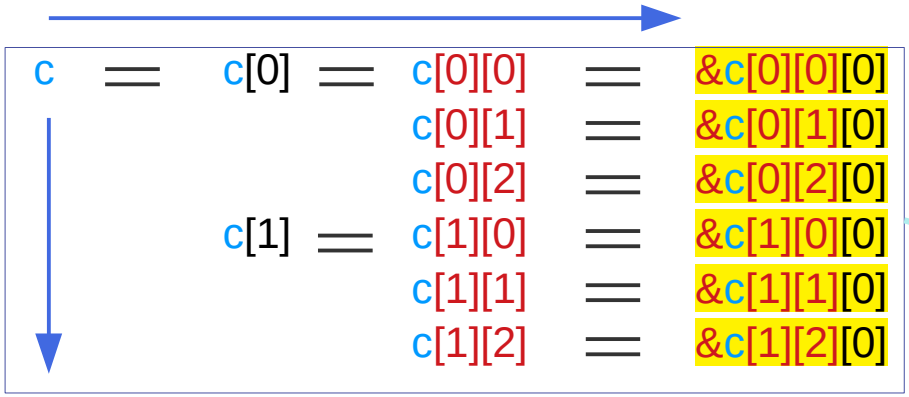
```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
int     c [2][3][4];
```

virtual array pointers

in each row in the following figure
have the same value (address value)



Horizontal displacements are not counted
only **vertical displacements** are considered
for address values

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

Finding address values of **c**, **c[i]**, **c[i][j]**

```
c [i][j][k];
```

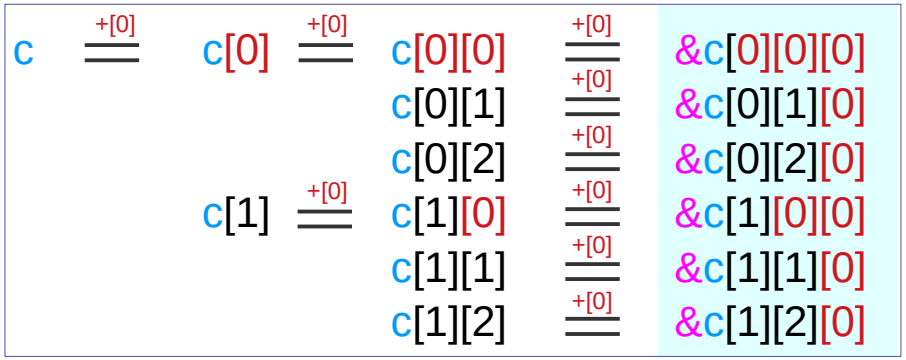
```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
int     c [2][3][4];
```

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

append [0] to the right



int (*) [3][4] int (*) [4] int [4] int

c[i][j][0] : leading elements of c[i][j]

```
&c[0][0][0]
&c[0][1][0]
&c[0][2][0]
&c[1][0][0]
&c[1][1][0]
&c[1][2][0]
```

c[i][0][0] : leading elements of c[i]

```
&c[0][0][0]

&c[1][0][0]
```

c[0][0][0] : leading elements of c

```
&c[0][0][0]
```

Finding sub arrays for the leading elements **c[i][j][0]**

```
c [i][j][k];
```

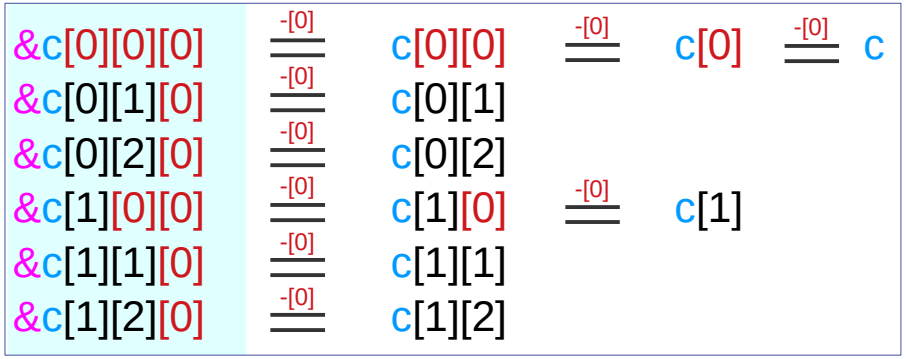
```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
int     c [2][3][4];
```

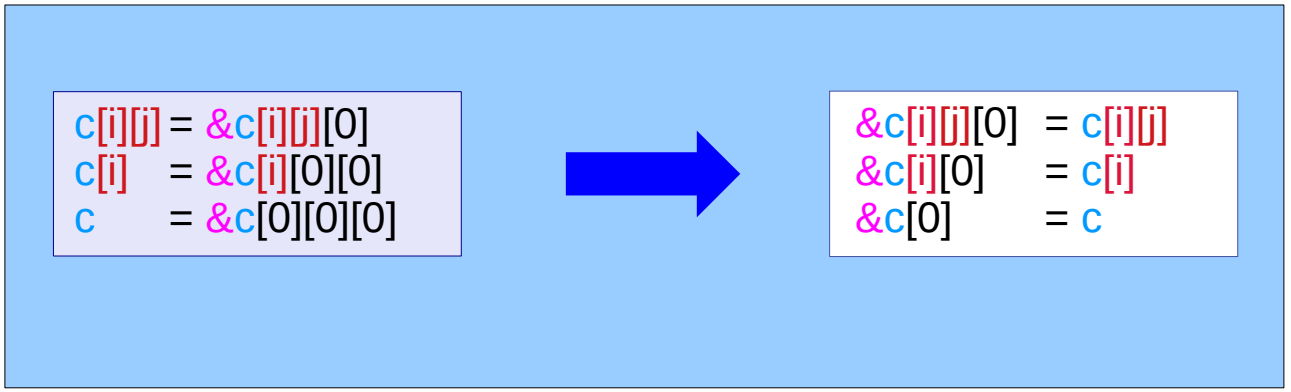
```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

delete [0] from the right



c[0][0][0] is the leading element of c[0][0], c[0], c
c[0][1][0] is the leading element of c[0][1]
c[0][2][0] is the leading element of c[0][2]
c[1][0][0] is the leading element of c[1][0], c[1]
c[1][1][0] is the leading element of c[1][1]
c[1][2][0] is the leading element of c[1][2]

multi-dimensional arrays



Pointer reference and dereference relationship

```
c [i][j][k];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
int     c [2][3][4];
```

```
c → c[0] → c[0][0] → c[0][0][0]
           c[0][1] → c[0][1][0]
           c[0][2] → c[0][2][0]
   c[1] → c[1][0] → c[1][0][0]
           c[1][1] → c[1][1][0]
           c[1][2] → c[1][2][0]
```

```
                c[i][j]    =    &c[i][j][0]
c → c[0] → c[0][0]  ==  &c[0][0][0]
           c[0][1]  ==  &c[0][1][0]
           c[0][2]  ==  &c[0][2][0]
   c[1] → c[1][0]  ==  &c[1][0][0]
           c[1][1]  ==  &c[1][1][0]
           c[1][2]  ==  &c[1][2][0]
```

```
          c[i]    =    &c[i][0]
c →  c[0]  ==  &c[0][0]
     c[1]  ==  &c[1][0]
```

```
c   =    &c[0]
c   ==    c[0]
```

General requirements for **c[i][j][k]**

```
c [i][j][k];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
int      c [2][3][4];
```

c[i][j] virtual array pointer of the type int (*)
c[i][j][0] : leading element of a 4-integer array int

```
*(c[0][0]+0) = c[0][0][0]
*(c[0][1]+0) = c[0][1][0]
*(c[0][2]+0) = c[0][2][0]
*(c[1][0]+0) = c[1][0][0]
*(c[1][1]+0) = c[1][1][0]
*(c[1][2]+0) = c[1][2][0]
```

c[0][0] is the address of c[0][0][0]
c[0][1] is the address of c[0][1][0]
c[0][2] is the address of c[0][2][0]
c[1][0] is the address of c[1][0][0]
c[1][1] is the address of c[1][1][0]
c[1][2] is the address of c[1][2][0]

c[i] virtual array pointer of the type int (*) [4]
c[i][j] : a 4-element 1-d array name int [4]

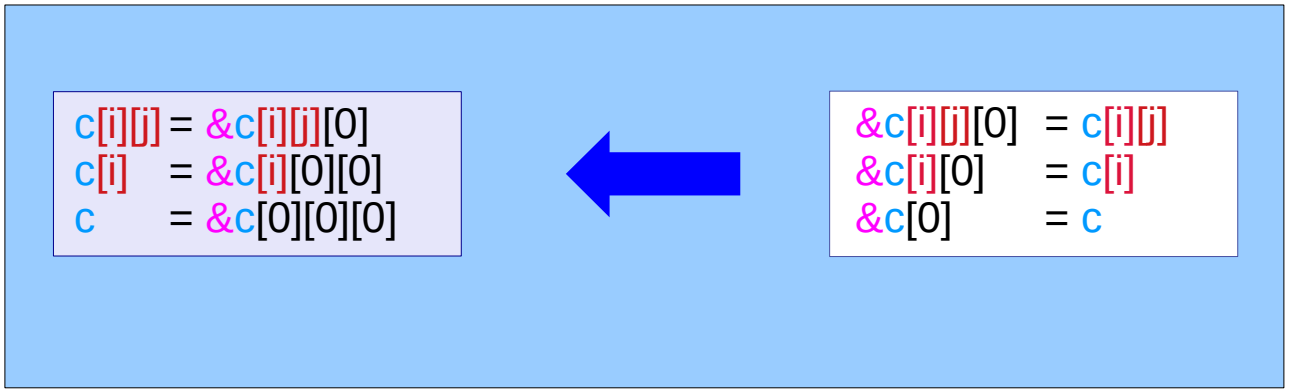
```
*(c[0]+0)  = c[0][0]
*(c[1]+0)  = c[1][0]
```

c[0] is the address of c[0][0]
c[1] is the address of c[1][0]

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

multi-dimensional arrays



c[0] = c[0][0] relation

```
c [i][j][k];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

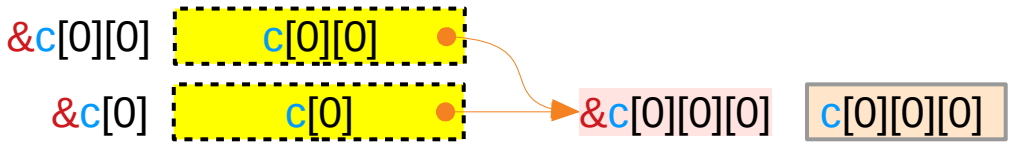
```
int      c [2][3][4];
```

c == c[0] == c[0][0] == &c[0][0][0]

value(c[0]) = &c[0][0][0]
value(c[0][0]) = &c[0][0][0]

type(c[0]) = int (*)[4]
type(c[0][0]) = int [4]

