Laurent Series and z-Transform - Geometric Series Causality B

20180613

2 formulas of z

① $$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\Updownarrow z^{-1}$$

② $$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$$\boxed{\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \frac{3}{2}\frac{2}{3}\left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$= \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

z^{-1}

$$\frac{3}{2}\frac{-1}{(z^{-1}-0.5)(z^{-1}-2)} = \frac{3}{2}\frac{2}{3}\left(\frac{1}{z^{-1}-0.5} - \frac{1}{z^{-1}-2}\right)$$

$$= \left(\frac{2}{2z^{-1}-1} - \frac{0.5}{0.5z^{-1}-1}\right)$$

$$= \left(\frac{2z}{2-z} - \frac{0.5z}{0.5-z}\right)$$

$$= \left(\frac{-2z}{z-2} + \frac{0.5z}{z-0.5}\right)$$

$$= z\left(\frac{-2}{z-2} + \frac{0.5}{z-0.5}\right)$$

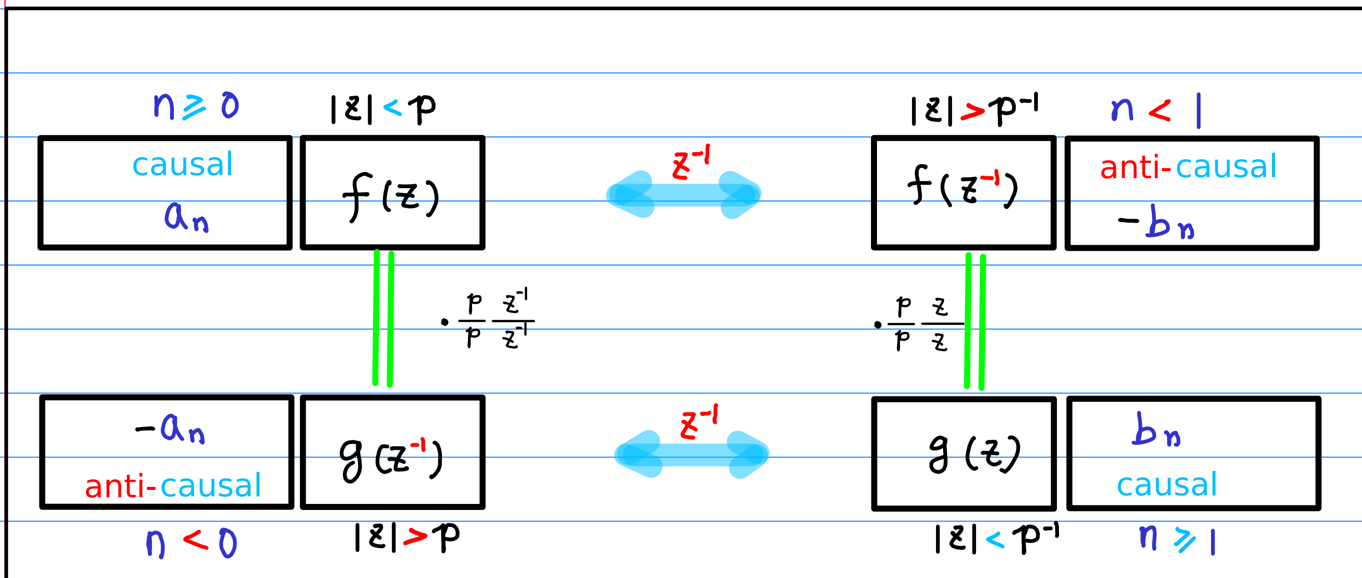
$$= z\left(\frac{-\frac{3}{2}z}{(z-2)(z-0.5)}\right)$$

$$= \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

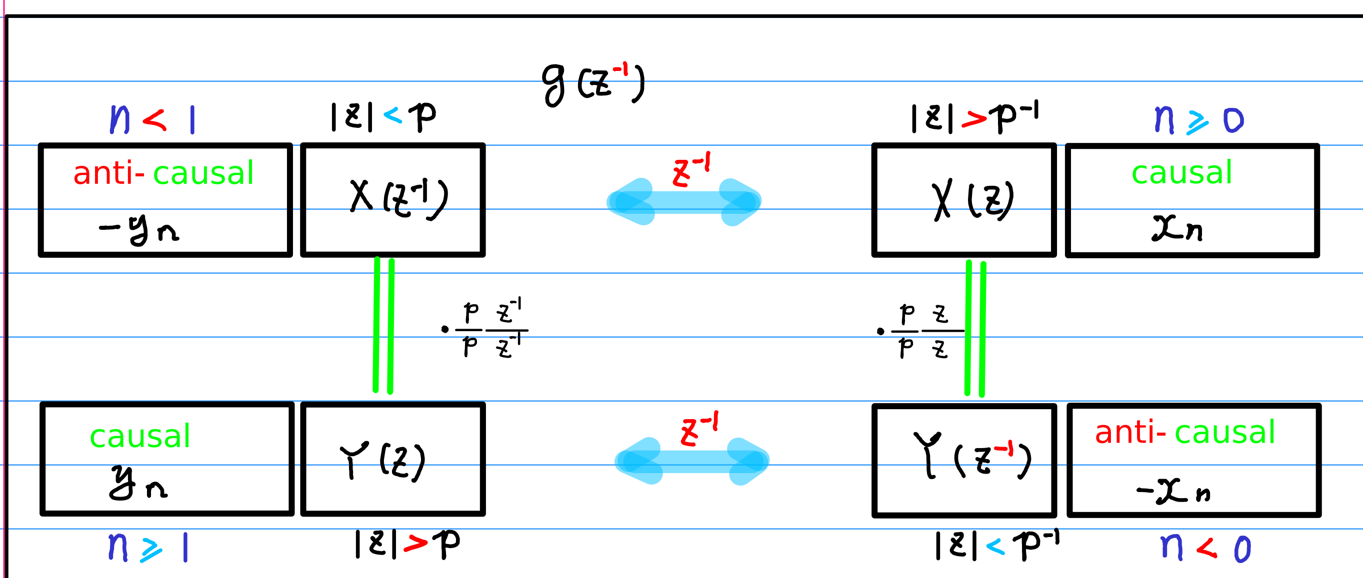
$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \frac{3}{2}\frac{2}{3}\left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

Laurent Series & Z-Transform (1)

