# Truth Table (2A)

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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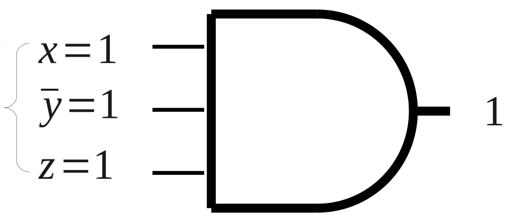
# Truth Table and minterms (1)

| x | y | z | | |
|---|---|---|---|---|
| 0 | 0 | 0 | the case when x=0 **and** y=0 **and** z=0 ⟺ | $\bar{x}\,\bar{y}\,\bar{z} = 1$ |
| 0 | 0 | 1 | the case when x=0 **and** y=0 **and** z=1 ⟺ | $\bar{x}\,\bar{y}\,z = 1$ |
| 0 | 1 | 0 | the case when x=0 **and** y=1 **and** z=0 ⟺ | $\bar{x}\,y\,\bar{z} = 1$ |
| 0 | 1 | 1 | the case when x=0 **and** y=1 **and** z=1 ⟺ | $\bar{x}\,y\,z = 1$ |
| 1 | 0 | 0 | the case when x=1 **and** y=0 **and** z=0 ⟺ | $x\,\bar{y}\,\bar{z} = 1$ |
| 1 | 0 | 1 | the case when x=1 **and** y=0 **and** z=1 ⟺ | $x\,\bar{y}\,z = 1$ |
| 1 | 1 | 0 | the case when x=1 **and** y=1 **and** z=0 ⟺ | $x\,y\,\bar{z} = 1$ |
| 1 | 1 | 1 | the case when x=1 **and** y=1 **and** z=1 ⟺ | $x\,y\,z = 1$ |

inputs

All possible combination of inputs

$x\,\bar{y}\,z = 1$ ⟺




⟺ $x=1$, $y=0$, $z=1$

For the output of an **and** gate to be 1, all inputs must be 1

# Truth Table and minterms (2)

| index | x | y | z | | minterm |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | the case when the minterm | $m_0 = \bar{x}\,\bar{y}\,\bar{z} = 1$ |
| 1 | 0 | 0 | 1 | the case when the minterm | $m_1 = \bar{x}\,\bar{y}\,z = 1$ |
| 2 | 0 | 1 | 0 | the case when the minterm | $m_2 = \bar{x}\,y\,\bar{z} = 1$ |
| 3 | 0 | 1 | 1 | the case when the minterm | $m_3 = \bar{x}\,y\,z = 1$ |
| 4 | 1 | 0 | 0 | the case when the minterm | $m_4 = x\,\bar{y}\,\bar{z} = 1$ |
| 5 | 1 | 0 | 1 | the case when the minterm | $m_5 = x\,\bar{y}\,z = 1$ |
| 6 | 1 | 1 | 0 | the case when the minterm | $m_6 = x\,y\,\bar{z} = 1$ |
| 7 | 1 | 1 | 1 | the case when the minterm | $m_7 = x\,y\,z = 1$ |

index

inputs

All possible combination of inputs

minterm $m_5$ → index 5

binary 101

$x\,\bar{y}\,z = 1$

0 → bar

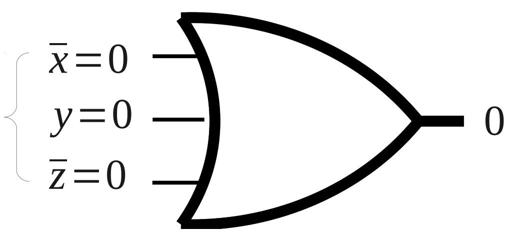
# Truth Table and MAXterms (1)