c[0] = c[0][0] *means*
value(c[0]) = value(c[0][0])




```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

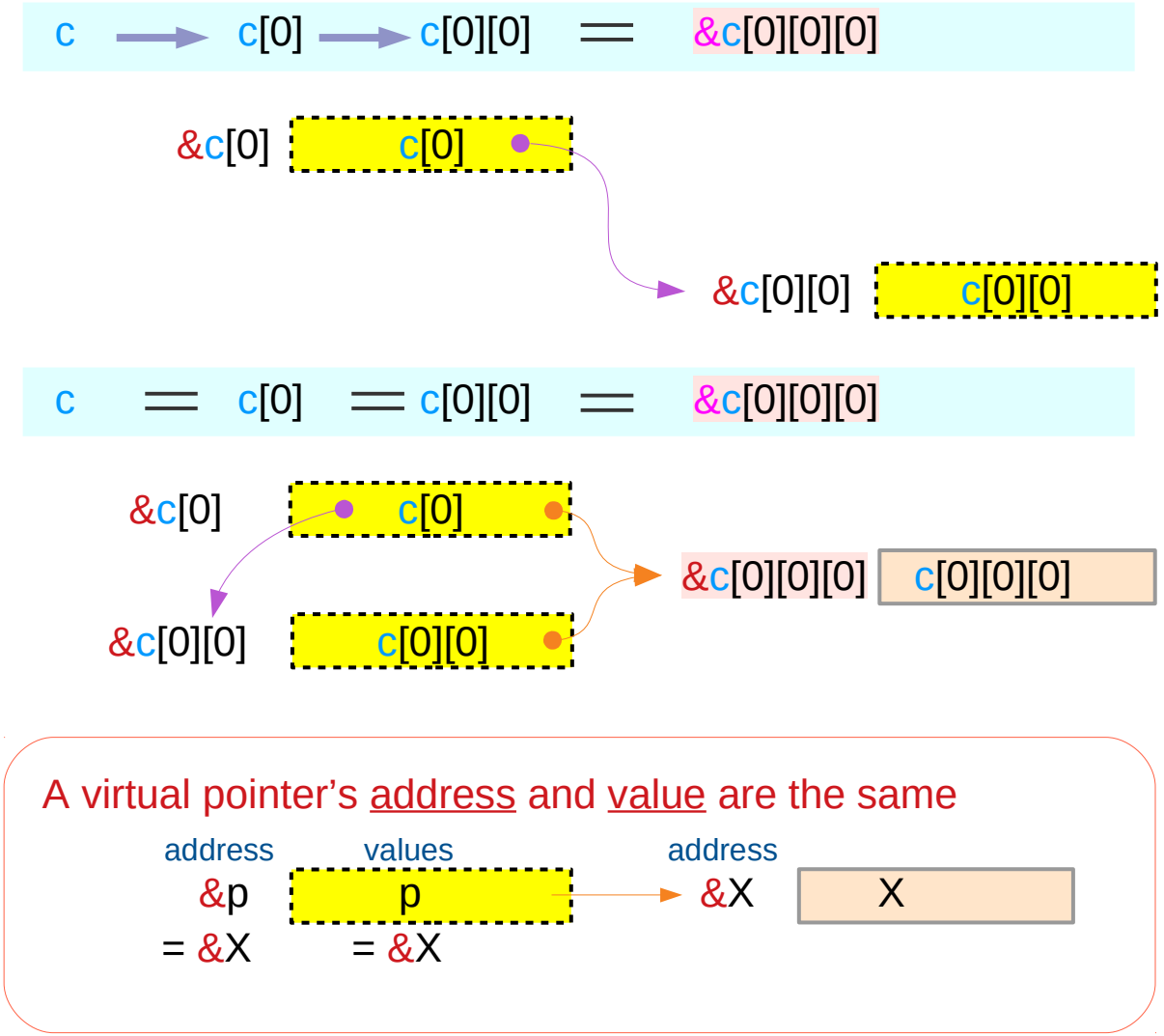
Addresses and Values of **c[0]** and **c[0][0]**

```
c [i][j][k];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
int     c [2][3][4];
```



c[0] and c[0][0] point to the same c[i][0][0]

```
c [i][j][k];
```

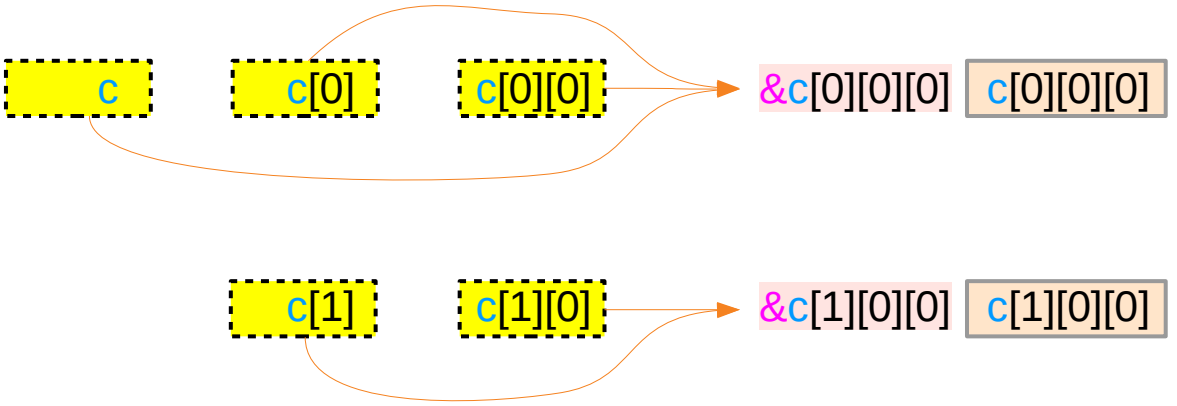
```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
int     c [2][3][4];
```

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

```
c =    c[0] =    c[0][0] = &c[0][0][0]   ⬅ value
int(*)[3][4]  int(*)[4]    int(*)         int   ⬅ type

       c[1] =    c[1][0] = &c[1][0][0]   ⬅ value
              int(*)[4]    int(*)         int   ⬅ type
```



These virtual pointers have different types
but the same value (address)

&c[i][0] and &c[i][0][0] – equivalence relations

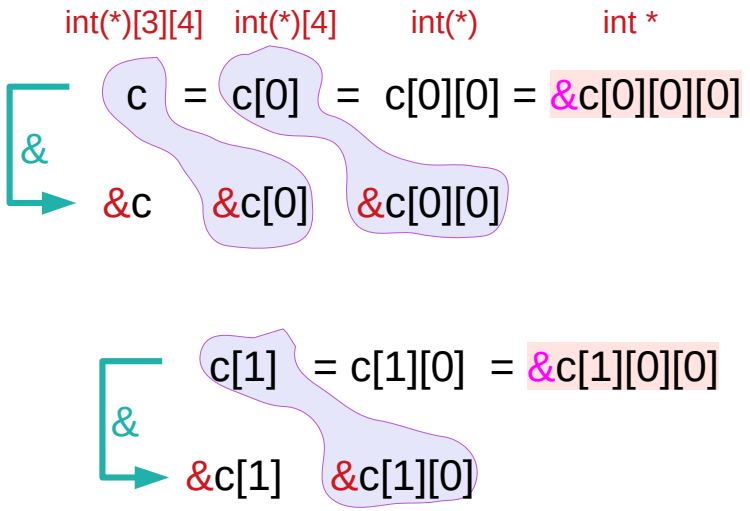
```
c [i][j][k];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
int     c [2][3][4];
```

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```



equivalences

```
c    ≡ &c[0],
c[0] ≡ &c[0][0]
c[0][0] ≡ &c[0][0][0]
```

equivalences

```
c[1] ≡ &c[1][0]
c[1][0] ≡ &c[1][0][0]
```

Horizontal displacements are not counted
only **vertical displacements** are considered
for **address values**

equivalences

```
c    ≡ &c[0],
c[i] ≡ &c[i][0]
c[i][0] ≡ &c[i][0][0]
```

c[i] = &c[i] and c[i][0] = &c[i][0]

```
c [i][j][k];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

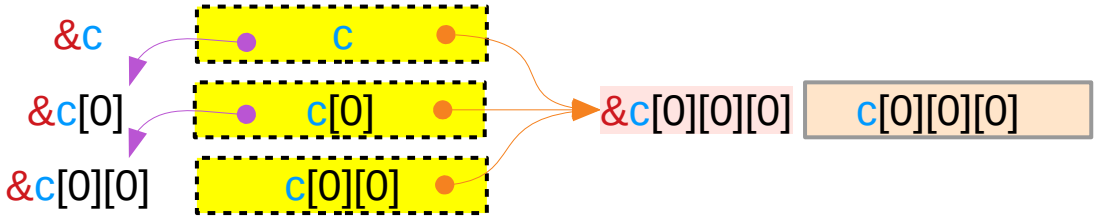
```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
int      c [2][3][4];
```

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

c = c[0] = c[0][0] = &c[0][0][0]
|| || ||
&c = &c[0] = &c[0][0]

c[1] = c[1][0] = &c[1][0][0]
|| ||
&c[1] = &c[1][0]



c[i] = &c[i] and c[i][0] = &c[i][0]

```
c [i][j][k];
```

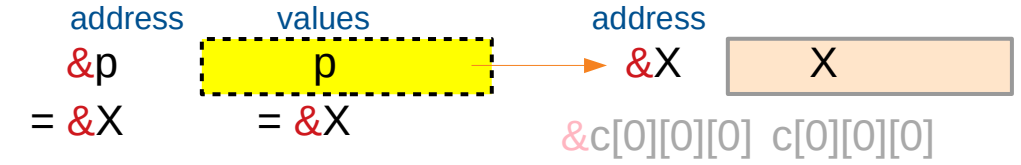
```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

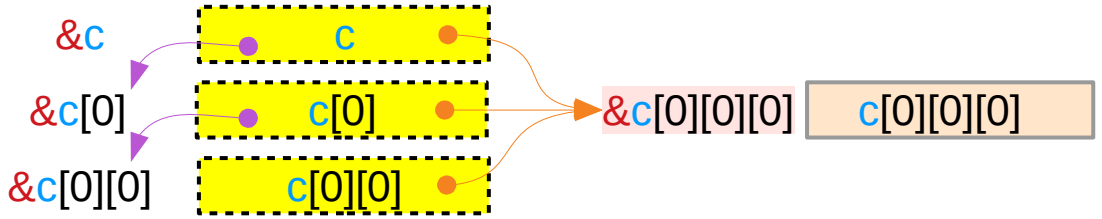
```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
int     c [2][3][4];
```

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

A virtual pointer's address and value are the same





Leading elements and array pointers

c[0][0][0] is the leading element of c[0][0], c[0], c
c[0][1][0] is the leading element of c[0][1]
c[0][2][0] is the leading element of c[0][2]
c[1][0][0] is the leading element of c[1][0], c[1]
c[1][1][0] is the leading element of c[1][1]
c[1][2][0] is the leading element of c[1][2]

Array Pointers to **c[i][0][0]**

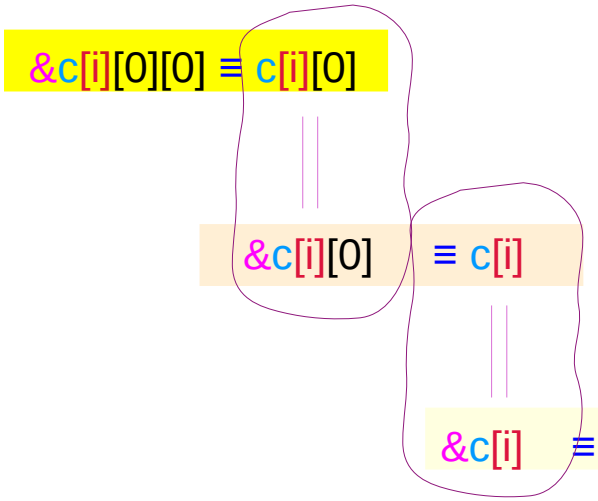
```
c [i][j][k];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
int     c [2][3][4];
```

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```



virtual pointers:
the address of a pointer is
the same as its value

= c + i*sizeof(*c)

= &c[0][0][0] + i*3*4

delete [0] from the right

```
&c[0][0][0]  -[0]  c[0][0]  -[0]  c[0]  -[0]  c
&c[1][0][0]  -[0]  c[1][0]  -[0]  c[1]
```

Array Pointers to **c[i][j][0]**