Laurent Series $a_n \leftrightarrow f(z)$ $b_n \leftrightarrow g(z)$

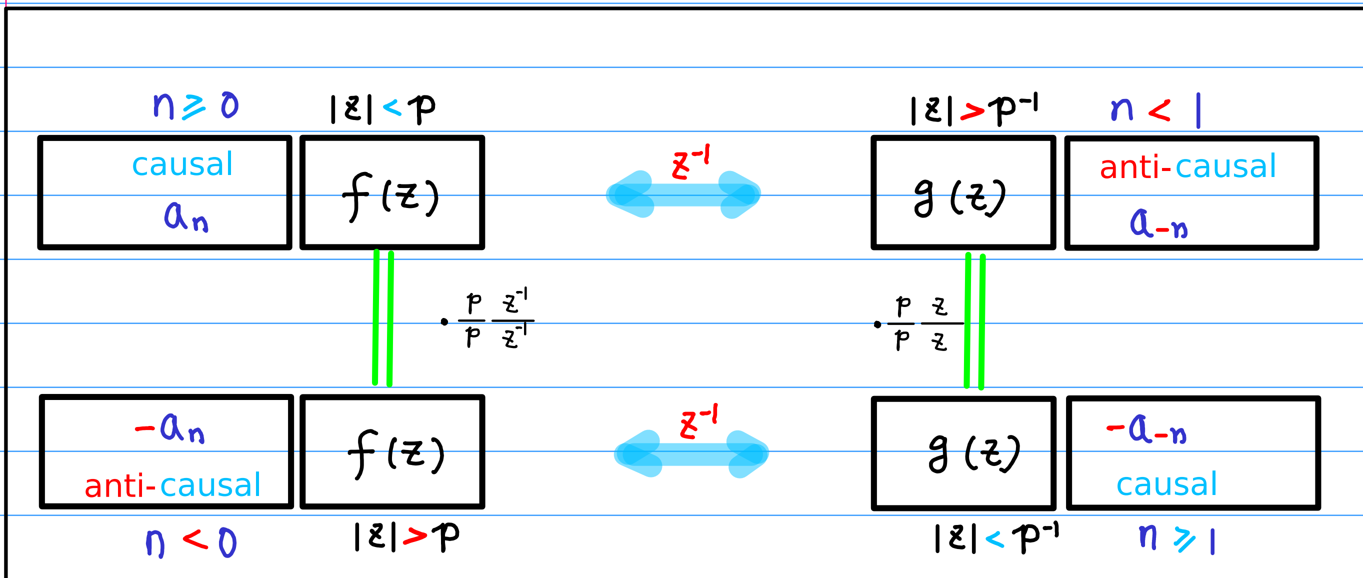


Z- Transform $X(z) \leftrightarrow x_n$ $Y(z) \leftrightarrow y_n$

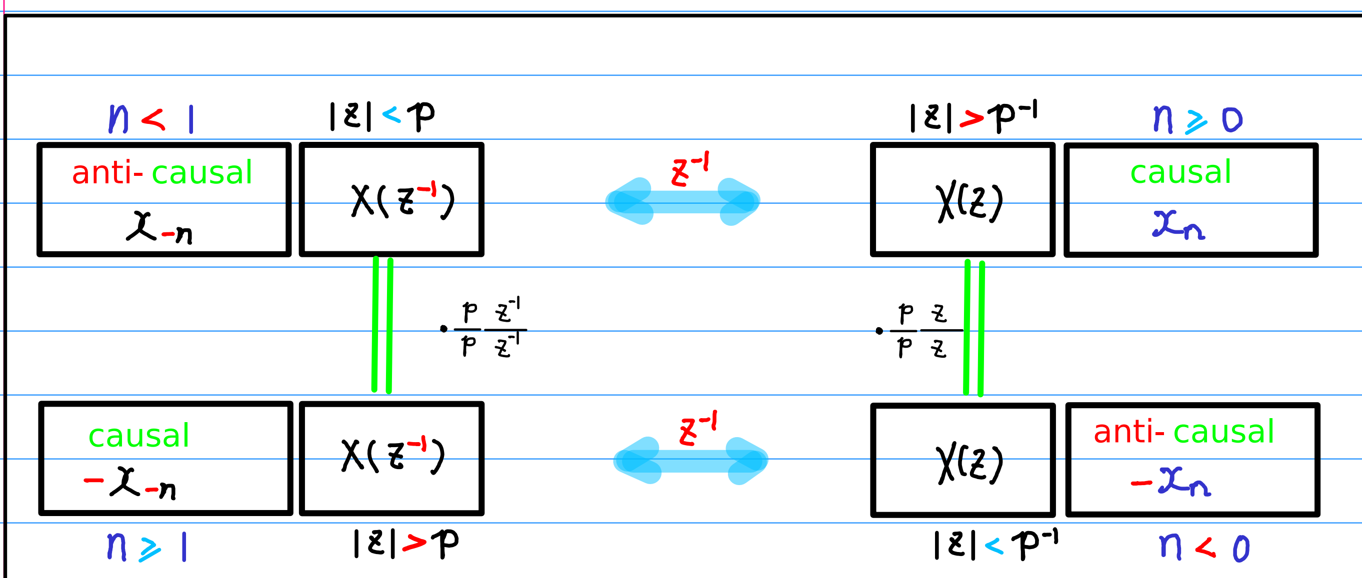


Laurent Series & Z-Transform (2)

Laurent Series $a_n \leftrightarrow f(z)$



Z-Transform $X(z) \leftrightarrow x_n$



$f(z)$

①

causal $f(z)$ $(|z|<p)$ anti-causal $f(z)$ $(|z|>p)$

$f(z) \leftrightarrow a_n$ $(n\geq 0)$ $g(z^{-1}) \leftrightarrow -a_n$ $(n<0)$

$n\geq 0$ $|z|<p$

causal f(z): $-(p^{-1})^{n+1}$ $\quad -\dfrac{p^{-1}}{1-p^{-1}z}$

$$-(p^{-1}+p^{-2}z^1+p^{-3}z^2+\cdots)=\sum_{n=0}^{\infty}-(p^{-1})^{n+1}z^n \qquad n\geq 0$$

$\cdot(-1)$ $\quad \cdot\dfrac{p}{p}\dfrac{z^{-1}}{z^{-1}}$

$(p)^{-n-1}$ $\quad \dfrac{z^{-1}}{1-pz^{-1}}$

anti-causal f(z)

$n<0$ $|z|>p$

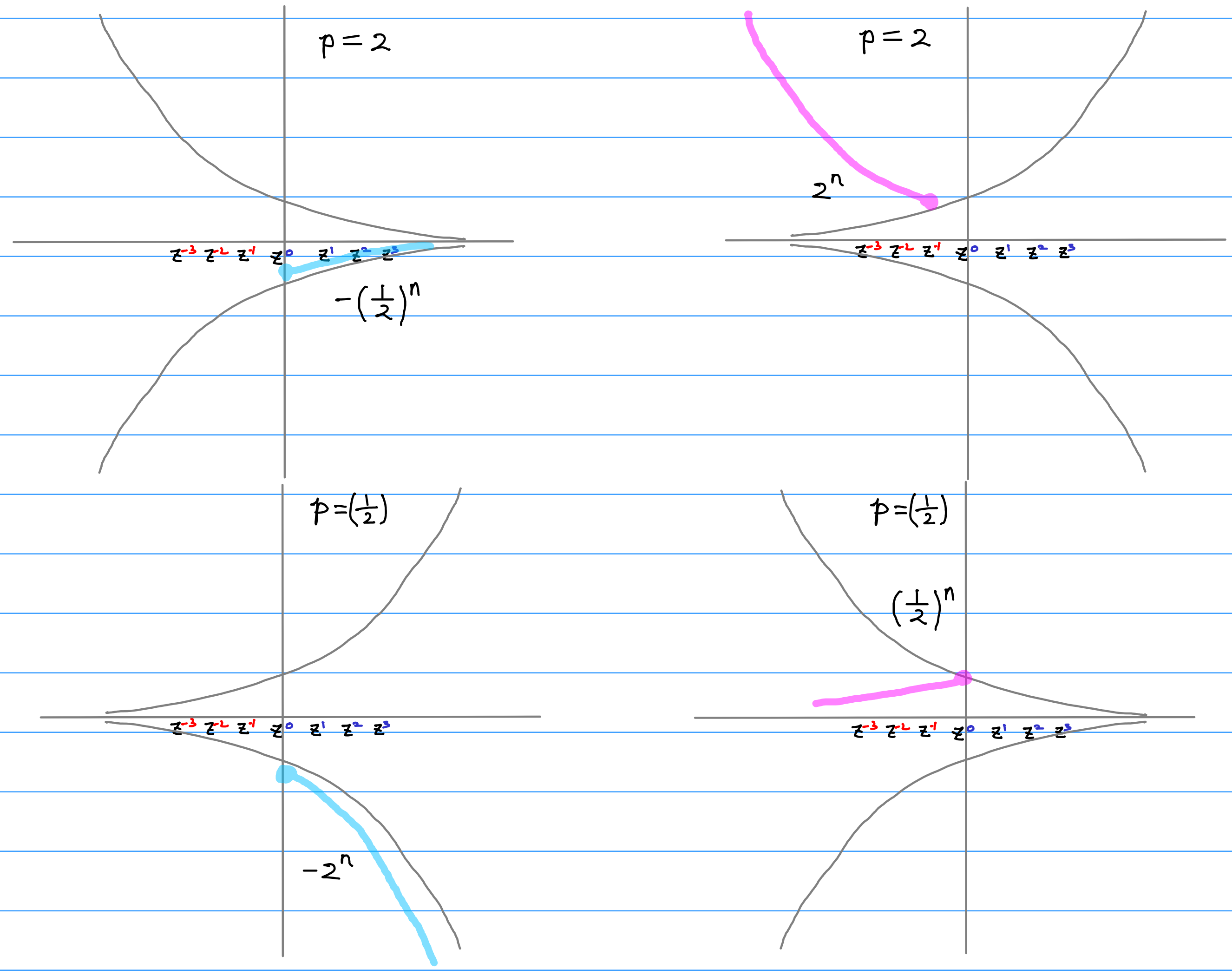
$$p^0z^{-1}+p^1z^{-2}+p^2z^{-3}+\cdots=\sum_{n=-1}^{-\infty}(p)^{-n-1}z^n \qquad n<0$$

causal n=0,+1,+2,+3,....

$-(p^{-1},p^{-2},p^{-3},\cdots)$

anti-causal n=-1,-2,-3,....

$(p^0,p^1,p^2,\cdots)$



$f(z)$

②

anti-causal $g(z)$ $(|z|>p^{-1})$

$f(z^{-1}) \leftrightarrow -b_n$ $(n<1)$

causal $g(z)$ $(|z|<p^{-1})$

$g(z) \leftrightarrow b_n$ $(n\geq 1)$

$n<1$ $\quad -(p^{-1}+p^{-2}z^{-1}+p^{-3}z^{-2}+\cdots) = \sum_{n=0}^{-\infty} -(p)^{n-1} z^n$

$|z|>p^{-1}$ $\quad n<1$

$\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}}$ $\quad$ anti-causal f(z) $\boxed{-(p^{-1})^{-n+1}}$

$\cdot\frac{p}{p}\frac{z}{z}$ $\quad \cdot(-1)$

$n\geq 1$ $\quad p^0z^1+p^1z^2+p^2z^3+\cdots = \sum_{n=1}^{\infty}(p)^{n-1}z^n$

$\boxed{\frac{z}{1-pz}}$ $\quad \boxed{(p)^{n-1}}$ causal f(z)