| x | y | z | | |
|---|---|---|---|---|
| 0 | 0 | 0 | the case when x=0 **and** y=0 **and** z=0 ⟺ | $x+y+z = 0$ |
| 0 | 0 | 1 | the case when x=0 **and** y=0 **and** z=1 ⟺ | $x+y+\bar{z} = 0$ |
| 0 | 1 | 0 | the case when x=0 **and** y=1 **and** z=0 ⟺ | $x+\bar{y}+z = 0$ |
| 0 | 1 | 1 | the case when x=0 **and** y=1 **and** z=1 ⟺ | $x+\bar{y}+\bar{z} = 0$ |
| 1 | 0 | 0 | the case when x=1 **and** y=0 **and** z=0 ⟺ | $\bar{x}+y+z = 0$ |
| 1 | 0 | 1 | the case when x=1 **and** y=0 **and** z=1 ⟺ | $\bar{x}+y+\bar{z} = 0$ |
| 1 | 1 | 0 | the case when x=1 **and** y=1 **and** z=0 ⟺ | $\bar{x}+\bar{y}+z = 0$ |
| 1 | 1 | 1 | the case when x=1 **and** y=1 **and** z=1 ⟺ | $\bar{x}+\bar{y}+\bar{z} = 0$ |

inputs

All possible combination of inputs

$\bar{x}+y+\bar{z} = 0$ ⟺




⟺ $x=1$, $y=0$, $z=1$

For the output of an **or** gate to be 0, all inputs must be 0

# Truth Table and MAXterms (2)

| index | x | y | z | | MAXterm |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | the case when the MAXterm | $M_0 = x + y + z = 0$ |
| 1 | 0 | 0 | 1 | the case when the MAXterm | $M_1 = x + y + \bar{z} = 0$ |
| 2 | 0 | 1 | 0 | the case when the MAXterm | $M_2 = x + \bar{y} + z = 0$ |
| 3 | 0 | 1 | 1 | the case when the MAXterm | $M_3 = x + \bar{y} + \bar{z} = 0$ |
| 4 | 1 | 0 | 0 | the case when the MAXterm | $M_4 = \bar{x} + y + z = 0$ |
| 5 | 1 | 0 | 1 | the case when the MAXterm | $M_5 = \bar{x} + y + \bar{z} = 0$ |
| 6 | 1 | 1 | 0 | the case when the MAXterm | $M_6 = \bar{x} + \bar{y} + z = 0$ |
| 7 | 1 | 1 | 1 | the case when the MAXterm | $M_7 = \bar{x} + \bar{y} + \bar{z} = 0$ |

index

inputs

All possible
combination of inputs

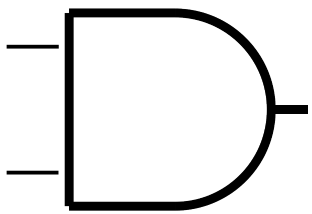
minterm $M_5$ → index 5

binary 101

$\bar{x} + y + \bar{z}$

1 → bar

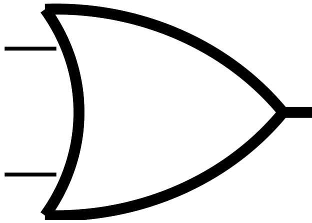
# Maxterm and minterm Conditions



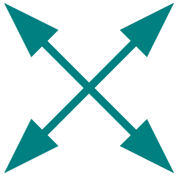
1

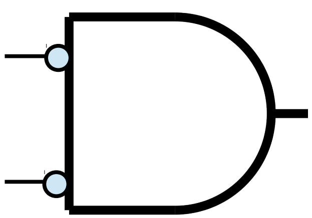
| $x$ | $y$ | $xy$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| $x$ | $y$ | $x+y$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |



0

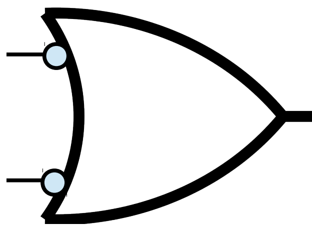




1

| $\overline{x}$ | $\overline{y}$ | $\overline{x}\overline{y}$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

| $\overline{x}$ | $\overline{y}$ | $\overline{x}+\overline{y}$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |



0

# Boolean Function with minterms (1)

| index | x | y | z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| ➡ 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| ➡ 3 | 0 | 1 | 1 | 1 |
| ➡ 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 0 |
| 6 | 1 | 1 | 0 | 0 |
| 7 | 1 | 1 | 1 | 0 |

index — inputs — output

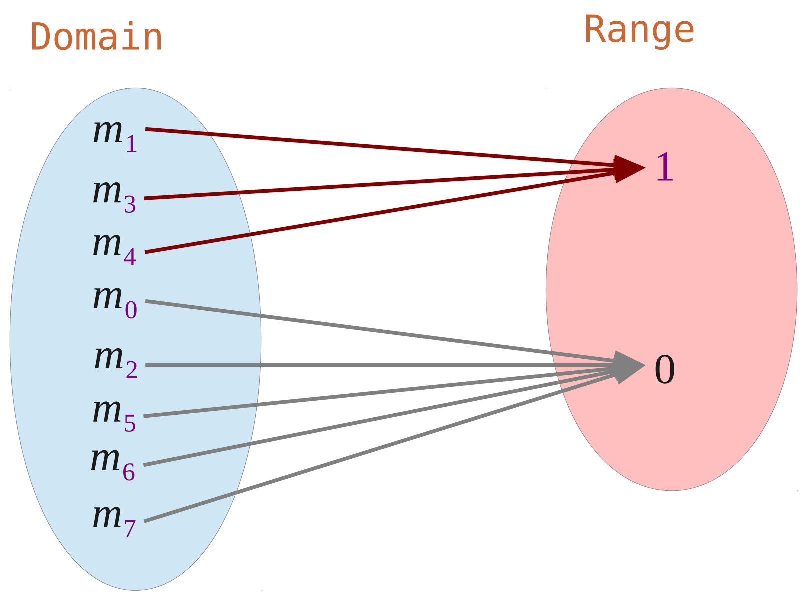
All possible combination of inputs

The output F becomes 1,
for one of the three following cases

(the case when x=0 **and** y=0 **and** z=1) ⟺ $m_1 = \bar{x}\,\bar{y}\,z = 1$

**or** (the case when x=0 **and** y=1 **and** z=1) ⟺ $m_3 = \bar{x}\,y\,z = 1$

**or** (the case when x=1 **and** y=0 **and** z=0) ⟺ $m_4 = x\,\bar{y}\,\bar{z} = 1$



# Boolean Function with minterms (2)

| index | x | y | z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| ➡ 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| ➡ 3 | 0 | 1 | 1 | 1 |
| ➡ 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 0 |
| 6 | 1 | 1 | 0 | 0 |
| 7 | 1 | 1 | 1 | 0 |

index — inputs — output

All possible
combination of inputs

The output F becomes 1,

either $m_1 = 1$ or $m_3 = 1$ or $m_4 = 1$

$m_1 + m_3 + m_4 = 1$ ➡ $F = 1$

⟺ $F = m_1 + m_3 + m_4$

The output F becomes 0,

either $m_0 = 1$ or $m_2 = 1$ or $m_5 = 1$ or $m_6 = 1$ or $m_7 = 1$

$m_0 + m_2 + m_5 + m_6 + m_7 = 1$ ➡ $F = 0$

⟺ $\overline{F} = m_0 + m_2 + m_5 + m_6 + m_7$

For the output of an **or** gate to be 1,
at least one must be 1

# Boolean Function with Maxterms (1)

The output F becomes 0,
for one of the five following cases

| | index | x | y | z | F |
|---|---|---|---|---|---|
| ➡ | 0 | 0 | 0 | 0 | 0 |
| | 1 | 0 | 0 | 1 | 1 |
| ➡ | 2 | 0 | 1 | 0 | 0 |
| | 3 | 0 | 1 | 1 | 1 |
| | 4 | 1 | 0 | 0 | 1 |
| ➡ | 5 | 1 | 0 | 1 | 0 |
| ➡ | 6 | 1 | 1 | 0 | 0 |
| ➡ | 7 | 1 | 1 | 1 | 0 |

index — inputs (x y z) — output (F)