```
c [i][j][k];
```

```
&c[i][j][0]  = c[i][j]
&c[i][0]     = c[i]
&c[0]        = c
```

```
&c[i][j][k]  = c[i][j]+k
&c[i][j]     = c[i]+j
&c[i]        = c+i
```

```
int     c [2][3][4];
```

```
c[i][j] = &c[i][j][0]
c[i]    = &c[i][0][0]
c       = &c[0][0][0]
```

&c[i][j][0] ≡ c[i][j]

||

&c[i][j] ≡ c[i] + j

= c[i] + j*sizeof(*c[i])

= c + i*sizeof(*c) + j*4

= &c[0][0][0] + i*3*4 + j*4

delete [0] from the right

&c[0][0][0]	-[0] =	c[0][0]	-[0] =	c[0]	-[0] =	c
&c[0][1][0]	-[0] =	c[0][1]				
&c[0][2][0]	-[0] =	c[0][2]				
&c[1][0][0]	-[0] =	c[1][0]	-[0] =	c[1]		
&c[1][1][0]	-[0] =	c[1][1]				
&c[1][2][0]	-[0] =	c[1][2]				

Contiguity Constraints

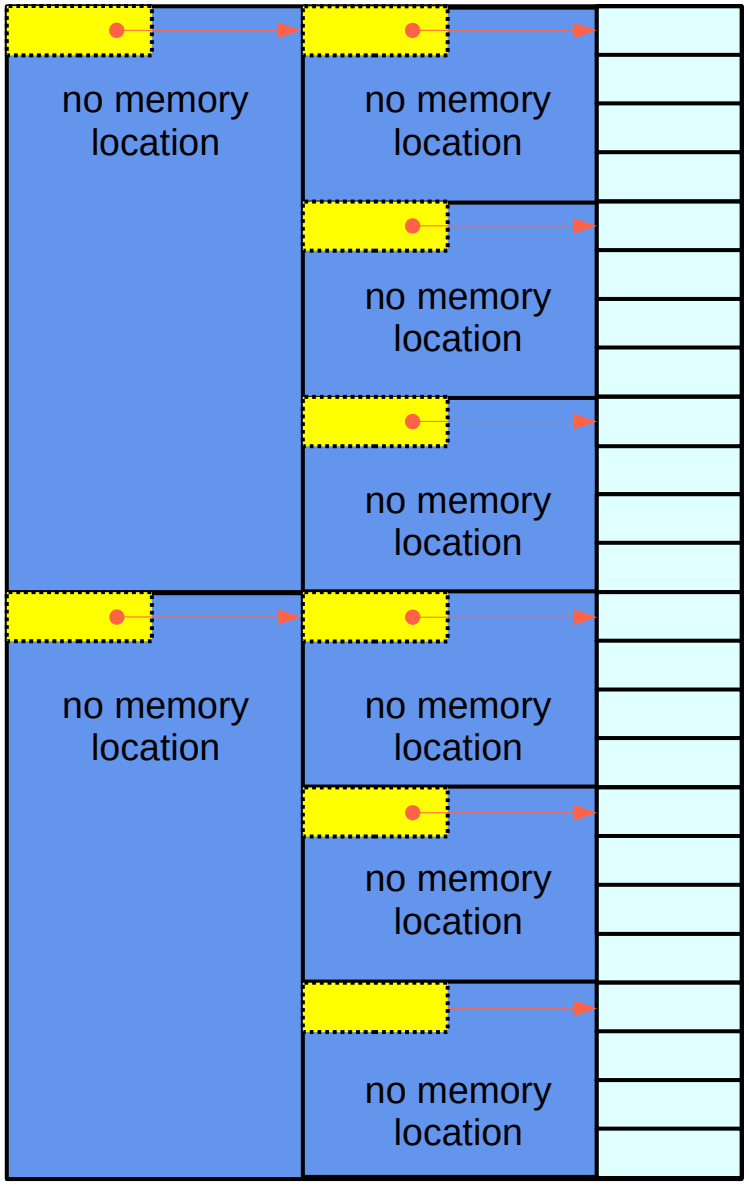
```
c [i][j][k];
```

Virtual Array Pointers and Contiguity

Using array pointers

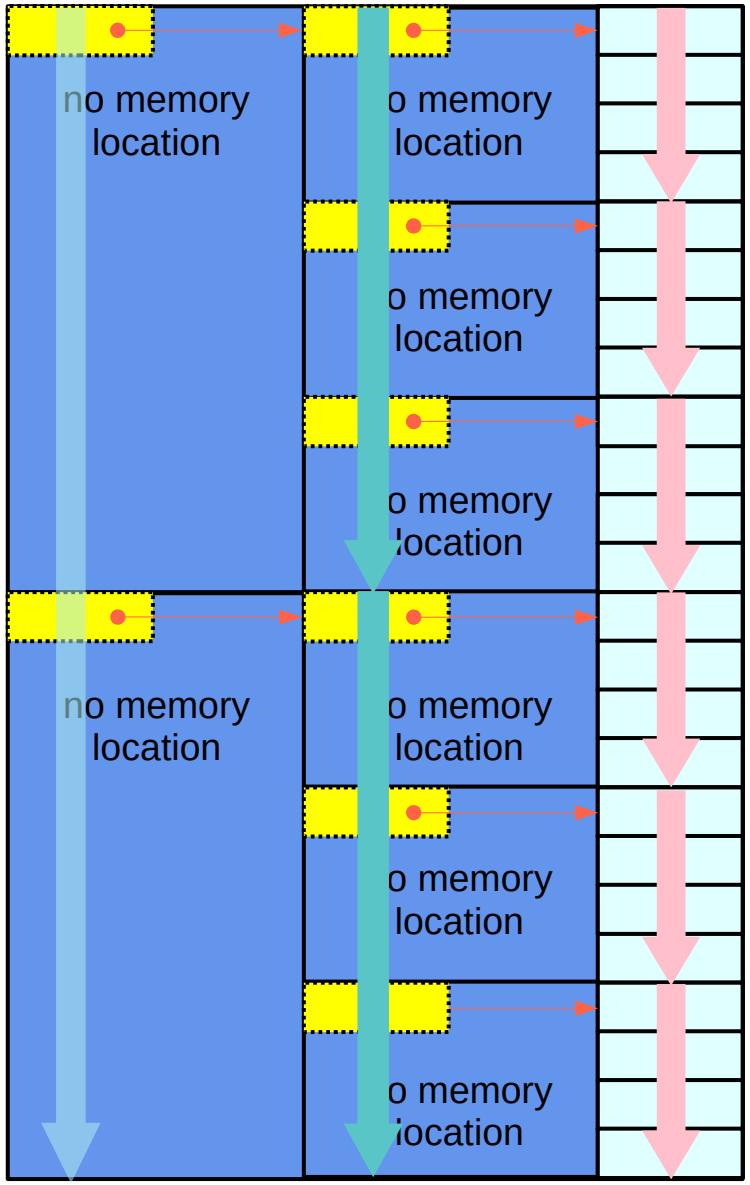
int (*) [N], int (*) [M][N], int (*) [L][M][N], …

Array pointer approach for 3-d access patterns



Array Pointer Approach
(pointer to arrays)

Array pointer approach – contiguity constraints



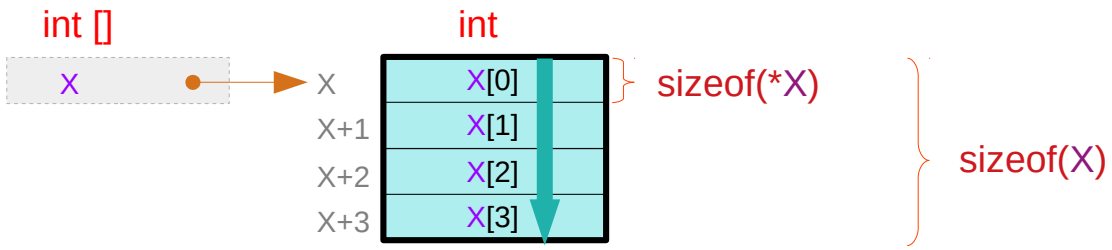
Array Pointer Approach
(pointer to arrays)

Equivalence and contiguity

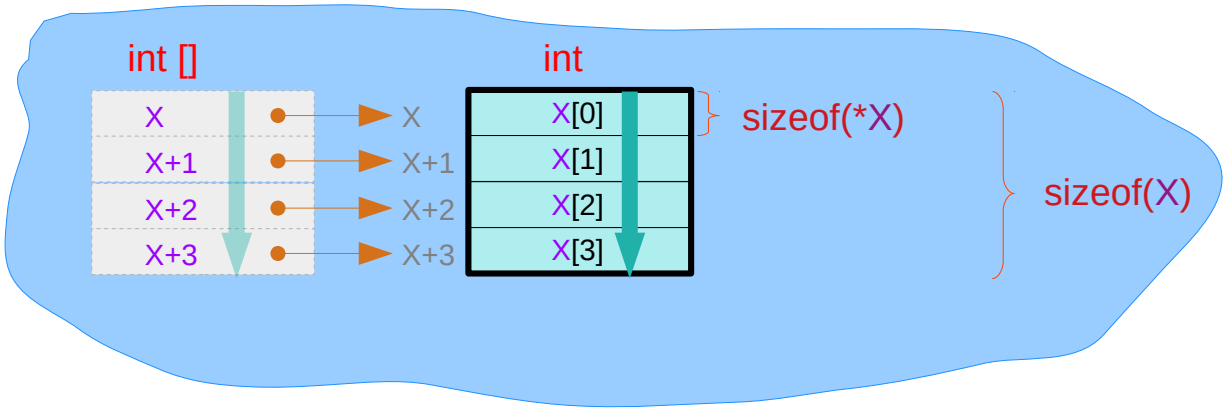
consecutive address　　consecutive data

*(X+n) ≡ X[n]

contiguous index : n



int X[4]; contiguous X[i] for a given X : **primitive types**

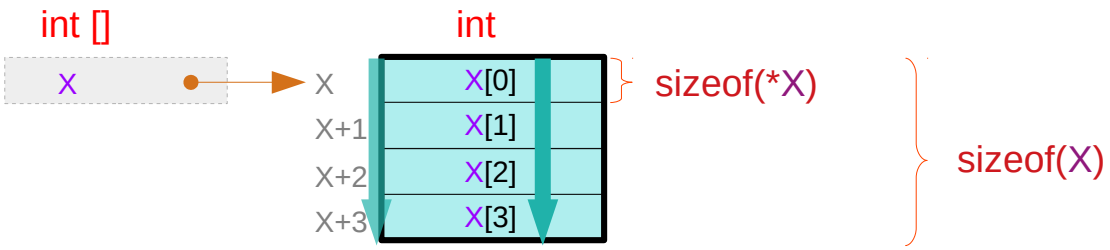


Equivalence and contiguity

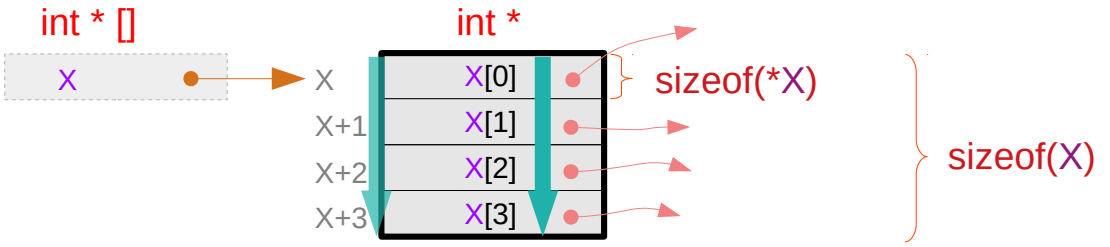
consecutive address　　consecutive data

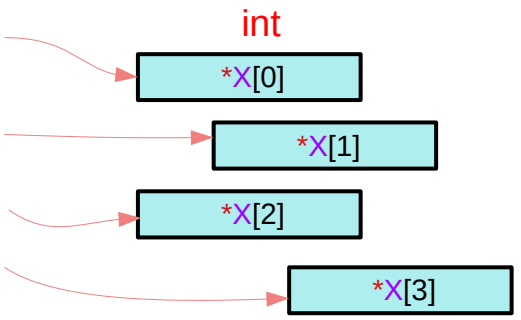
*(X+n) ≡ X[n]