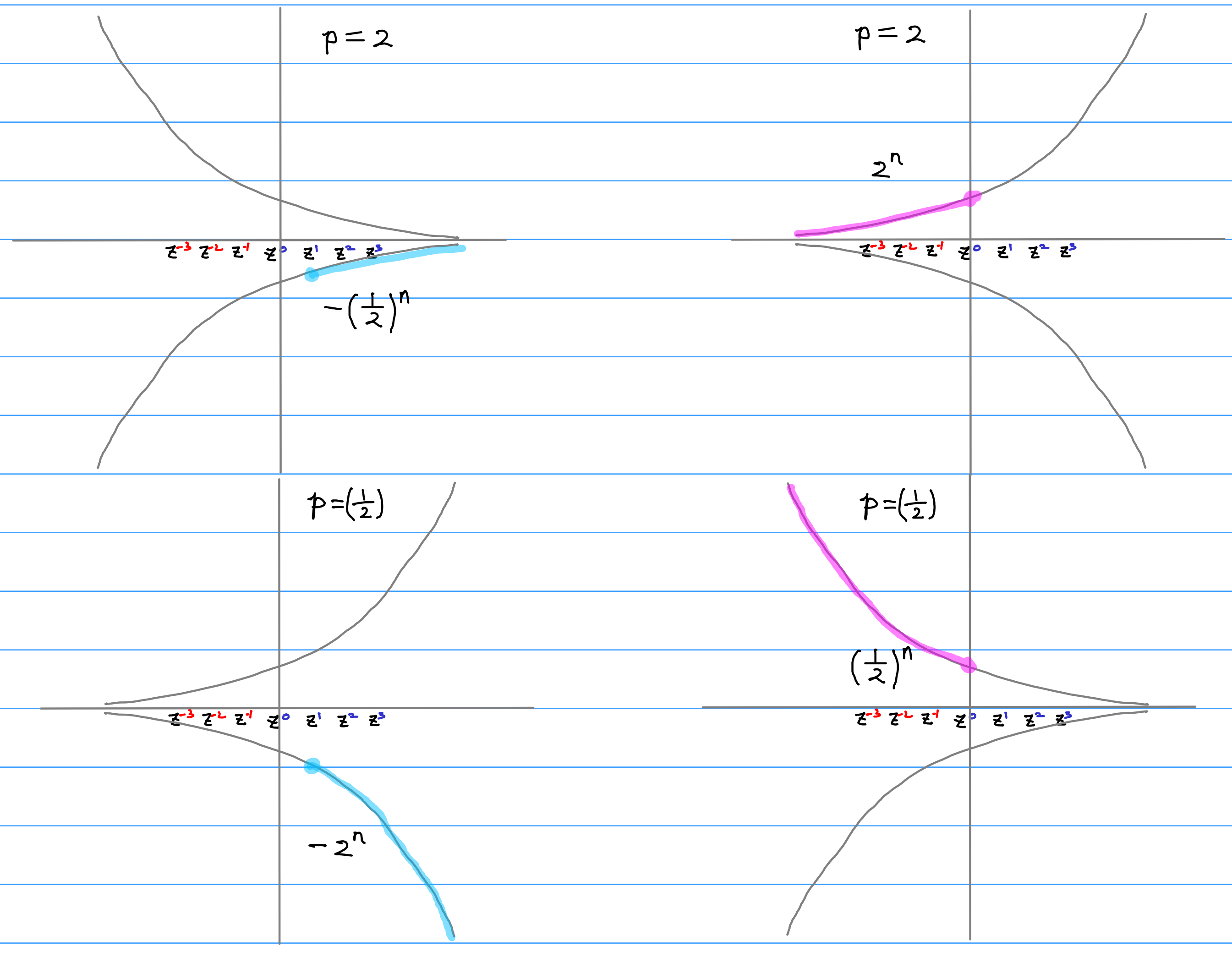
$|z|<p^{-1}$ $\quad n\geq 1$

causal $\quad$ n=+1,+2,+3,....

$-(p^0, p^1, p^2, \cdots)$

anti-causal $\quad$ n=0,-1,-2,-3,....

$(p^{-1}, p^{-2}, p^{-3}, \cdots)$

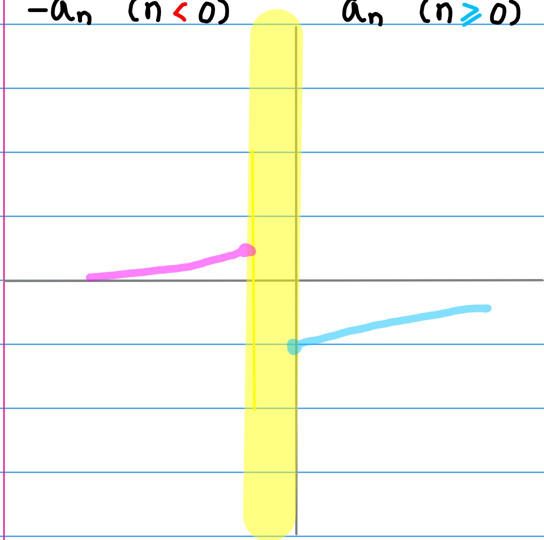


$f(z)$

③

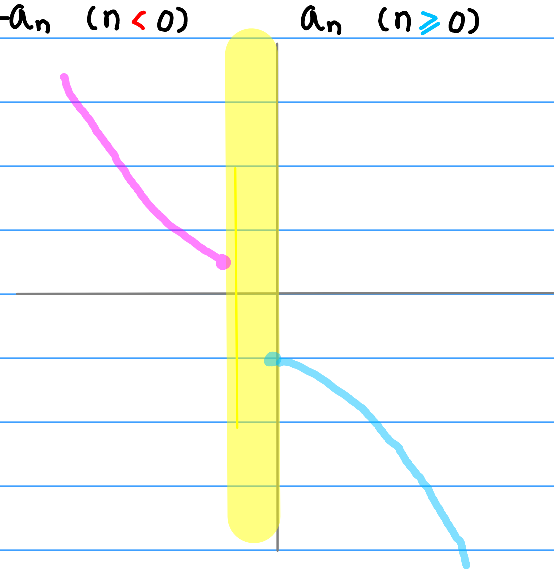
$f(z)\ (|z|>p)$ $f(z)\ (|z|<p)$

$-a_n\ (n<0)$ $a_n\ (n\geq 0)$



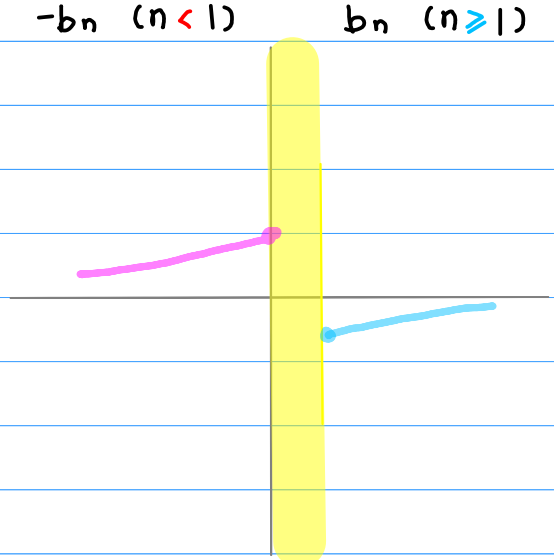
$f(z)\ (|z|>p)$ $f(z)\ (|z|<p)$

$-a_n\ (n<0)$ $a_n\ (n\geq 0)$



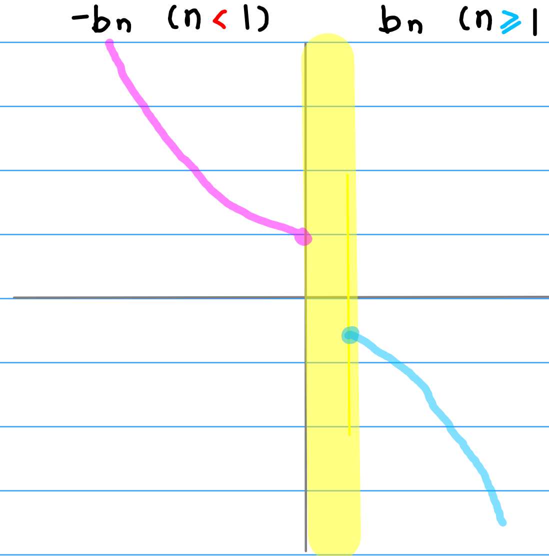
$g(z)\ (|z|>p^{-1})$ $g(z)\ (|z|<p^{-1})$

$-b_n\ (n<1)$ $b_n\ (n\geq 1)$



$g(z)\ (|z|>p^{-1})$ $g(z)\ (|z|<p^{-1})$

$-b_n\ (n<1)$ $b_n\ (n\geq 1)$



$X(z)$ ①

anti-causal $X(z^{-1})$ $(|z|<p)$ causal $Y(z)$ $(|z|>p)$

$X(z^{-1}) \leftrightarrow x_{-n}$ $(n<1)$ $Y(z) \leftrightarrow y_n$ $(n \geq 0)$

	$n<1$	$\lvert z\rvert<p$
x_{-n}	anti-causal X(z) $-(p^{-1})^{-n+1}$	$-\dfrac{p^{-1}}{1-p^{-1}z}$
	$\cdot(-1)$	$\cdot\dfrac{p}{p}\dfrac{z^{-1}}{z^{-1}}$
y_n	$(p)^{n-1}$ causal X(z)	$\dfrac{z^{-1}}{1-pz^{-1}}$
	$n \geq 1$	$\lvert z\rvert>p$

$$-(p^{-1}+p^{-2}z^{1}+p^{-3}z^{2}+\cdots)=\sum_{n=0}^{-\infty}(p^{-1})^{-n+1}z^{-n} \quad n<1$$

