

# Applicatives Methods (3B)

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# The definition of Applicative

```
class (Functor f) => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

The class has a two methods :

**pure** brings arbitrary values into the functor

**(<\*>)** takes a function wrapped in a functor **f**  
and a value wrapped in a functor **f**  
and returns the result of the application  
which is also wrapped in a functor **f**

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

# The Maybe instance of Applicative

```
instance Applicative Maybe where
```

```
  pure          = Just
```

```
  (Just f) <*> (Just x) = Just (f x)
```

```
  _          <*> _      = Nothing
```

**pure** wraps the value with **Just**;

**(<\*>)** applies

the function wrapped in **Just**

to the value wrapped in **Just** if both exist,

and results in **Nothing** otherwise.

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

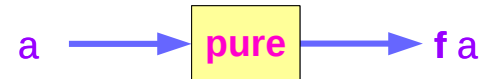
# The Applicative Typeclass

```
class (Functor f) => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

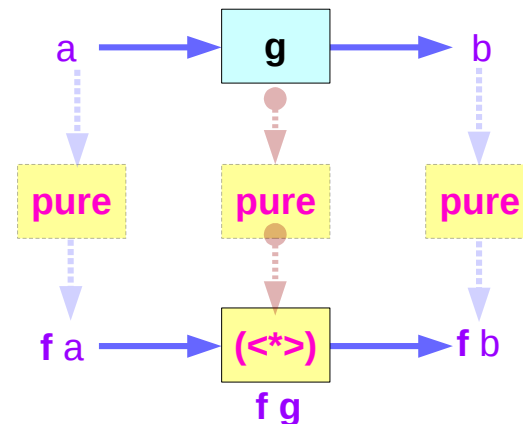
**f** : **Functor**, **Applicative**

```
instance Applicative Maybe where
  pure = Just
  Nothing <*> _ = Nothing
  (Just f) <*> something = fmap f something
```

**f** : function in a context



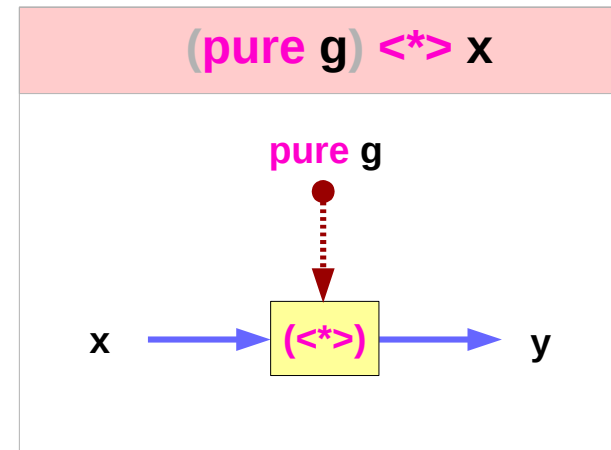
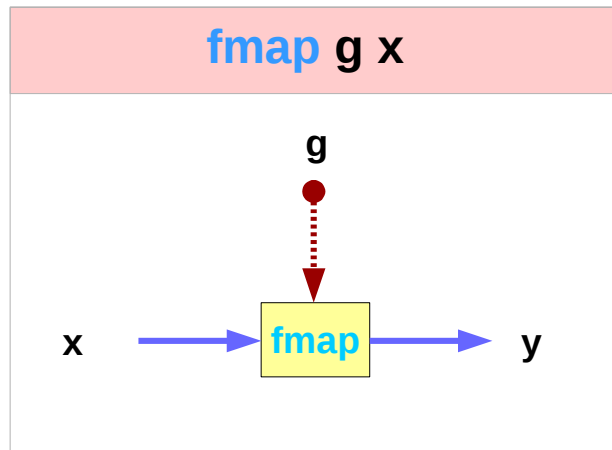
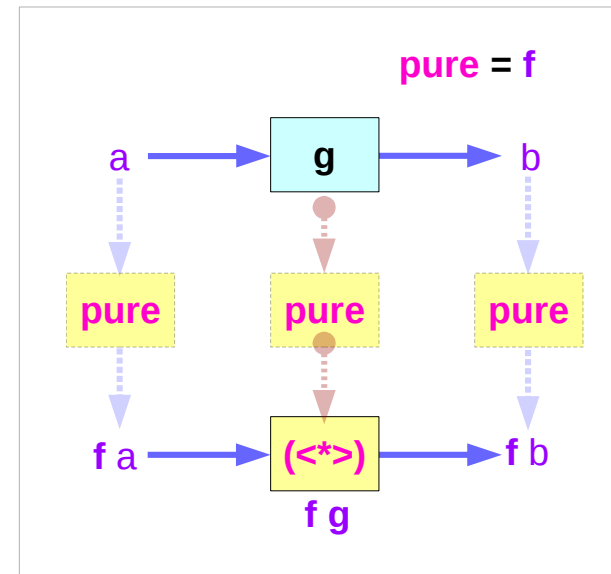
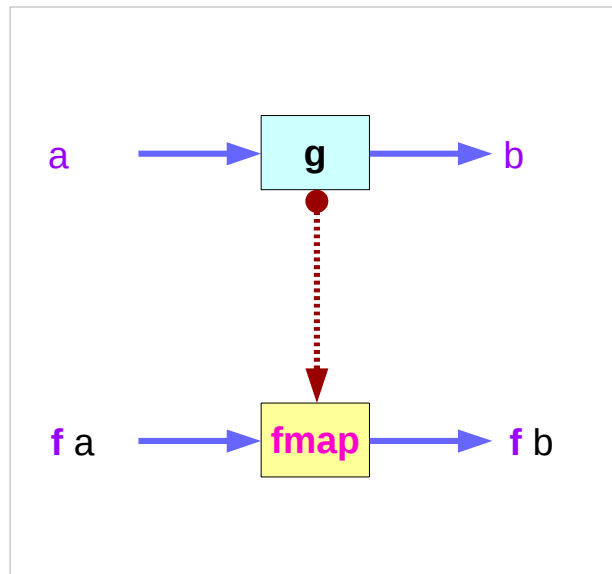
(Functor f) => Applicative f