contiguous index : n



int X[4]; contiguous X[i] for a given X : **primitive types**





int * X[4]; contiguous X[i] for a given X : **pointer types**

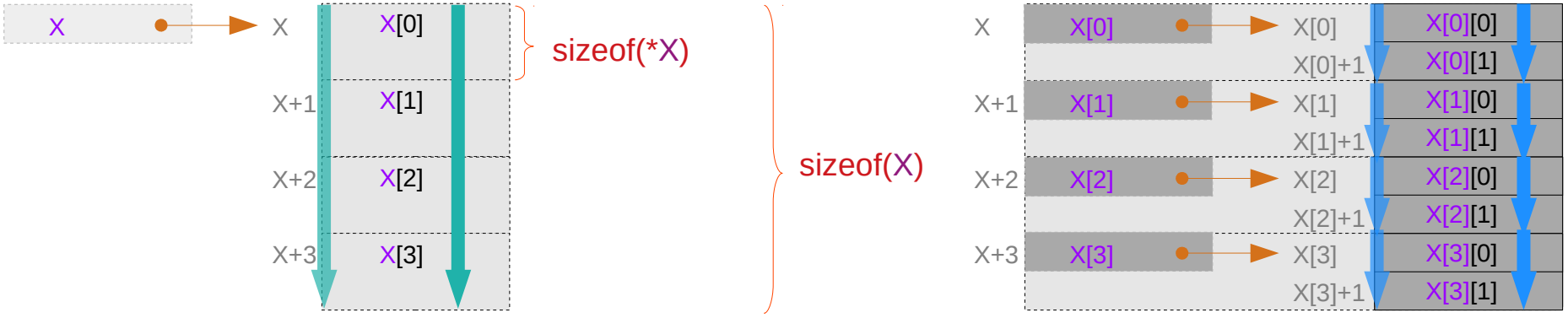
Equivalence and contiguity

consecutive address　　consecutive data

*(X+n) ≡ X[n]

contiguous index : n

can be recursively applied



atype * X[4]; contiguous X[i] for a given X : **abstract data types**

Equivalence

By definition, contiguous memory locations are assumed

consecutive address　　consecutive data

*(X+n) ≡ X[n]

contiguous index : n

*(p[m]+n) ⟺ p[m][n]

X = p[m]　contiguous index : **n**

(*(p+m))[n]; ⟺ p[m][n];

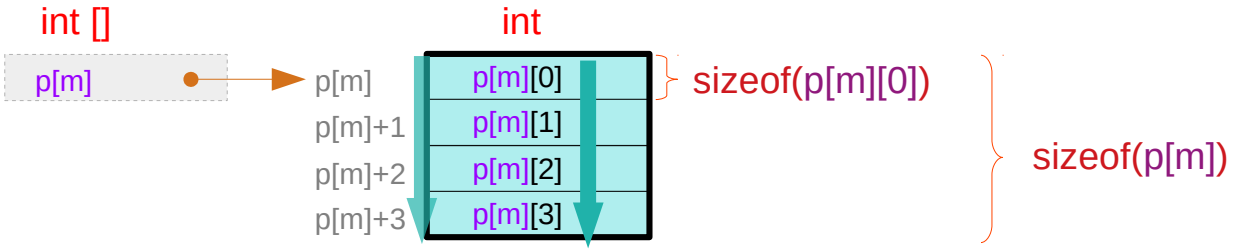
X = p　contiguous index : **m**

For a given p[m] –

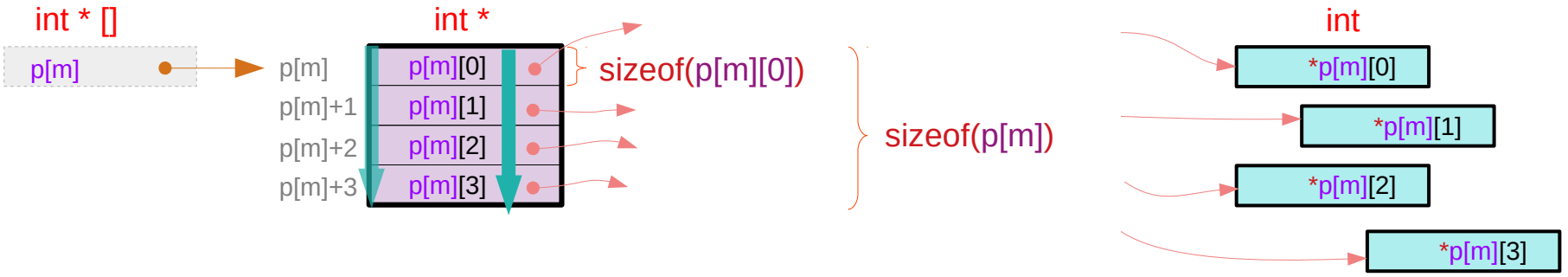
*(p[m]+n) ⟺ p[m][n]

for a given p[m] contiguous index : **n**

int p[M][4]; contiguous p[m][n] for a given p[m] : **primitive types** m = 0, 1, …, M-1



int * p[M][4]; contiguous p[m][n] for a given p[m] : **pointer types** m = 0, 1, …, M-1

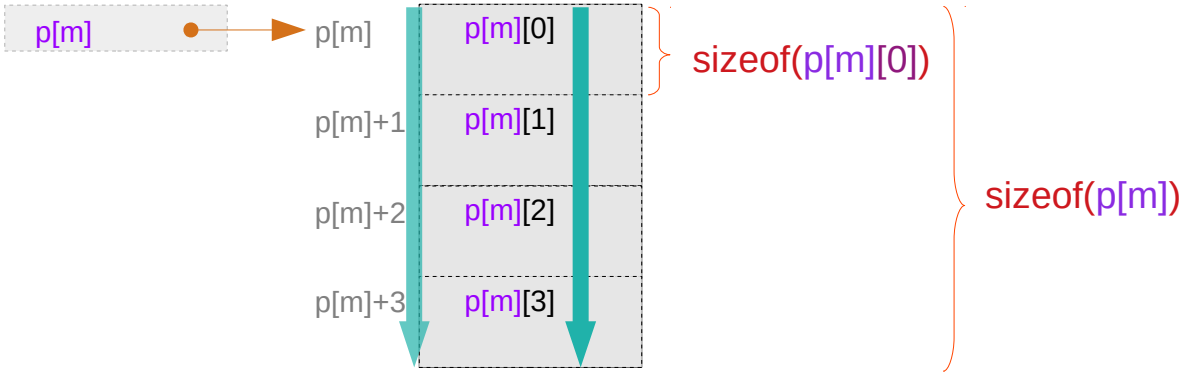


For a given p[m] –

*(p[m]+n) ⟺ p[m][n]

for a given p[m] contiguous index : **n**

atype * p[M][4]; contiguous p[m][n] for a given p[m] : **abstract data types** m = 0, 1, …, M-1



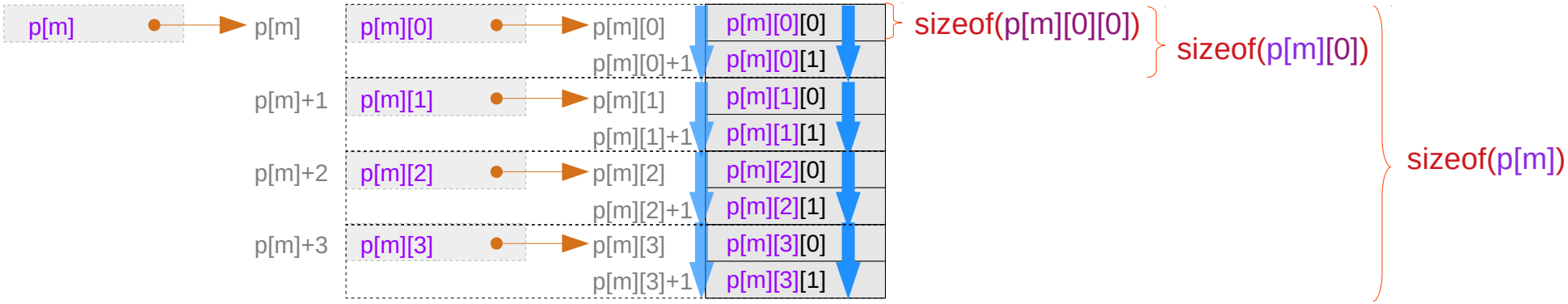
can be recursively applied

For a given p[m][n] –

*(p[m][n]+k) ⟺ p[m][n][k]

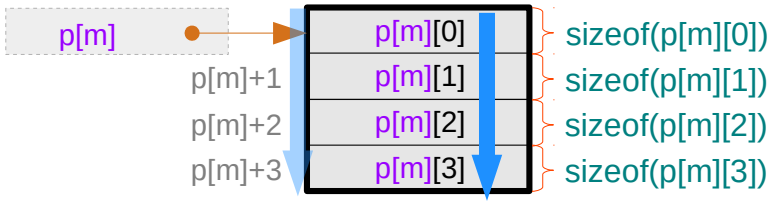
for a given p[m][n] contiguous index : **k**

atype * p[M][4][2]; contiguous p[m][n][k] for a given p[m][n] : **abstract data types** m = 0, 1, …, M-1