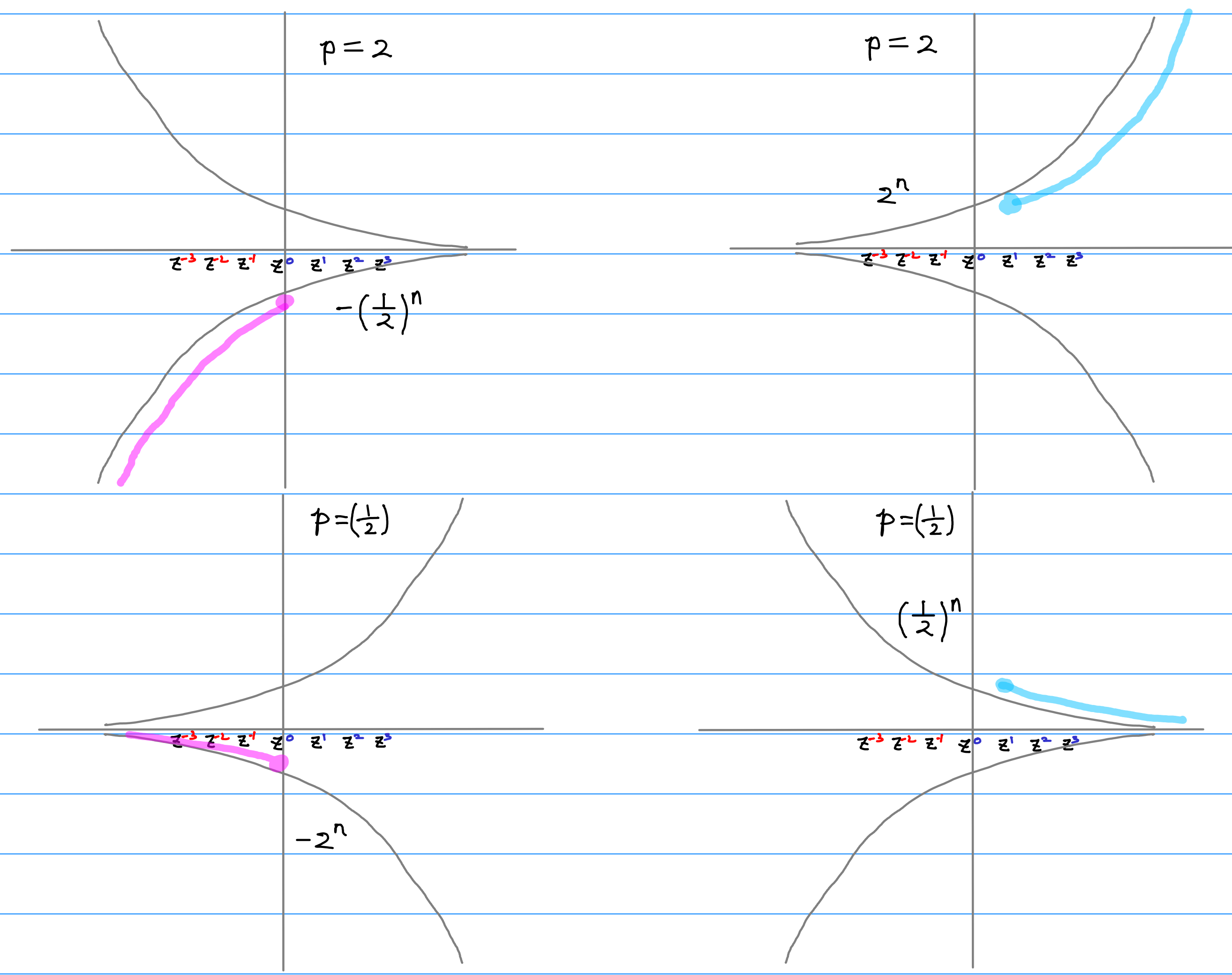
$$p^{0}z^{-1}+p^{1}z^{-2}+p^{2}z^{-3}+\cdots=\sum_{n=1}^{\infty}(p)^{n-1}z^{-n} \quad n \geq 1$$

anti-causal n=0,-1,-2,-3,....

$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

causal n=+1,+2,+3,....

$(p^{0}, p^{1}, p^{2}, \cdots)$



$X(z)$

②

anti-causal $g(z)$ $(|z| > p^{-1})$

$f(z^{-1}) \longleftrightarrow -b_n \quad (n < 1)$

causal $g(z)$ $(|z| < p^{-1})$

$g(z) \longleftrightarrow b_n \quad (n \geq 1)$

$n \geq 0$ $\quad -(p^{-1} + p^{-2}z^{-1} + p^{-3}z^{-2} + \cdots) = \sum_{n=0}^{\infty} -(p^{-1})^{n+1} z^{-n}$

$n < 0$ $\quad p^0 z^1 + p^1 z^2 + p^2 z^3 + \cdots = \sum_{n=-1}^{-\infty} (p^{-1})^{n+1} z^{-n}$

$|z| > p^{-1}$ $\qquad$ $n \geq 0$

$\boxed{-\dfrac{p^{-1}}{1 - p^{-1}z^{-1}}}$ $\qquad$ $\boxed{\text{causal } X(z) \quad -(p^{-1})^{n+1}}$ $\quad x_n$

$\cdot \dfrac{p}{p} \cdot \dfrac{z}{z}$ $\qquad\qquad$ $\cdot(-1)$

$\boxed{\dfrac{z}{1 - pz}}$ $\qquad$ $\boxed{(p)^{-n-1} \quad \text{anti-causal } X(z)}$ $\quad y_{-n}$

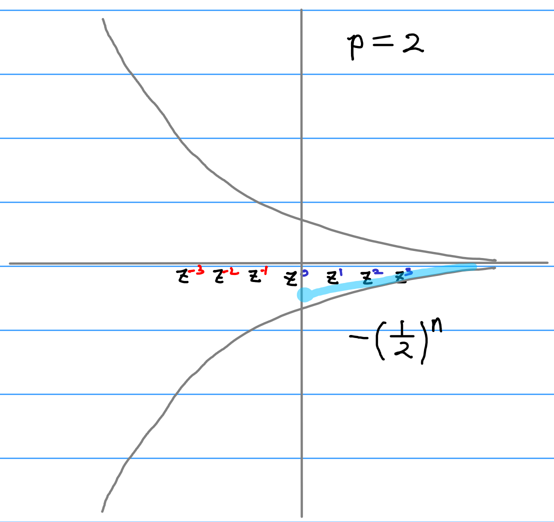
$|z| < p^{-1}$ $\qquad$ $n < 0$

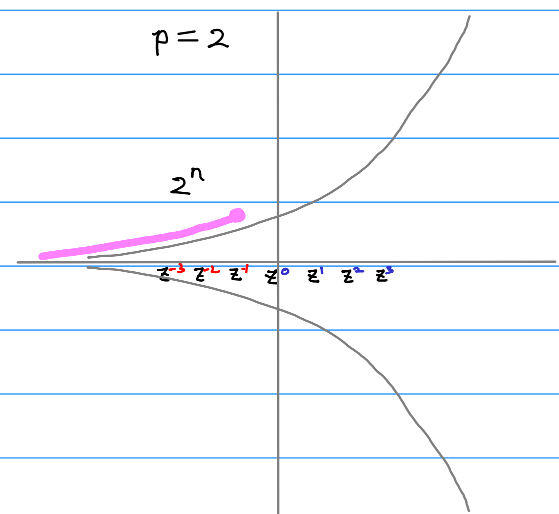
causal $\quad$ $n = 0, +1, +2, +3, \ldots.$

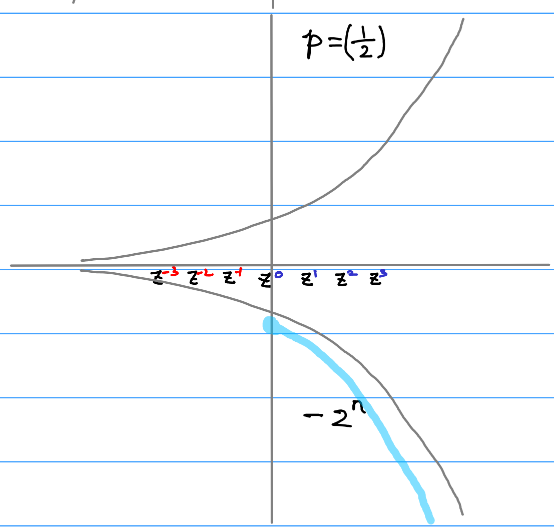
$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

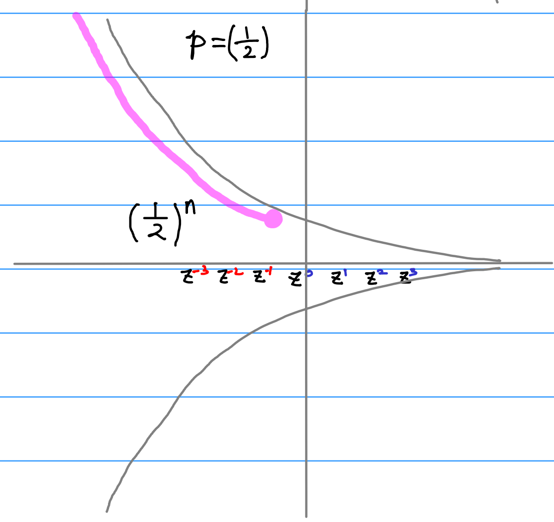
anti-causal $\quad$ $n = -1, -2, -3, \ldots.$