All possible combination of inputs

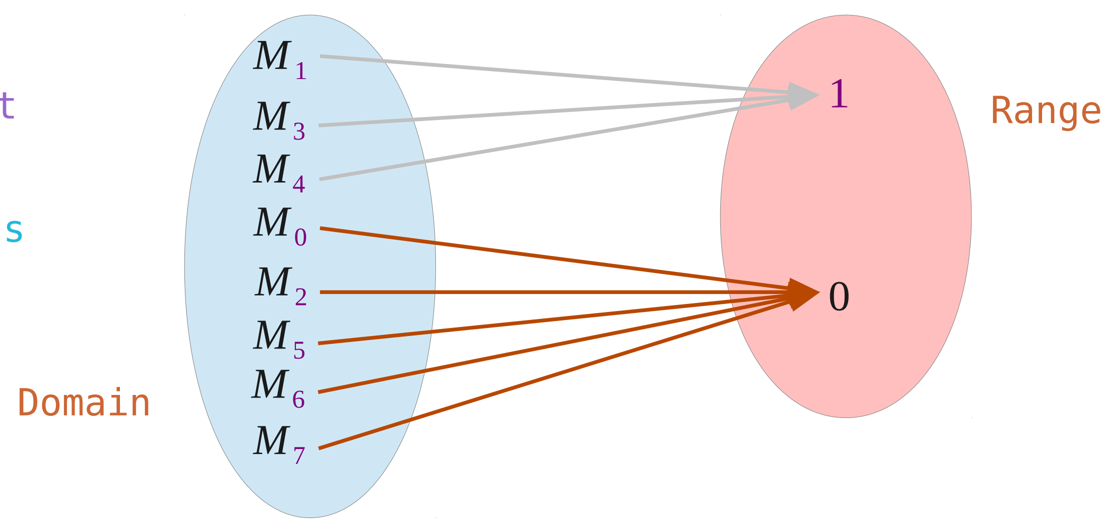
(the case when x=0 **and** y=0 **and** z=0) ⟺ $x + y + z = 0$

**or** (the case when x=0 **and** y=1 **and** z=0) ⟺ $x + \bar{y} + z = 0$

**or** (the case when x=1 **and** y=0 **and** z=1) ⟺ $\bar{x} + y + \bar{z} = 0$

**or** (the case when x=1 **and** y=1 **and** z=0) ⟺ $\bar{x} + \bar{y} + z = 0$

**or** (the case when x=1 **and** y=1 **and** z=1) ⟺ $\bar{x} + \bar{y} + \bar{z} = 0$



# Boolean Function with Maxterms (2)

| index | x | y | z | F |
|---|---|---|---|---|
| ➡ 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| ➡ 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1 |
| 4 | 1 | 0 | 0 | 1 |
| ➡ 5 | 1 | 0 | 1 | 0 |
| ➡ 6 | 1 | 1 | 0 | 0 |
| ➡ 7 | 1 | 1 | 1 | 0 |

index — inputs — output

All possible combination of inputs

The output F becomes 0,

either $M_0=0$ or $M_2=0$ or $M_5=0$ or $M_6=0$ or $M_7=0$

$M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 = 0$ ➡ $F = 0$

⟺ $F = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$

The output F becomes 1,

either $M_1=0$ or $M_3=0$ or $M_4=0$

$M_1 \cdot M_3 \cdot M_4 = 0$ ➡ $F = 1$

⟺ $\overline{F} = M_1 \cdot M_3 \cdot M_4$

For the output of an **and** gate to be 0,
at least one input must be 0

# Complimentary Relations

| index | x | y | z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 0 |
| 6 | 1 | 1 | 0 | 0 |
| 7 | 1 | 1 | 1 | 0 |