Contiguity constraints

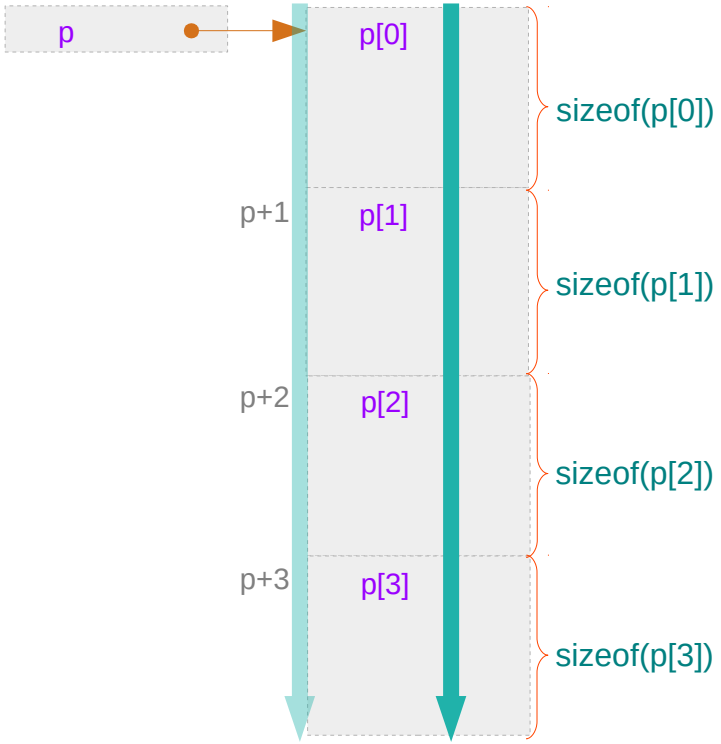
*(p[m]+n) ⟺ p[m][n]



for a given p[m], thus for a given p and m,
p[m][n]'s must be contiguous for all n.
p[m][0], p[m][1], … , p[m][N-1]

contiguous index : n

*(p+m) ⟺ p[m]



for a given p,
p[m]'s must be contiguous for all m.
p[0], p[1], … , p[M-1]

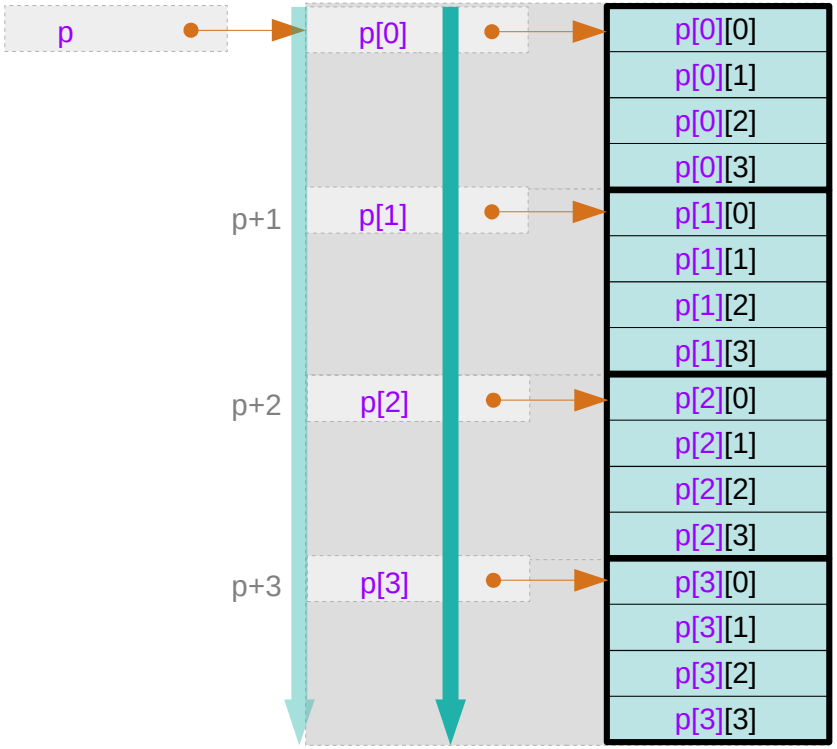
contiguous index : m

Contiguity constraints

*(p+m) ⟺ p[m]

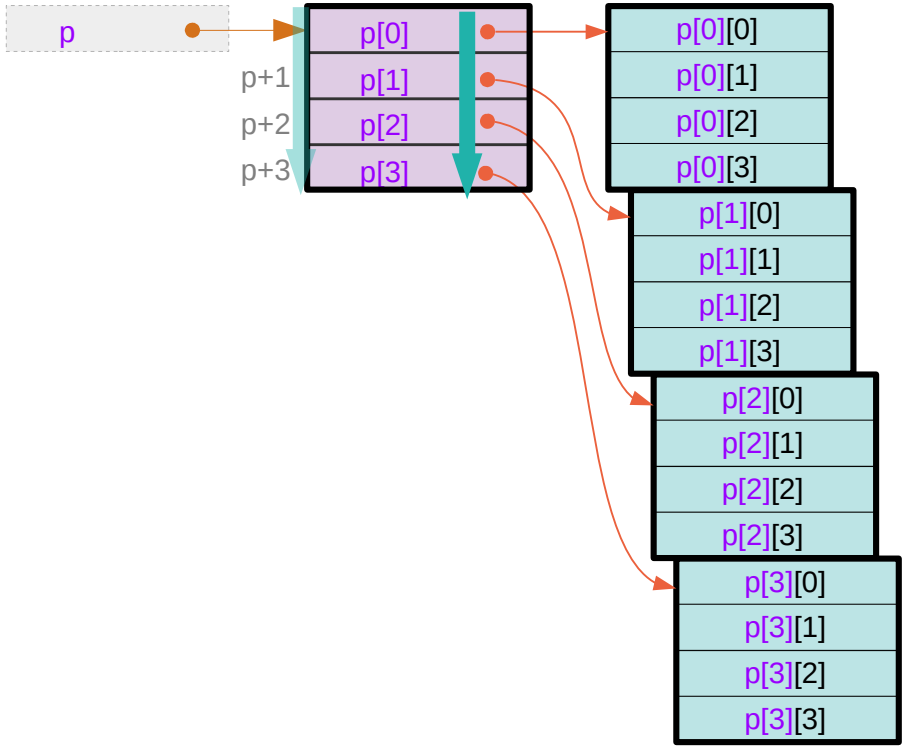
for a given p contiguous index : **m**

2-d array name **1-d** array names



contiguous p[m] ➡ contiguous p[m][n]

1-d array of pointers



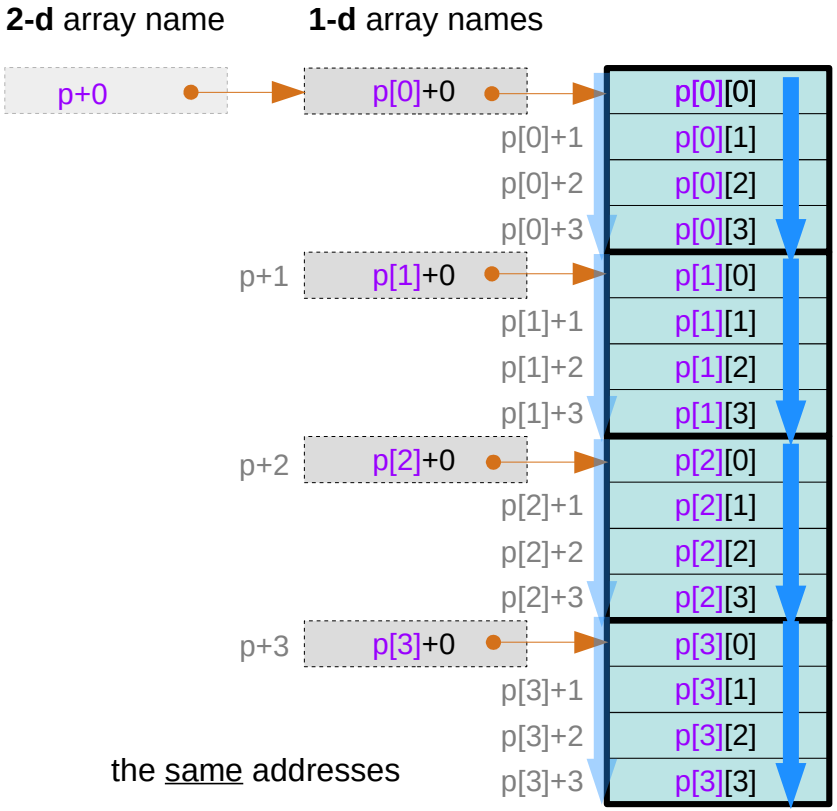
contiguous p[m] ➡ contiguous p[m][n]

Not necessarily

Contiguity constraints – using array pointers

*(p[m]+n) ⟺ p[m][n]

for a given p[m] contiguous index : **n**



contiguous p[m] ➡ contiguous p[m][n]

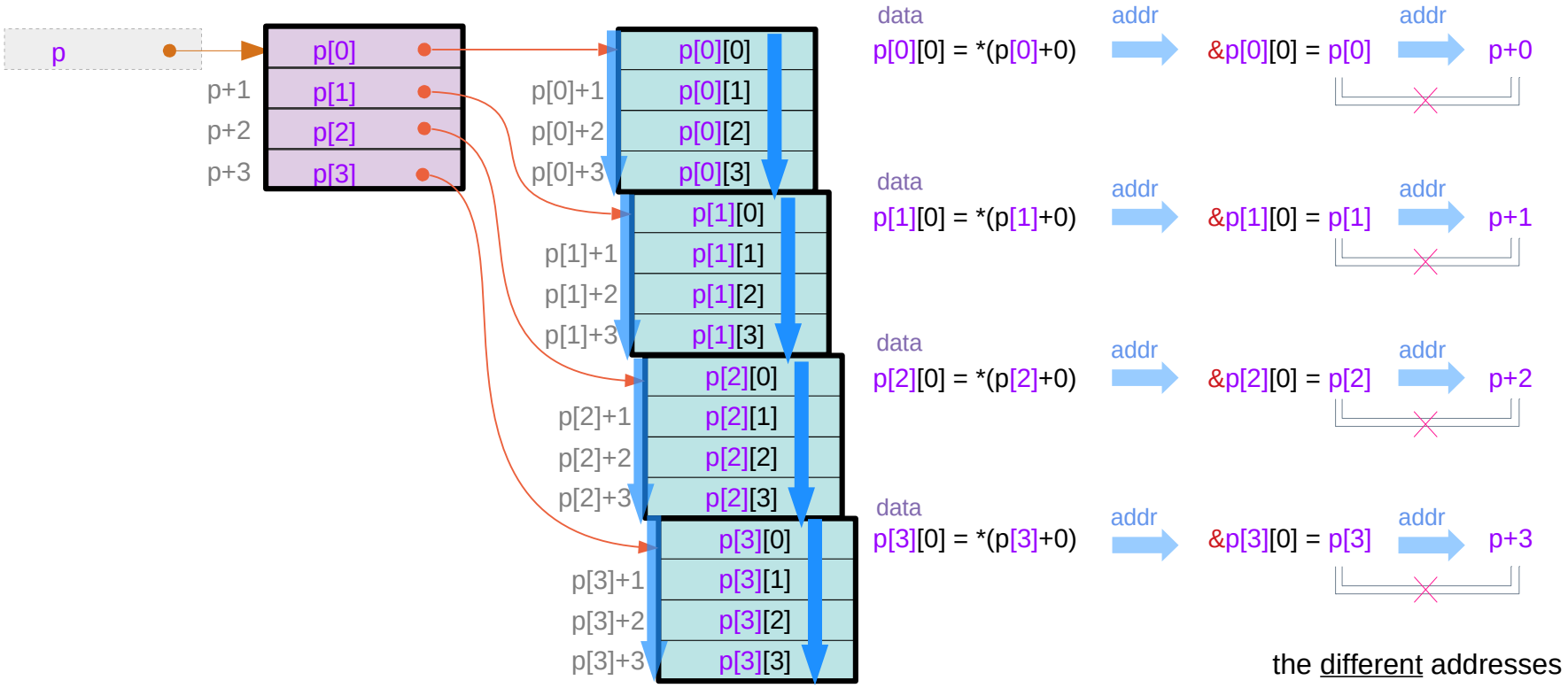
virtual array pointer ⟺ no real memory locations

Contiguity constraints – using pointer arrays

*(p[m]+n) ⟺ p[m][n]

for a given p[m] contiguous index : **n**

1-d array of pointers



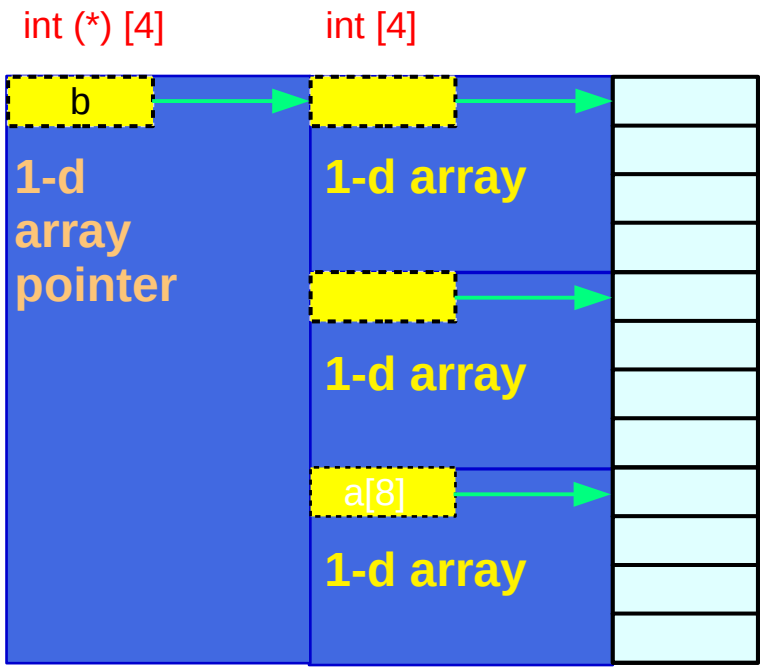
contiguous p[m] ➡ contiguous p[m][n]

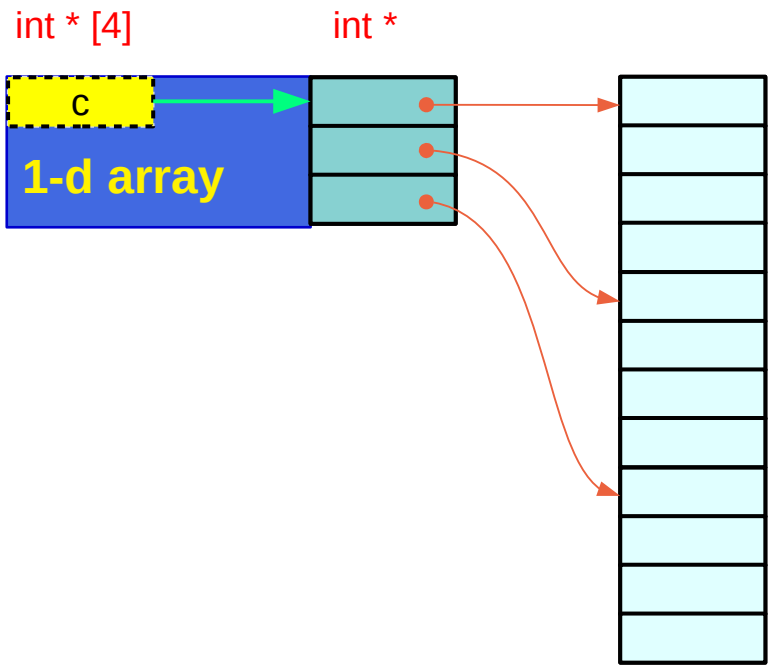
Not necessarily

Contiguity constraints

int a[M][N] ;	int (*b)[N] ;	int * c[M] ;
*(a+m) ⟺ a[m]	*(b+m) ⟺ b[m]	*(c+m) ⟺ c[m]
a[0], a[1], … , a[M-1] are contiguous	b[0], b[1], … , b[M-1] are contiguous	c[0], c[1], … , c[M-1] are contiguous
*(a[m]+n) ⟺ a[m][n]	*(b[m]+n) ⟺ b[m][n]	*(c[m]+n) ⟺ c[m][n]