$(p^0, p^1, p^2, \cdots)$









causal $f(z)$

$f(z) \leftrightarrow a_n \quad (n \geq 0)$

anti-causal $f(z)$

$f(z^{-1}) \leftrightarrow a_{-n} \quad (n < 1)$

$n \geq 0$ $\quad |z| < p$

$\boxed{\text{causal } f(z) \quad -(p^{-1})^{n+1}}$ $\quad \boxed{-\dfrac{p^{-1}}{1 - p^{-1} z}}$

$$-(p^{-1} + p^{-2} z^{1} + p^{-3} z^{2} + \cdots) = \sum_{n=0}^{\infty} -(p^{-1})^{n+1} z^{n} \qquad n \geq 0$$

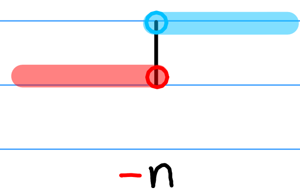
$|z| > p^{-1}$ $\quad n < 1$

$$n < 1 \qquad -(p^{-1} + p^{-2} z^{-1} + p^{-3} z^{-2} + \cdots) = \sum_{n=0}^{-\infty} -(p)^{n-1} z^{n}$$

$\boxed{-\dfrac{p^{-1}}{1 - p^{-1} z^{-1}}}$ $\quad \boxed{\text{anti-causal } f(z) \quad -(p^{-1})^{-n+1}}$

$n \geq 0$

$\boxed{\text{causal } f(z) \quad -p^{-n-1}}$



$n < 1$

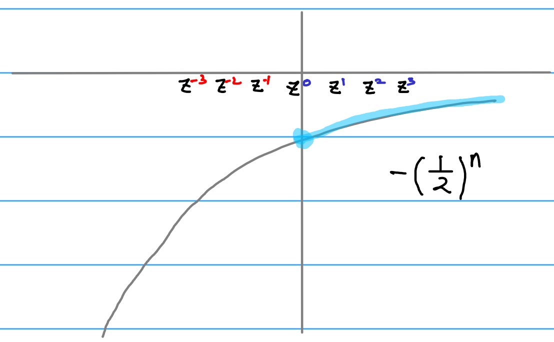
$\boxed{\text{anti-causal } f(z) \quad -p^{+n-1}}$

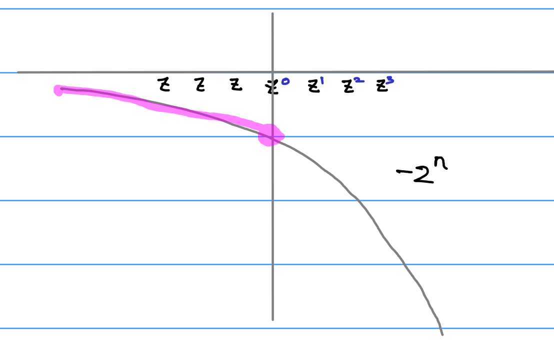
causal $\quad$ n=0,+1,+2,+3,....

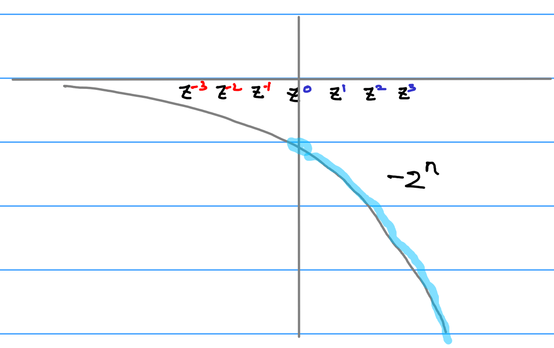
$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$

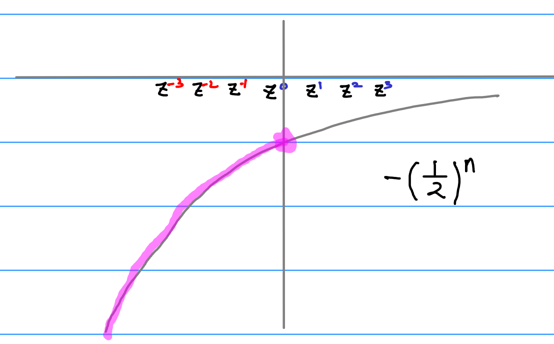
anti-causal $\quad$ n=0,-1,-2,-3,....

$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$









anti-causal $g(z)$

$g(z^{-1}) \leftrightarrow b_{-n}$ $(n<0)$

causal $g(z)$

$g(z) \leftrightarrow b_n$ $(n \geq 1)$

$n<0$ $\quad |z|>p$

$\boxed{(p)^{-n-1} \text{ anti-causal } f(z)}$ $\quad \boxed{\dfrac{z^{-1}}{1-pz^{-1}}}$

$$p^0z^{-1}+p^1z^{-2}+p^2z^{-3}+\dots = \sum_{n=-1}^{-\infty}(p)^{-n-1}z^n \qquad n<0$$

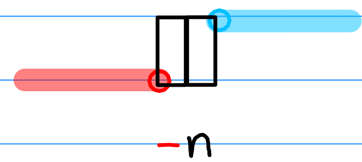
$|z|<p^{-1}$ $\quad n \geq 1$

$$n \geq 1 \qquad p^0z^1+p^1z^2+p^2z^3+\dots = \sum_{n=1}^{\infty}(p)^{n-1}z^n$$

$\boxed{\dfrac{z}{1-pz}}$ $\quad \boxed{(p)^{n-1} \text{ causal } f(z)}$

$n<0$

$\boxed{p^{-n-1} \text{ anti-causal } f(z)}$



$n \geq 1$

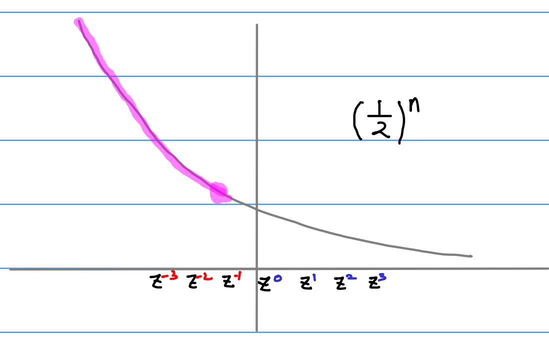
$\boxed{p^{n-1} \text{ causal } f(z)}$

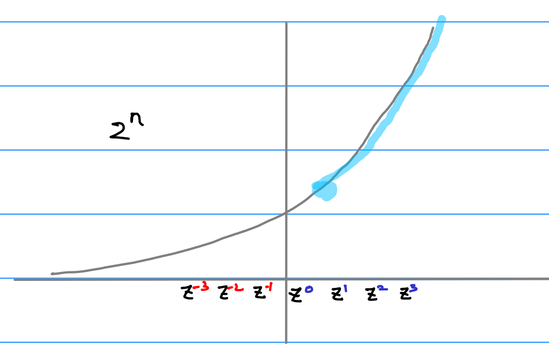
anti-causal $\quad n=-1,-2,-3,\dots.$

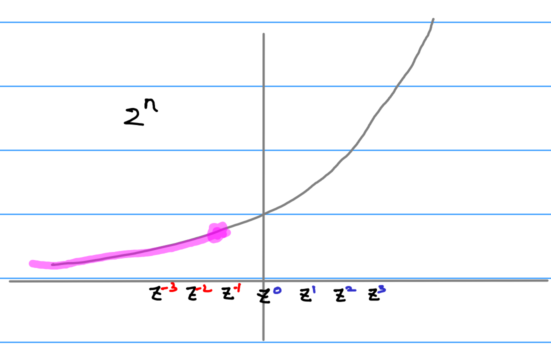
$(p^{-2}, p^{-3}, p^{-4}, \dots)$

causal $\quad n=+1,+2,+3,\dots.$

$(p^{-2}, p^{-3}, p^{-4}, \dots)$





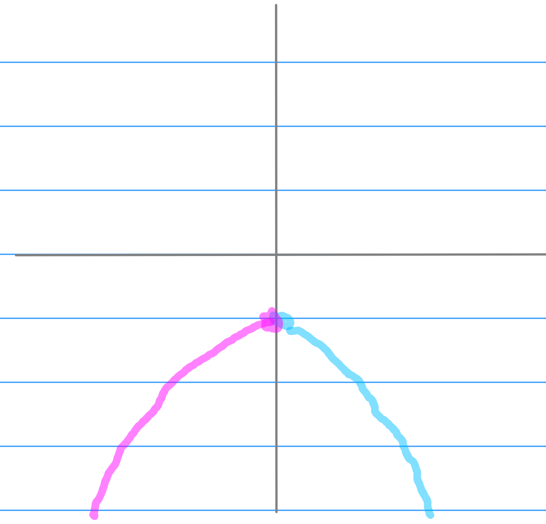


$f(z^{-1})$ $(|z|>p^{-1})$ $f(z)$ $(|z|<p)$

a_{-n} $(n<1)$ a_n $(n\geq 0)$

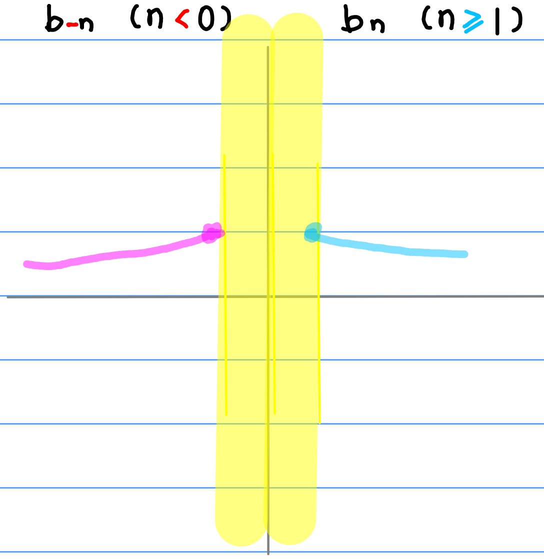
$f(z^{-1})$ $(|z|>p^{-1})$ $f(z)$ $(|z|<p)$