(Functor f) => Applicative f

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

$$\text{fmap } g \ x = (\text{pure } g) \langle * \rangle x$$



<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

# Left Associative <\*>

```
ghci> pure (+) <*> Just 3 <*> Just 5  
Just 8
```

pure (+) <\*> Just 3 <\*> Just 5

pure (+3) <\*> Just 5

Just 8

```
ghci> pure (+) <*> Just 3 <*> Nothing  
Nothing
```

```
ghci> pure (+) <*> Nothing <*> Just 5  
Nothing
```

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

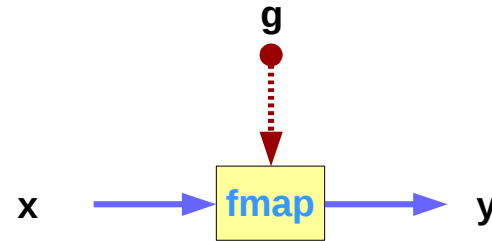
# Infix Operator $\langle \$ \rangle$

`pure f  $\langle * \rangle$  x  $\langle * \rangle$  y  $\langle * \rangle$  z`

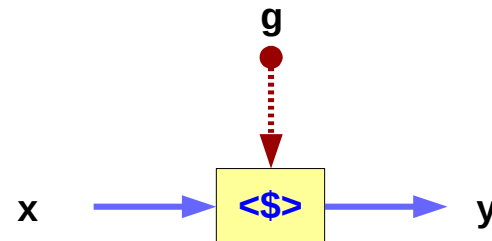
`fmap f x  $\langle * \rangle$  y  $\langle * \rangle$  z`