a[m][0], a[m][1], … , a[m][N-1] are contiguous	b[m][0], b[m][1], … , b[m][N-1] are contiguous	c[m][0], c[m][1], … , c[m][N-1] are contiguous
		a set of assignments of pointers are necessary for this contiguity

Pointer Arrays vs Array Pointers





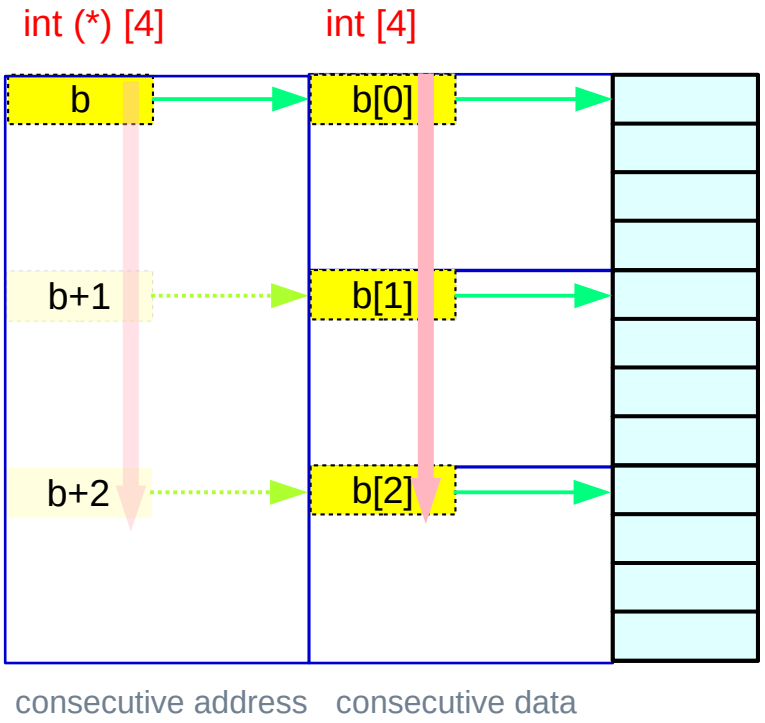
int (*b)[N] ;

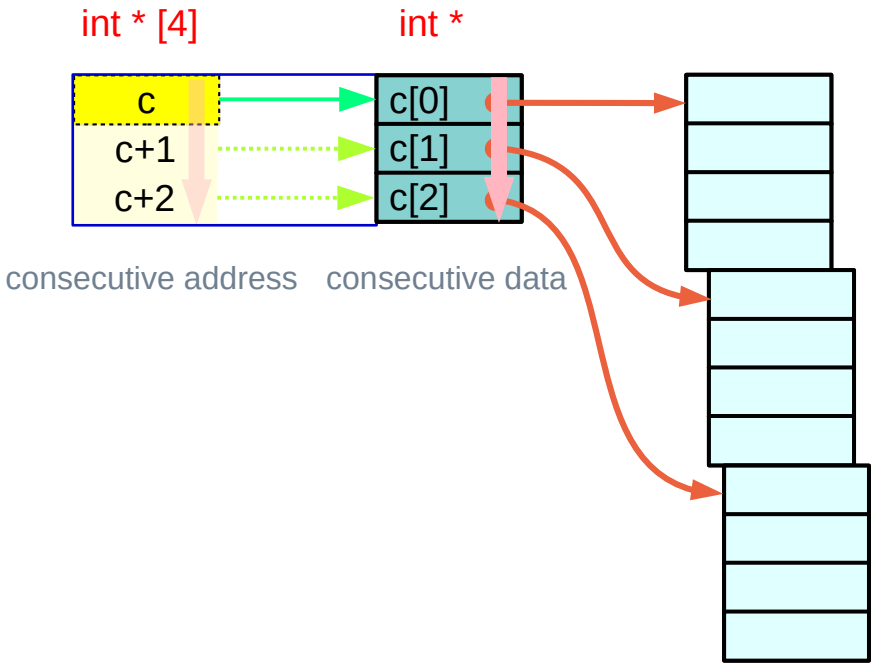
int * c[M] ; with proper assignments

*(b+m) ⟺ b[m]
*(b[m]+n) ⟺ b[m][n]

*(c+m) ⟺ c[m] or
(*(c+m))[n] ⟺ c[m][n]

Pointer Arrays vs Array Pointers





int (*b)[N] ;

(*(b+m))[n] ⟺ b[m][n]

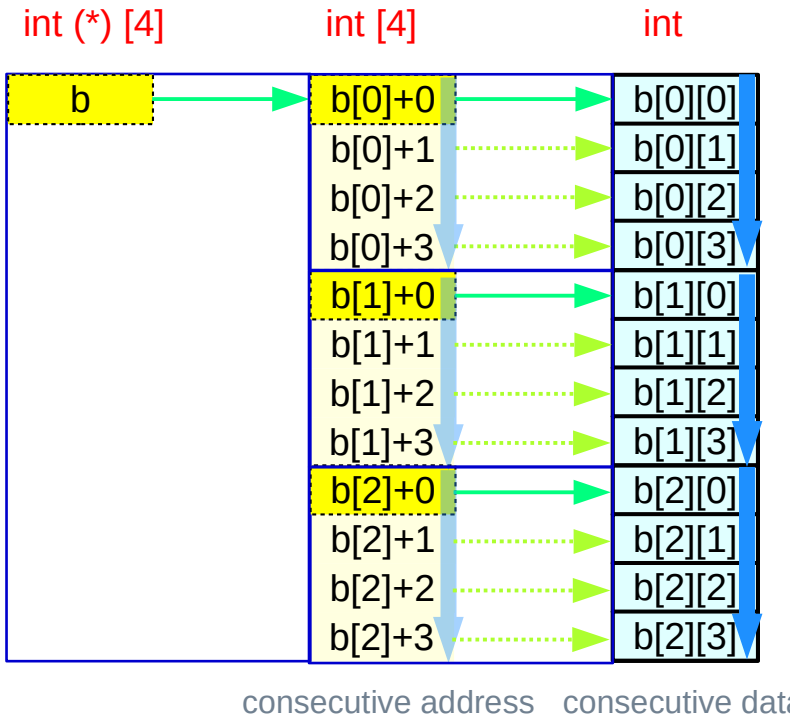
*(b[m]+n) ⟺ b[m][n]

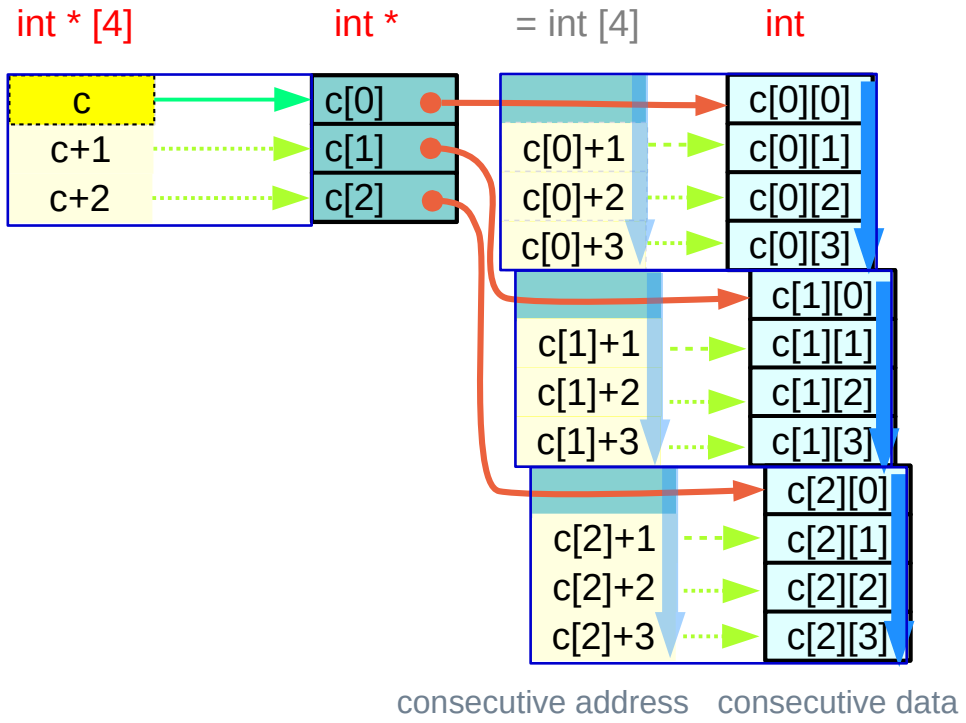
int * c[M] ; with proper assignments

(*(c+m)) ⟺ c[m] or

*(c[m]+n) ⟺ c[m][n]

Pointer Arrays vs Array Pointers





int (*b)[N] ;

(*(b+m))[n] ⟺ b[m][n]

*(b[m]+n) ⟺ b[m][n]

int * c[M] ; with proper assignments

(*(c+m)) ⟺ c[m]

*(c[m]+n) ⟺ c[m][n]

Three contiguity constraints

Pointer Array Approach (array of pointers)

c[i][j][k]	➡	*(c[i][j] +k)
*(c[i][j] +k)	➡	*(*(c[i] +j) +k)
((c[i] +j) +k)	➡	*(*(*(c +i) +j) +k)

contiguous **int**	int
contiguous pointers to **int**	int *
contiguous double pointers to **int**	int **

the contiguity constraints are satisfied by allocating arrays of pointers

Array Pointer Approach (pointer to arrays)

c[i][j][k]	➡	*(c[i][j] +k)
*(c[i][j] +k)	➡	*(*(c[i] +j) +k)
((c[i] +j) +k)	➡	*(*(*(c +i) +j) +k)

contiguous **1-d** array elements	int
contiguous **1-d** array names	int [4]
contiguous **1-d** array pointers	int (*) [4]

The contiguity constraints are satisfied by row major ordered linear data layout

c[i][j][k] ≡ *(c[i][j] +k)

c[0][0][0] = *(c[0][0] + 0)
c[0][0][1] = *(c[0][0] + 1)
c[0][0][2] = *(c[0][0] + 2)
c[0][0][3] = *(c[0][0] + 3)
c[0][1][0] = *(c[0][1] + 0)
c[0][1][1] = *(c[0][1] + 1)
c[0][1][2] = *(c[0][1] + 2)
c[0][1][3] = *(c[0][1] + 3)

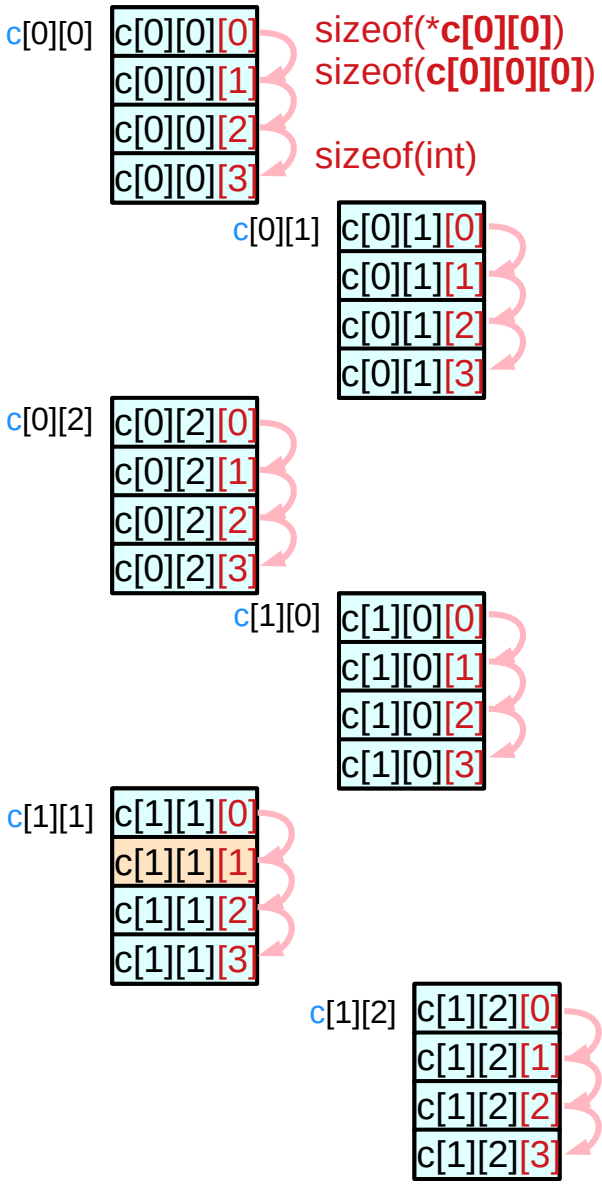
contiguous 1-d
array elements

c[i][j] **int *** or **int [4]**
contiguous 0-d arrays
4 **int** elements
points to the 1st 0-d array

sizeof(**c[i][j]**)
sizeof(**c[i][j][k]**) * 4
sizeof(**int**) * 4

int c[2][3][4];

Address Value
c[i][j] + k
&c[i][j][0] + k * sizeof(*c[i][j])
&c[i][j][0] + k * sizeof(c[i][j][0])
&c[i][j][0] + k * 4

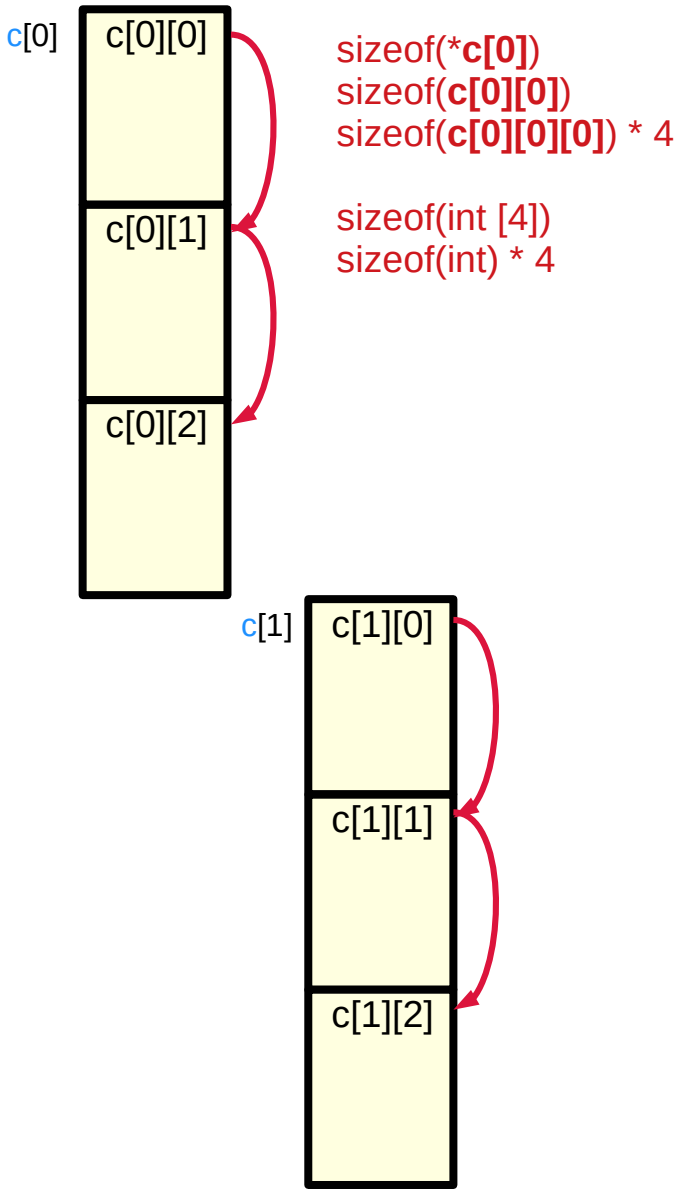


c[i][j] ≡ *(c[i] +j)

c[0][0] = *(c[0] + 0)
c[0][1] = *(c[0] + 1)
c[0][2] = *(c[0] + 2)
c[1][0] = *(c[1] + 0)
c[1][1] = *(c[2] + 1)
c[1][2] = *(c[3] + 2)

c[i] **int (*) [4]** or **int [3][4]**
contiguous 1-d arrays
3 **int[4]** arrays
points to the 1st 1-d array

sizeof(**c[i]**)
sizeof(**c[i][j]**) * 3
sizeof(**c[i][j][k]**) * 3 * 4
sizeof(**int**) * 3 * 4



int c[2][3][4];

Address Value
c[i] + j
&c[i][0][0] + j * sizeof(*c[i])
&c[i][0][0] + j * sizeof(c[i][0])
&c[i][0][0] + j * 4 * 4

c[i] ≡ *(c +i)

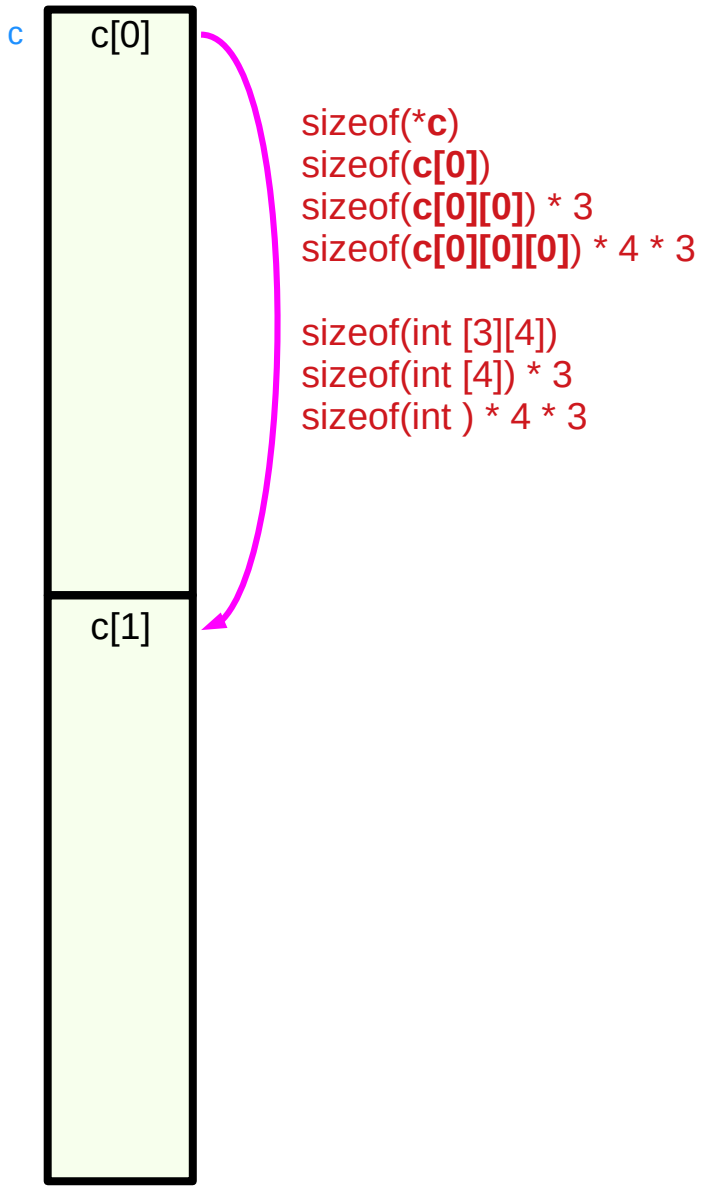
c[0] = *(c + 0)
c[1] = *(c + 1)

c **int (*) [3][4]** or **int [2][3][4]**
contiguous 2-d arrays
2 **int [3][4]** arrays
points to the 1st 2-d array

sizeof(**c**)
sizeof(**c[i]**) * 2
sizeof(**c[i][j]**) * 2 * 3
sizeof(**c[i][j][k]**) * 2 * 3 * 4
sizeof(**int**) * 2 * 3 * 4

```
int c[2][3][4];
```

Address Value
c + i
&c[0][0][0] + i * sizeof(*c)
&c[0][0][0] + i * sizeof(c[0])
&c[0][0][0] + i * 4 * 4 * 3



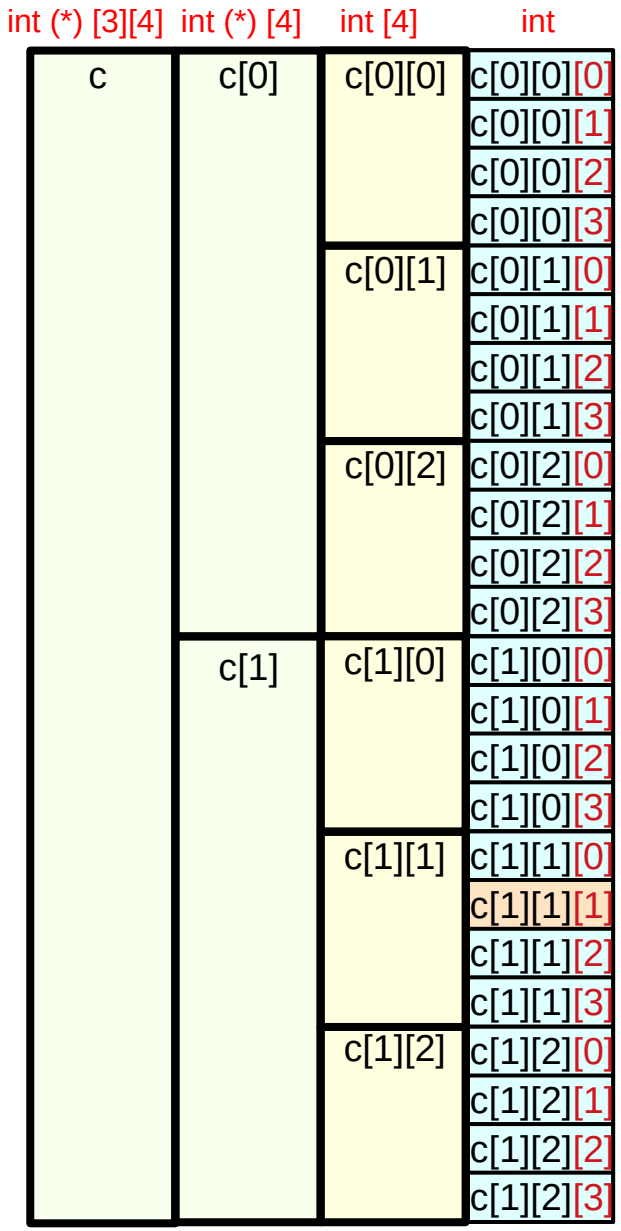
c[i][j][k] ≡ *(c[i][j] +k)

c[i][j][k]	➡	*(c[i][j] +k)
*(c[i][j] +k)	➡	*(*(c[i] +j) +k)
((c[i] +j) +k)	➡	*(*(*(c +i) +j) +k)

int c[2][3][4];

contiguous **1-d** array elements int
contiguous **1-d** array names int [4]
contiguous **1-d** array pointers int (*) [4]

The contiguity constraints are satisfied by row major ordered linear data layout



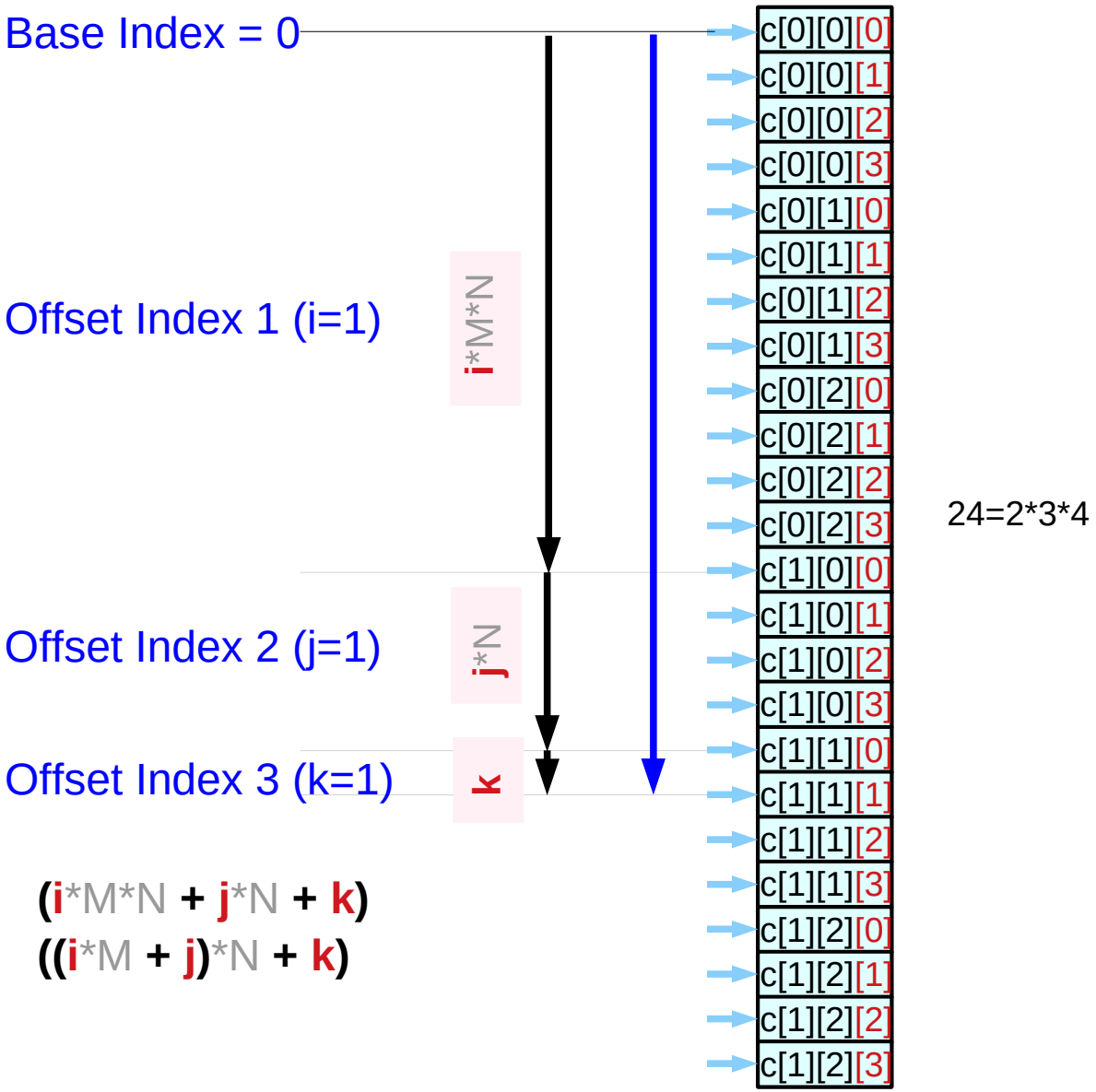
c[i] ≡ *(c +i)

c[i] ≡ *(c +i)	c[i][j] ≡ *(c[i] +j)	c[i][j][k]≡ *(c[i][j] +k)
2-d array pointer c int (*) [3][4]	1-d array pointers c[i] int (*) [4]	0-d array pointers c[i][j] int (*)
address value c + i &c[0][0][0] + i * sizeof(*c) &c[0][0][0] + i * sizeof(c[0]) &c[0][0][0] + i * 4 * 4 * 3	**address value** c[i] + j &c[i][0][0] + j * sizeof(*c[i]) &c[i][0][0] + j * sizeof(c[i][0]) &c[i][0][0] + j * 4 * 4	**address value** c[i][j] + k &c[i][j][0] + k * sizeof(*c[i][j]) &c[i][j][0] + k * sizeof(c[i][j][0]) &c[i][j][0] + k * 4
leading elements c[0][0][0]	**leading elements** c[0][0][0] c[1][0][0]	**leading elements** c[0][0][0] c[0][1][0] c[0][2][0] c[1][0][0] c[1][1][0] c[1][2][0]

Contiguous linear layout

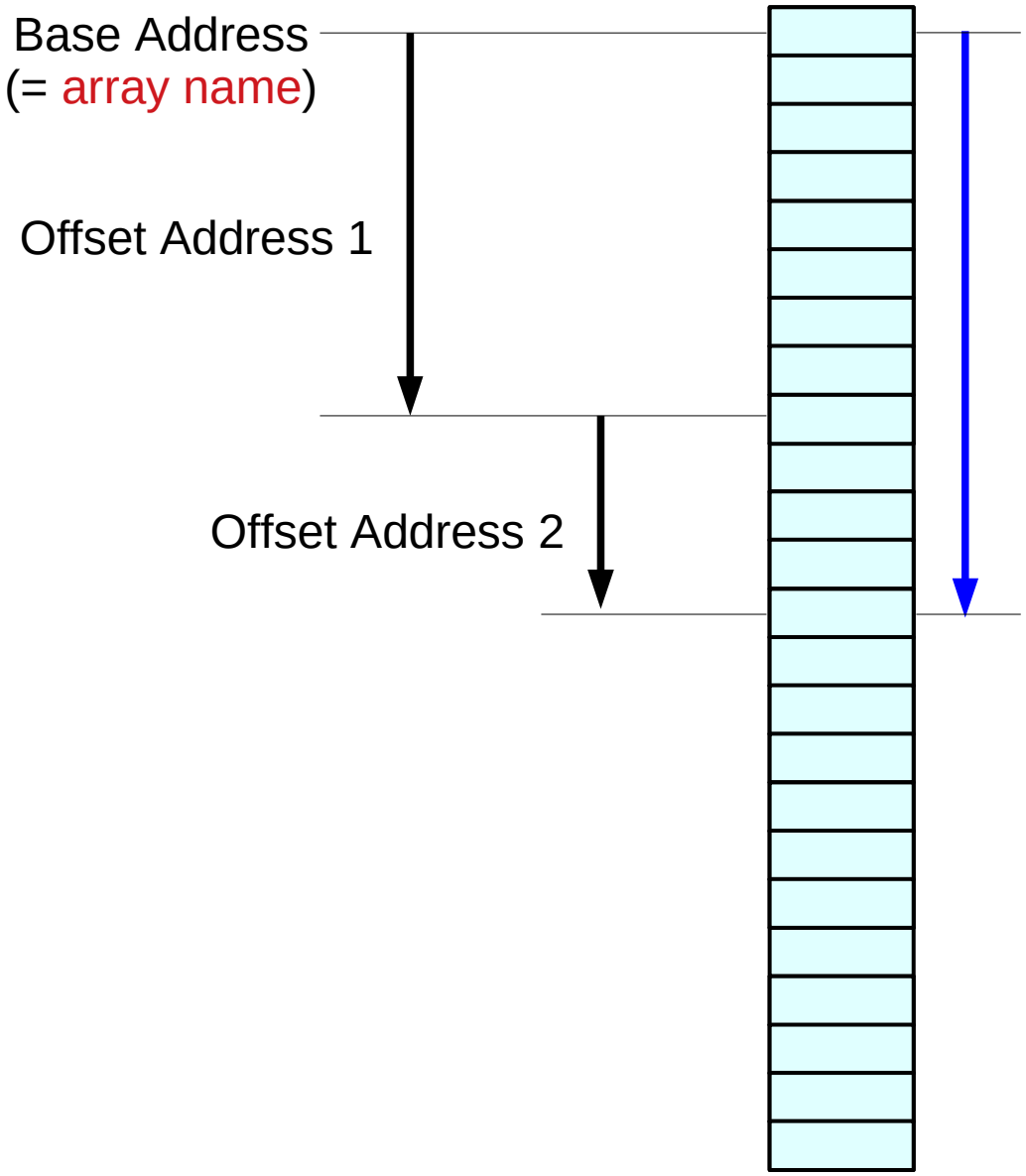
```
int     c [L][M][N];
```

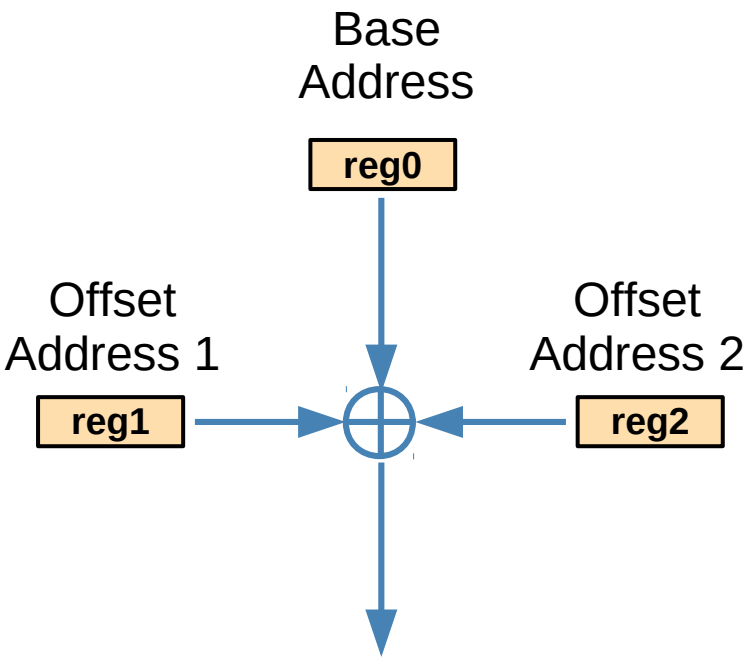
L	M	N
i	j	k
i*M*N	j*N	k



(i*M*N + j*N + k)
((i*M + j)*N + k)

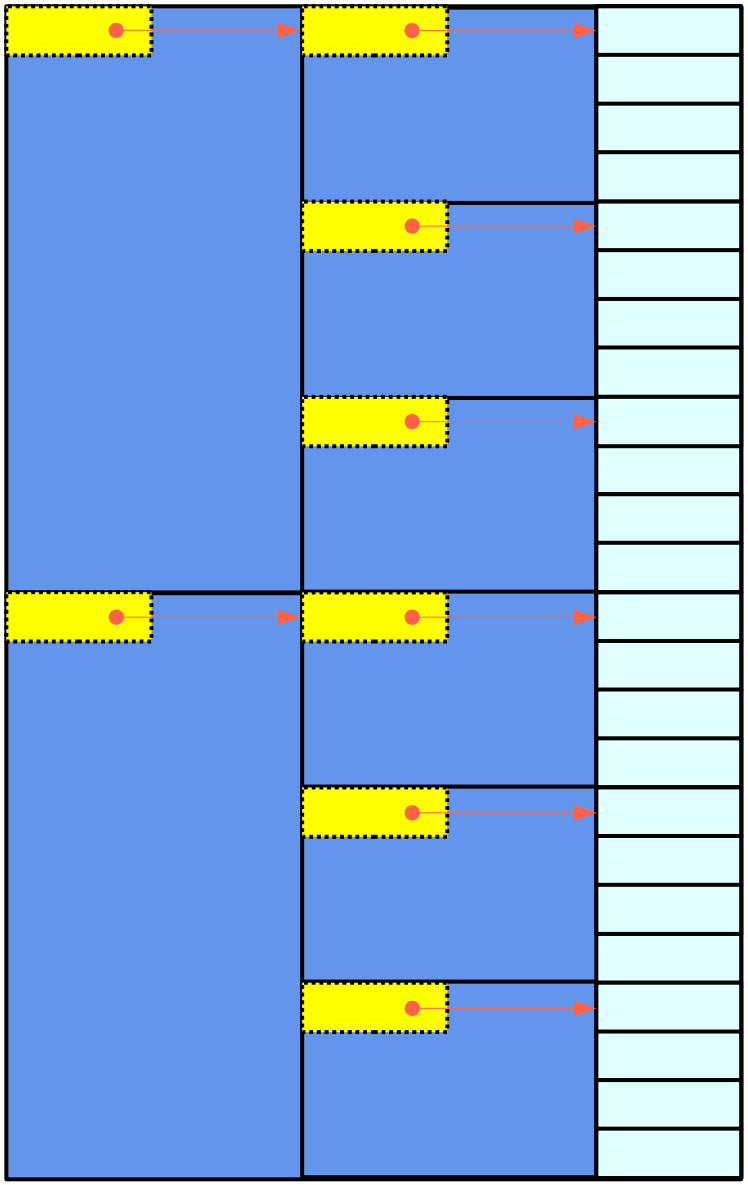
Base and Offset Addressing

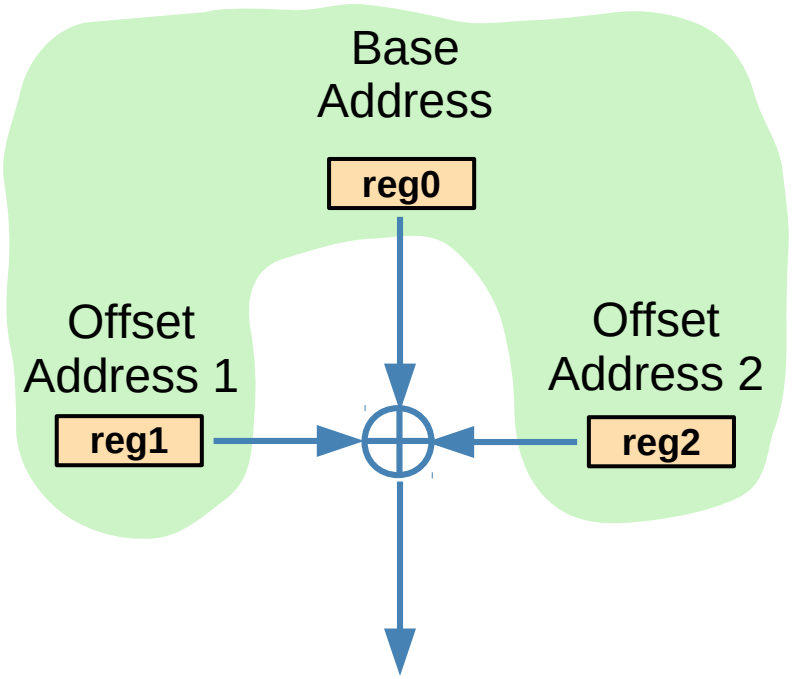




compiler
assembly instruction
registers in the CPU

Array Pointer Approach





register based address **computations**
eliminate the pointer arrays – by a compiler

Array Pointer Approach
(pointer to arrays)

References

[1] Essential C, Nick Parlante
[2] Efficient C Programming, Mark A. Weiss
[3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
[4] C Language Express, I. K. Chun