a_{-n} $(n<1)$ a_n $(n\geq 0)$



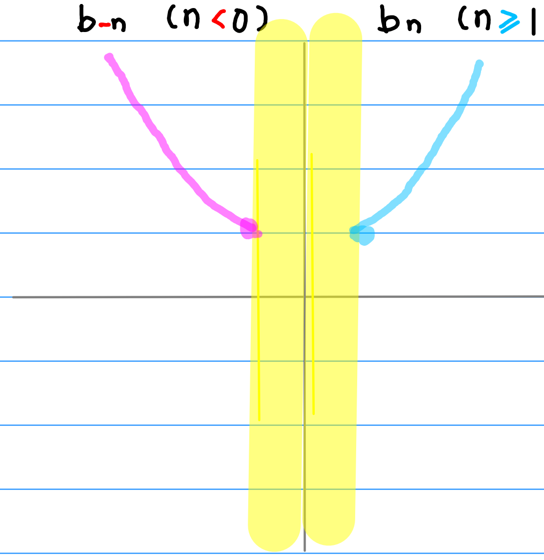
$g(z^{-1})$ $(|z|>p)$ $g(z)$ $(|z|<p^{-1})$

b_{-n} $(n<0)$ b_n $(n\geq 1)$



$g(z^{-1})$ $(|z|>p)$ $g(z)$ $(|z|<p^{-1})$

b_{-n} $(n<0)$ b_n $(n\geq 1)$



x_n		a_{-n}	
y_n		b_{-n}	
causal		anti-causal	
$n \geq 0$	$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$	$n < 1$	
$n \geq 1$	$(p^{-2}, p^{-3}, p^{-4}, \cdots)$	$n < 0$	
anti-causal		causal	
$n < 1$	$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$	$n \geq 0$	
$n < 0$	$(p^{-2}, p^{-3}, p^{-4}, \cdots)$	$n \geq 1$	

causal $f(z)$ $(|z|<p)$ anti-causal $f(z)$ $(|z|>p)$

$f(z) \leftrightarrow a_n$ $(n \geq 0)$ $g(z^{-1}) \leftrightarrow -a_n$ $(n<0)$

$n \geq 0$ $|z|<p$

$$\boxed{\begin{matrix}\text{causal } f(z)\\ -(p^{-1})^{n+1}\end{matrix}} \quad \boxed{-\frac{p^{-1}}{1-p^{-1}z}} \qquad -(p^{-1}+p^{-2}z^{1}+p^{-3}z^{2}+\cdots)=\sum_{n=0}^{\infty}-(p^{-1})^{n+1}z^{n} \qquad n \geq 0$$

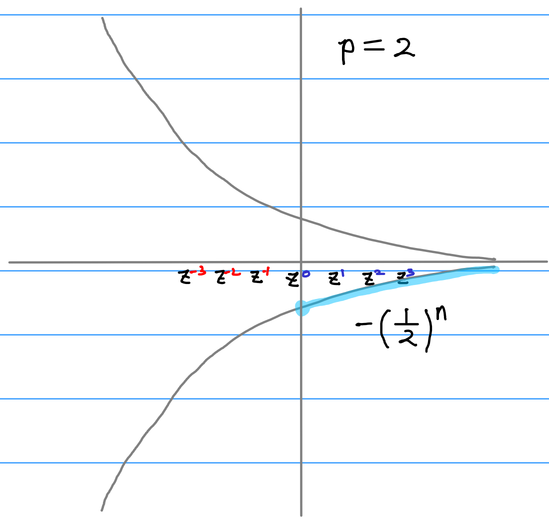
$\cdot(-1)$ $\cdot \frac{p}{p}\frac{z^{-1}}{z^{-1}}$

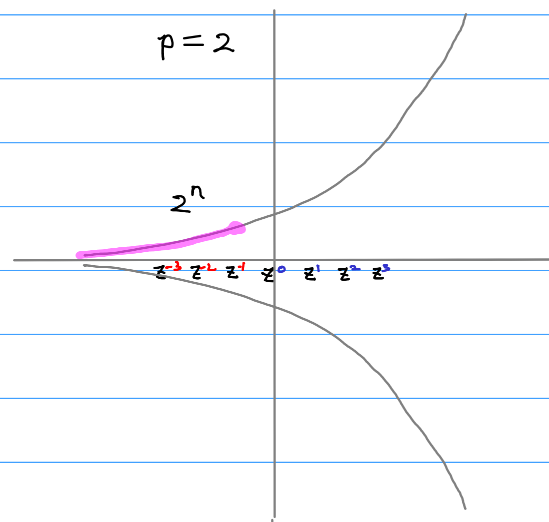
$$\boxed{\begin{matrix}(p)^{-n-1}\\ \text{anti-causal } f(z)\end{matrix}} \quad \boxed{\frac{z^{-1}}{1-pz^{-1}}} \qquad p^{0}z^{-1}+p^{1}z^{-2}+p^{2}z^{-3}+\cdots=\sum_{n=-1}^{-\infty}(p)^{-n-1}z^{n} \qquad n<0$$

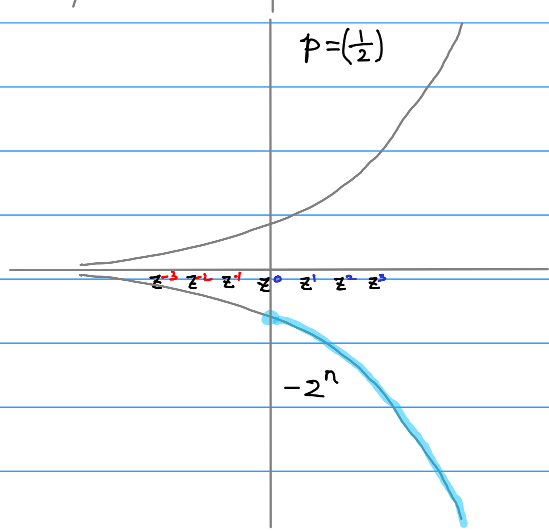
$n<0$ $|z|>p$

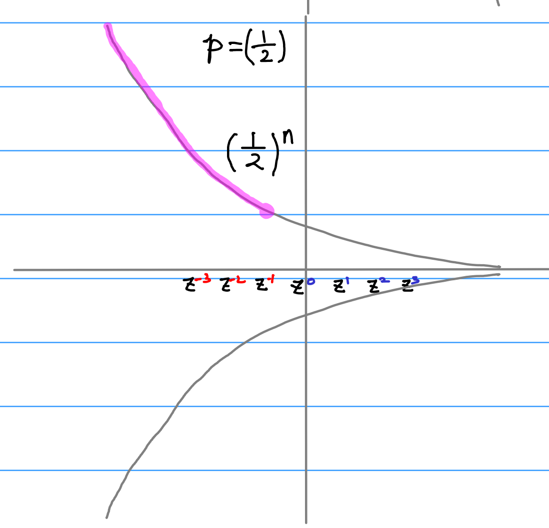
causal $n=0,+1,+2,+3,\ldots.$ anti-causal $n=-1,-2,-3,\ldots.$