index
inputs output

All possible combination of inputs

$$m_i = \overline{M_i}$$

$$M_i = \overline{m_i}$$

$$F(x,y,z) = m_1 + m_3 + m_4$$

The output F becomes 1,
either $m_1 = 1$ or $m_3 = 1$ or $m_4 = 1$

For the output of an **or** gate to be 1,
at least one must be 1

$$\bar{F}(x,y,z) = m_0 + m_2 + m_5 + m_6 + m_7$$

$$\Leftrightarrow F(x,y,z) = \overline{m_0 + m_2 + m_5 + m_6 + m_7}$$

$$= \overline{m_0} \cdot \overline{m_2} \cdot \overline{m_5} \cdot \overline{m_6} \cdot \overline{m_7}$$

$$F(x,y,z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7$$

The output F becomes 0,
either $M_0 = 0$ or $M_2 = 0$ or $M_5 = 0$ or $M_6 = 0$ or $M_7 = 0$

For the output of an **and** gate to be 0,
at least one input must be 0

# Boolean Function Summary

| | x | y | z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | |
| 3 | 0 | 1 | 1 | 1 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | |
| 6 | 1 | 1 | 0 | |
| 7 | 1 | 1 | 1 | |

The output F becomes 1,

for the cases

1) when $m_1 = 1$ or $m_3 = 1$ or $m_4 = 1$

$$F(x, y, z) = m_1 + m_3 + m_4 \Rightarrow F = 1$$

2) when $M_1 = 0$ or $M_3 = 0$ or $M_4 = 0$

$$\overline{F}(x, y, z) = M_1 \cdot M_3 \cdot M_4 \Rightarrow F = 1 \; (\overline{F} = 0)$$

| | x | y | z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | |
| 4 | 1 | 0 | 0 | |
| 5 | 1 | 0 | 1 | 0 |
| 6 | 1 | 1 | 0 | 0 |
| 7 | 1 | 1 | 1 | 0 |

The output F becomes 0,

for the cases

1) when $m_0 = 1$ or $m_2 = 1$ or $m_5 = 1$ or $m_6 = 1$ or $m_7 = 1$

$$\overline{F}(x, y, z) = m_0 + m_2 + m_5 + m_6 + m_7 \Rightarrow F = 0 \; (\overline{F} = 1)$$

2) when $M_0 = 0$ or $M_2 = 0$ or $M_5 = 0$ or $M_6 = 0$ or $M_7 = 0$

$$F(x, y, z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 \Rightarrow F = 0$$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# Boolean Function Summary

| | x | y | z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 0 |
| 6 | 1 | 1 | 0 | 0 |
| 7 | 1 | 1 | 1 | 0 |

$$F(x,y,z) = m_1 + m_3 + m_4 \Rightarrow F=1$$

$$F(x,y,z) = M_0 \cdot M_2 \cdot M_5 \cdot M_6 \cdot M_7 \Rightarrow F=0$$

| | x | y | z | F |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 2 | 0 | 1 | 0 | 0 |
| 3 | 0 | 1 | 1 | 1 |
| 4 | 1 | 0 | 0 | 1 |
| 5 | 1 | 0 | 1 | 0 |
| 6 | 1 | 1 | 0 | 0 |
| 7 | 1 | 1 | 1 | 0 |

$$\overline{F}(x,y,z) = m_0 + m_2 + m_5 + m_6 + m_7 \Rightarrow F=0 \; (\overline{F}=1)$$

$$\overline{F}(x,y,z) = M_1 \cdot M_3 \cdot M_4 \Rightarrow F=1 \; (\overline{F}=0)$$

# Truth Table

## References

[1] http://en.wikipedia.org/