`f  $\langle \$ \rangle$  x  $\langle * \rangle$  y  $\langle * \rangle$  z`

`fmap g x`



`g  $\langle \$ \rangle$  x`



<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>



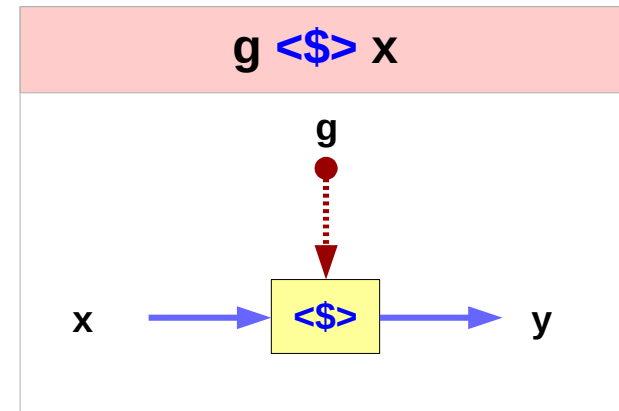
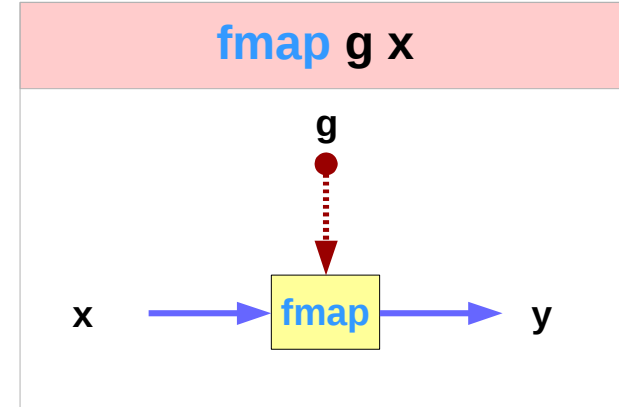
# Infix Operator `<$>` : not a class method

```
class (Functor f) => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

*not a class method*

```
(<$>) :: (Functor f) => (a -> b) -> f a -> f b
f <$> x = fmap f x
```

```
instance Applicative Maybe where
  pure = Just
  Nothing <*> _ = Nothing
  (Just f) <*> something = fmap f something
```



<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

# The Applicative Typeclass

**Applicative** is a superclass of **Monad**.

every **Monad** is also a **Functor** and an **Applicative**

**fmap**, **pure**, **(<\*>)** can all be used with **monads**.

a **Monad** instance also requires  
**Functor** and **Applicative** instances.

the types and roles of **return** and **(>>)**

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

# (\*> v.s. >>) and (pure v.s. return)

(\*>) :: **Applicative** f => f a -> f b -> f b

(>>) :: **Monad** m => m a -> m b -> m b

**pure** :: **Applicative** f => a -> f a

**return** :: **Monad** m => a -> m a

the constraint changes from **Applicative** to **Monad**.

(\*>) in **Applicative**

(>>) in **Monad**

**pure** in **Applicative**

**return** in **Monad**

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

# The Applicative Laws

The identity law:  $\text{pure id} \langle * \rangle v = v$

Homomorphism:  $\text{pure f} \langle * \rangle \text{pure x} = \text{pure (f x)}$

Interchange:  $u \langle * \rangle \text{pure y} = \text{pure (\$ y)} \langle * \rangle u$

Composition:  $u \langle * \rangle (v \langle * \rangle w) = \text{pure (.)} \langle * \rangle u \langle * \rangle v \langle * \rangle w$

# The Identity Law

The identity law

**pure id**  $\langle * \rangle$   $v = v$

**pure** to inject values into the functor  
in a default, featureless way,  
so that the result is as close as possible to the plain value.

applying the **pure id** morphism does nothing,  
exactly like with the plain **id** function.

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

# The Homomorphism Law

The homomorphism law

$$\text{pure } f \text{ <*> pure } x = \text{pure } (f \ x)$$

applying a "**pure**" function to a "**pure**" value is the same as  
applying the function to the value in the normal way  
and then using **pure** on the result.  
means **pure** preserves function application.

**applying** a non-effectful function **f**  
to a non-effectful argument **x** in an effectful context **pure**  
is the same as just **applying** the function **f** to the argument **x**  
and then injecting the result **(f x)** into the context with **pure**.

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

# The Interchange Law

The interchange law

$$u \text{ <*> pure } y = \text{pure } (\$ y) \text{ <*> } u$$

applying a morphism  $u$  to a "pure" value  $\text{pure } y$   
is the same as applying  $\text{pure } (\$ y)$  to the morphism  $u$

$(\$ y)$  is the function that supplies  $y$  as argument to another function  
– the higher order functions

when evaluating the application of  
an effectful function  $u$  to a pure argument  $\text{pure } y$ ,  
the order in which we evaluate  
the function  $u$  and its argument  $\text{pure } y$  doesn't matter.

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

# The Composition Law

The composition law

$$\text{pure } (.) \langle * \rangle u \langle * \rangle v \langle * \rangle w = u \langle * \rangle (v \langle * \rangle w)$$

**pure** (.) composes morphisms similarly  
to how (.) composes functions:

$$(f . g) x = f (g x)$$

$$\begin{aligned} &\text{pure } (.) \langle * \rangle \text{pure } f \langle * \rangle \text{pure } g \langle * \rangle \text{pure } x \\ &= \text{pure } f \langle * \rangle (\text{pure } g \langle * \rangle \text{pure } x) \end{aligned}$$

$$\begin{aligned} u &= \text{pure } f \\ v &= \text{pure } g \\ w &= \text{pure } x \end{aligned}$$

applying the composed morphism **pure** (.)  $\langle * \rangle$  **u**  $\langle * \rangle$  **v** to **w**  
gives the same result as applying **u**  
to the result of applying **v** to **w**

$$\begin{aligned} &u \\ &(v \langle * \rangle w) \end{aligned}$$

it is expressing a sort of associativity property of ( $\langle * \rangle$ ).

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)



# Sequencing of Effects

Functor map <\$>

$(\<\$) :: \text{Functor } f \Rightarrow (a \rightarrow b) \rightarrow f\ a \rightarrow f\ b$

$(\<\$) :: \text{Functor } f \Rightarrow a \rightarrow f\ b \rightarrow f\ a$

$(\$) :: \text{Functor } f \Rightarrow f\ a \rightarrow b \rightarrow f\ b$

The <\$> operator is just a synonym for the fmap function from the Functor typeclass. This function generalizes the map function for lists to many other data types, such as Maybe, IO, and Map.

<https://haskell-lang.org/tutorial/operators>

# Sequencing of Effects

Functor map <\$>

(<\$>) :: Functor f => (a -> b) -> f a -> f b

(<\$) :: Functor f => a -> f b -> f a

(\$>) :: Functor f => f a -> b -> f b

The <\$> operator is just a synonym for the fmap function from the Functor typeclass. This function generalizes the map function for lists to many other data types, such as Maybe, IO, and Map.

<https://haskell-lang.org/tutorial/operators>

# Sequencing of Effects