$-(p^{-1}, p^{-2}, p^{-3}, \cdots)$ $(p^{0}, p^{1}, p^{2}, \cdots)$









getting causal sequence

$$
\begin{tikzcd}
\boxed{\dfrac{1}{z - p}} \arrow[r, leftrightarrow, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, leftrightarrow, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1 - p^{-1} z}} = f(z) \leftrightarrow \boxed{?} \\
\boxed{\dfrac{z^{-1}}{1 - p z^{-1}}} &
\end{tikzcd}
$$

$$\Vert$$

$$Y(z) \leftrightarrow \boxed{?}$$

$$
\begin{tikzcd}
\boxed{\dfrac{1}{z^{-1} - p}} \arrow[r, leftrightarrow, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, leftrightarrow, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1 - p^{-1} z^{-1}}} = X(z) \leftrightarrow \boxed{?} \\
\boxed{\dfrac{z}{1 - p z}} &
\end{tikzcd}
$$

$$\Vert$$

$$g(z) \leftrightarrow \boxed{?}$$

getting causal sequence w/o memorizing

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z}}$$

$$\Vert$$

$$f(z) \longleftrightarrow \boxed{-(p^{-1})^{n+1}}$$

$$\boxed{\frac{z}{1-pz}}$$

Left shift

$$\Vert$$

$$g(z) \longleftrightarrow \boxed{(p)^{n-1}}$$

$$\boxed{\frac{z^{-1}}{1-pz^{-1}}}$$

Left shift

$$\Vert$$

$$Y(z) \longleftrightarrow \boxed{(p)^{n-1}}$$

$$\boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}}$$

$$\Vert$$

$$X(z) \longleftrightarrow \boxed{-(p^{-1})^{n+1}}$$

getting anti-causal sequence

$$
\begin{tikzcd}
\boxed{\dfrac{1}{z-p}} \arrow[r, leftrightarrow, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, leftrightarrow, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} = f(z) \\
\boxed{\dfrac{z^{-1}}{1-pz^{-1}}} &
\end{tikzcd}
$$

$$f(z) \longleftrightarrow -(p^{-1})^{n+1}$$

$$X(z^{-1}) \longleftrightarrow \boxed{?}$$

$$\dfrac{z^{-1}}{1-pz^{-1}} = g(z^{-1}) \longleftrightarrow \boxed{?}$$

$$Y(z) \longleftrightarrow (p)^{n-1}$$

$$
\begin{tikzcd}
\boxed{\dfrac{1}{z^{-1}-p}} \arrow[r, leftrightarrow, "\frac{p^{-1}}{p^{-1}}"] \arrow[d, leftrightarrow, "\frac{z^{-1}}{z^{-1}}"'] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} = f(z^{-1}) \\
\boxed{\dfrac{z}{1-pz}} &
\end{tikzcd}
$$

$$f(z^{-1}) \longleftrightarrow \boxed{?}$$

$$X(z) \longleftrightarrow -(p^{-1})^{n+1}$$

$$\dfrac{z}{1-pz} = g(z) \longleftrightarrow (p)^{n-1}$$

$$Y(z^{-1}) \longleftrightarrow \boxed{?}$$

① $z \leftarrow z^{-1}$ ② $a_n \leftarrow a_{-n}$

$$\boxed{\frac{1}{z-p}} \xrightleftharpoons{\frac{p^{-1}}{p^{-1}}} \boxed{-\frac{p^{-1}}{1-p^{-1}z}} = f(z) \leftrightarrow -(p^{-1})^{n+1}$$

$$X(z^{-1}) \leftrightarrow \boxed{-(p^{-1})^{-n+1}}$$

$$\boxed{\frac{1}{z-p}} \xrightleftharpoons{\frac{z^{-1}}{z^{-1}}} \boxed{\frac{z^{-1}}{1-pz^{-1}}}$$

$$\boxed{\frac{z^{-1}}{1-pz^{-1}}} = g(z^{-1}) \leftrightarrow \boxed{(p)^{-n-1}}$$

$$Y(z) \leftrightarrow (p)^{n-1}$$

z^{-1}

z^{-1}

$$\boxed{\frac{1}{z^{-1}-p}} \xrightleftharpoons{\frac{p^{-1}}{p^{-1}}} \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} = f(z^{-1}) \leftrightarrow \boxed{-(p^{-1})^{-n+1}}$$

$$X(z) \leftrightarrow -(p^{-1})^{n+1}$$

$$\boxed{\frac{1}{z^{-1}-p}} \xrightleftharpoons{\frac{z^{-1}}{z^{-1}}} \boxed{\frac{z}{1-pz}}$$

$$\boxed{\frac{z}{1-pz}} = g(z) \leftrightarrow (p)^{n-1}$$

$$Y(z^{-1}) \leftrightarrow \boxed{(p)^{-n-1}}$$

Getting anti-causal sequence w/o memorizing

$$\begin{tikzcd}
f(z^{-1}) = \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} \arrow[r, "z^{-1}"] \arrow[d, leftrightarrow] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} = f(z) \arrow[d, leftrightarrow] \\
a_{-n} = \boxed{-(p^{-1})^{-n+1}} & \boxed{-(p^{-1})^{n+1}} = a_n \arrow[l, "-n"']
\end{tikzcd}$$

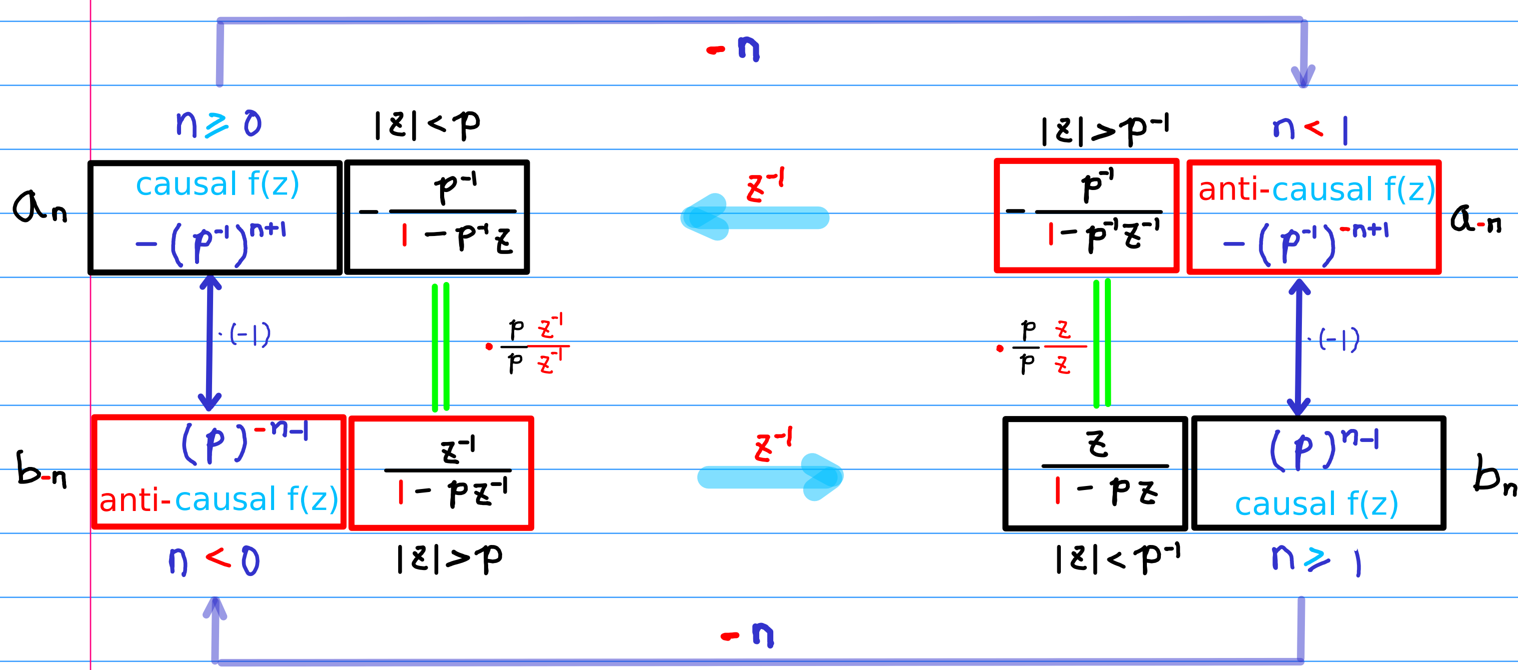
$$\begin{tikzcd}
g(z^{-1}) = \boxed{\dfrac{z^{-1}}{1-pz^{-1}}} \arrow[r, "z^{-1}"] \arrow[d, leftrightarrow] & \boxed{\dfrac{z}{1-pz}} = g(z) \arrow[d, leftrightarrow] \\
b_{-n} = \boxed{(p)^{-n-1}} & \boxed{(p)^{n-1}} = b_n \arrow[l, "-n"']
\end{tikzcd}$$

$$\begin{tikzcd}
Y(z^{-1}) = \boxed{\dfrac{z}{1-pz}} \arrow[r, "z^{-1}"] \arrow[d, leftrightarrow] & \boxed{\dfrac{z^{-1}}{1-pz^{-1}}} = Y(z) \arrow[d, leftrightarrow] \\
y_{-n} = \boxed{(p)^{-n-1}} & \boxed{(p)^{n-1}} = y_n \arrow[l, "-n"']
\end{tikzcd}$$

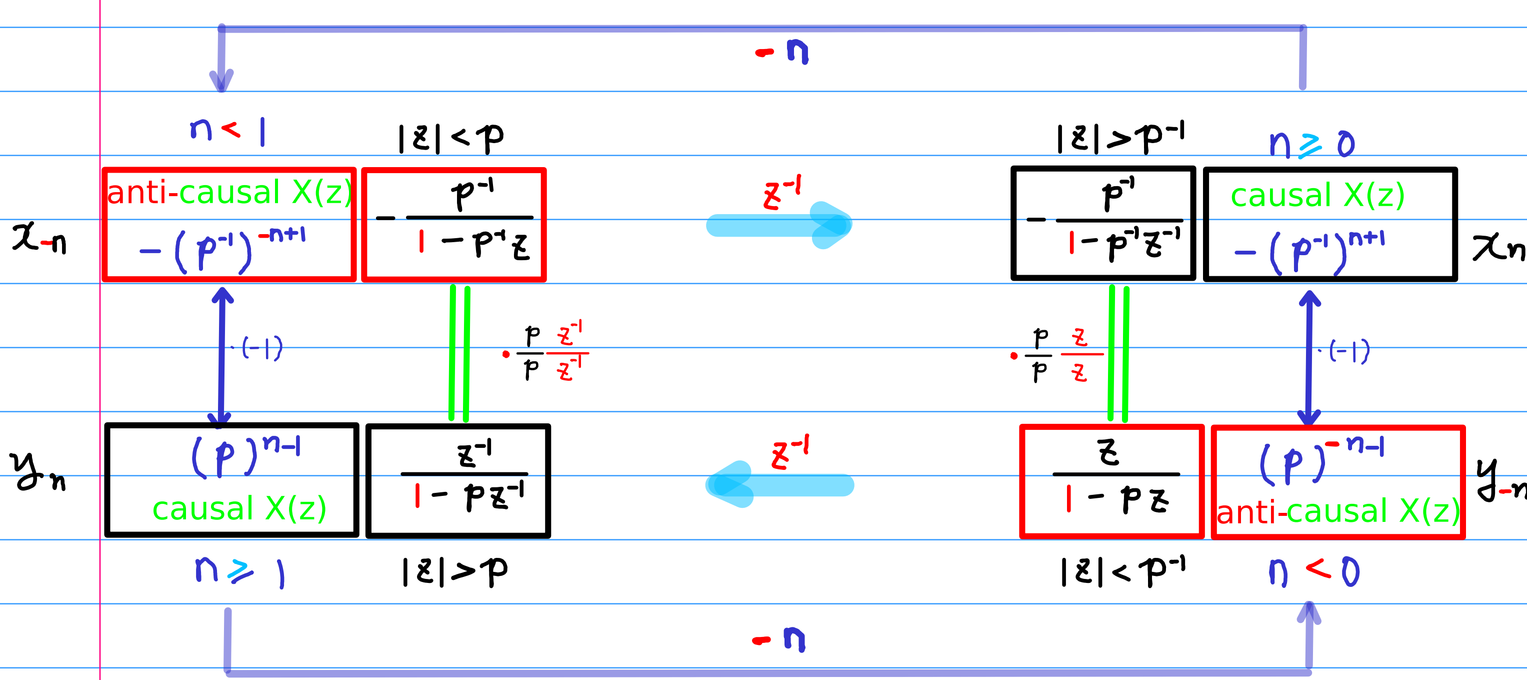
$$\begin{tikzcd}
X(z^{-1}) = \boxed{-\dfrac{p^{-1}}{1-p^{-1}z}} \arrow[r, "z^{-1}"] \arrow[d, leftrightarrow] & \boxed{-\dfrac{p^{-1}}{1-p^{-1}z^{-1}}} = X(z) \arrow[d, leftrightarrow] \\
x_{-n} = \boxed{-(p^{-1})^{-n+1}} & \boxed{-(p^{-1})^{n+1}} = x_n \arrow[l, "-n"']
\end{tikzcd}$$

Getting anti-causal sequence

Laurent Series



Z-Transform



$f(z^{-1})$ $\quad g(z^{-1})$

① $\boxed{z^{-1} \to z}$ $\quad f(z),\ g(z)$

② $f(z) \leftrightarrow a_n \quad g(z) \leftrightarrow b_n$

③ $\boxed{n \to -n}$ $\quad a_{-n},\ b_{-n}$

$X(z^{-1})$ $\quad Y(z^{-1})$

① $\boxed{z^{-1} \to z}$ $\quad X(z),\ Y(z)$

② $X(z) \leftrightarrow x_n \quad Y(z) \leftrightarrow y_n$

③ $\boxed{n \to -n}$ $\quad x_{-n},\ x_{-n}$

$$f(z^{-1}) = \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} \qquad g(z^{-1}) = \boxed{\frac{z^{-1}}{1-pz^{-1}}}$$

anti-causal

$$f(z) = \boxed{-\frac{p^{-1}}{1-p^{-1}z}} \qquad g(z) = \boxed{\frac{z}{1-pz}}$$

$$Y(z^{-1}) = \boxed{\frac{z}{1-pz}} \qquad X(z^{-1}) = \boxed{-\frac{p^{-1}}{1-p^{-1}z}}$$

anti-causal

$$Y(z) = \boxed{\frac{z^{-1}}{1-pz^{-1}}} \qquad X(z) = \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}}$$

$f(z^{-1})$ $g(z^{-1})$

① $\boxed{z^{-1} \to z}$ $f(z), g(z)$

② $f(z) \leftrightarrow a_n$ $g(z) \leftrightarrow b_n$

③ $\boxed{n \to -n}$ a_{-n}, b_{-n}

$$f(z^{-1}) = \boxed{-\frac{p^{-1}}{1-p^{-1}z^{-1}}} \qquad g(z^{-1}) = \boxed{\frac{z^{-1}}{1-pz^{-1}}}$$

① $$f(z) = \boxed{-\frac{p^{-1}}{1-p^{-1}z}} \qquad g(z) = \boxed{\frac{z}{1-pz}}$$

② $$a_n = \boxed{-(p^{-1})^{n+1}} \qquad b_n = \boxed{(p)^{n-1}}$$

③ $$a_{-n} = \boxed{-(p^{-1})^{-n+1}} \qquad b_{-n} = \boxed{(p)^{-n-1}}$$

$X(z^{-1})$ $Y(z^{-1})$

1. $\boxed{z^{-1} \to z}$ $X(z),\ Y(z)$

2. $X(z) \leftrightarrow x_n$ $Y(z) \leftrightarrow y_n$

3. $\boxed{n \to -n}$ $x_{-n},\ x_{-n}$

$$Y(z^{-1}) = \boxed{\frac{z}{1 - pz}} \qquad X(z^{-1}) = \boxed{-\frac{p^{-1}}{1 - p^{-1}z}}$$

1. $$Y(z) = \boxed{\frac{z^{-1}}{1 - pz^{-1}}} \qquad X(z) = \boxed{-\frac{p^{-1}}{1 - p^{-1}z^{-1}}}$$

2. $$y_n = \boxed{(p)^{n-1}} \qquad x_n = \boxed{-(p^{-1})^{n+1}}$$

3. $$y_{-n} = \boxed{-(p^{-1})^{-n+1}} \qquad x_{-n} = \boxed{(p)^{-n-1}}$$

① $\dfrac{-1}{(z-1)(z-2)}$

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$f(z)$ $|z|<1$ causal

$X(z)$ $|z|<1$ anti-causal

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$f(z)$ $|z|>1$ anti-causal

$X(z)$ $|z|>1$ causal

② $\dfrac{-0.5z^{2}}{(z-1)(z-0.5)}$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$f(z)$ $|z|<0.5$ causal

$X(z)$ $|z|<0.5$ anti-causal

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$f(z)$ $|z|>2$ anti-causal

$X(z)$ $|z|>2$ causal

$$-\frac{1}{1-z}+\frac{0.5}{1-0.5z}$$

$f(z) \quad |z|<1$

$$+\frac{z}{1-z}-\frac{z}{1-2z}$$

$f(z) \quad |z|<0.5$

$\boxed{\cdot z \quad n-1}$

$$+\frac{1}{1-z}-\frac{1}{1-2z}$$

$g(z) \quad |z|<0.5$

$$+\frac{z^{-1}}{1-z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$X(z) \quad |z|>1$

$\boxed{\cdot z^{-1} \quad n-1}$

$$+\frac{1}{1-z^{-1}}-\frac{1}{1-2z^{-1}}$$

$V(z) \quad |z|>1$

$$-\frac{1}{1-z^{-1}}+\frac{1}{1-0.5z^{-1}}$$

$X(z) \quad |z|>2$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$X(z)$ $\quad |z| < 1$

$\boxed{z^{-1} \quad -n}$

$$-\frac{1}{1-z^{-1}} + \frac{1}{1-0.5z^{-1}}$$

$V(z)$ $\quad |z| > 2$

$$+\frac{z}{1-z} - \frac{z}{1-2z}$$

$X(z)$ $\quad |z| < 0.5$

$\boxed{z^{-1} \quad -n}$

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$V(z)$ $\quad |z| > 1$

$\boxed{\cdot z^{-1} \quad n-1}$

$$+\frac{1}{1-z^{-1}} - \frac{1}{1-2z^{-1}}$$

$W(z)$ $\quad |z| > 1$

$$+\frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$f(z)$ $\quad |z| > 1$

$\boxed{z^{-1} \quad -n}$

$$+\frac{z}{1-z} - \frac{z}{1-2z}$$

$g(z)$ $\quad |z| < 0.5$

$\boxed{\cdot z \quad n-1}$

$$+\frac{1}{1-z} - \frac{1}{1-2z}$$

$h(z)$ $\quad |z| < 0.5$

$$-\frac{1}{1-z^{-1}} + \frac{1}{1-0.5z^{-1}}$$

$f(z)$ $\quad |z| > 2$

$\boxed{z^{-1} \quad -n}$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$g(z)$ $\quad |z| < 1$