```
#!/usr/bin/env stack
-- stack --resolver ghc-7.10.3 runghc
import Data.Monoid ((<>))

main :: IO ()
main = do
    putStrLn "Enter your year of birth"
    year <- read <$> getLine
    let age :: Int
        age = 2020 - year
    putStrLn $ "Age in 2020: " <> show age
```

<https://haskell-lang.org/tutorial/operators>

# Sequencing of Effects

In addition, there are two additional operators provided which replace a value inside a Functor instead of applying a function. This can be both more convenient in some cases, as well as for some Functors be more efficient. In terms of definition:

`value <$ functor = const value <$> functor`

`functor $> value = const value <$> functor`

`x <$ y = y $> x`

`x $> y = y <$ x`

<https://haskell-lang.org/tutorial/operators>

# Sequencing of Effects

Applicative function application `<*>`

`(<*>) :: Applicative f => f (a -> b) -> f a -> f b`

`(<*>) :: Applicative f => f a -> f b -> f b`

`(<*>) :: Applicative f => f a -> f b -> f a`

Commonly seen with `<$>`, `<*>` is an operator that applies a wrapped function to a wrapped value. It is part of the `Applicative` typeclass, and is very often seen in code like the following:

```
foo <$> bar <*> baz
```

<https://haskell-lang.org/tutorial/operators>

# Sequencing of Effects

For cases when you're dealing with a Monad, this is equivalent to:

```
do x <- bar
  y <- baz
  return (foo x y)
```

Other common examples including parsers and serialization libraries. Here's an example you might see using the `aeson` package:

```
data Person = Person { name :: Text, age :: Int } deriving Show

-- We expect a JSON object, so we fail at any non-Object value.
instance FromJSON Person where
  parseJSON (Object v) = Person <$> v .: "name" <*> v .: "age"
  parseJSON _ = empty
```

<https://haskell-lang.org/tutorial/operators>

# Sequencing of Effects

To go along with this, we have two helper operators that are less frequently used:

`*>` ignores the value from the first argument. It can be defined as:

```
a1 *> a2 = (id <$ a1) <*> a2
```

Or in do-notation:

```
a1 *> a2 = do
  _ <- a1
  a2
```

For Monads, this is completely equivalent to `>>`.

<https://haskell-lang.org/tutorial/operators>

# Sequencing of Effects

$<^*$  is the same thing in reverse: perform the first action then the second, but only take the value from the first action. Again, definitions in terms of  $<^*>$  and `do`-notation:

$(<^*) = \text{liftA2 } \text{const}$

$a1 <^* a2 = \text{do}$

`res <- a1`

`_ <- a2`

`return res`

<https://haskell-lang.org/tutorial/operators>



## References

- [1] <ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf>
- [2] <https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf>