Applications of Array Pointers (1A)

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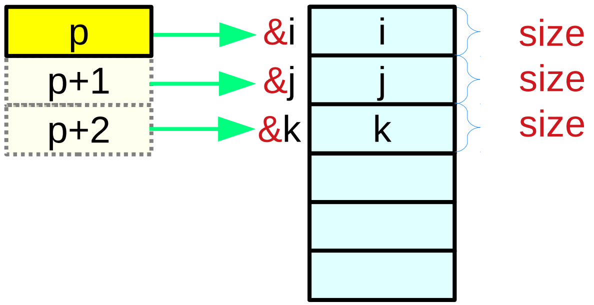
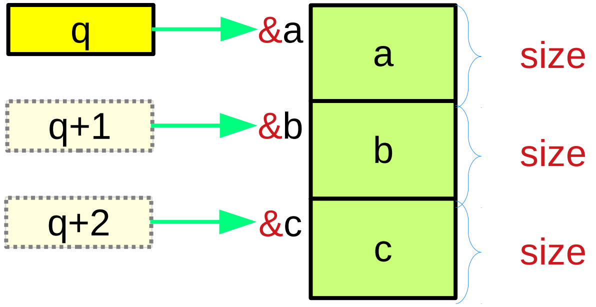
Virtual Array Pointers in Multi-dimensional Arrays

Pointers to various data types

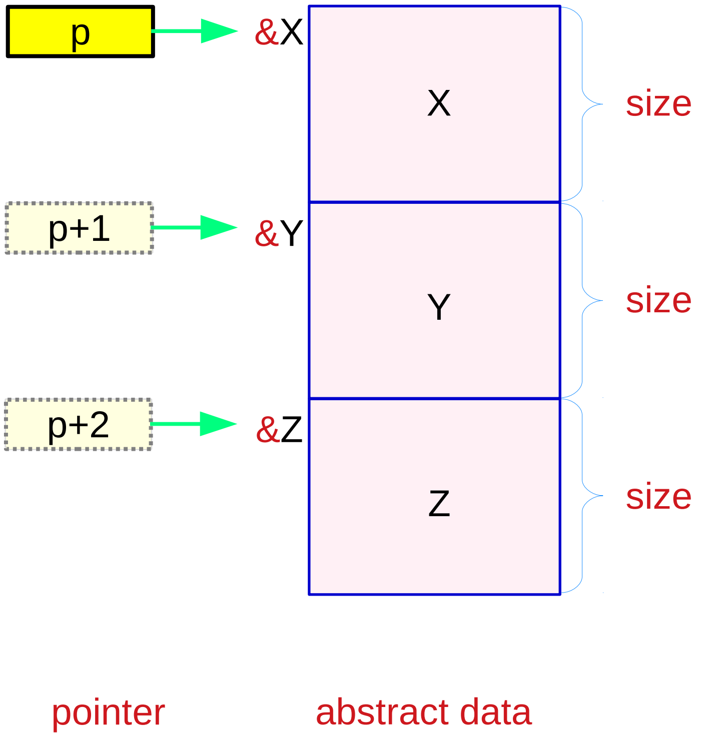
int *p; int i, j, k;



double *q; double a, b, c;

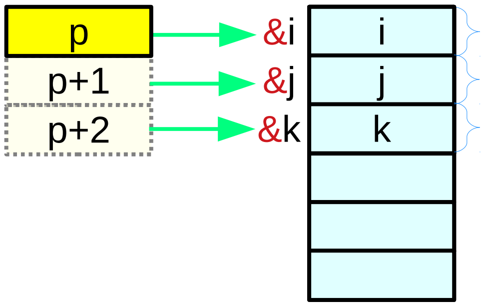


T *p; T X, Y, Z;



Pointers to primitive data

int *p; **int i, j, k;** **sizeof(int) = 4 bytes**



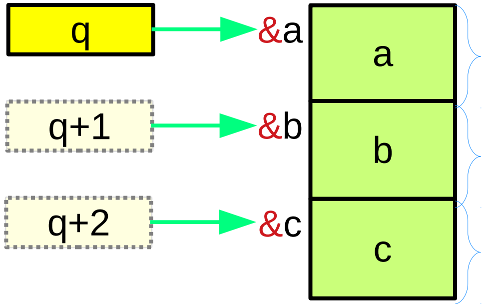
size = sizeof(i) = sizeof(*p) ≠ sizeof(p)

size = sizeof(j) = sizeof(*(p+1)) ≠ sizeof(p+1)

size = sizeof(k) = sizeof(*(p+2)) ≠ sizeof(p+2)

pointer size
4 or 8 bytes

double *q; **double a, b, c;** **sizeof(double) = 8 bytes**



size = sizeof(a) = sizeof(*q) ≠ sizeof(q)

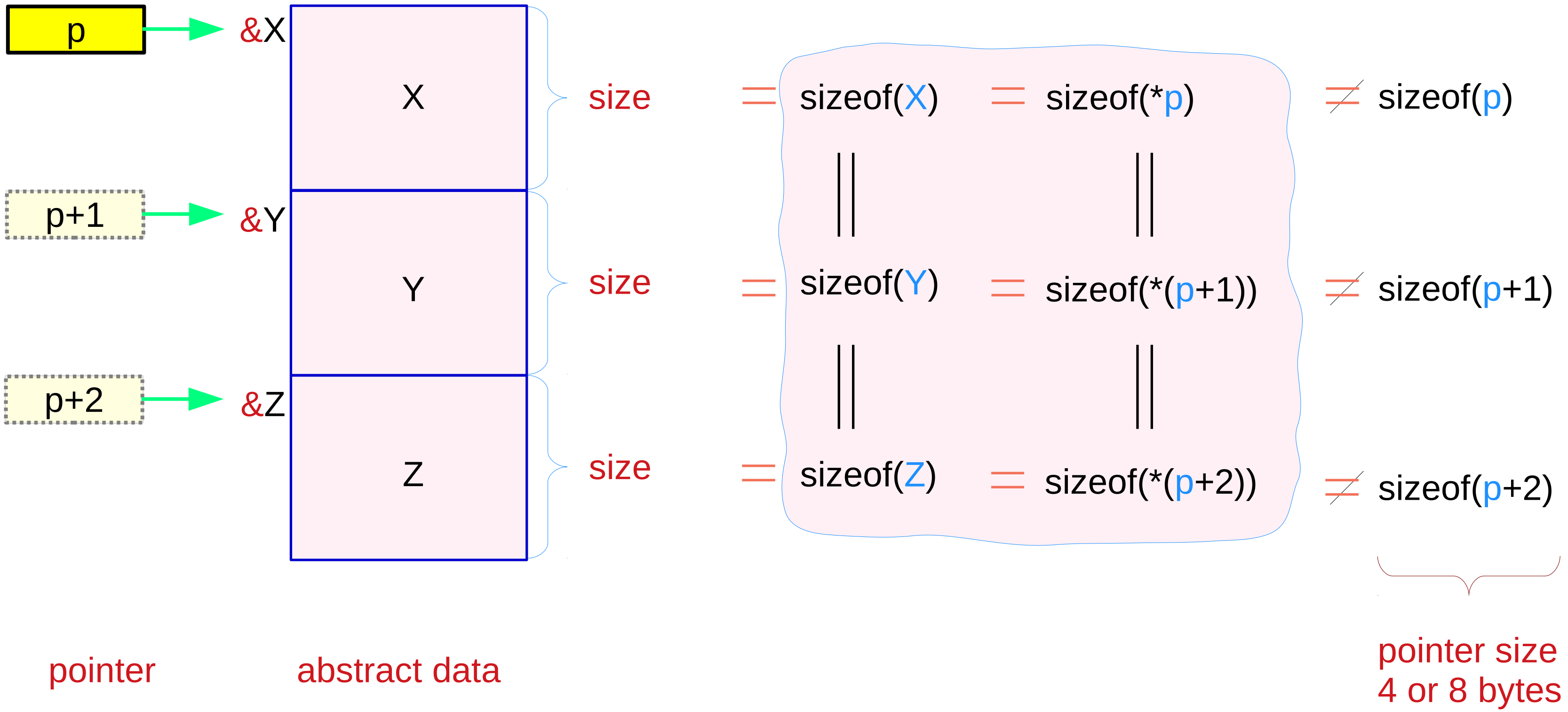
size = sizeof(b) = sizeof(*(q+1)) ≠ sizeof(q+1)

size = sizeof(c) = sizeof(*(q+2)) ≠ sizeof(q+2)

pointer size
4 or 8 bytes

Pointers to abstract data

T *p; **T X, Y, Z;** **sizeof(T)**



p → &X
p+1 → &Y
p+2 → &Z

X size = sizeof(X) = sizeof(*p) ≠ sizeof(p)

Y size = sizeof(Y) = sizeof(*(p+1)) ≠ sizeof(p+1)

Z size = sizeof(Z) = sizeof(*(p+2)) ≠ sizeof(p+2)

sizeof(X) || sizeof(Y) || sizeof(Z)

sizeof(*p) || sizeof(*(p+1)) || sizeof(*(p+2))

pointer

abstract data

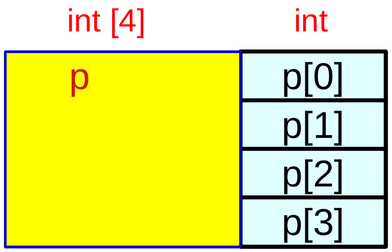
pointer size
4 or 8 bytes

type -------- array
value -------- start address
increment size -------- size

Virtual pointers in an array of integers

int p[3];

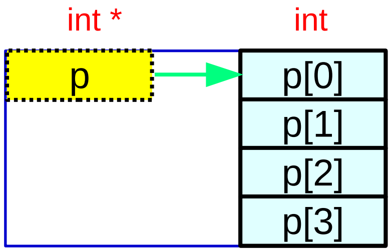
p is an abstract data (array)



p is the name of an array
p has the size of the whole array

p has an array type (abstract data)

p can also be viewed as a pointer

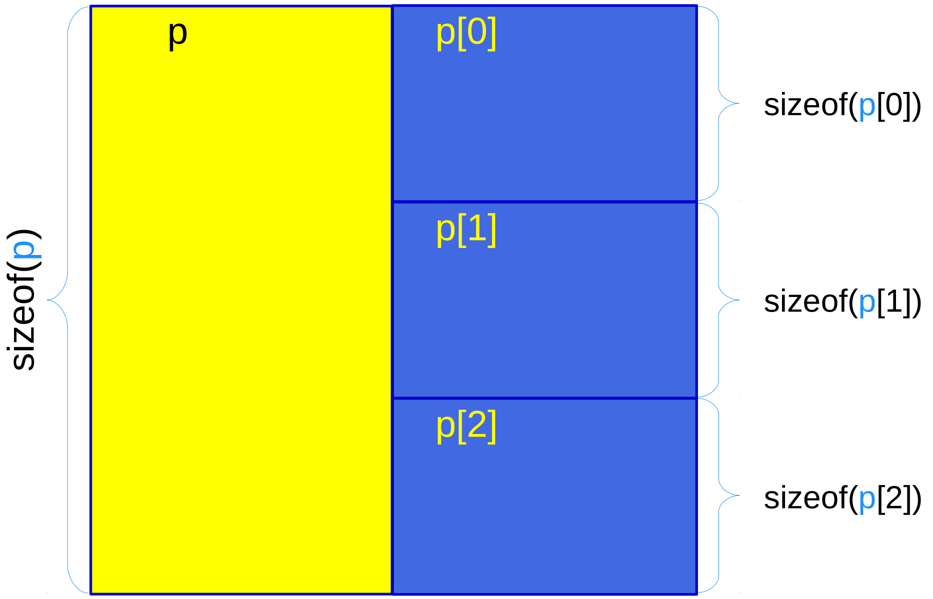


p also has pointer characteristics
p has the value of the starting address

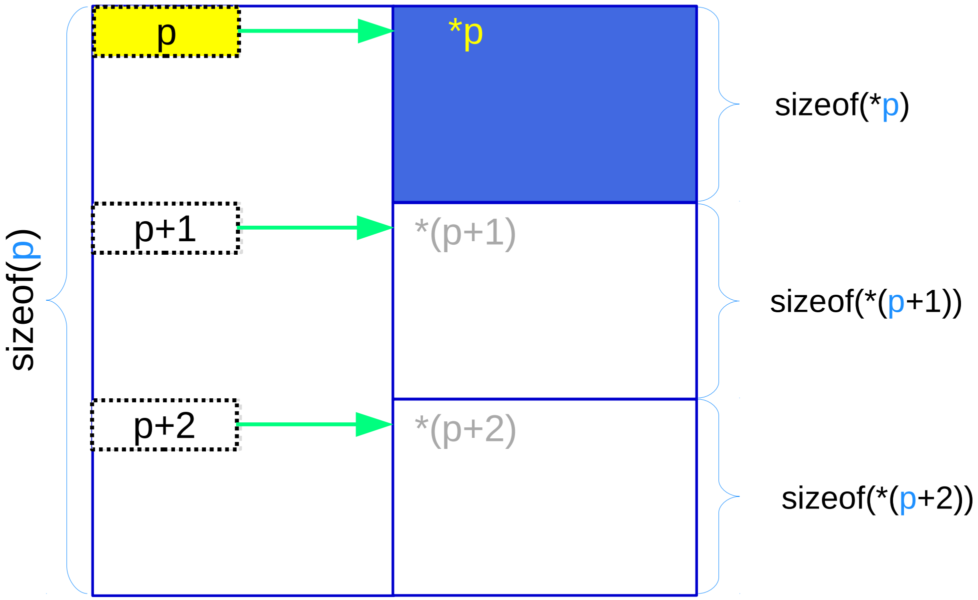
p is a virtual pointer

Virtual pointers in an array of abstract data

Abstract data array p



Virtual pointer p

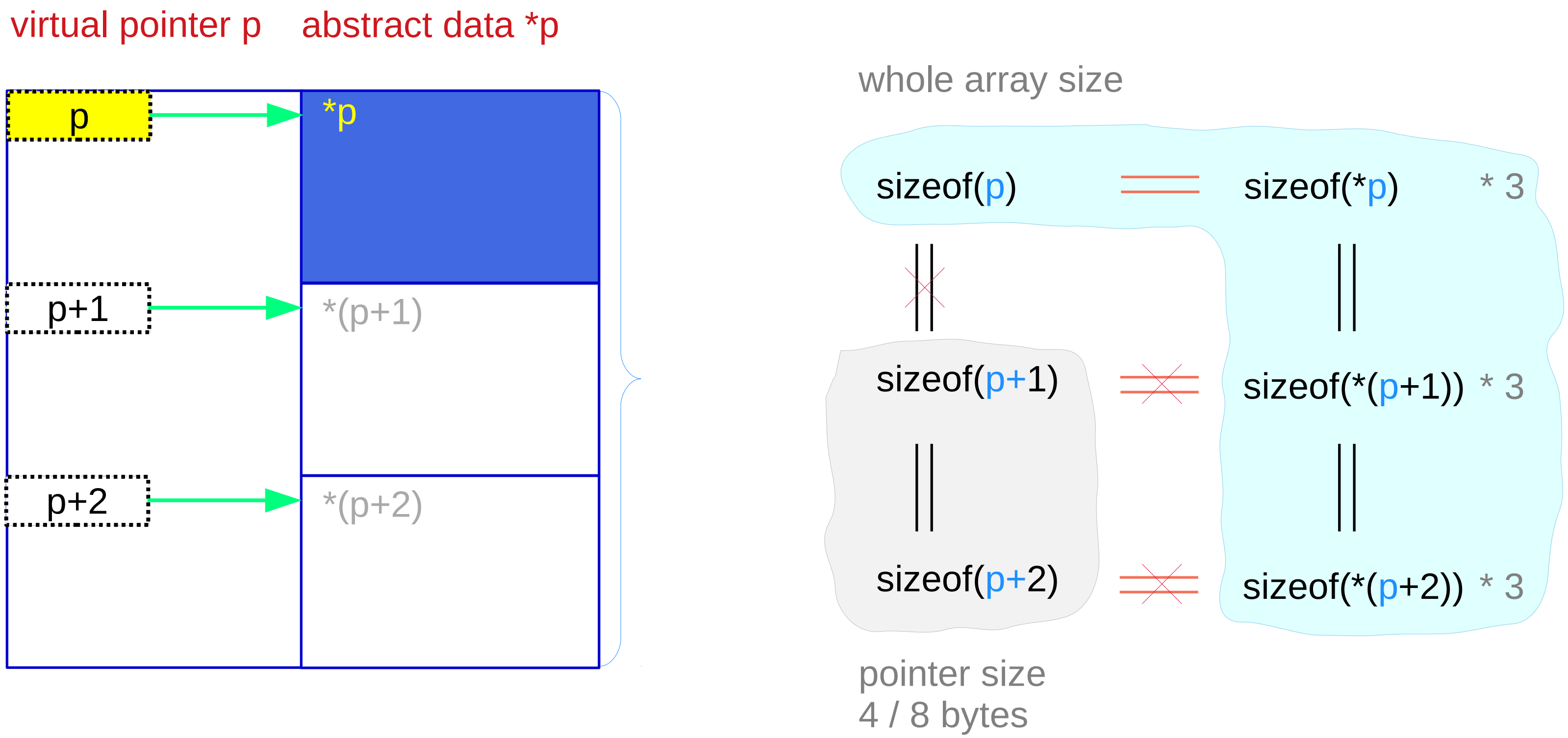


p has an array type (abstract data element)
p is the name of an array
p has the size of the whole array

p also has a pointer type
p has the value of the starting address

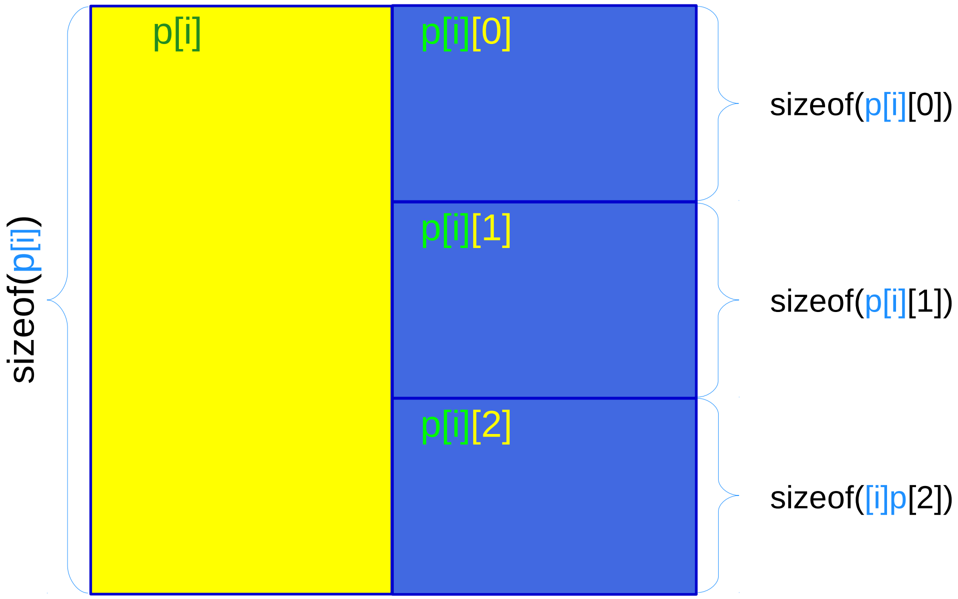
p is a virtual array pointer

Virtual pointer to abstract data

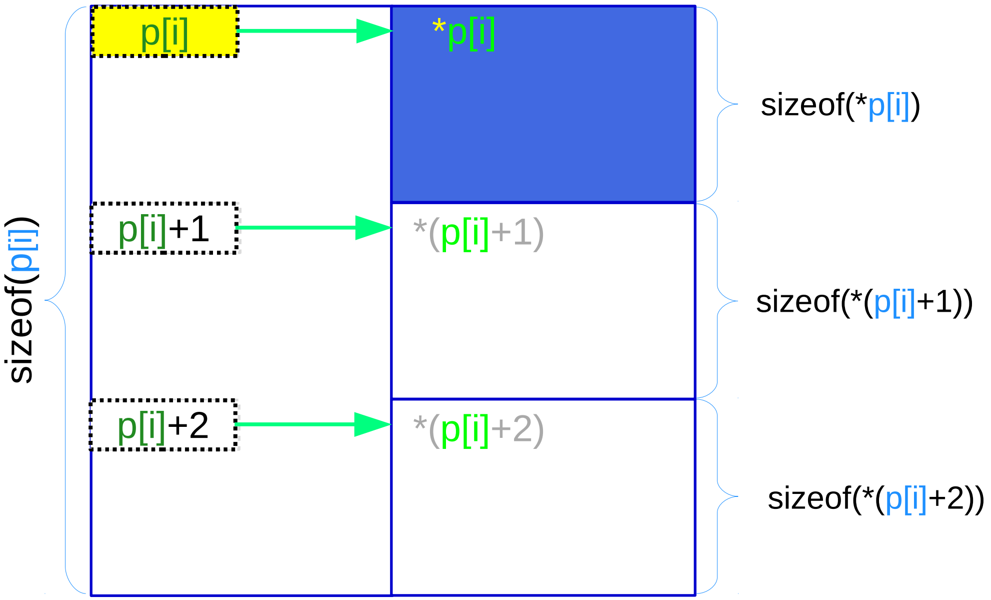


Virtual array pointers in a multi-dimensional array

Abstract data (array) p[i]



Virtual array pointer p[i]



p[i] has an array type (abstract data)
p[i] is the name of an array
p[i] has the size of the whole array

p[i] also has an array pointer type
p[i] has the value of the starting address

p[i] is a virtual array pointer

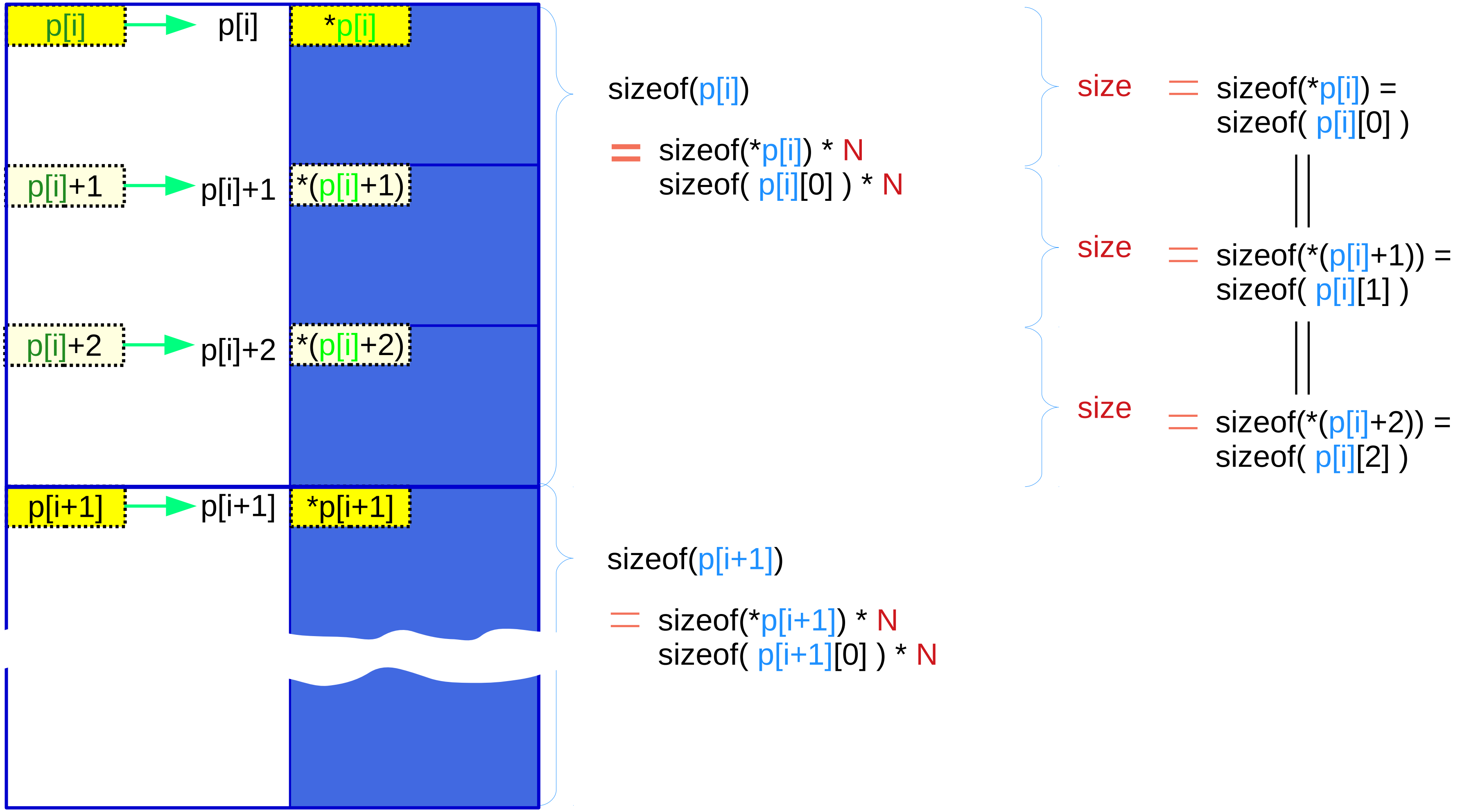
Virtual pointers to a sub array

p[i] :: T *

array pointer type

***p[i], *p[i+1] :: T**

array type



sizeof(p[i])

= sizeof(*p[i]) * N
sizeof(p[i][0]) * N

size = sizeof(*p[i]) =
sizeof(p[i][0])

||

size = sizeof(*(p[i]+1)) =
sizeof(p[i][1])

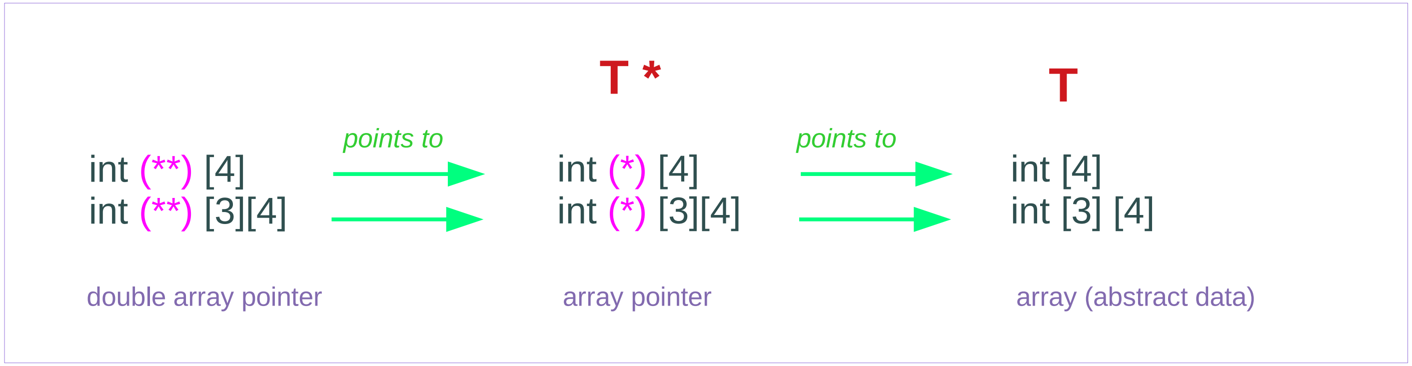
||

size = sizeof(*(p[i]+2)) =
sizeof(p[i][2])

sizeof(p[i+1])

= sizeof(*p[i+1]) * N
sizeof(p[i+1][0]) * N

Array pointer types



int (**) [4]
int (**) [3][4]

double array pointer

points to

T *

int (*) [4]
int (*) [3][4]

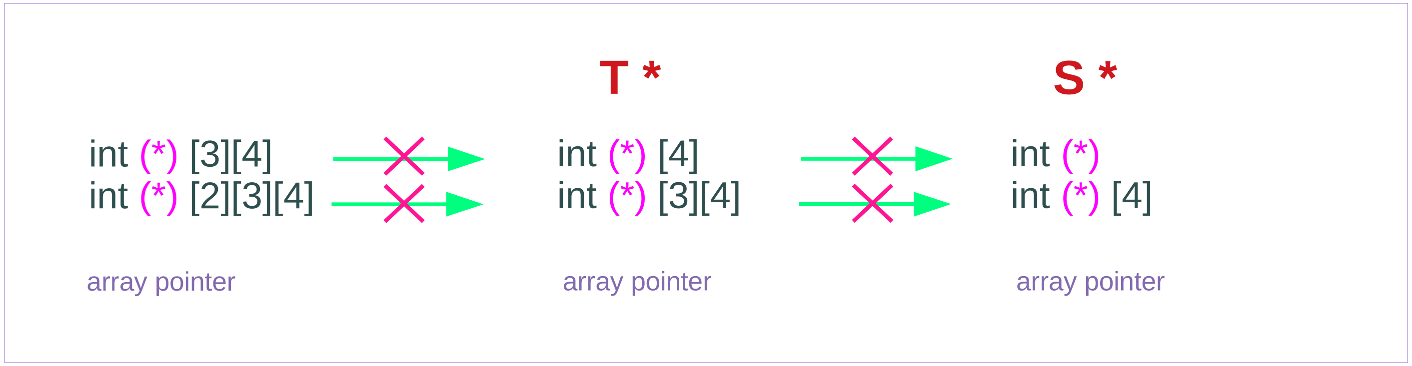
array pointer

points to

T

int [4]
int [3] [4]

array (abstract data)



int (*) [3][4]
int (*) [2][3][4]

array pointer

T *

int (*) [4]
int (*) [3][4]

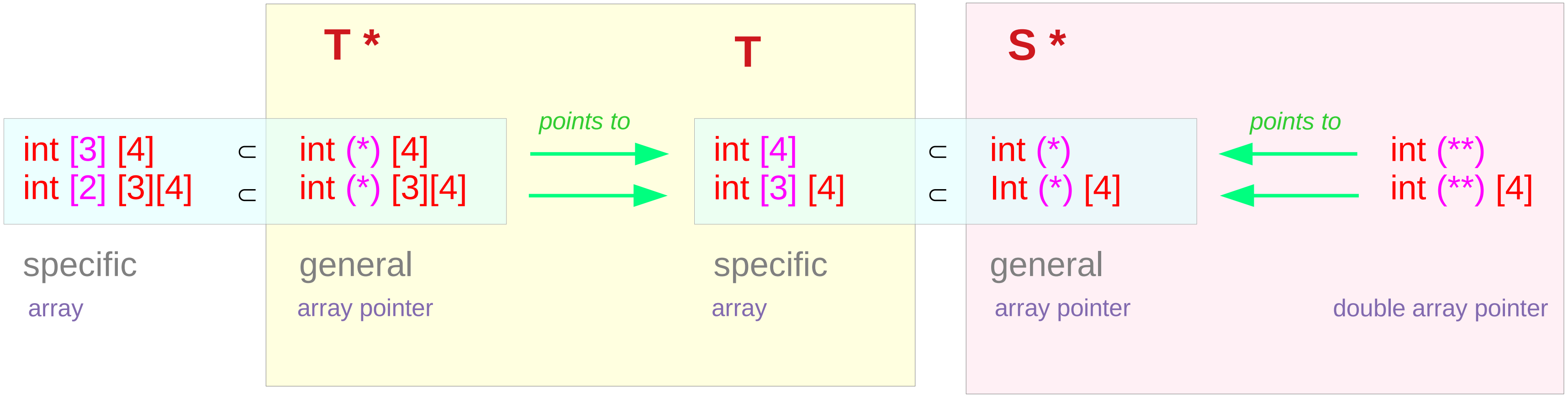
array pointer

S *

int (*)
int (*) [4]

array pointer

Array pointer types point to array types



T *

T

S *

int [3] [4] ⊂ int (*) [4]
int [2] [3][4] ⊂ int (*) [3][4]

points to

int [4] ⊂ int (*)
int [3] [4] ⊂ Int (*) [4]

points to

int (**)
int (**) [4]

specific
array

general
array pointer

specific
array

general
array pointer

double array pointer

Virtual array pointers pointing to arrays

```
typedef int (*T1) [4];          typedef int T2[4];
typedef int (*T1) [3][4];       typedef int T2[3][4];

      int (*) [4]                     int [4]
      int (*) [3][4]                  int [3] [4]
      general                         specific
```

```
T1  a;
T2  b;
```

T1 is a pointer type
T2 is an array type
T1 has one more dimension than T2

```
 a  = &b;
*a  =  b;
```

a references **b**

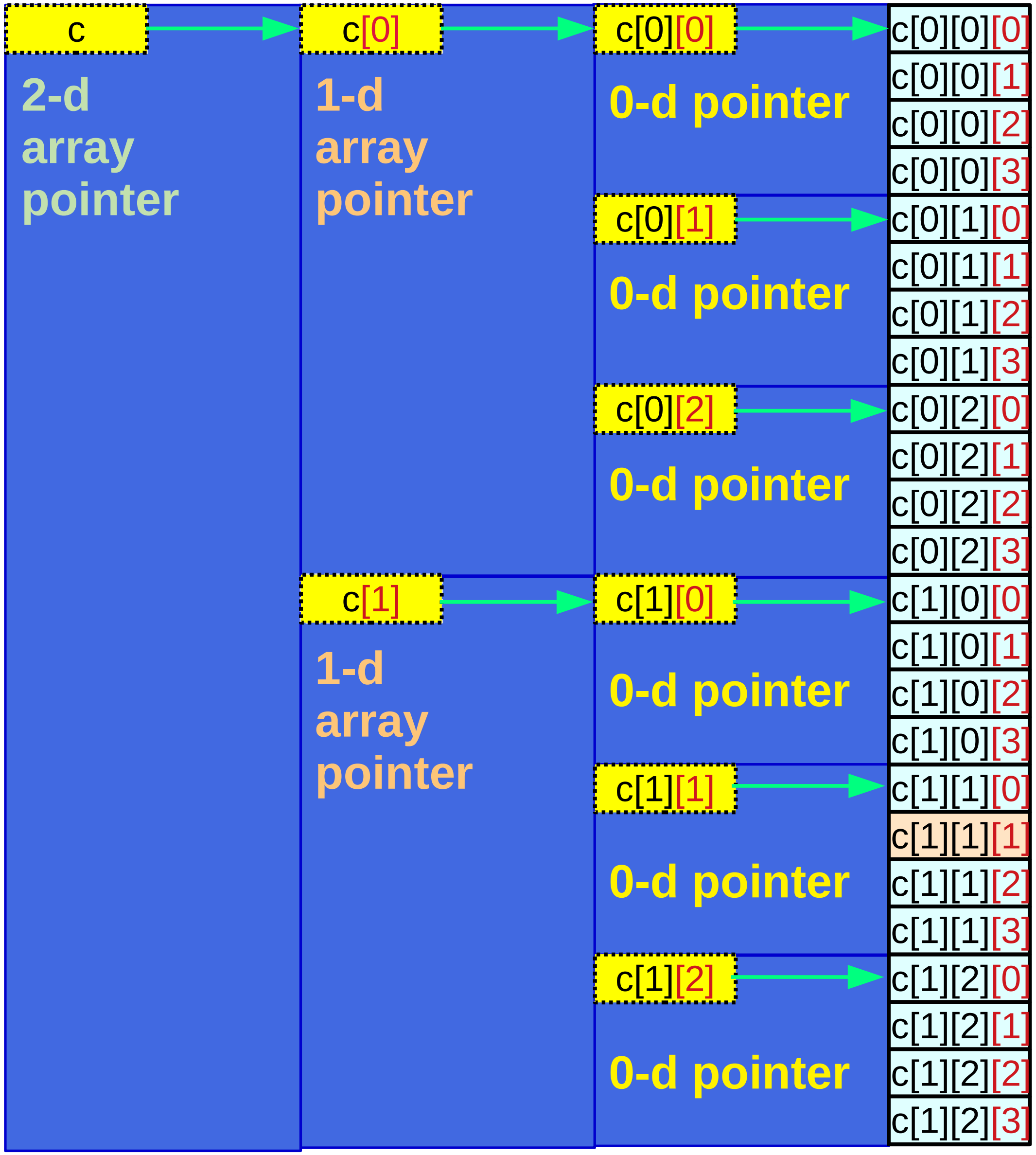
b is the dereference of **a**

3-d array structure – pointer representation

int c[2][3][4];

((*(c +i) +j) +k)

- Hierarchical
- Nested Structure
- Virtual Array Pointers to abstract data (subarrays)
- Contiguous and Linear Data Layout
- Row Major Order



3-d array structure – abstract data representation

int c[2][3][4];

((c [i])[j])[k]

- Hierarchical
- Nested Structure
- Virtual Array Pointers to abstract data (subarrays)
- Contiguous and Linear Data Layout
- Row Major Order

c
3-d array name

c[0]
2-d array name

c[1]
2-d array name

c[0][0]
1-d array name

c[0][1]
1-d array name

c[0][2]
1-d array name

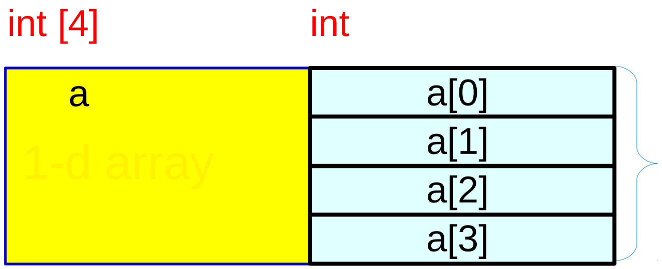
c[1][0]
1-d array name

c[1][1]
1-d array name

c[1][2]
1-d array name

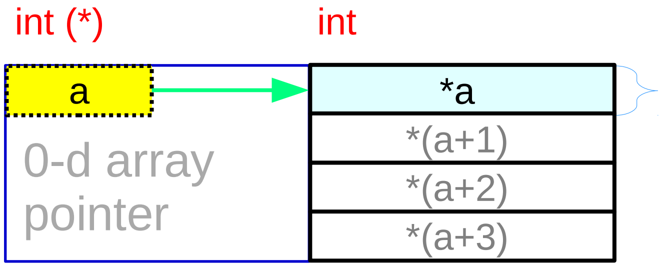
c[0][0][0]
c[0][0][1]
c[0][0][2]
c[0][0][3]
c[0][1][0]
c[0][1][1]
c[0][1][2]
c[0][1][3]
c[0][2][0]
c[0][2][1]
c[0][2][2]
c[0][2][3]
c[1][0][0]
c[1][0][1]
c[1][0][2]
c[1][0][3]
c[1][1][0]
c[1][1][1]
c[1][1][2]
c[1][1][3]
c[1][2][0]
c[1][2][1]
c[1][2][2]
c[1][2][3]

Array **a** and pointer **a**



1-d array a specific array type

sizeof(a)



pointer a general pointer type

sizeof(a) = sizeof(*a) * 4

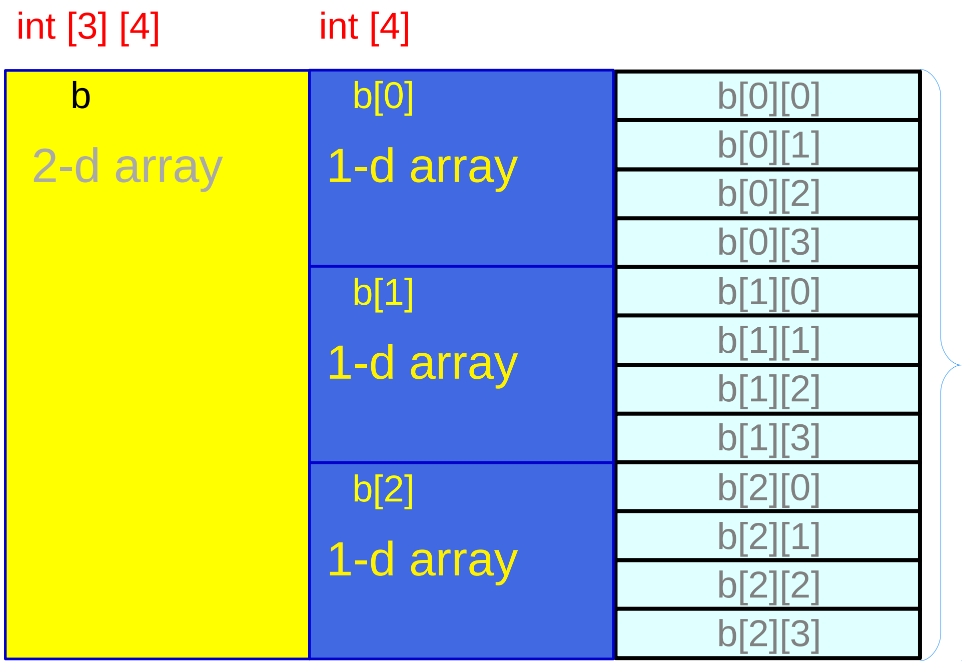
a is the name of a 1-d array
a also has a pointer type
a has the size of the array
a has the value of the starting address

a is a virtual array pointer

Array **b** and pointer **b**

2-d array b specific array type

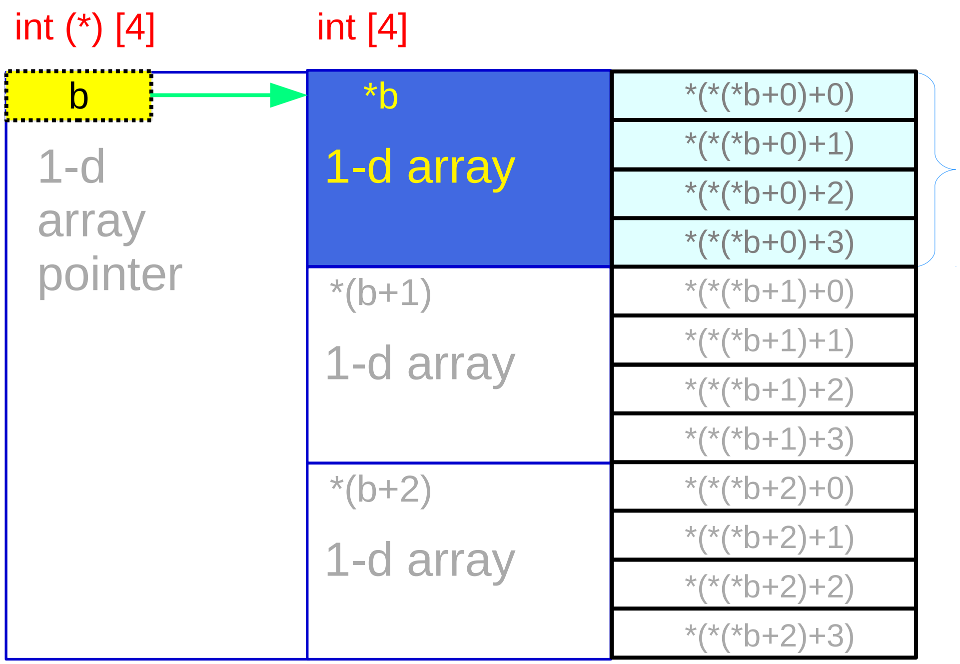
sizeof(b)



b is the name of a 2-d array
b has the size of the array

1-d array pointer b general pointer type

sizeof(b) = sizeof(*b) * 3



b also has a 1-d array pointer type
b has the value of the starting address

b is a virtual array pointer

Array **c**

3-d array c

specific array type

sizeof(c)

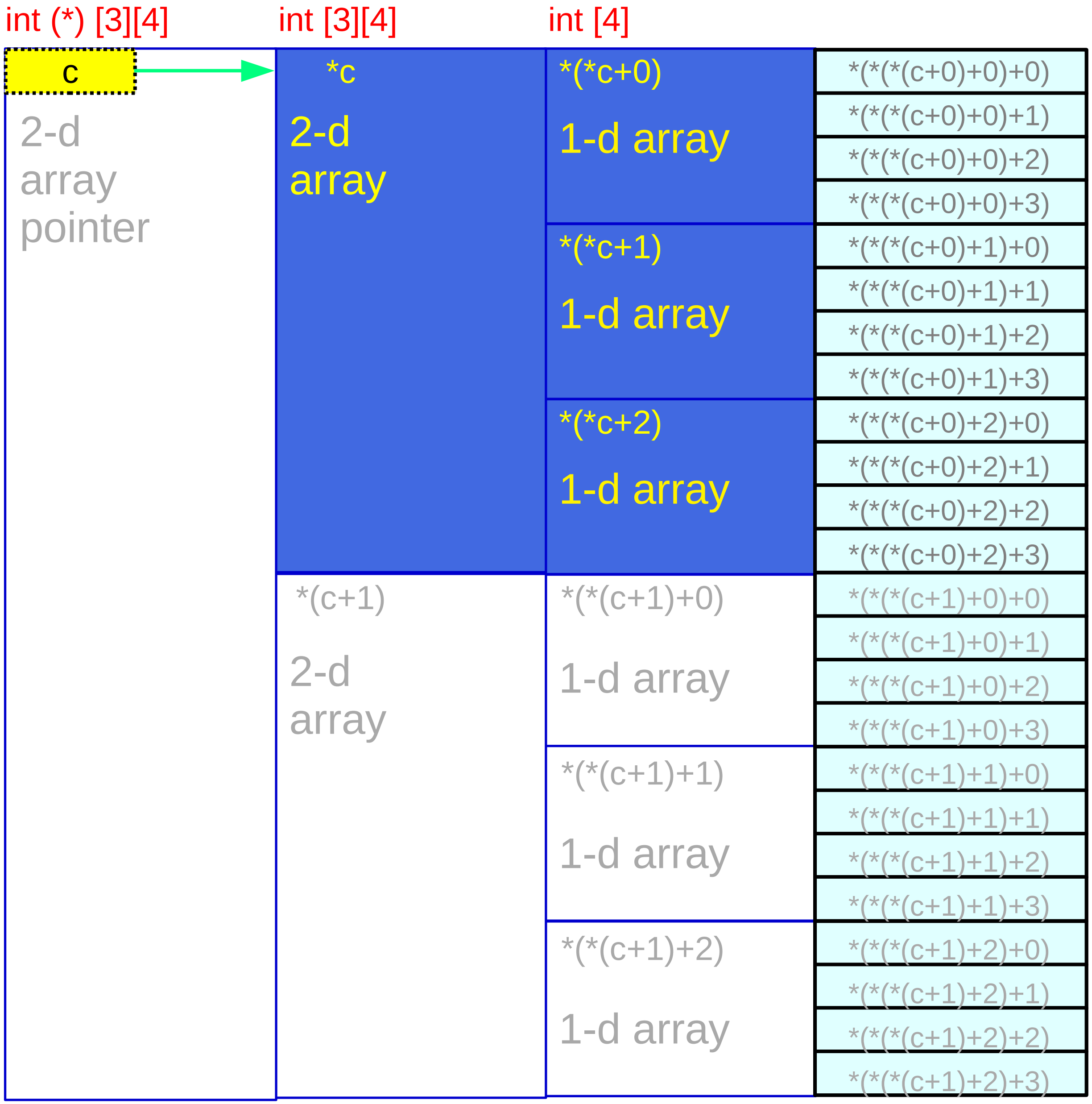
c is the name of a 3-d array
c has the size of the array

Pointer **c**

2-d array pointer c

general pointer type

sizeof(c) = sizeof(*c) * 2



int (*) [3][4]

c

2-d
array
pointer

int [3][4]

*c

2-d
array

*(c+1)

2-d
array

int [4]

*(*c+0)

1-d array

*(*c+1)

1-d array

*(*c+2)

1-d array

((c+1)+0)

1-d array

((c+1)+1)

1-d array

((c+1)+2)

1-d array

((*(c+0)+0)+0)
((*(c+0)+0)+1)
((*(c+0)+0)+2)
((*(c+0)+0)+3)
((*(c+0)+1)+0)
((*(c+0)+1)+1)
((*(c+0)+1)+2)
((*(c+0)+1)+3)
((*(c+0)+2)+0)
((*(c+0)+2)+1)
((*(c+0)+2)+2)
((*(c+0)+2)+3)
((*(c+1)+0)+0)
((*(c+1)+0)+1)
((*(c+1)+0)+2)
((*(c+1)+0)+3)
((*(c+1)+1)+0)
((*(c+1)+1)+1)
((*(c+1)+1)+2)
((*(c+1)+1)+3)
((*(c+1)+2)+0)
((*(c+1)+2)+1)
((*(c+1)+2)+2)
((*(c+1)+2)+3)

c also has a 2-d array pointer type
c has the value of the starting address

c is a virtual array pointer

Types of virtual array pointers in a 3-d array

`int c[2][3][4];`

c[i][j][k]	int	int
c[i][j] [k]	int [4] [k]	int (*) [k]
c[i] [j] [k]	int [3][4] [j] [k]	int (*)[4] [j] [k]
c [i] [j] [k]	int [2][3][4] [i] [j] [k]	int (*)[3][4] [i] [j] [k]
	array type (name)	**array pointer type**

Sizes of virtual array pointers in a 3-d array

```
int c[2][3][4];
```

$\text{sizeof}(c[i][j][k]) = \text{sizeof}(\text{int})$

$\text{sizeof}(c[i][j])\underset{[k]}{} = \text{sizeof}(\text{int}) * \underset{[k]}{4}$

$\text{sizeof}(c[i])\underset{[j]\quad[k]}{} = \text{sizeof}(\text{int}) * \underset{[j]}{3} * \underset{[k]}{4}$

$\text{sizeof}(c)\underset{[i]\quad[j]\quad[k]}{} = \text{sizeof}(\text{int}) * \underset{[i]}{2} * \underset{[j]}{3} * \underset{[k]}{4}$

Address values of virtual array pointers in a 3-d array

```
int c[2][3][4];
```

c[i][j][k] = &c[i][j][k]

c[i][j]+k = &c[i][j][0] + k * sizeof(c[i][j][k])

sizeof(*c[i][j])
= sizeof(c[i][j][0]) = sizeof(int)

c[i]+j = &c[i][0][0] + j * sizeof(c[i][j])

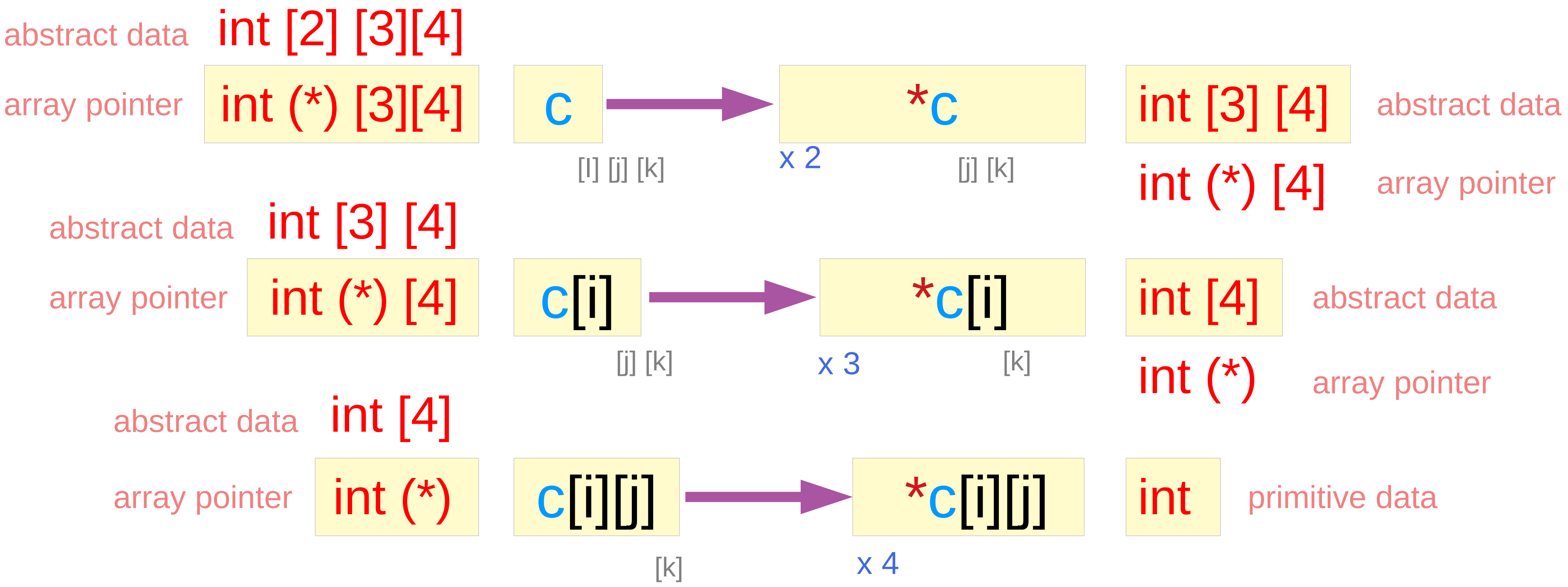
sizeof(*c[i])
= sizeof(c[i][0]) = sizeof(int) * $\underset{[k]}{4}$

c+i = &c[0][0][0] + i * sizeof(c[i])

sizeof(*c)
= sizeof(c[0]) = sizeof(int) * $\underset{[j]}{3}$ * $\underset{[k]}{4}$

Types in a multi-dimensional **3-d** array

int c [2][3][4];



abstract data int [2] [3][4]

array pointer int (*) [3][4]

c

[I] [j] [k]

*c

x 2

[j] [k]

int [3] [4] abstract data

int (*) [4] array pointer

abstract data int [3] [4]

array pointer int (*) [4]

c[i]

[j] [k]

*c[i]

x 3

[k]

int [4] abstract data

int (*) array pointer

abstract data int [4]

array pointer int (*)

c[i][j]

[k]

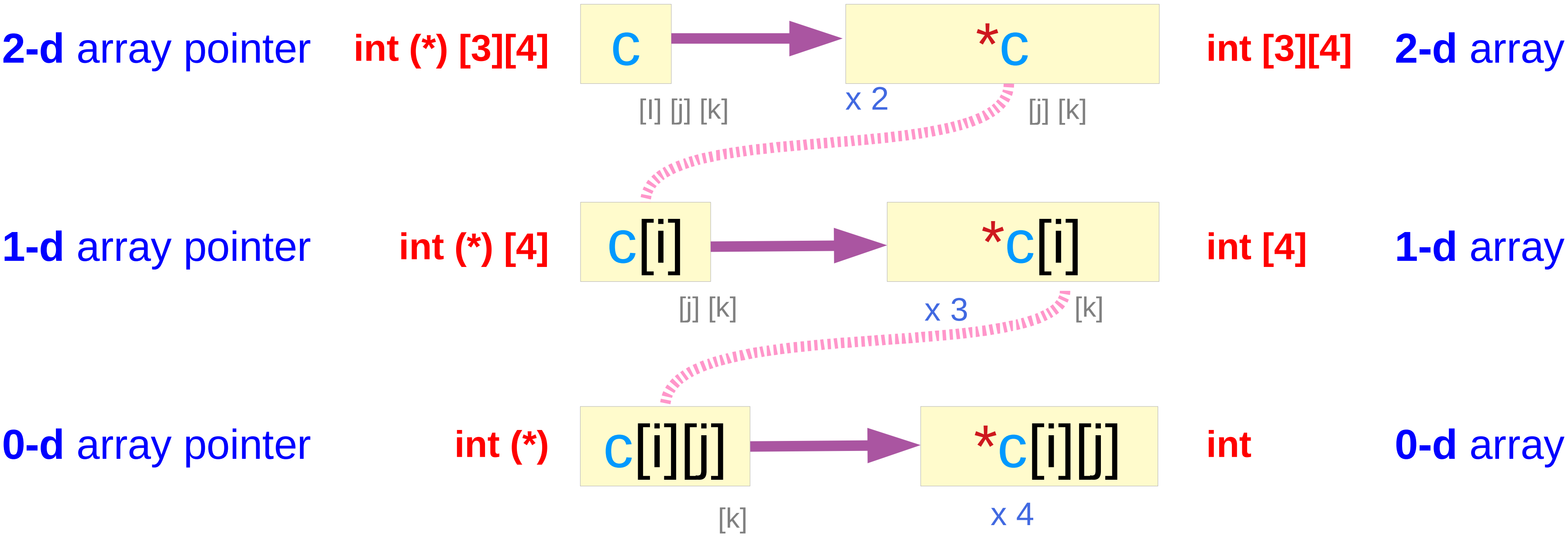
*c[i][j]

x 4

int primitive data

Virtual array pointers and abstract data

```
int      c [2][3][4];
```



2-d array pointer **int (*) [3][4]** c → *c **int [3][4]** **2-d** array

[I] [j] [k] x 2 [j] [k]

1-d array pointer **int (*) [4]** c[i] → *c[i] **int [4]** **1-d** array

[j] [k] x 3 [k]

0-d array pointer **int (*)** c[i][j] → *c[i][j] **int** **0-d** array

[k] x 4

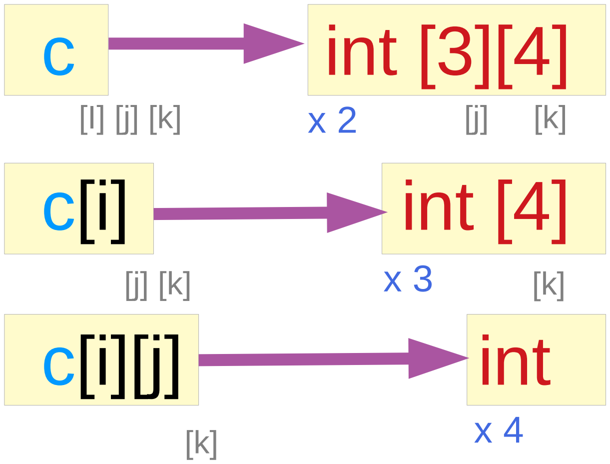
all these pointers are virtual, and take no actual memory locations

exploiting the **contiguity** of allocated memory locations

Abstract data sizes

int c [2][3][4];

the size of a pointer type is fixed
Here, the sizes of virtual pointers are shown
i.e, the sizes of different abstract data types


c → int [3][4]
[I] [j] [k] x 2 [j] [k]
c[i] → int [4]
[j] [k] x 3 [k]
c[i][j] → int
[k] x 4

sizeof(c) = sizeof(int [**2**][3][4])
sizeof(*c) = sizeof(int [3][4])

sizeof(c[i]) = sizeof(int [**3**][4])
sizeof(*c[i]) = sizeof(int [4])

sizeof(c[i][j]) = sizeof(int [**4**])
sizeof(*c[i][j]) = sizeof(int)

all are sizes of arrays

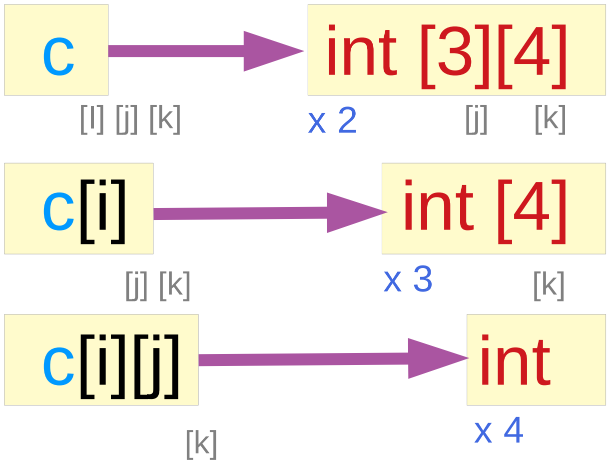
c, c[i], c[i][j] are virtual array pointers
[k]
and they are also abstract data (arrays)

when sizes are considered,
view them as abstract data (arrays)

Virtual array pointer sizes and abstract data sizes

int c [2][3][4];

size of a virtual array pointer = size of the pointed abstract data type * the number of such types



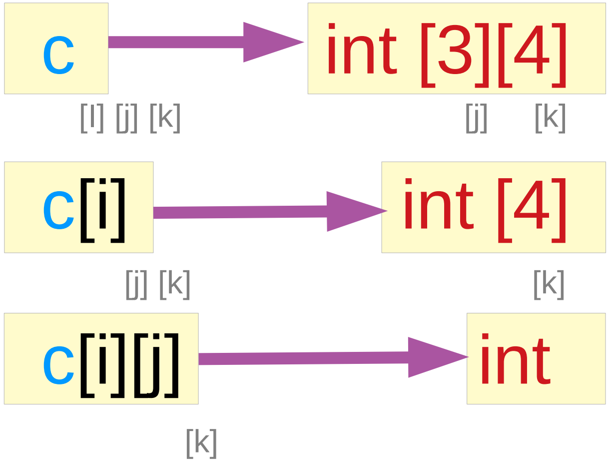
sizeof(c) = sizeof(*c) * **2**

sizeof(c[i]) = sizeof(*c[i]) * **3**

sizeof(c[i][j]) = sizeof(*c[i][j]) * **4**

Sizes of array pointer types

int c [2][3][4];



not real array pointers
virtual array pointers

c int (*)[3][4] = sizeof(c)
sizeof(int (*) [3][4]) = pointer size ≠ sizeof(c)

c[i] int (*) [4] = sizeof(c[i])
sizeof(int (*) [4]) = pointer size ≠ sizeof(c[i])

c[i][j] int [4] = sizeof(c[i][j])
sizeof(int [4]) = pointer size ≠ sizeof(c[i][j])

4 bytes for 32-bit machines
8 bytes for 64-bit machines

Virtual array pointer increment size

`int c [2][3][4];`

c points to a **2-d** array
increment size: sizeof(int[2][3][4])

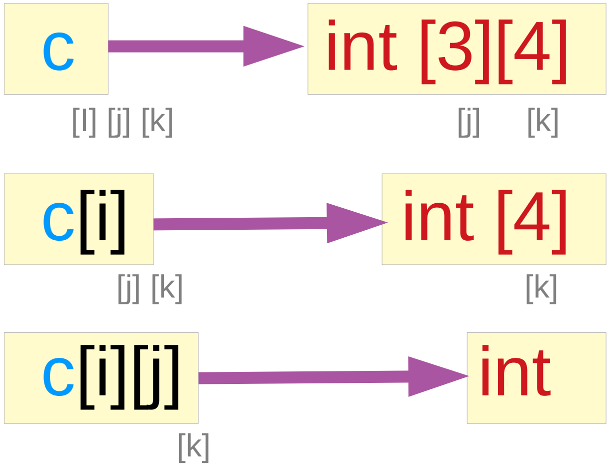
int (*) [3][4]

c[i] points to an **1-d** array
increment size: sizeof(int[3][4])

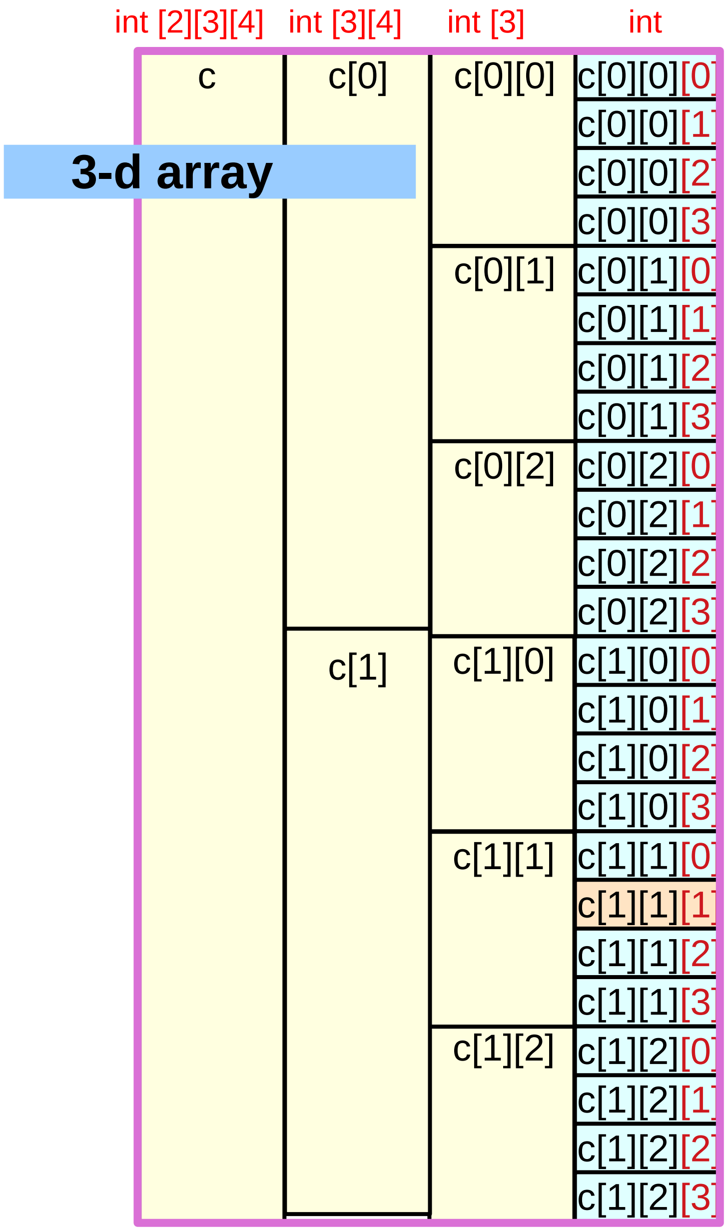
int (*) [4]

c[i][j] points to an integer
increment size: sizeof(int[4])

int (*)

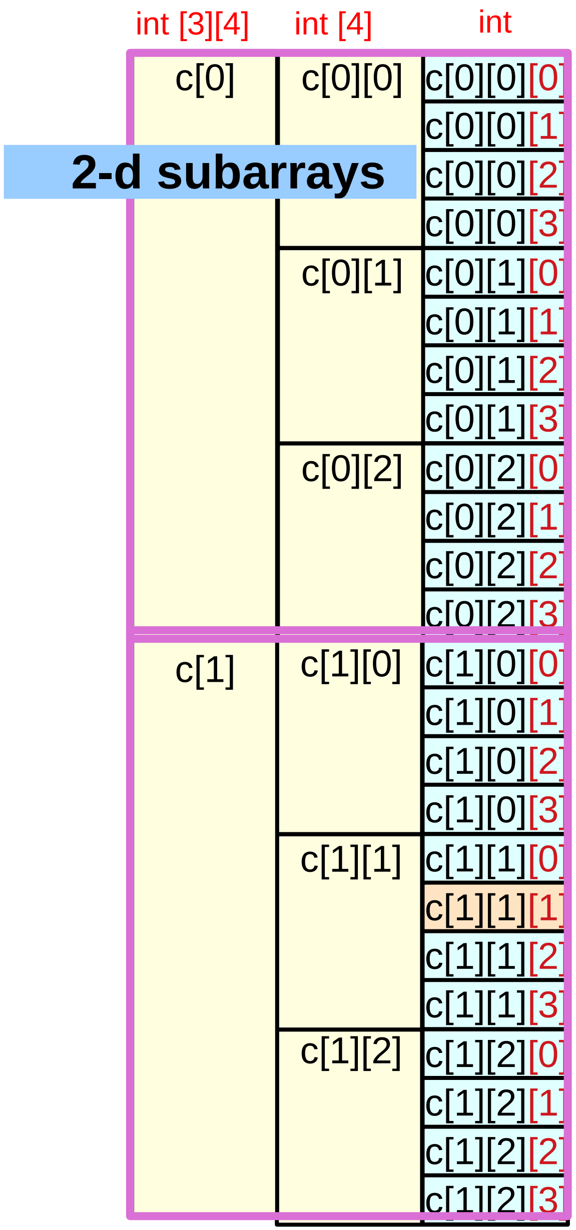


Subarrays in a 3-d array



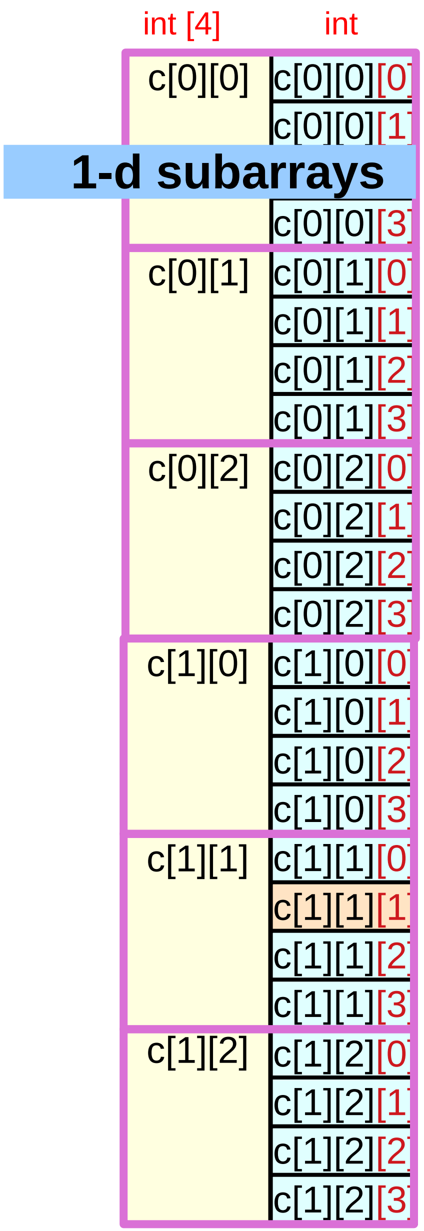
3-d array

int [2][3][4]	int [3][4]	int [3]	int
c	c[0]	c[0][0]	c[0][0][0]
			c[0][0][1]
			c[0][0][2]
			c[0][0][3]
		c[0][1]	c[0][1][0]
			c[0][1][1]
			c[0][1][2]
			c[0][1][3]
		c[0][2]	c[0][2][0]
			c[0][2][1]
			c[0][2][2]
			c[0][2][3]
	c[1]	c[1][0]	c[1][0][0]
			c[1][0][1]
			c[1][0][2]
			c[1][0][3]
		c[1][1]	c[1][1][0]
			c[1][1][1]
			c[1][1][2]
			c[1][1][3]
		c[1][2]	c[1][2][0]
			c[1][2][1]
			c[1][2][2]
			c[1][2][3]



2-d subarrays

int [3][4]	int [4]	int
c[0]	c[0][0]	c[0][0][0]
		c[0][0][1]
		c[0][0][2]
		c[0][0][3]
	c[0][1]	c[0][1][0]
		c[0][1][1]
		c[0][1][2]
		c[0][1][3]
	c[0][2]	c[0][2][0]
		c[0][2][1]
		c[0][2][2]
		c[0][2][3]
c[1]	c[1][0]	c[1][0][0]
		c[1][0][1]
		c[1][0][2]
		c[1][0][3]
	c[1][1]	c[1][1][0]
		c[1][1][1]
		c[1][1][2]
		c[1][1][3]
	c[1][2]	c[1][2][0]
		c[1][2][1]
		c[1][2][2]
		c[1][2][3]

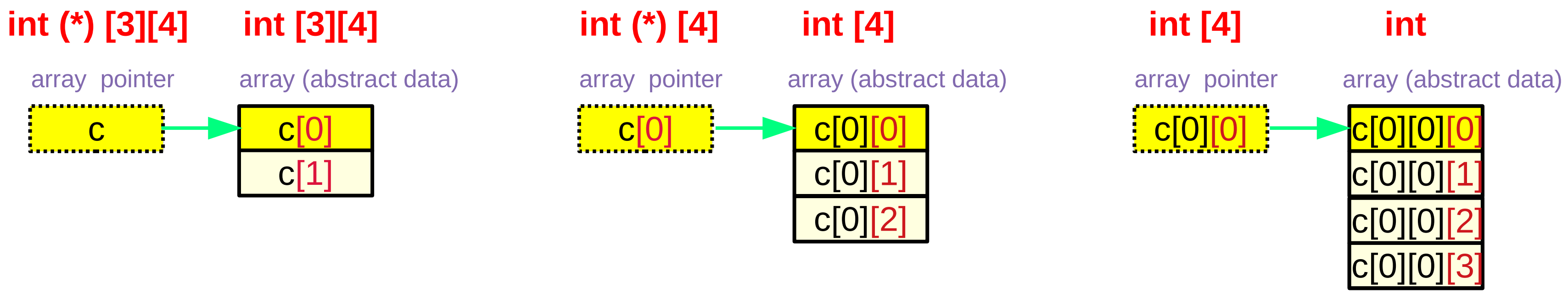


1-d subarrays

int [4]	int
c[0][0]	c[0][0][0]
	c[0][0][1]
	c[0][0][2]
	c[0][0][3]
c[0][1]	c[0][1][0]
	c[0][1][1]
	c[0][1][2]
	c[0][1][3]
c[0][2]	c[0][2][0]
	c[0][2][1]
	c[0][2][2]
	c[0][2][3]
c[1][0]	c[1][0][0]
	c[1][0][1]
	c[1][0][2]
	c[1][0][3]
c[1][1]	c[1][1][0]
	c[1][1][1]
	c[1][1][2]
	c[1][1][3]
c[1][2]	c[1][2][0]
	c[1][2][1]
	c[1][2][2]
	c[1][2][3]

Virtual array pointer c, c[0], c[0][0] – types and sizes

Types – array pointers



int (*) [3][4] **int [3][4]**

array pointer array (abstract data)

c → c[0] c[1]

int (*) [4] **int [4]**

array pointer array (abstract data)

c[0] → c[0][0] c[0][1] c[0][2]

int [4] **int**

array pointer array (abstract data)

c[0][0] → c[0][0][0] c[0][0][1] c[0][0][2] c[0][0][3]

Sizes – abstract data

sizeof(**c**)
sizeof(**int [2][3][4]**)
sizeof(**int**) * **2** * **3** * **4**

sizeof(**c[0]**)
sizeof(**int [3][4]**)
sizeof(**int**) * **3** * **4**

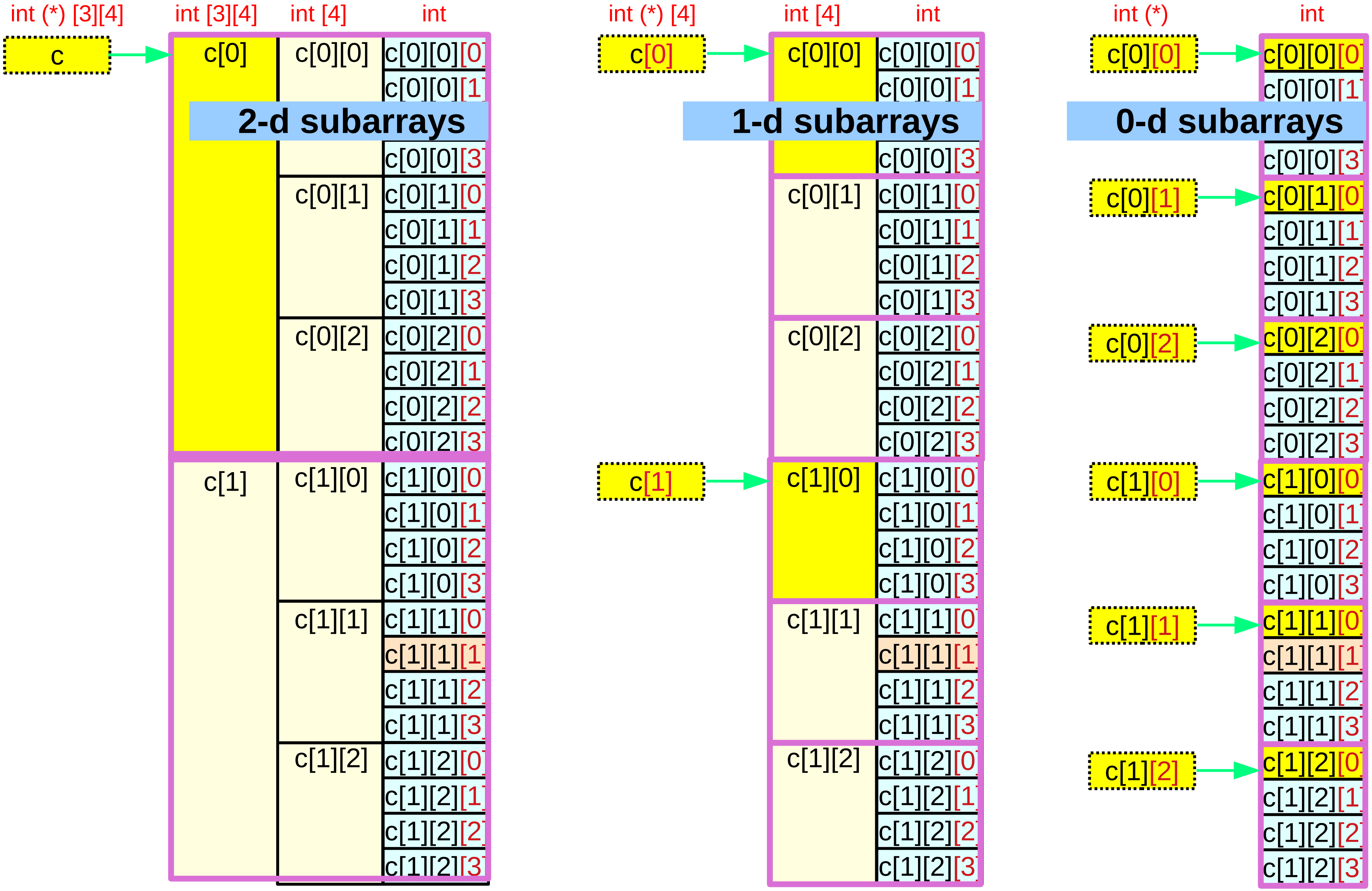
sizeof(**c[0][0]**)
sizeof(**int [4]**)
sizeof(**int**) * **4**

sizeof(**int [2][3][4]**) = 96
sizeof(**int (*)[3][4]**) = 4 / 8

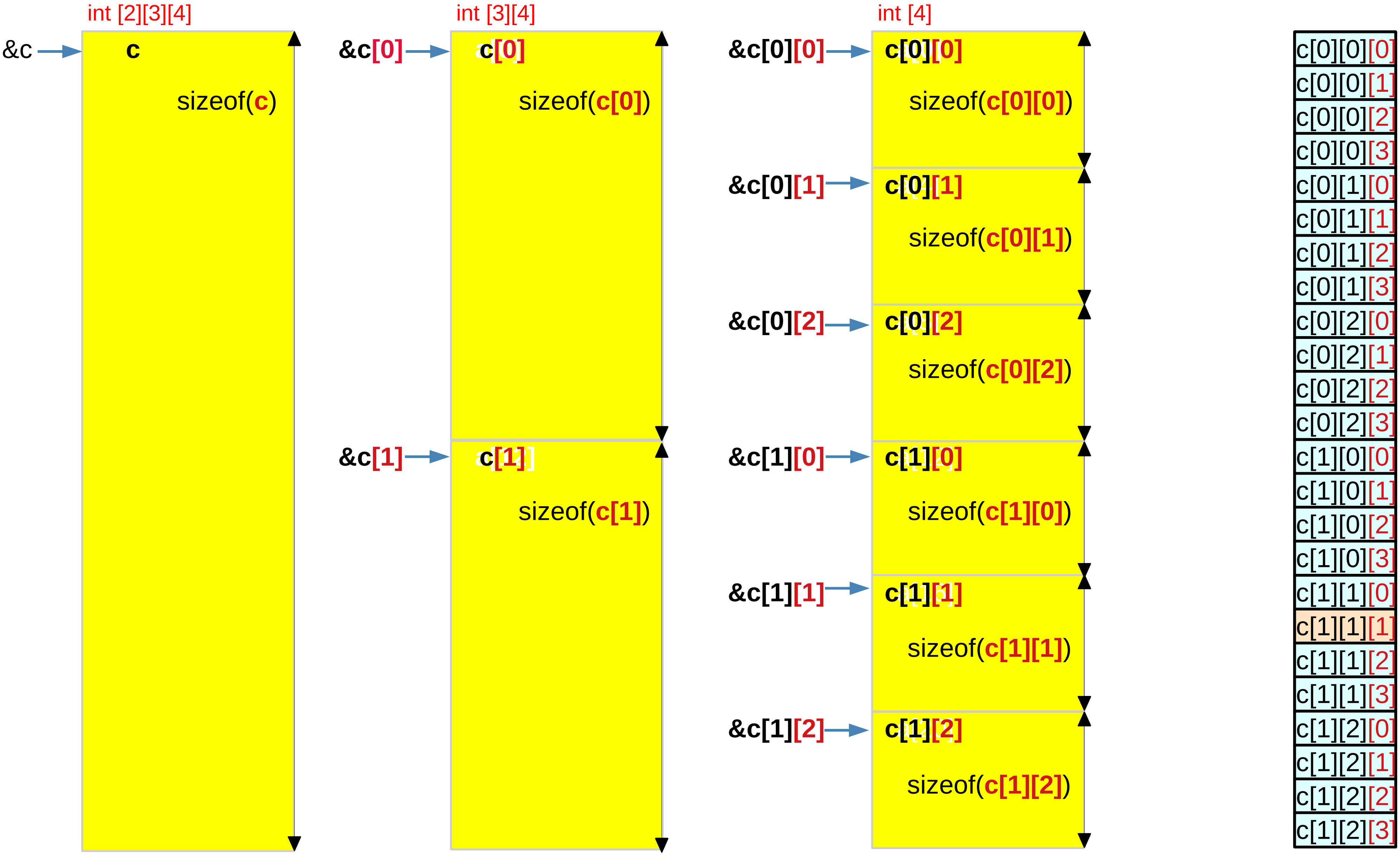
sizeof(**int [3][4]**) = 48
sizeof(**int (*)[4]**) = 4 / 8

sizeof(**int [4]**) = 16
sizeof(**int (*)**) = 4 / 8

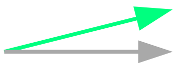
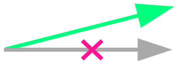
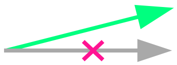
Pointers to subarrays in a 3-d array



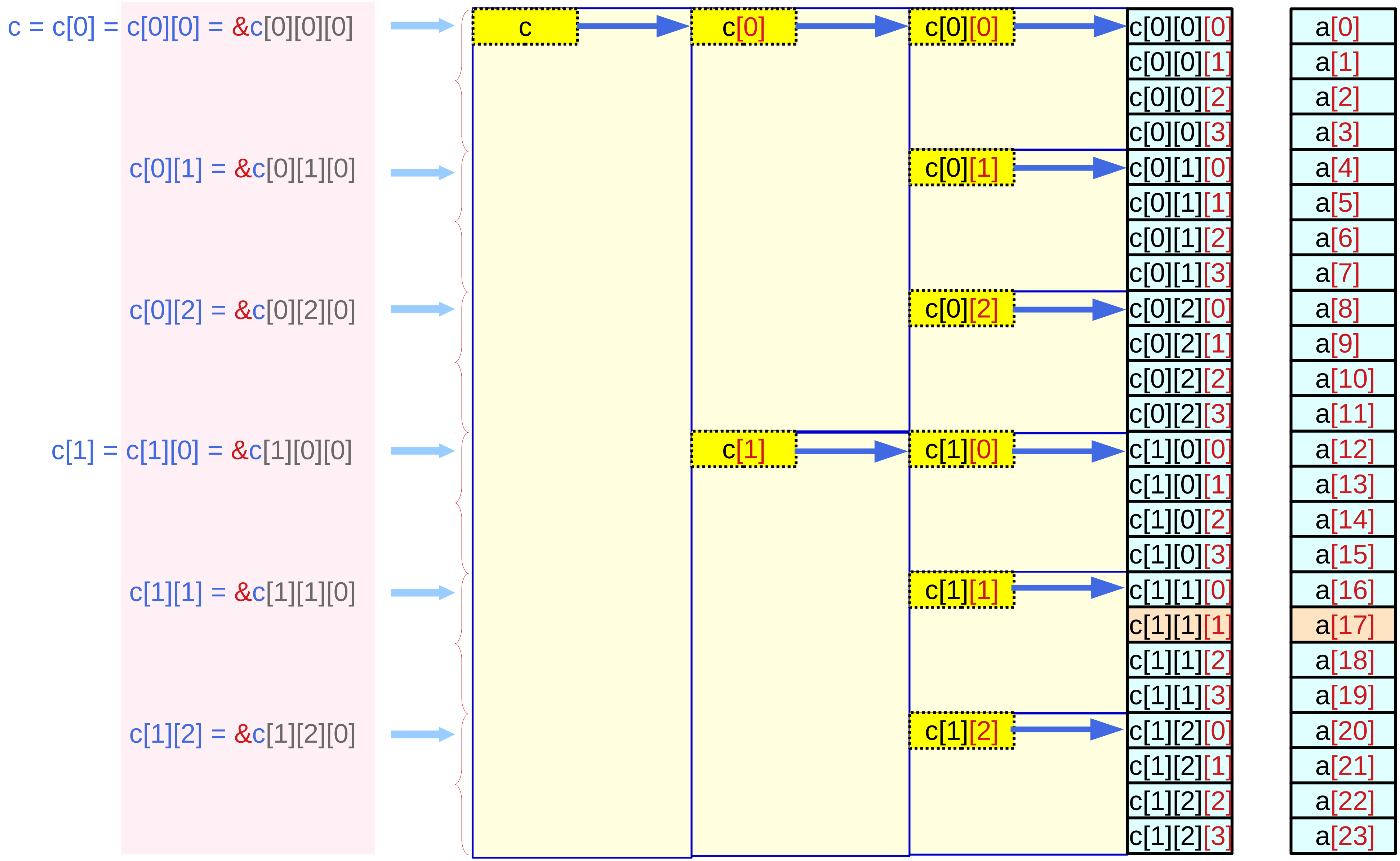
Abstract Data c, c[i], c[i][j] – start addresses and sizes



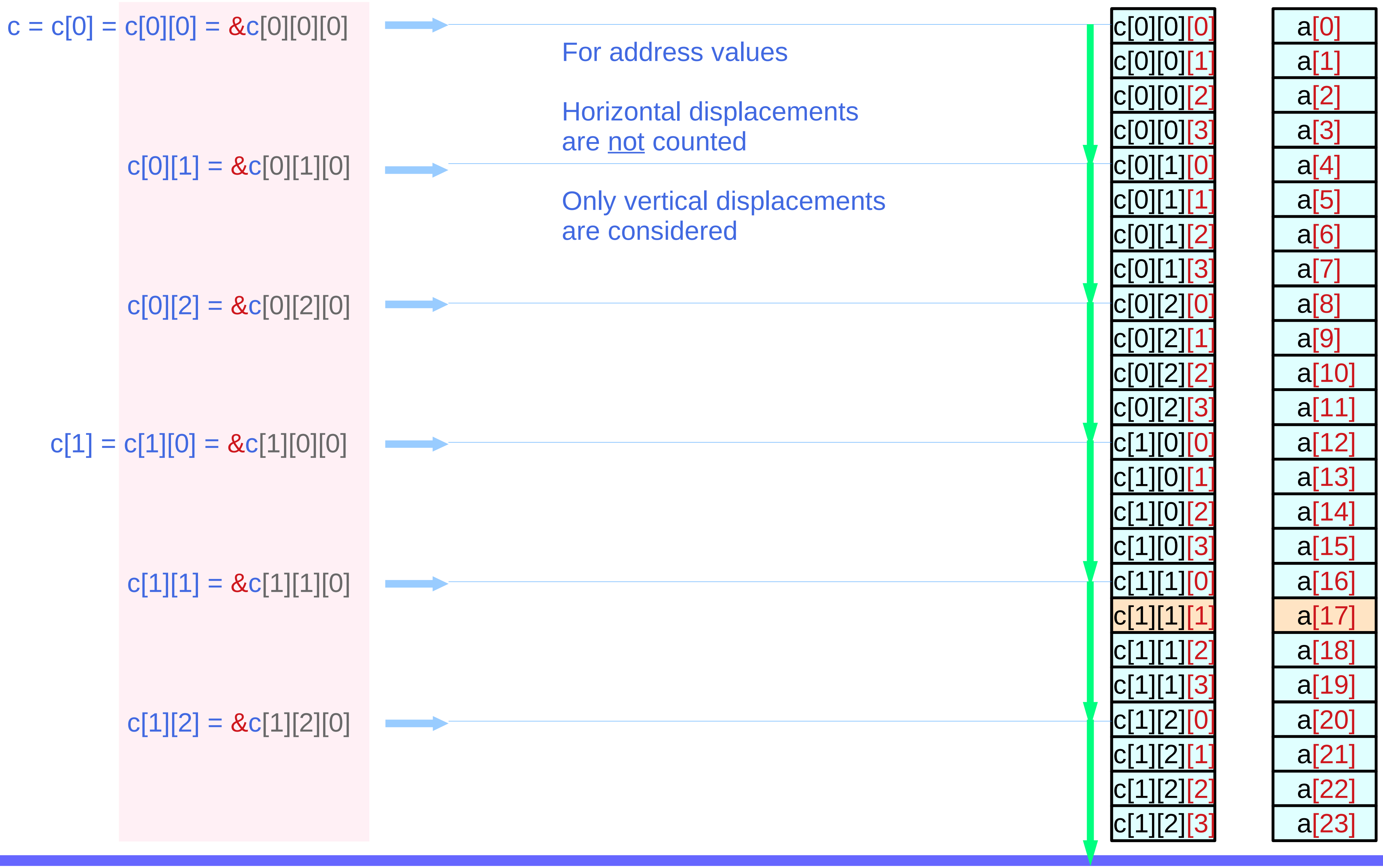
Virtual array pointers – types, sizes, and values

int c[2][3][4];	**c[i][j]**		**c[i][j][0]**	
type	int **[4]** int (*)		int int	• abstract data type • array pointer type
size	sizeof(c[i][j]) =		sizeof(c[i][j][0]) * **4**	= sizeof(int) * **4**
value (address)	c[i][j] =		&c[i][j][0]	
int c[2][3][4];	**c[i]**		**c[i][0]**	
type	int **[3]**[4] int (*)[4]		int [4] int (*)	• abstract data type • array pointer type
size	sizeof(c[i]) =		sizeof(c[i][0]) * **3**	= sizeof(int) * **4** * **3**
value (address)	c[i] =		&c[i][0][0]	
int c[2][3][4];	**c**		**c[0]**	
type	int **[2]**[3][4] int (*)[3][4]		int [3][4] int (*)[4]	• abstract data type • array pointer type
size	sizeof(c) =		sizeof(c[0]) * **2**	= sizeof(int) * **4** * **3** * **2**
value (address)	c =		&c[0][0][0]	

Virtual array pointer c, c[i], c[i][j] – values (addresses)



Virtual array pointer c, c[i], c[i][j] – vertical displacement



c = c[0] = c[0][0] = &c[0][0][0]

c[0][1] = &c[0][1][0]

c[0][2] = &c[0][2][0]

c[1] = c[1][0] = &c[1][0][0]

c[1][1] = &c[1][1][0]

c[1][2] = &c[1][2][0]

For address values

Horizontal displacements
are not counted

Only vertical displacements
are considered

c[0][0][0]	a[0]
c[0][0][1]	a[1]
c[0][0][2]	a[2]
c[0][0][3]	a[3]
c[0][1][0]	a[4]
c[0][1][1]	a[5]
c[0][1][2]	a[6]
c[0][1][3]	a[7]
c[0][2][0]	a[8]
c[0][2][1]	a[9]
c[0][2][2]	a[10]
c[0][2][3]	a[11]
c[1][0][0]	a[12]
c[1][0][1]	a[13]
c[1][0][2]	a[14]
c[1][0][3]	a[15]
c[1][1][0]	a[16]
c[1][1][1]	a[17]
c[1][1][2]	a[18]
c[1][1][3]	a[19]
c[1][2][0]	a[20]
c[1][2][1]	a[21]
c[1][2][2]	a[22]
c[1][2][3]	a[23]

Virtual array pointer c, c[i], c[i][j] – values and types

c = c[0] = c[0][0] = &c[0][0][0]

means

value(c) = value(c[0]) = value(c[0][0]) = value(&c[0][0][0])
type(c) ≠ type(c[0]) ≠ type(c[0][0]) = type(&c[0][0][0])
int (*) [3][4] int (*) [4] int * int *

c[0][1] = &c[0][1][0]

means

value(c[0][1]) = value(&c[0][1][0])
type(c[0][1]) = type(&c[0][1][0])
int * int *

c[0][2] = &c[0][2][0]

means

value(c[0][2]) = value(&c[0][2][0])
type(c[0][2]) = type(&c[0][2][0])
int * int *

c[1] = c[1][0] = &c[1][0][0]

means

value(c[1]) = value(c[1][0]) = value(&c[1][0][0])
type(c[1]) ≠ type(c[1][0]) = type(&c[1][0][0])
int (*) [4] int * int *

c[1][1] = &c[1][1][0]

means

value(c[1][1]) = value(&c[1][1][0])
type(c[1][1]) = type(&c[1][1][0])
int * int *

c[1][2] = &c[1][2][0]

means

value(c[1][2]) = value(&c[1][2][0])
type(c[1][2]) = type(&c[1][2][0])
int * int *

Summary of virtual array pointers in a 3-d array

c[i] ≡ *(c +i)	c[i][j] ≡ *(c[i] +j)	c[i][j][k]≡ *(c[i][j] +k)
int (*) [3][4] 2-d array pointer c int [2] [3][4] 3-d array name c	int (*) [4] 1-d array pointers c[i] Int [3] [4] 2-d array names c[i]	int (*) 0-d array pointers c[i][j] int [4] 1-d array names c[i][j]
address value c + i &c[0][0][0] + i * sizeof(*c) &c[0][0][0] + i * sizeof(c[0]) &c[0][0][0] + i * 4 * 3 * 4	**address value** c[i] + j &c[i][0][0] + j * sizeof(*c[i]) &c[i][0][0] + j * sizeof(c[i][0]) &c[i][0][0] + j * 4 * 4	**address value** c[i][j] + k &c[i][j][0] + k * sizeof(*c[i][j]) &c[i][j][0] + k * sizeof(c[i][j][0]) &c[i][j][0] + k * 4
leading elements c[0][0][0]	**leading elements** c[0][0][0] c[1][0][0]	**leading elements** c[0][0][0] c[0][1][0] c[0][2][0] c[1][0][0] c[1][1][0] c[1][2][0]

Sub-array properties in multi-dimensional arrays

int c [2][3][4]; → 3-d access c **[i][j][k]**

2-d array pointer	c	int (*) [3][4]
1-d array pointers	c[i]	int (*) [4]
0-d array pointers	c[i][j]	int (*)

Hierarchical sub-arrays in a 3-d array

```
int     c [L][M][N];

        c [i][j][k]
```

left-to-right associativity

c	[i]	[j][k]
c[i]	[j]	[k]
c[i][j]	[k]	

Array Names and Types

Pointers to hierarchical sub-arrays

c	3-d array names	int (*) [M][N]	2-d array pointer
c[i]	2-d array names	int (*) [N]	1-d array pointer
c[i][j]	1-d array names	int (*)	0-d array pointer

Associativity and Equivalence Relations

left-to-right associativity		left-to-right associativity
((c [i])[j])[k]	≡	*(*(*(c +i) +j) +k)

X[n] ≡ *(X+n)

given c[i][j]	c[i][j][k]	≡	*(c[i][j]+k)	for all k
given c[i]	c[i][j]	≡	*(c[i]+j)	for all j
given c	c[i]	≡	*(c+i)	for all i

Requirements for the expression **c[i][j][k]**

for a given c[i][j],	for all k	c[i][j][k] = *(c[i][j]+k)
for a given c[i],	for all j	c[i][j] = *(c[i]+j)
for a given c,	for all i	c[i] = *(c+i)

3 contiguity requirements

for a given c[i][j],	contiguous c[i][j][k]’s
for a given c[i],	contiguous c[i][j]’s
for a given c,	contiguous c[i]’s
for a given subarray pointer	contiguous subarrays

Equivalent requirements for the expression **c[i][j][k]**

for all k c[i][j][k] = *(c[i][j]+k)
for all j c[i][j] = *(c[i]+j)
for all i c[i] = *(c+i)

&c[i][j][k] = c[i][j]+k for all k
&c[i][j] = c[i]+j for all j
&c[i] = c+i for all i

c[i][j][0] = *(c[i][j])
c[i][0] = *(c[i])
c[0] = *(c)

with contiguous subarrays

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

3-d access pattern **c[i][j][k]**

General requirements

```
c [i][j][k]
```

↕

```
&c[i][j][k]  = c[i][j]+k     for all k
&c[i][j]     = c[i]+j        for all j
&c[i]        = c+i           for all i
```

Pointer array approach

```
int** c[2];
int*  b[2*3];
int   c[2*3*4];
```

```
c[i][j][k]    :: int
c[i][j]       :: int *
c[i]          :: int **
c             :: int ***
```

```
c[i]   ⟸   &b[i*3]
b[j]   ⟸   &a[j*4]
```

with contiguous a, b, c

Explicit
Arrays of pointers with
Multiple Indirection

N-dim Array approach

```
int c[2][3][4];
```

```
c[i][j][k] :: int
c[i][j]    :: int [4]
c[i]       :: int [3][4]
c          :: int [2][3][4]
```

```
c[i][j]   ⟸   &c[i][j][0]
c[i]      ⟸   &c[i][0][0]
c         ⟸   &c[0][0][0]
```

with contiguous c

Implicit
Nested
Virtual Array Pointers

3-d access pattern **c[i][j][k]** – array pointer approach

General requirements

c [i][j][k]

&c[i][j][k] = c[i][j]+k   for all k
&c[i][j] = c[i]+j   for all j
&c[i] = c+i   for all i

N-dim array approach

int c[2][3][4];

c[i][j][k] :: int
c[i][j] :: int [4]
c[i] :: int [3][4]
c :: int [2][3][4]

c[i][j] ← &c[i][j][0]
c[i] ← &c[i][0][0]
c ← &c[0][0][0]

with contiguous c

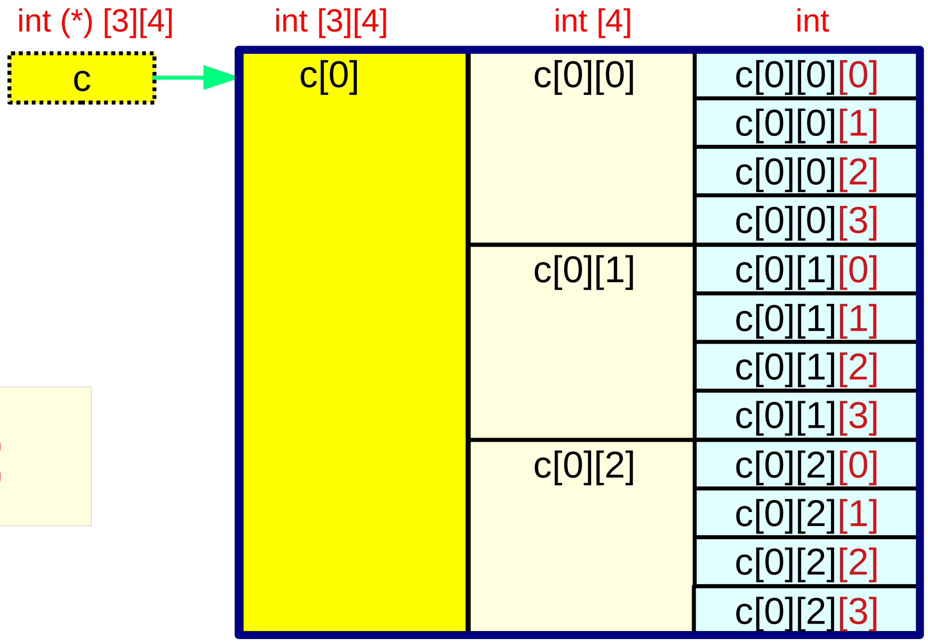
Implicit
Nested
Virtual Array Pointers

Virtual pointers to subarrays in a 3-d array

virtual 2-d array pointer

sizeof(**c**) =
sizeof(**c[0]**) * **2**

int c [2][3][4];

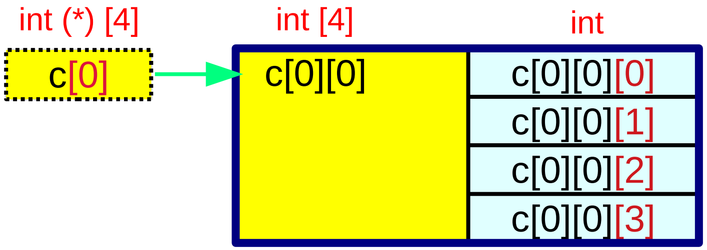


the first 2-d subarray

sizeof(**c[0]**) =
sizeof(**int [3][4]**)

virtual 1-d array pointer

sizeof(**c[0]**) =
sizeof(**c[0][0]**) * **3**

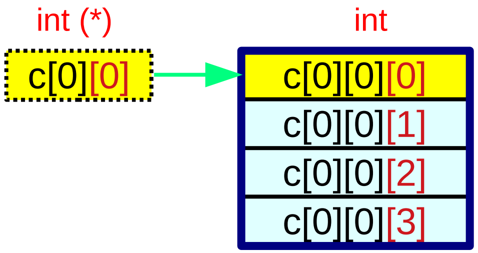


the first 1-d subarray

sizeof(**c[0][0]**) =
sizeof(**int [4]**)

virtual 0-d array pointer

sizeof(**c[0][0]**) =
sizeof(**c[0][0][0]**) * **4**



the first 0-d subarray

sizeof(**c[0][0][0]**) =
sizeof(**int**)

Virtual pointer values in a 3-d array

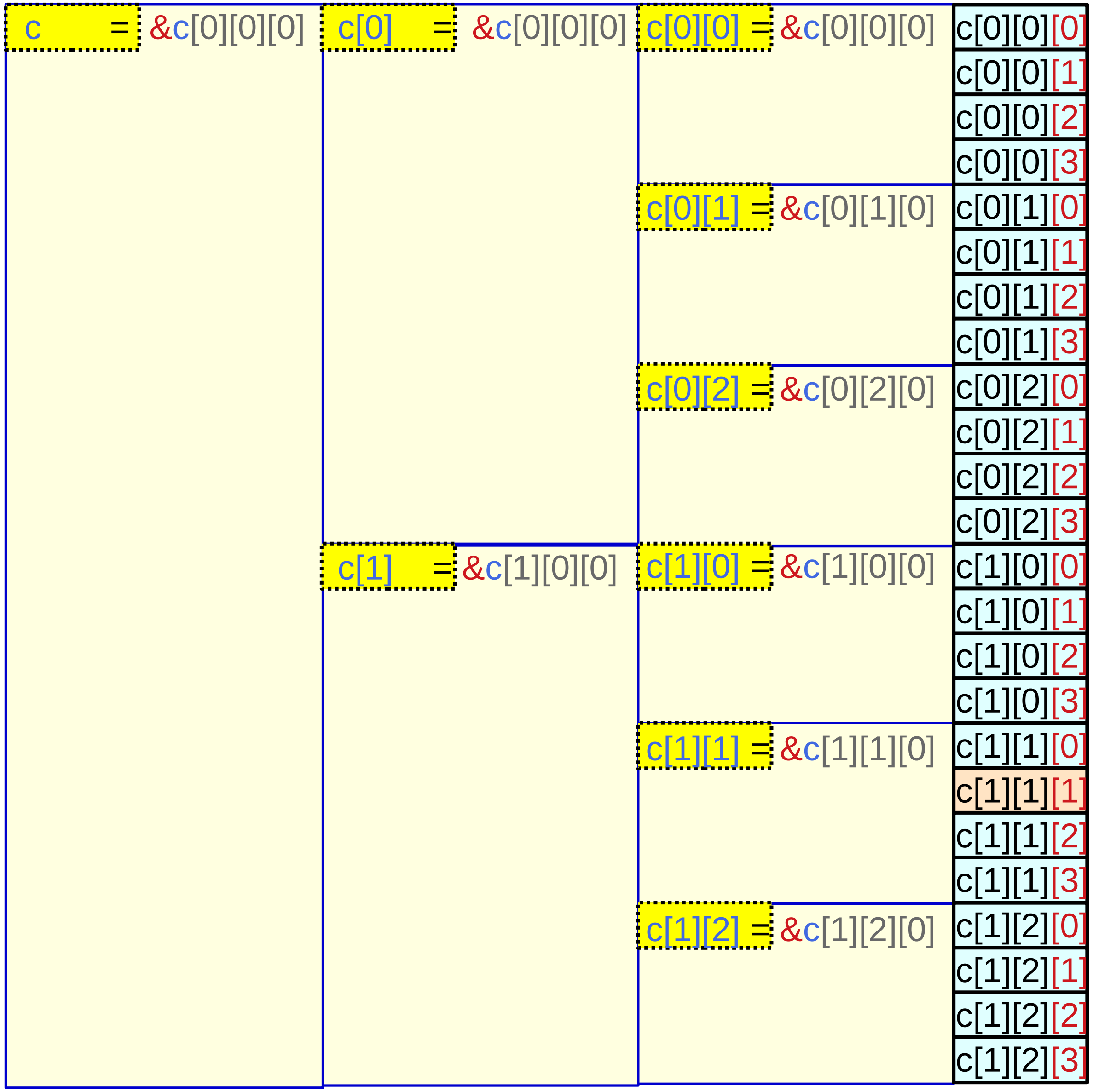
```
int    c [2][3][4];
```

```
       c [i][j][k];
```

virtual assignments

c ← &c[0][0][0]
c[i] ← &c[i][0][0]
c[i][j] ← &c[i][j][0]

row major ordering
contiguous linear layout



c = &c[0][0][0]

c[0] = &c[0][0][0]

c[1] = &c[1][0][0]

c[0][0] = &c[0][0][0]

c[0][1] = &c[0][1][0]

c[0][2] = &c[0][2][0]

c[1][0] = &c[1][0][0]

c[1][1] = &c[1][1][0]

c[1][2] = &c[1][2][0]

c[0][0][0]
c[0][0][1]
c[0][0][2]
c[0][0][3]
c[0][1][0]
c[0][1][1]
c[0][1][2]
c[0][1][3]
c[0][2][0]
c[0][2][1]
c[0][2][2]
c[0][2][3]
c[1][0][0]
c[1][0][1]
c[1][0][2]
c[1][0][3]
c[1][1][0]
c[1][1][1]
c[1][1][2]
c[1][1][3]
c[1][2][0]
c[1][2][1]
c[1][2][2]
c[1][2][3]

Virtual assignments

int c [2][3][4];

⇓

c [i][j][k];

virtual assignments

c ⇐ &c[0][0][0]
c[i] ⇐ &c[i][0][0]
c[i][j] ⇐ &c[i][j][0]

c ⇐ (int (*)[3][4]) &c[0][0][0]
c[i] ⇐ (int (*)[4]) &c[i][0][0]
c[i][j] ⇐ (int (*)) &c[i][j][0]

type casts **address values**

if c, c[i], c[i][j] were real pointer variables,
type casts would be needed

row major ordering
contiguous linear layout

Dual types of **c**, **c[i]**, **c[i][j]**

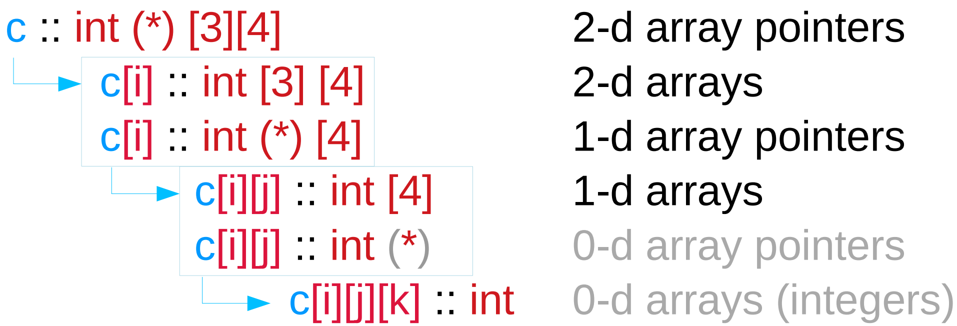
```
int     c [2][3][4];
```

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

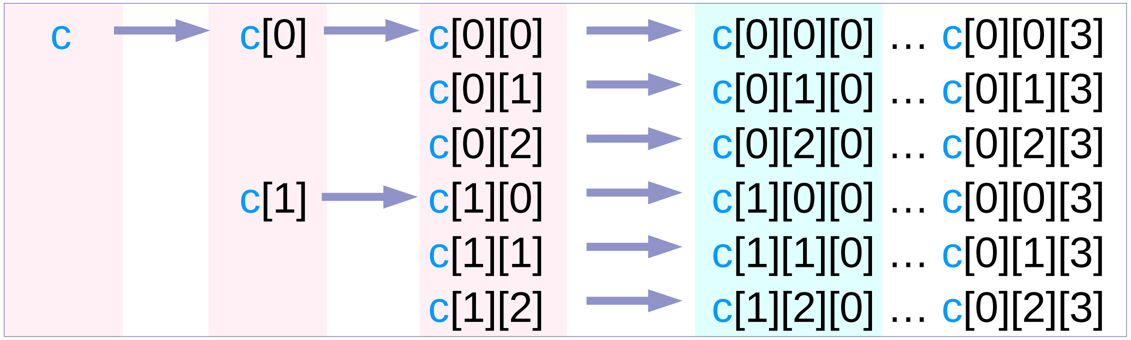
with contiguous subarrays

&c[i][j][k]	= c[i][j]+k	for all k
&c[i][j]	= c[i]+j	for all j
&c[i]	= c+i	for all i

c [i][j][k]



c :: int (*) [3][4]	2-d array pointers
c[i] :: int [3] [4]	2-d arrays
c[i] :: int (*) [4]	1-d array pointers
c[i][j] :: int [4]	1-d arrays
c[i][j] :: int (*)	0-d array pointers
c[i][j][k] :: int	0-d arrays (integers)



int [2] [3][4]	int [3] [4]	int [4]	int	…	int
int (*) [3][4]	int (*) [4]	int (*)	int	…	int
pointers to a 2-d array	pointers to a 1-d array	1-d array names	leading element of 4-integer array		

Values of **c**, **c[i]**, **c[i][j]**

int c [2][3][4];

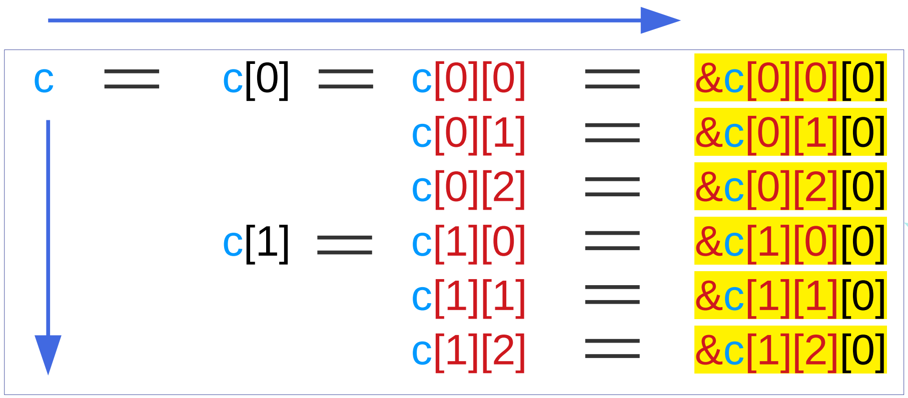
&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k for all k
&c[i][j] = c[i]+j for all j
&c[i] = c+i for all i

c [i][j][k]

virtual array pointers have address values
in each row in the following figure
have the <u>same</u> <u>address value</u>



c = c[0] = c[0][0] = &c[0][0][0]
c[0][1] = &c[0][1][0]
c[0][2] = &c[0][2][0]
c[1] = c[1][0] = &c[1][0][0]
c[1][1] = &c[1][1][0]
c[1][2] = &c[1][2][0]

Horizontal displacements are <u>not</u> counted
only **vertical displacements** are considered
for address values

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

Finding address values of **c**, **c[i]**, **c[i][j]**

```
int     c [2][3][4];
```

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k	for all k
&c[i][j] = c[i]+j	for all j
&c[i] = c+i	for all i

c [i][j][k]

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

append [0] to the right

c	$\overset{+[0]}{=}$	c[0]	$\overset{+[0]}{=}$	c[0][0]	$\overset{+[0]}{=}$	&c[0][0][0]
				c[0][1]	$\overset{+[0]}{=}$	&c[0][1][0]
				c[0][2]	$\overset{+[0]}{=}$	&c[0][2][0]
		c[1]	$\overset{+[0]}{=}$	c[1][0]	$\overset{+[0]}{=}$	&c[1][0][0]
				c[1][1]	$\overset{+[0]}{=}$	&c[1][1][0]
				c[1][2]	$\overset{+[0]}{=}$	&c[1][2][0]
int (*) [3][4]		int (*) [4]		int [4]		int

c[0][0][0] **:** leading elements of **c**

&c[0][0][0]

c[i][0][0] **:** leading elements of **c[i]**

&c[0][0][0]
&c[1][0][0]

c[i][j][0] **:** leading elements of **c[i][j]**

&c[0][0][0]
&c[0][1][0]
&c[0][2][0]
&c[1][0][0]
&c[1][1][0]
&c[1][2][0]

Finding sub-array names with the address &c[i][j][0]

int c [2][3][4];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k for all k
&c[i][j] = c[i]+j for all j
&c[i] = c+i for all i

c [i][j][k]

virtual assignments

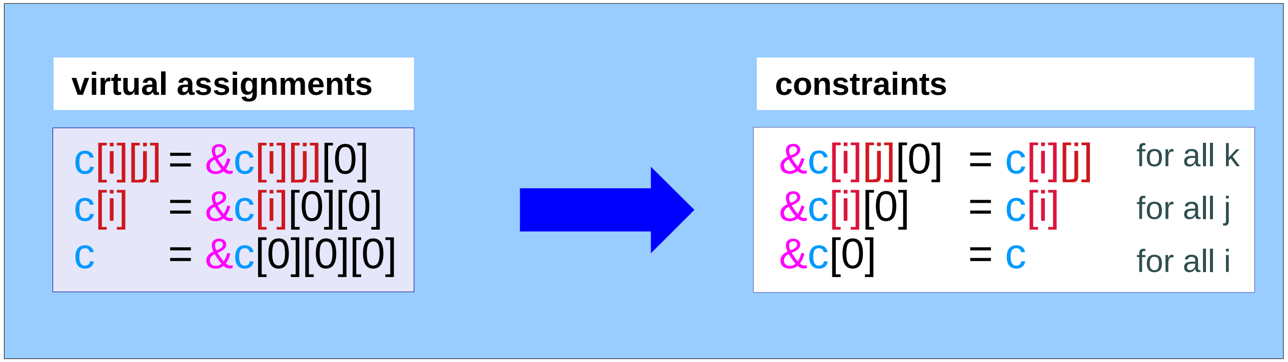
c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

delete [0] from the right

&c[0][0][0]	$\overset{-[0]}{\equiv}$	c[0][0]	$\overset{-[0]}{\equiv}$	c[0]	$\overset{-[0]}{\equiv}$	c
&c[0][1][0]	$\overset{-[0]}{\equiv}$	c[0][1]				
&c[0][2][0]	$\overset{-[0]}{\equiv}$	c[0][2]				
&c[1][0][0]	$\overset{-[0]}{\equiv}$	c[1][0]	$\overset{-[0]}{\equiv}$	c[1]		
&c[1][1][0]	$\overset{-[0]}{\equiv}$	c[1][1]				
&c[1][2][0]	$\overset{-[0]}{\equiv}$	c[1][2]				
int		int [4]		int (*) [4]		int (*) [3][4]

c[0][0][0] is the leading element of c[0][0], c[0], c
c[0][1][0] is the leading element of c[0][1]
c[0][2][0] is the leading element of c[0][2]
c[1][0][0] is the leading element of c[1][0], c[1]
c[1][1][0] is the leading element of c[1][1]
c[1][2][0] is the leading element of c[1][2]

multi-dimensional arrays

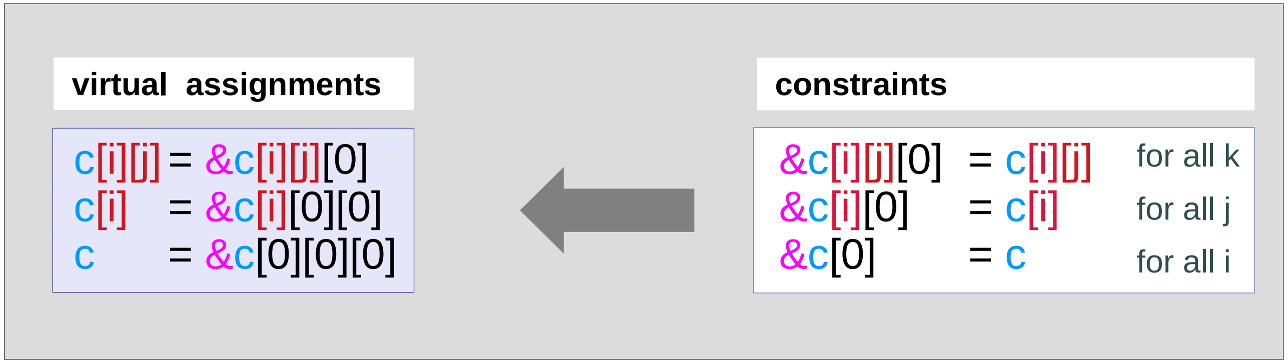


virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

constraints

&c[i][j][0] = c[i][j] for all k
&c[i][0] = c[i] for all j
&c[0] = c for all i



virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

constraints

&c[i][j][0] = c[i][j] for all k
&c[i][0] = c[i] for all j
&c[0] = c for all i

General requirements for **c[i][j][k]**

```
int     c [2][3][4];
```

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k   for all k
&c[i][j] = c[i]+j   for all j
&c[i] = c+i   for all i

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

Virtual assignments

int (*)	c[i][j]	=	(int (*))	&c[i][j][0]
int (*) [4]	c[i]	=	(int (*) [4])	&c[i][0][0]
int (*) [3][4]	c	=	(int (*) [3][4])	&c[0][0][0]

Pointer Types

Sizes of abstract data types

int [4]	c[i][j]	size =4*4
int [3][4]	c[i]	size =3*4*4
int [2][3][4]	c	size =2*3*4*4

Abstract Data Types

Strides of array elements

c[i][j][0]	stride =4*4
c[i][0][0]	stride =3*4*4
c[0][0][0]	stride =2*3*4*4

k=[0:3]	c[i][j][k]	4 integers
j=[0:2], k=[0:3]	c[i][j][k]	3*4 integers
I=[0:1], j=[0:2], k=[0:3]	c[i][j][k]	2*3*4 integers

General requirements for **c[i][j][k]**

int c [2][3][4];

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k for all k
&c[i][j] = c[i]+j for all j
&c[i] = c+i for all i

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

Each	c[i][j]	sub-array contains	4 integers
Each	c[i]	sub-array contains	3*4 integers
Each	c	sub-array contains	2*3*4 integers

Abstract Data Types

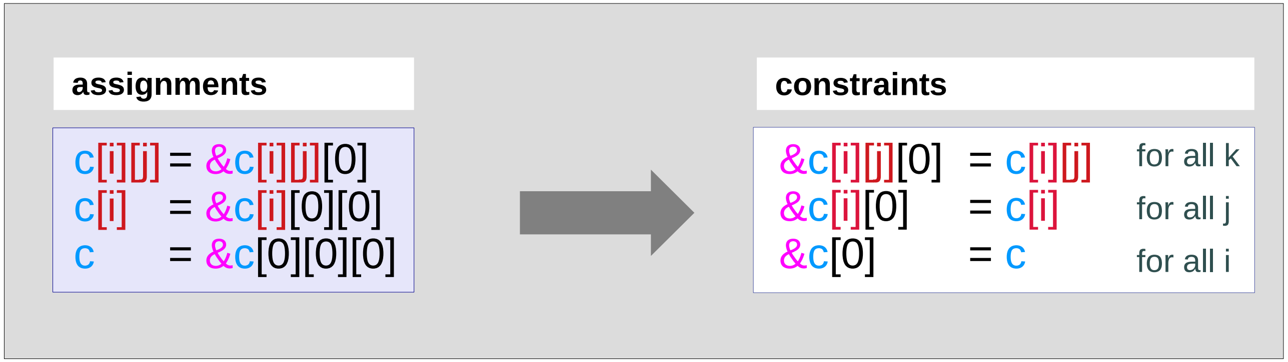
Each	c[i][j]+1	points to	c[i][j][1]	1 integer away
Each	c[i]+1	points to	c[i][1][0]	4 integers away
Each	c+1	points to	c[1][0][0]	3*4 integers away

Pointer Types

Each	c[i][j]+k	points to	c[i][j][k]	1*k integers away
Each	c[i]+j	points to	c[i][j][0]	4*j integers away
Each	c+i	points to	c[i][0][0]	3*4*i integers away

Pointer Types

multi-dimensional arrays

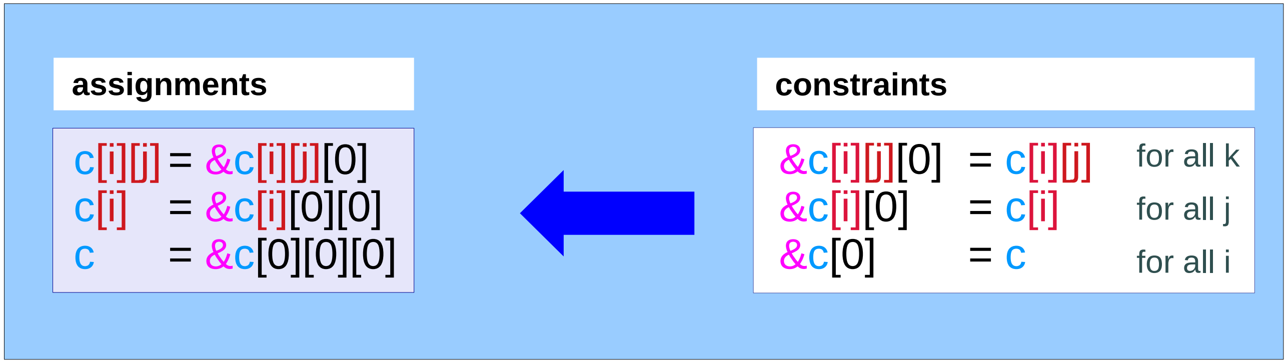


assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

constraints

&c[i][j][0] = c[i][j] for all k
&c[i][0] = c[i] for all j
&c[0] = c for all i



assignments

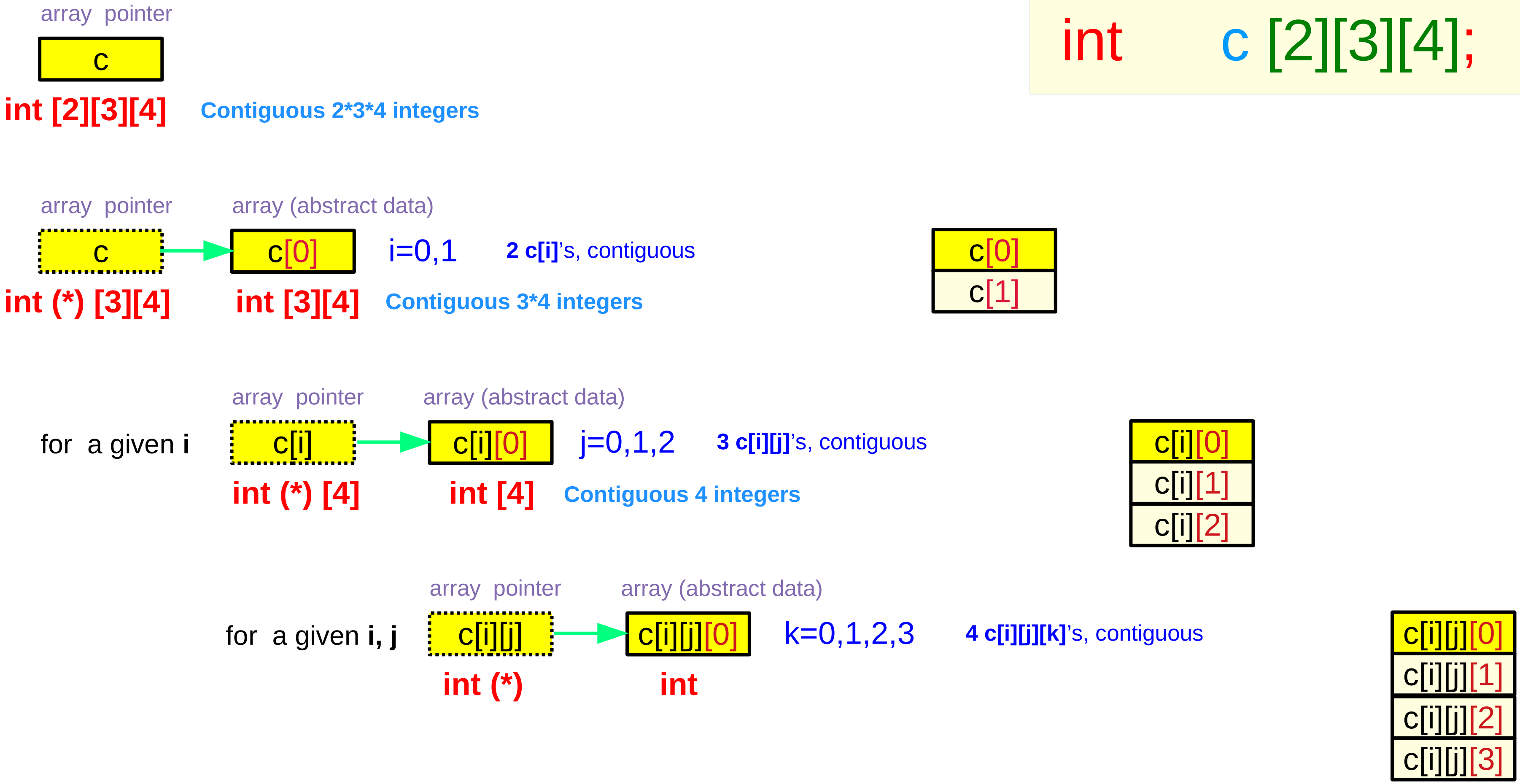
c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

constraints

&c[i][j][0] = c[i][j] for all k
&c[i][0] = c[i] for all j
&c[0] = c for all i

Contiguous subarrays

```
int     c [2][3][4];
```



array pointer

c

int [2][3][4]

Contiguous 2*3*4 integers

array pointer

c

int (*) [3][4]

array (abstract data)

c[0]

int [3][4]

i=0,1

2 c[i]'s, contiguous

Contiguous 3*4 integers

c[0]
c[1]

for a given **i**

array pointer

c[i]

int (*) [4]

array (abstract data)

c[i][0]

int [4]

j=0,1,2

3 c[i][j]'s, contiguous

Contiguous 4 integers

c[i][0]
c[i][1]
c[i][2]

for a given **i, j**

array pointer

c[i][j]

int (*)

array (abstract data)

c[i][j][0]

int

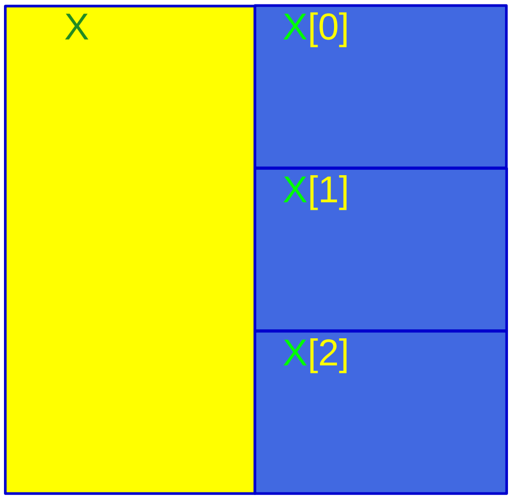
k=0,1,2,3

4 c[i][j][k]'s, contiguous

c[i][j][0]
c[i][j][1]
c[i][j][2]
c[i][j][3]

Virtual array pointers in a multi-dimensional array

Abstract data (array) p[i]

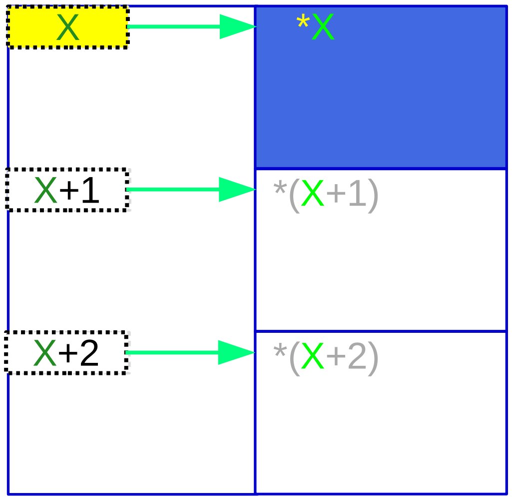


array (abstract data)   array (abstract data)

c[i][j] starts at c[i][j][0]
c[i] starts at c[i][0]
c starts at c[0]

&c[i][j] = &c[i][j][0]
&c[i] = &c[i][0]
&c = &c[0]

Virtual array pointer p[i]



array pointer   array (abstract data)

c[i][j] points to c[i][j][0]
c[i] points to c[i][0]
c points to c[0]

address value

c[i][j] = &c[i][j][0]
c[i] = &c[i][0]
c = &c[0]

General requirements for **c[i][j][k]**

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

array pointer — array (abstract data)

c[i][j] points to c[i][j][0]
c[i] points to c[i][0]
c points to c[0]

address value

&c[i][j][0] = &c[i][j]
&c[i][0] = &c[i]
&c[0] = &c

array (abstract data) — array (abstract data)

c[i][j] starts at c[i][j][0]
c[i] starts at c[i][0]
c starts at c[0]

c[i][j] = &c[i][j]
c[i] = &c[i]
c = &c

array (abstract data) — Address of an array pointer

c[i][j] pointer value = pointer address &c[i][j]
c[i] pointer value = pointer address &c[i]
c pointer value = pointer address &c

Pointer reference relationship

int c [2][3][4];

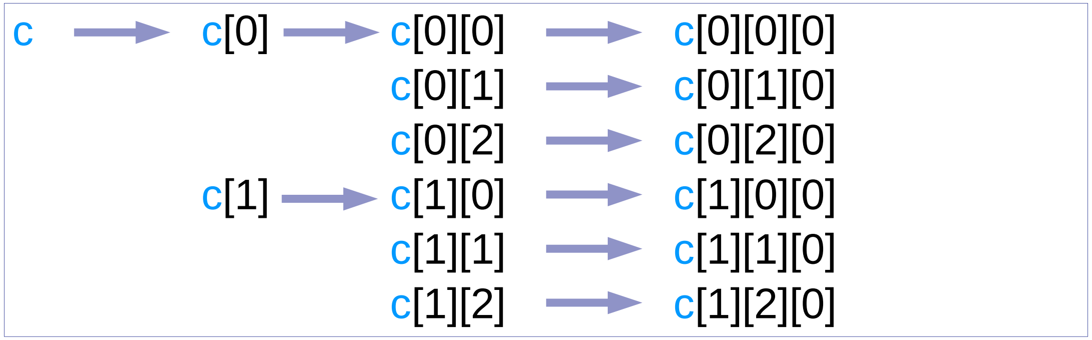
&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k for all k
&c[i][j] = c[i]+j for all j
&c[i] = c+i for all i

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]


c → c[0] → c[0][0] → c[0][0][0]
c[0][1] → c[0][1][0]
c[0][2] → c[0][2][0]
c[1] → c[1][0] → c[1][0][0]
c[1][1] → c[1][1][0]
c[1][2] → c[1][2][0]

c[i][j] = &c[i][j][0]

c[0][0] = &c[0][0][0]
c[0][1] = &c[0][1][0]
c[0][2] = &c[0][2][0]
c[1][0] = &c[1][0][0]
c[1][1] = &c[1][1][0]
c[1][2] = &c[1][2][0]

c[i] = &c[i][0]

c[0] = &c[0][0] = &c[0][0][0]
c[1] = &c[1][0] = &c[1][0][0]

c = &c[0]

c = &c[0] = &c[0][0] = &c[0][0][0]

c[0] = c[0][0] relation

```
int    c [2][3][4];
```

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k	for all k
&c[i][j] = c[i]+j	for all j
&c[i] = c+i	for all i

virtual assignments

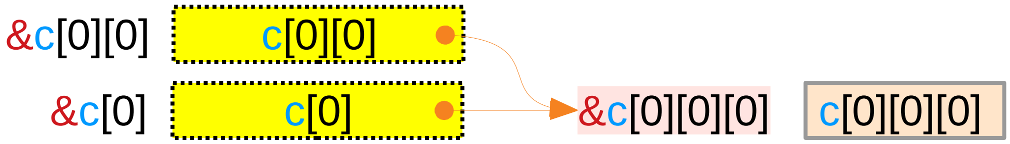
c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

c = c[0] = c[0][0] = &c[0][0][0]

value(c[0]**)** = &c[0][0][0]
value(c[0][0]**)** = &c[0][0][0]

type(c[0]**)** = int (*)[4]
type(c[0][0]**)** = int [4]

c[0] = c[0][0] *means*
value(c[0]**)** = **value(**c[0][0]**)**



c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

Addresses and Values of **c[0]** and **c[0][0]**

```
int    c [2][3][4];
```

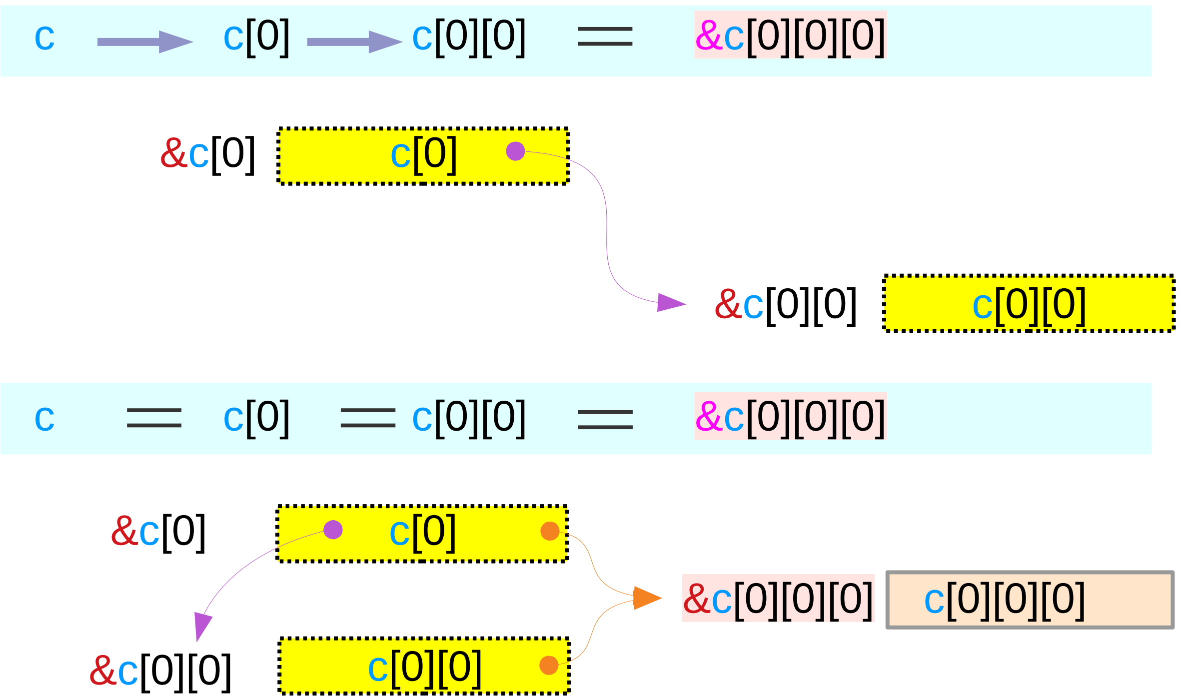
&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k   for all k
&c[i][j] = c[i]+j   for all j
&c[i] = c+i   for all i

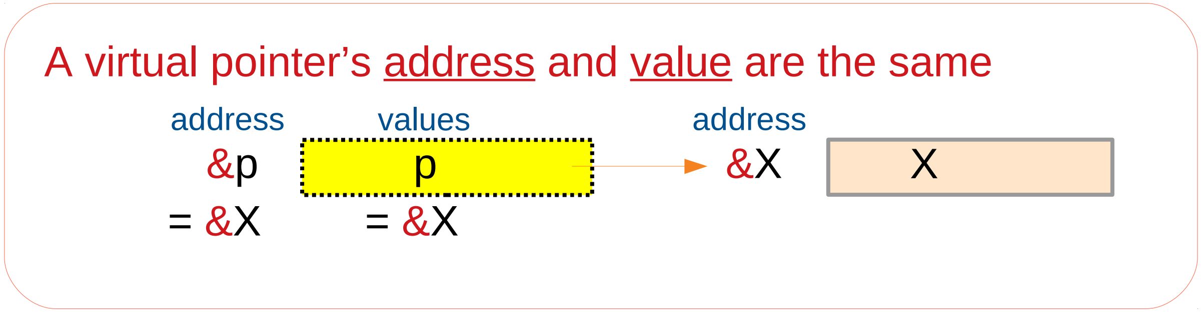
virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]



$c \rightarrow c[0] \rightarrow c[0][0] = \&c[0][0][0]$

$c = c[0] = c[0][0] = \&c[0][0][0]$



A virtual pointer's address and value are the same

c[0] and c[0][0] point to the same c[i][0][0]

```
int     c [2][3][4];
```

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

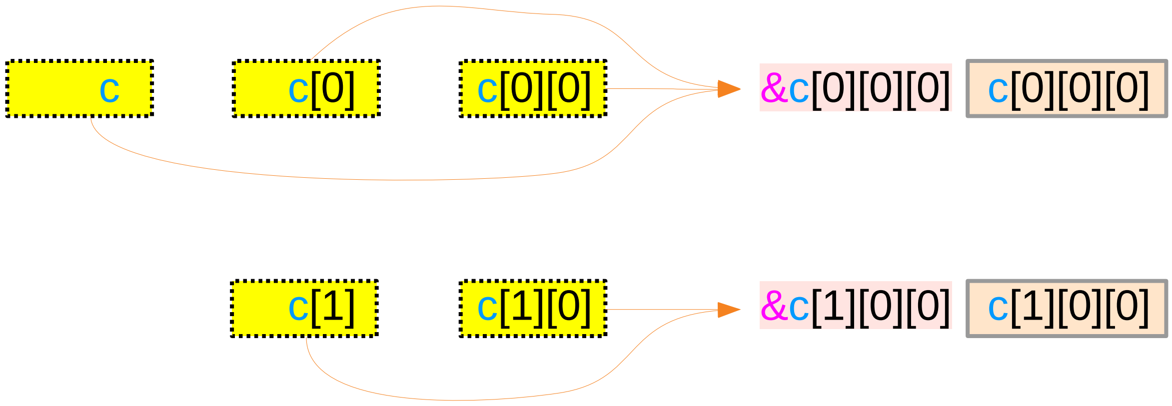
&c[i][j][k] = c[i][j]+k   for all k
&c[i][j] = c[i]+j   for all j
&c[i] = c+i   for all i

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

c =	c[0] =	c[0][0] =	&c[0][0][0]	← **value**
int(*)[3][4]	int(*)[4]	int(*)	int	← **type**

c[1] =	c[1][0] =	&c[1][0][0]	← **value**
int(*)[4]	int(*)	int	← **type**



These virtual pointers have different types
but the same value (address)

&c[i][0] and **&c[i][0][0]** – equivalence relations

int c [2][3][4];

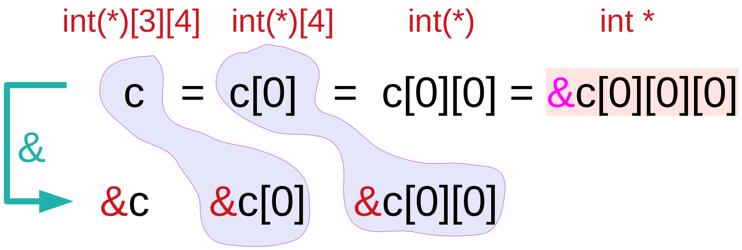
&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k for all k
&c[i][j] = c[i]+j for all j
&c[i] = c+i for all i

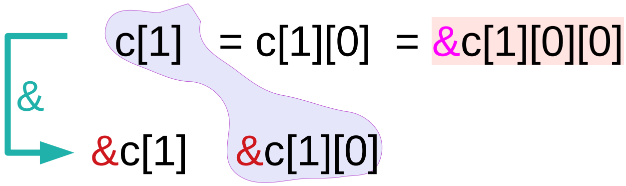
virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]



equivalences

c ≡ &c[0],
c[0] ≡ &c[0][0]
c[0][0] ≡ &c[0][0][0]



equivalences

c[1] ≡ &c[1][0]
c[1][0] ≡ &c[1][0][0]

Horizontal displacements are not counted
only **vertical displacements** are considered
for **address values**

equivalences

c ≡ &c[0],
c[i] ≡ &c[i][0]
c[i][0] ≡ &c[i][0][0]

c[i] = &c[i] and c[i][0] = &c[i][0]

```
int    c [2][3][4];
```

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

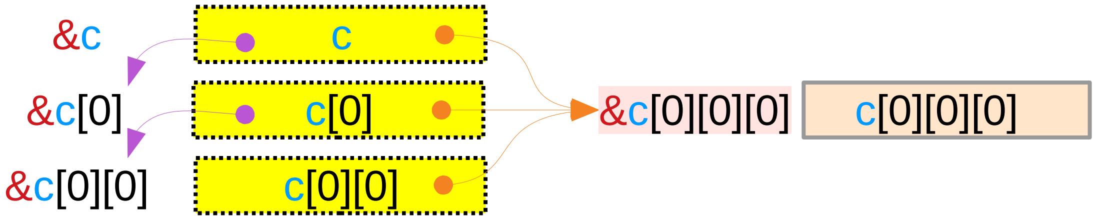
&c[i][j][k] = c[i][j]+k for all k
&c[i][j] = c[i]+j for all j
&c[i] = c+i for all i

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

c = c[0] = c[0][0]= &c[0][0][0]
|| || ||
&c = &c[0] = &c[0][0]

c[1] = c[1][0] = &c[1][0][0]
|| ||
&c[1] = &c[1][0]



c[i] = &c[i] and c[i][0] = &c[i][0]

```
int     c [2][3][4];
```

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

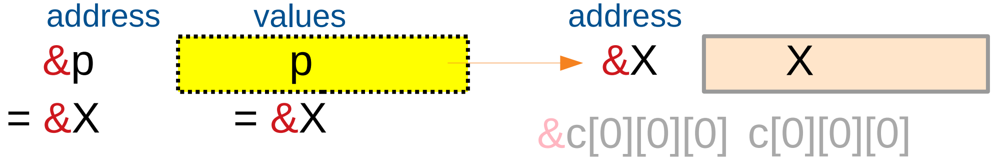
with contiguous subarrays

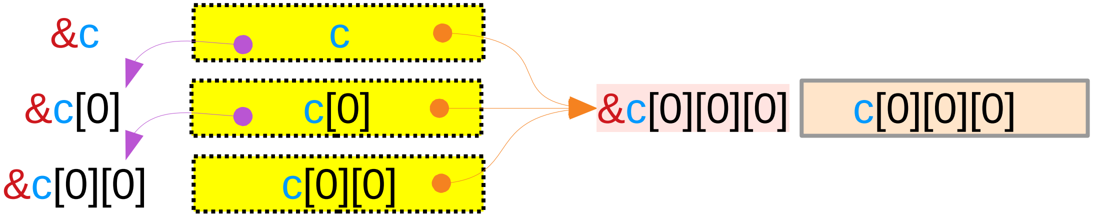
&c[i][j][k] = c[i][j]+k for all k
&c[i][j] = c[i]+j for all j
&c[i] = c+i for all i

virtual assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

A virtual pointer's <u>address</u> and <u>value</u> are the same





c[i]

Array Pointers to **c[i][0][0]**

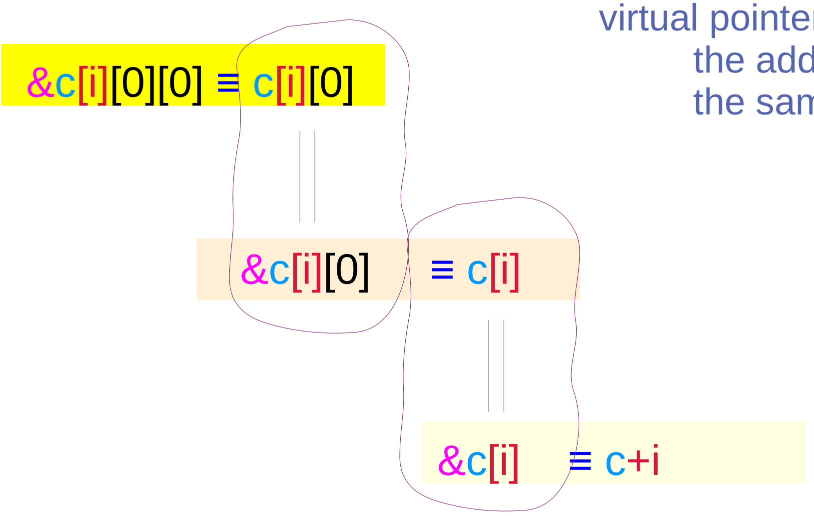
&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k]	= c[i][j]+k	for all k
&c[i][j]	= c[i]+j	for all j
&c[i]	= c+i	for all i

assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]



&c[i][0][0] ≡ c[i][0]

&c[i][0] ≡ c[i]

&c[i] ≡ c+i

virtual pointers:
the address of a pointer is
the same as its value

= c + i*sizeof(*c)

= &c[0][0][0] + i*3*4

delete [0] from the right

$$\&c[0][0][0] \overset{-[0]}{=} c[0][0] \overset{-[0]}{=} c[0] \overset{-[0]}{=} c$$

$$\&c[1][0][0] \overset{-[0]}{=} c[1][0] \overset{-[0]}{=} c[1]$$

Array Pointers to **c[i][j][0]**

&c[i][j][0] = c[i][j]
&c[i][0] = c[i]
&c[0] = c

with contiguous subarrays

&c[i][j][k] = c[i][j]+k for all k
&c[i][j] = c[i]+j for all j
&c[i] = c+i for all i

assignments

c[i][j] = &c[i][j][0]
c[i] = &c[i][0][0]
c = &c[0][0][0]

&c[i][j][0] ≡ c[i][j]

||

&c[i][j] ≡ c[i] + j

= c[i] + j*sizeof(*c[i])

= c + i*sizeof(*c) + j*4

= &c[0][0][0] + i*3*4 + j*4

delete [0] from the right

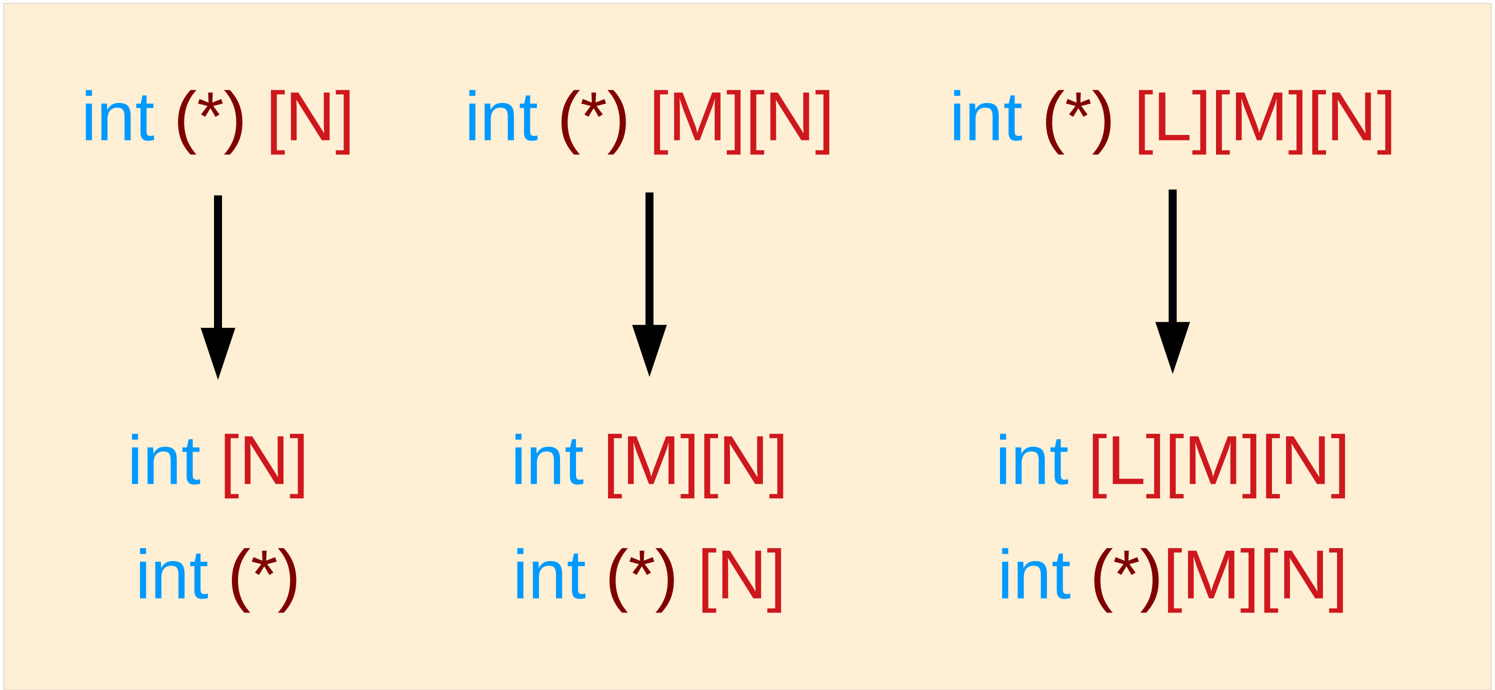
&c[0][0][0]	$\overset{-[0]}{\equiv}$	c[0][0]	$\overset{-[0]}{\equiv}$	c[0]	$\overset{-[0]}{\equiv}$	c
&c[0][1][0]	$\overset{-[0]}{\equiv}$	c[0][1]				
&c[0][2][0]	$\overset{-[0]}{\equiv}$	c[0][2]				
&c[1][0][0]	$\overset{-[0]}{\equiv}$	c[1][0]	$\overset{-[0]}{\equiv}$	c[1]		
&c[1][1][0]	$\overset{-[0]}{\equiv}$	c[1][1]				
&c[1][2][0]	$\overset{-[0]}{\equiv}$	c[1][2]				

Contiguity Constraints

c [i][j][k];

Virtual Array Pointers and Contiguity

Using array pointers

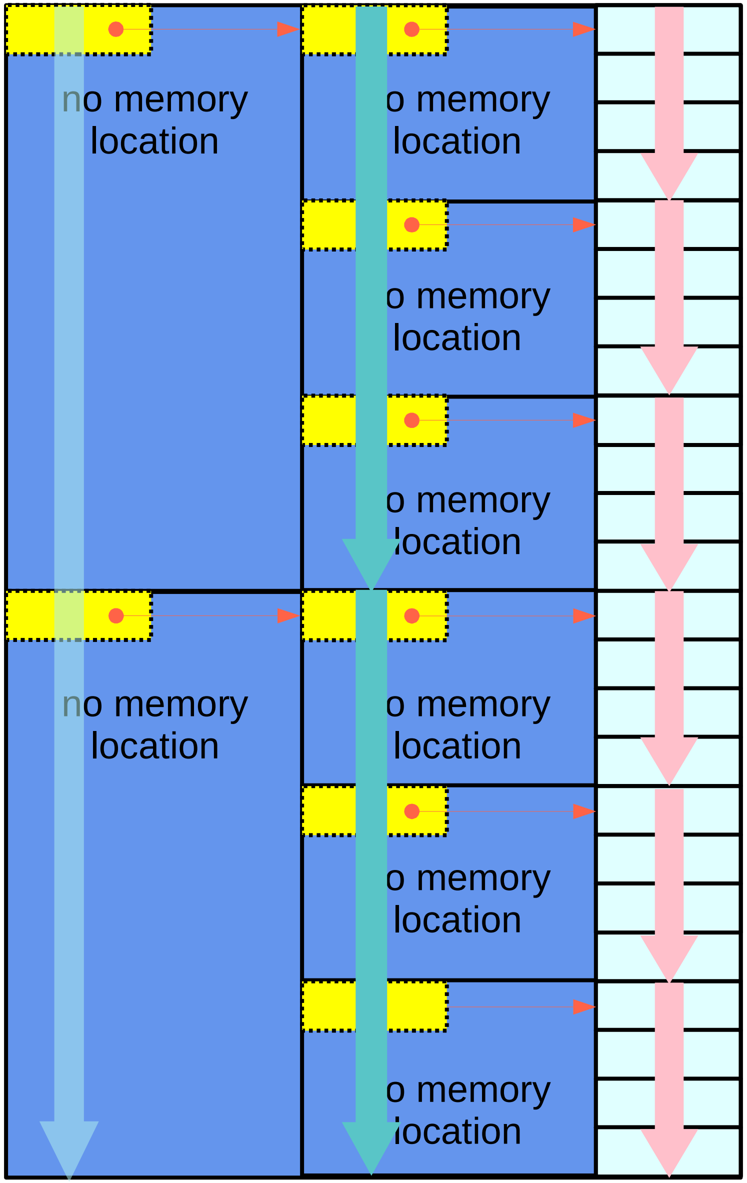


int (*) [N] int (*) [M][N] int (*) [L][M][N] Array Pointer

int [N] int [M][N] int [L][M][N] Array Name

int (*) int (*) [N] int (*)[M][N] Array Pointer

Array pointer approach – contiguity constraints



Array Pointer Approach
(pointer to arrays)

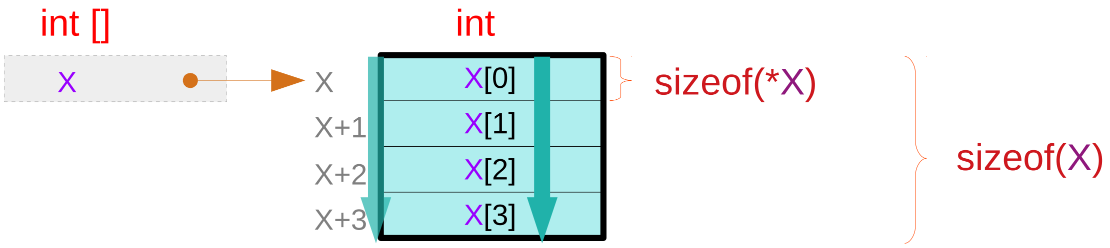
Equivalence and contiguity (1)

consecutive address

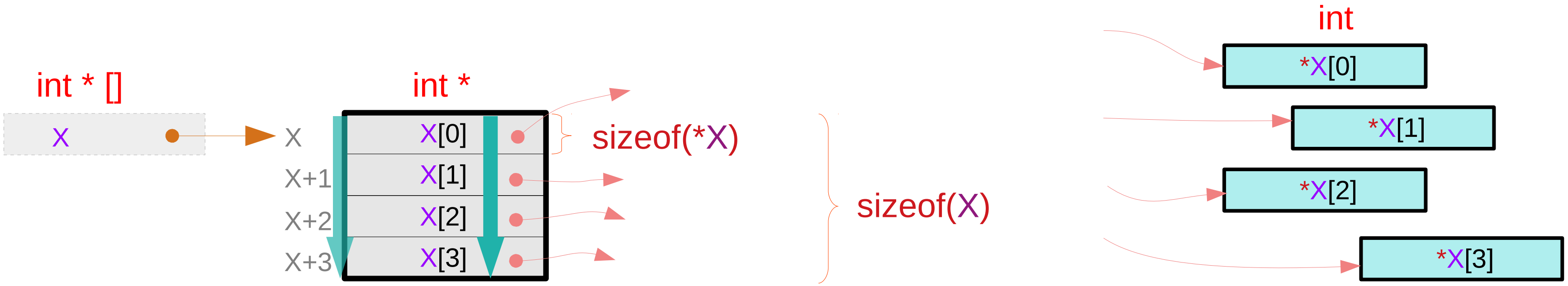
consecutive data

$$*(X+n) \equiv X[n]$$

contiguous index : n



int X[4]; contiguous X[i] for a given X : **primitive types**



int * X[4]; contiguous X[i] for a given X : **pointer types**

Equivalence and contiguity (2)

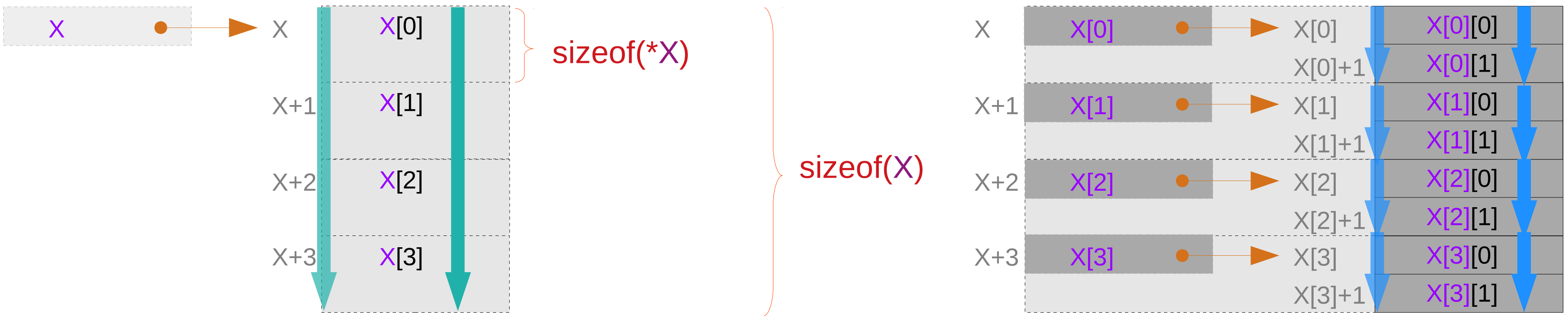
consecutive address

consecutive data

$$*(X+n) \equiv X[n]$$

contiguous index : n

can be recursively applied



atype * X[4]; contiguous X[i] for a given X : **abstract data types**

Recursive applications of equivalences

By definition, contiguous memory locations are assumed

consecutive address consecutive data

*(X+n) ≡ X[n]

contiguous index : n

*(p[m]+n) ⟺ p[m][n]

X = p[m] contiguous index : **n**

(*(p+m))[n]; ⟺ p[m][n];

X = p contiguous index : **m**

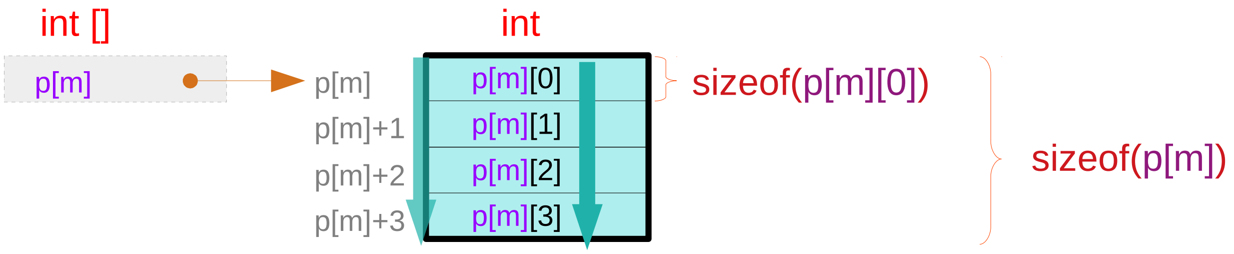
Equivalence for a given p[m] (1)

*(p[m]+n) ⟺ p[m][n]

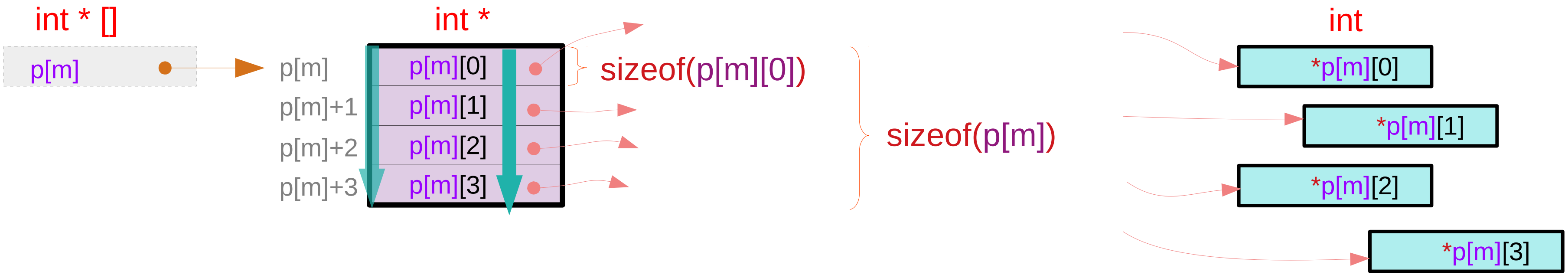
for a given p[m]

contiguous index : **n**

int p[M][4]; contiguous p[m][n] for a given p[m] : **primitive types** m = 0, 1, …, M-1



int * p[M][4]; contiguous p[m][n] for a given p[m] : **pointer types** m = 0, 1, …, M-1

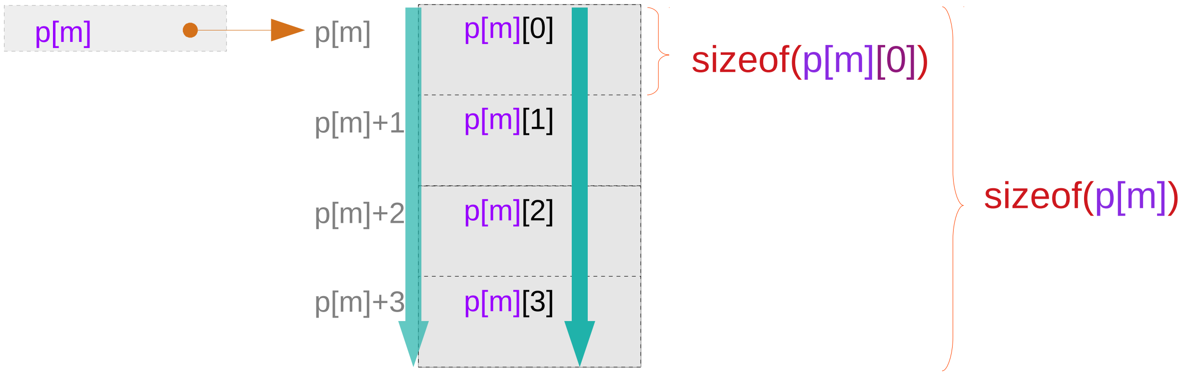


Equivalence for a given p[m] (2)

*(p[m]+n) ⟺ p[m][n]

for a given p[m]   contiguous index : **n**

atype * p[M][4]; contiguous p[m][n] for a given p[m] : **abstract data types**   m = 0, 1, …, M-1



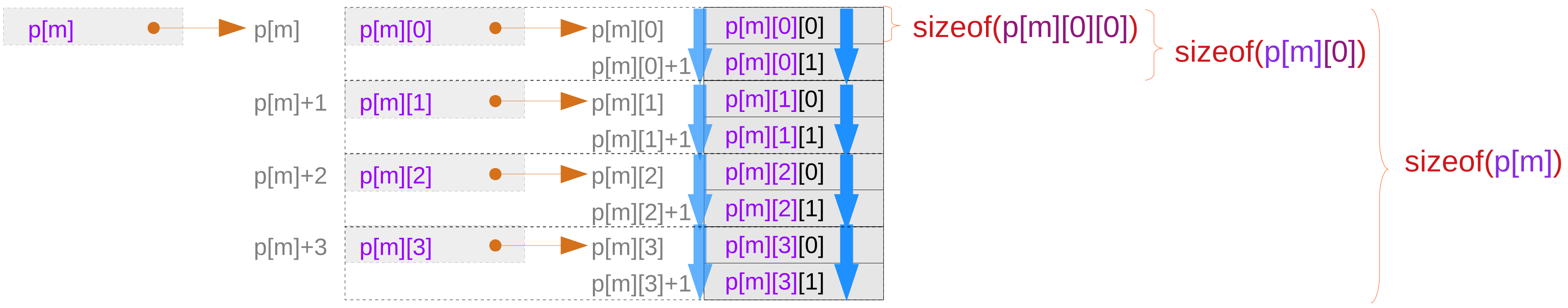
can be recursively applied

Equivalence for a given p[m][n]

*(p[m][n]+k) ⟷ p[m][n][k]

for a given p[m][n] contiguous index : **k**

atype * p[M][4][2]; contiguous p[m][n][k] for a given p[m][n] : **abstract data types** m = 0, 1, …, M-1

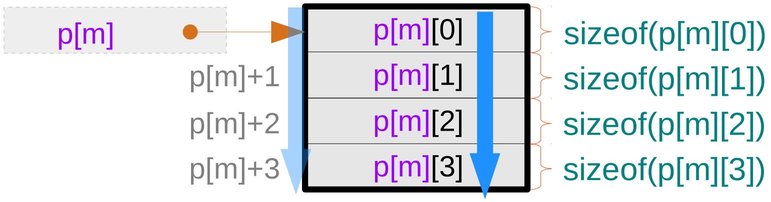


Contiguity constraints in multi-dimensional arrays

*(p[m]+n) ⟺ p[m][n]

for a given p[m], thus for a given p and m,
p[m][n]’s must be contiguous for all n.
p[m][0], p[m][1], … , p[m][N-1]

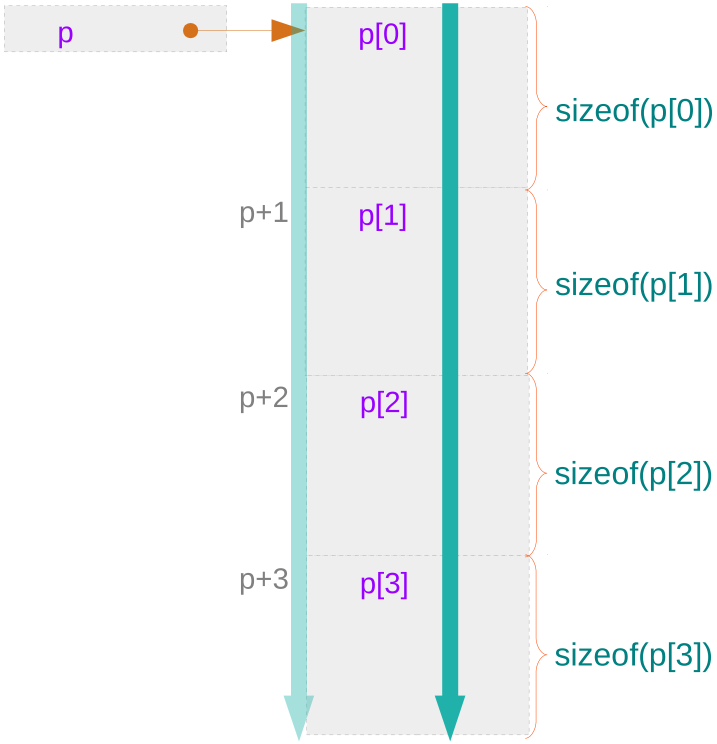
contiguous index : n



*(p+m) ⟺ p[m]

for a given p,
p[m]’s must be contiguous for all m.
p[0], p[1], … , p[M-1]

contiguous index : m

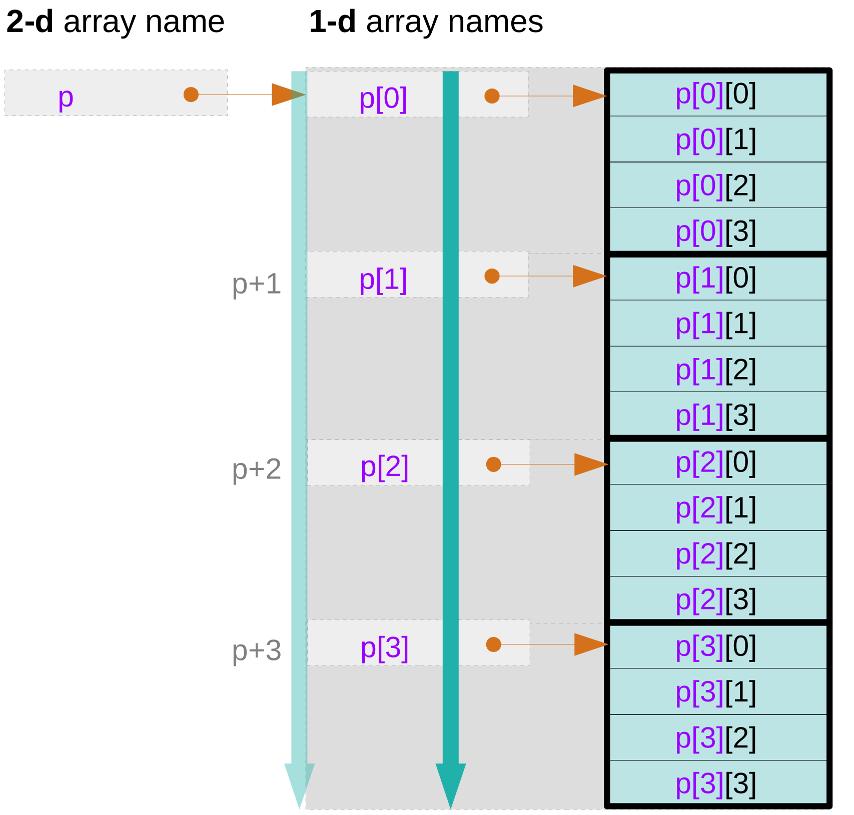


Contiguity constraints for p

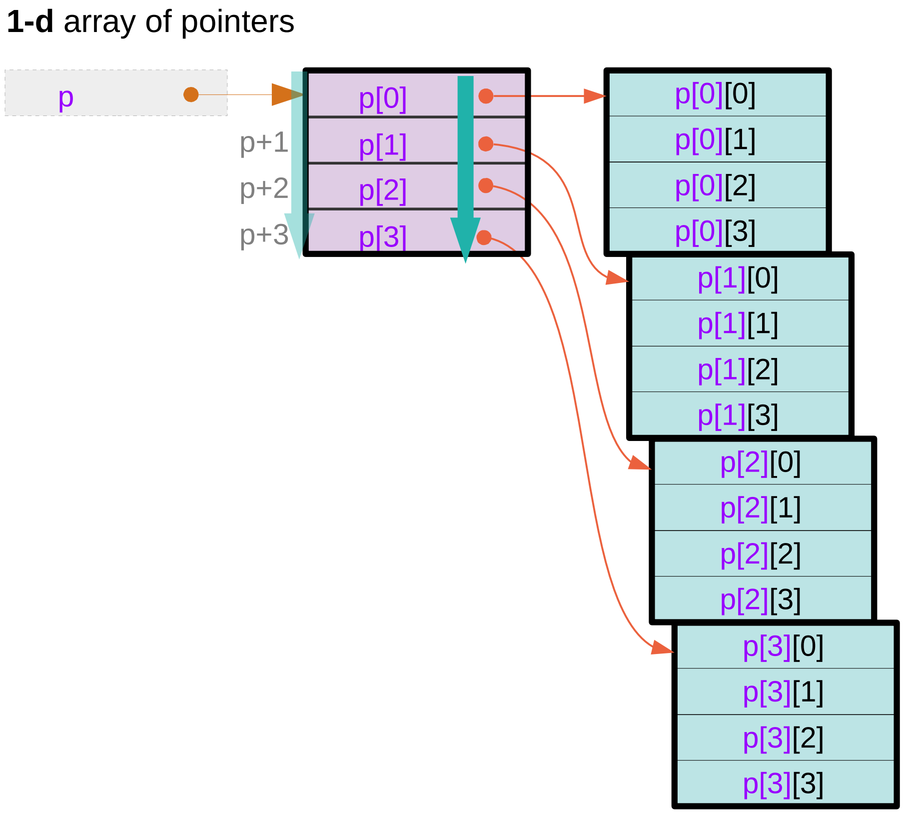
*(p+m) ⟺ p[m]

for a given p

contiguous index : **m**



contiguous p[m] ➡ contiguous p[m][n]



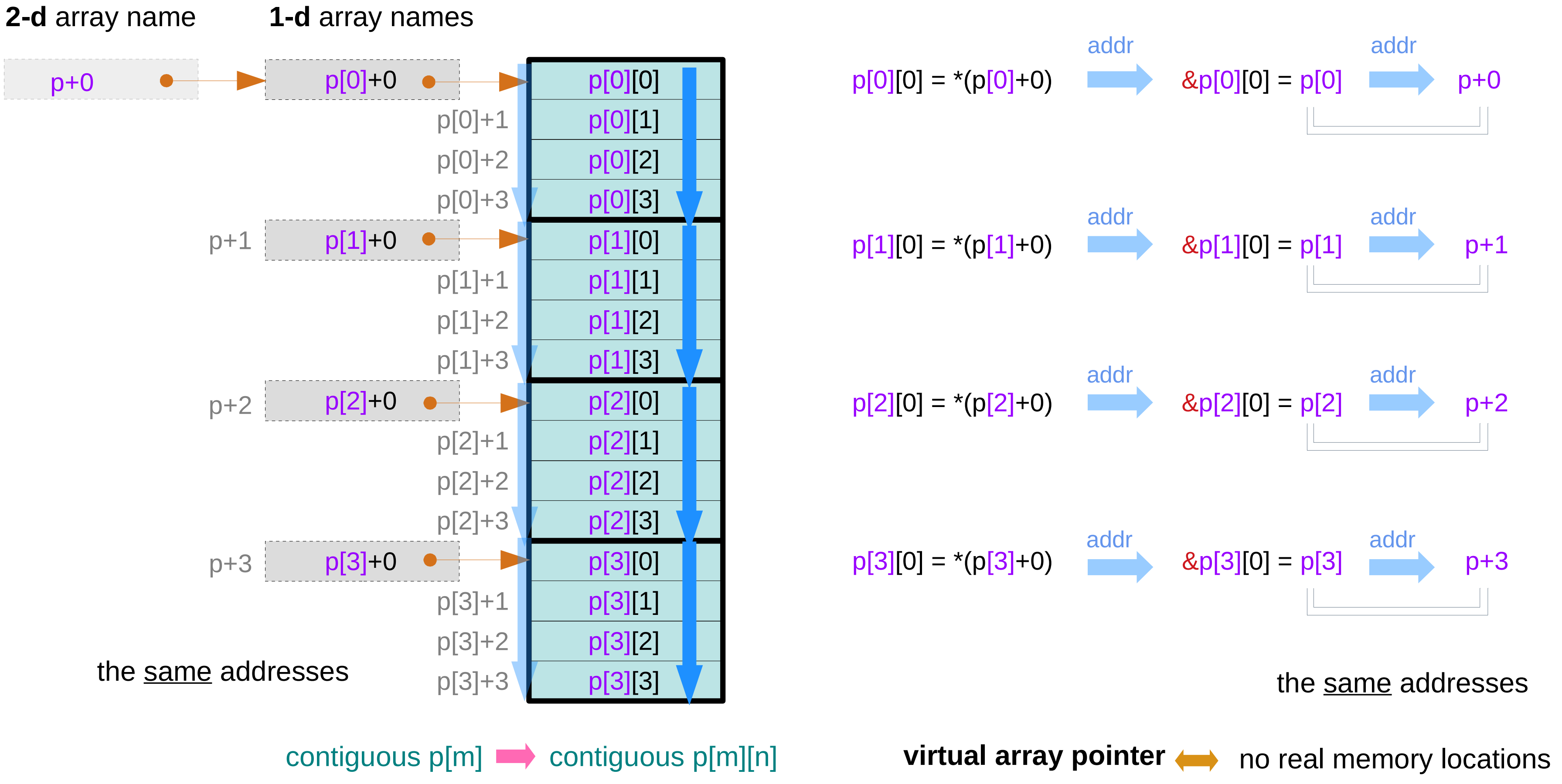
contiguous p[m] ➡ contiguous p[m][n]

Not necessarily

Contiguity constraints for p[m] – using array pointers

*(p[m]+n) ⟺ p[m][n]

for a given p[m] contiguous index : **n**

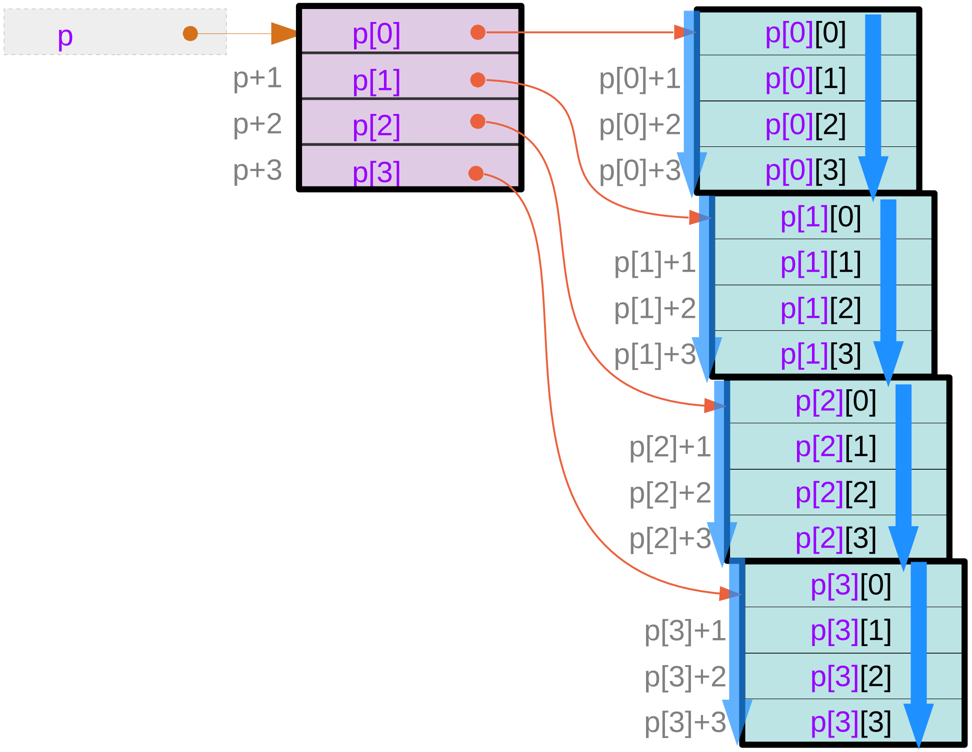


Contiguity constraints for p[m] – using pointer arrays

*(p[m]+n) ⟷ p[m][n]

for a given p[m]   contiguous index : **n**

1-d array of pointers



data   addr   addr

p[0][0] = *(p[0]+0) ➡ &p[0][0] = p[0] ➡ p+0

data   addr   addr

p[1][0] = *(p[1]+0) ➡ &p[1][0] = p[1] ➡ p+1

data   addr   addr

p[2][0] = *(p[2]+0) ➡ &p[2][0] = p[2] ➡ p+2

data   addr   addr

p[3][0] = *(p[3]+0) ➡ &p[3][0] = p[3] ➡ p+3

the different addresses

contiguous p[m] ➡ contiguous p[m][n]

Not necessarily

Contiguity constraints for 2-d arrays

int a[M][N] ;

*(a+m) ⟺ a[m]

a[0], a[1], … , a[M-1]
are contiguous

*(a[m]+n) ⟺ a[m][n]

a[m][0], a[m][1], … , a[m][N-1]
are contiguous

int (*b)[N] ;

*(b+m) ⟺ b[m]

b[0], b[1], … , b[M-1]
are contiguous

*(b[m]+n) ⟺ b[m][n]

b[m][0], b[m][1], … , b[m][N-1]
are contiguous

int * c[M] ;

*(c+m) ⟺ c[m]

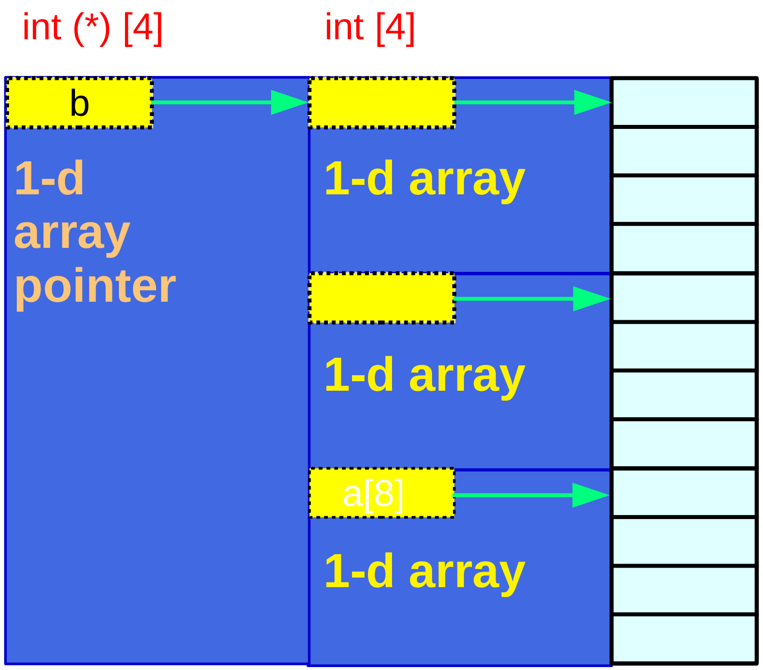
c[0], c[1], … , c[M-1]
are contiguous

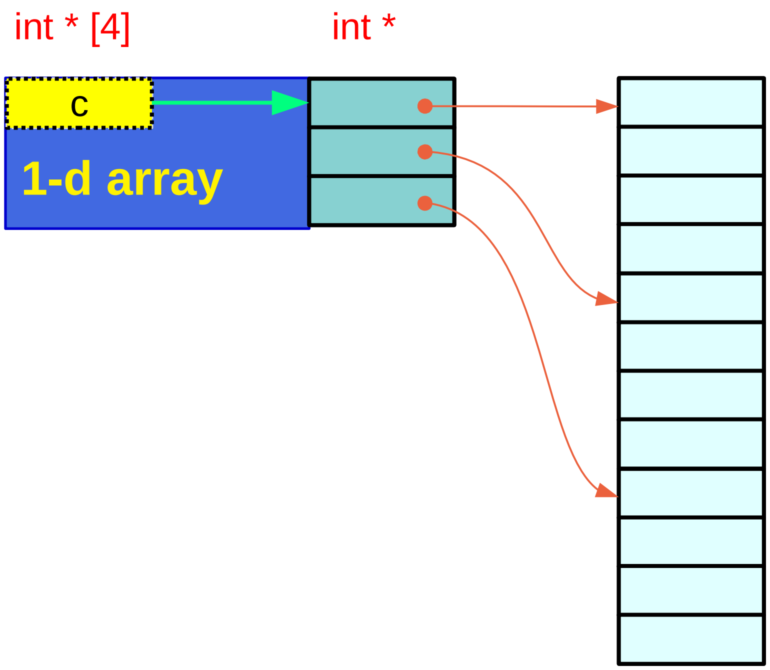
*(c[m]+n) ⟺ c[m][n]

c[m][0], c[m][1], … , c[m][N-1]
are contiguous

a set of assignments of pointers
are necessary for this contiguity

Pointer Arrays vs Array Pointers





int (*b)[N] ;

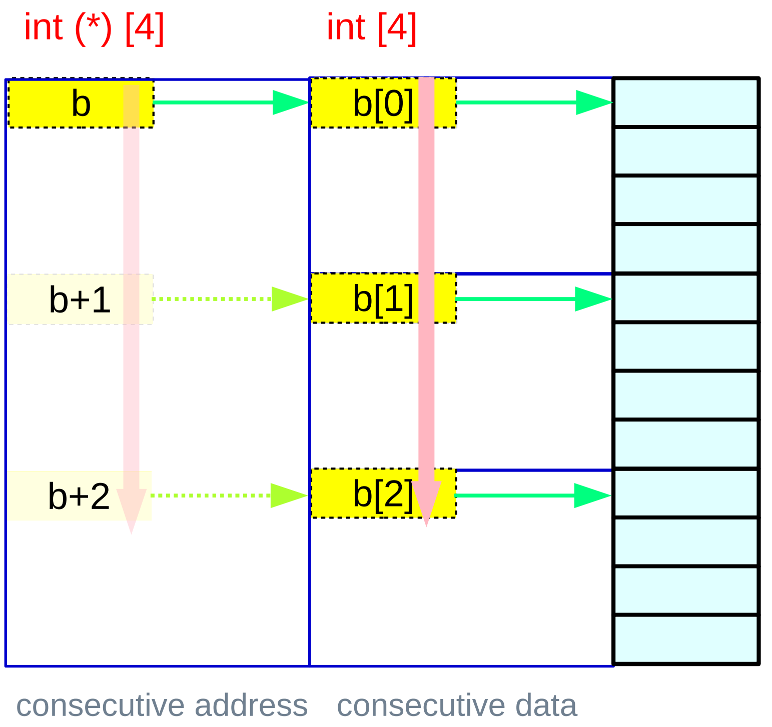
int * c[M] ;

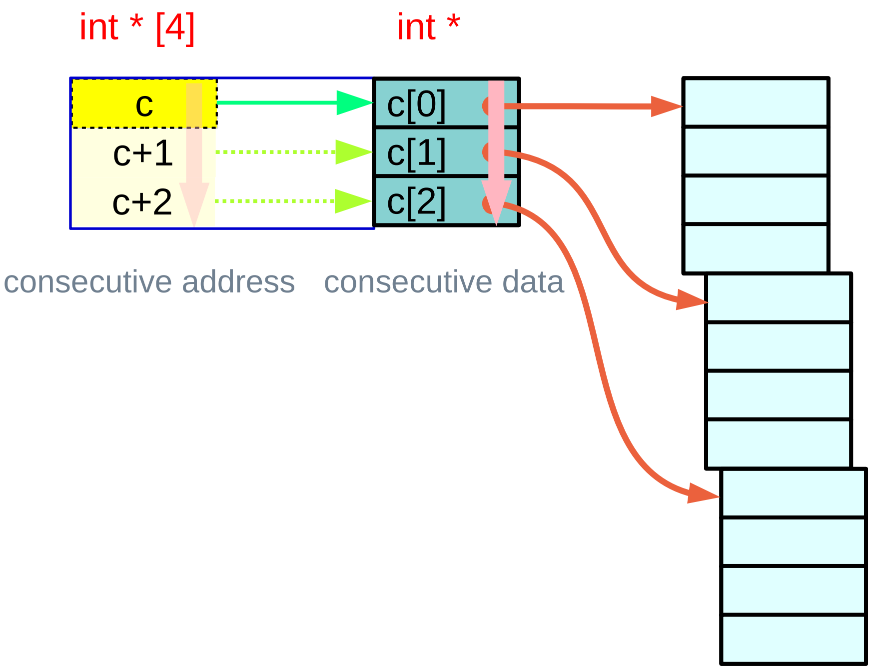
with proper assignments

*(b+m) ⟺ b[m]
*(b[m]+n) ⟺ b[m][n]

*(c+m) ⟺ c[m] or
*(c[m]+n) ⟺ c[m][n]

Pointer Arrays vs Array Pointers





int (*b)[N] ;

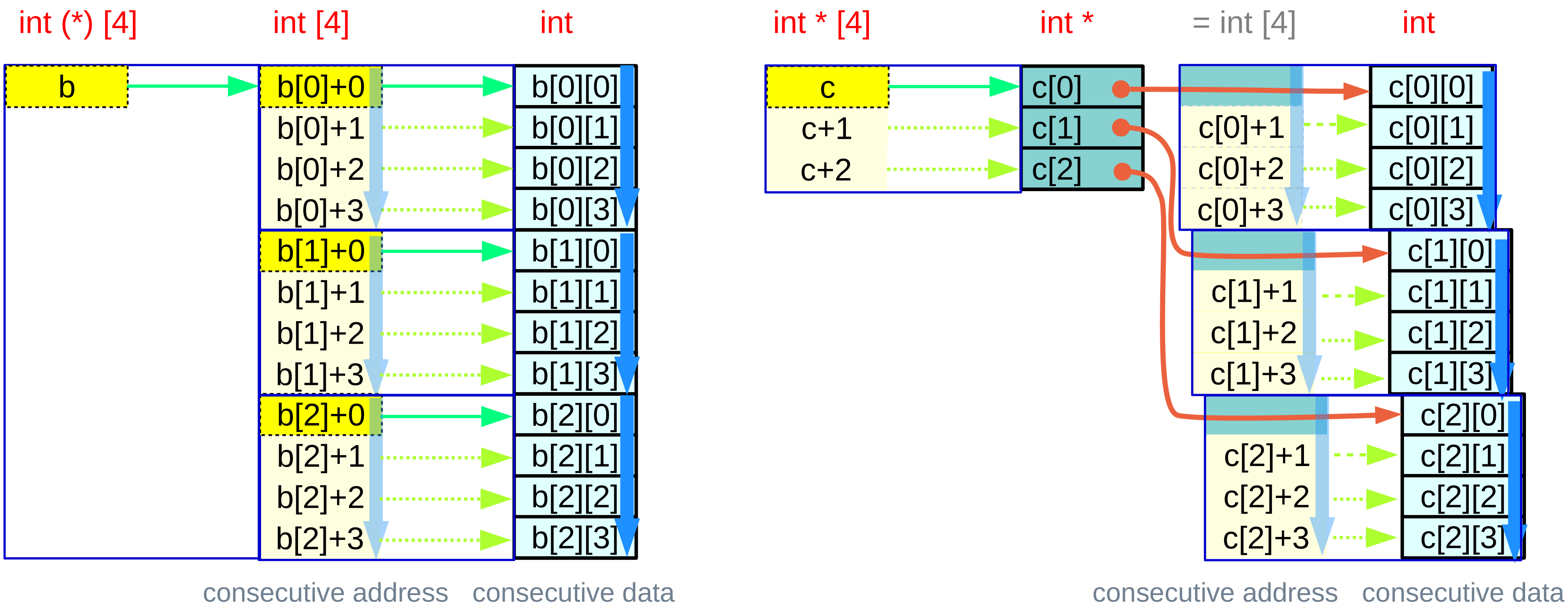
int * c[M] ;

with proper assignments

*(b+m)	⟺	b[m]
*(b[m]+n)	⟺	b[m][n]

*(c+m)	⟺	c[m] or
*(c[m]+n)	⟺	c[m][n]

Pointer Arrays vs Array Pointers



int (*b)[N] ;

int * c[M] ; with proper assignments

*(b+m)	⟺	b[m]
*(b[m]+n)	⟺	b[m][n]

*(c+m)	⟺	c[m]
*(c[m]+n)	⟺	c[m][n]

Three contiguity constraints for 3-d arrays

Pointer Array Approach (array of pointers)

c[i][j][k] → *(c[i][j] +k)
*(c[i][j] +k) → *(*(c[i] +j) +k)
((c[i] +j) +k) → *(*(*(c +i) +j) +k)

contiguous **int**	int
contiguous pointers to **int**	int *
contiguous double pointers to **int**	int **

the contiguity constraints are satisfied by allocating arrays of pointers

Array Pointer Approach (pointer to arrays)

c[i][j][k] → *(c[i][j] +k)
*(c[i][j] +k) → *(*(c[i] +j) +k)
((c[i] +j) +k) → *(*(*(c +i) +j) +k)

contiguous **0-d** arrays	int	int
contiguous **1-d** arrays	int [4]	int *
contiguous **2-d** arrays	int [3][4]	int (*) [4]

The contiguity constraints are satisfied by row major ordered linear data layout

Contiguous array pointers c[i][j][k] ≡ *(c[i][j] +k)

c[0][0][0] = *(c[0][0] + 0)
c[0][0][1] = *(c[0][0] + 1)
c[0][0][2] = *(c[0][0] + 2)
c[0][0][3] = *(c[0][0] + 3)
c[0][1][0] = *(c[0][1] + 0)
c[0][1][1] = *(c[0][1] + 1)
c[0][1][2] = *(c[0][1] + 2)
c[0][1][3] = *(c[0][1] + 3)

⋮ ⋮

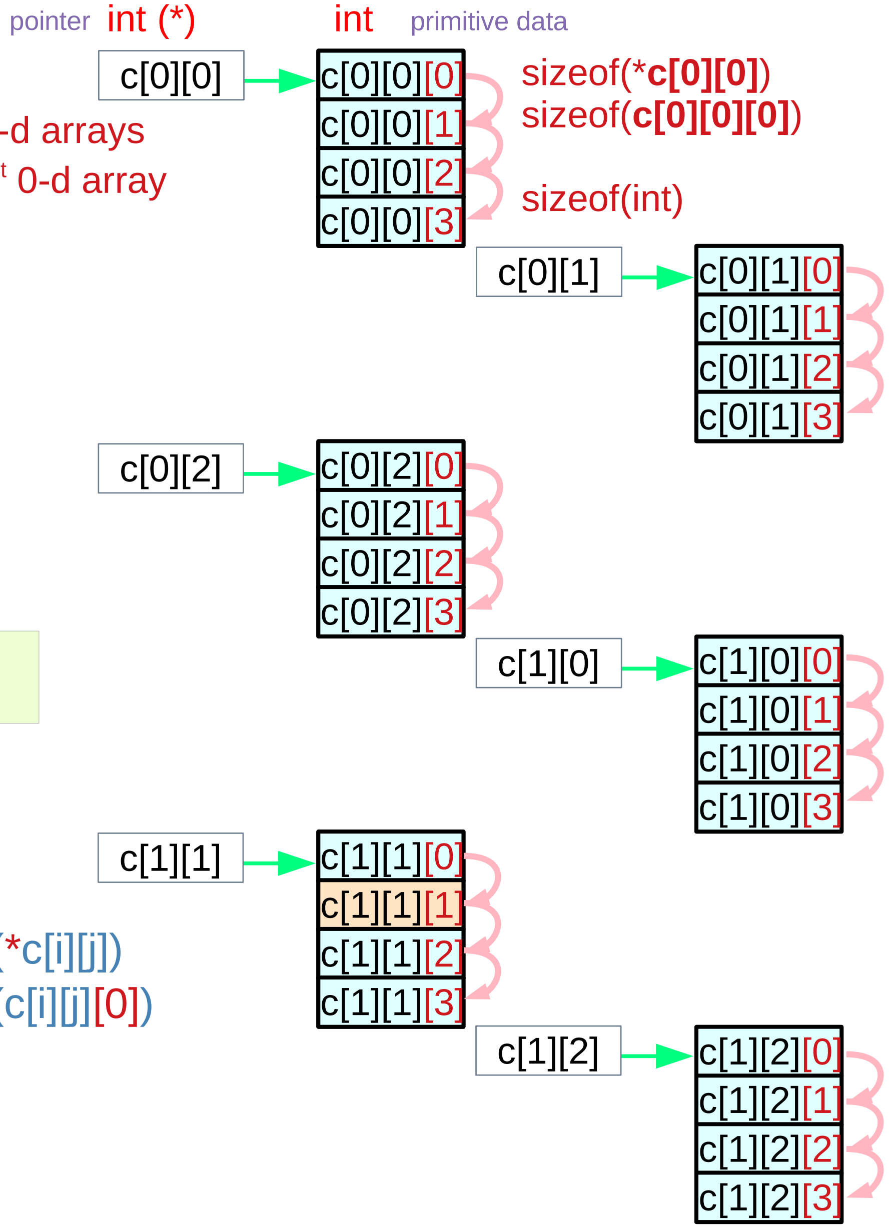
contiguous 1-d
array elements

c[i][j]
int [4] **4** contiguous 0-d arrays
int * points to the 1st 0-d array
int 0-d array

sizeof(**c[i][j]**) [k]
sizeof(**c[i][j][k]**) * 4
sizeof(**int**) * 4

int c[2][3][4];

Address Value
c[i][j] + k
&c[i][j][0] + k * sizeof(*c[i][j])
&c[i][j][0] + k * sizeof(c[i][j][0])
&c[i][j][0] + k * 4



pointer int (*)
int primitive data
sizeof(*c[0][0])
sizeof(c[0][0][0])
sizeof(int)

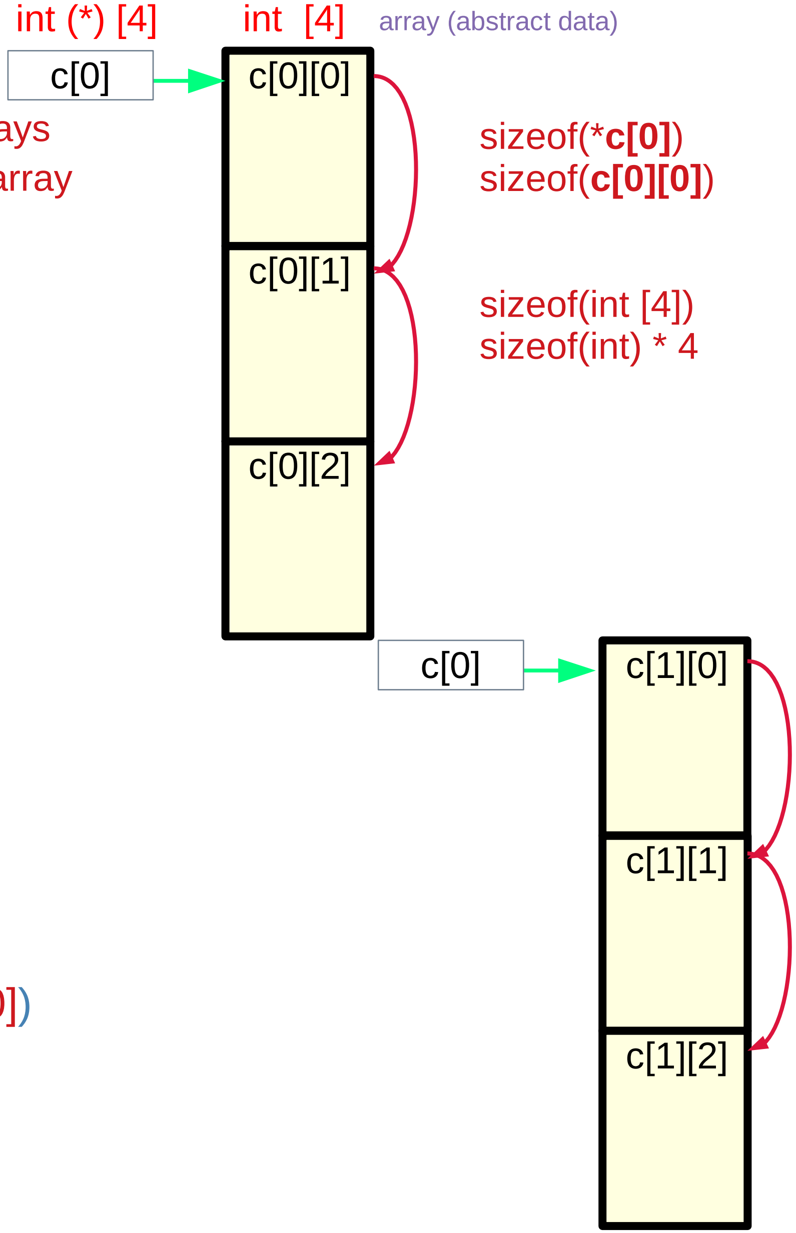
c[0][0] → c[0][0][0] c[0][0][1] c[0][0][2] c[0][0][3]
c[0][1] → c[0][1][0] c[0][1][1] c[0][1][2] c[0][1][3]
c[0][2] → c[0][2][0] c[0][2][1] c[0][2][2] c[0][2][3]
c[1][0] → c[1][0][0] c[1][0][1] c[1][0][2] c[1][0][3]
c[1][1] → c[1][1][0] c[1][1][1] c[1][1][2] c[1][1][3]
c[1][2] → c[1][2][0] c[1][2][1] c[1][2][2] c[1][2][3]

Contiguous array pointers c[i][j] ≡ *(c[i] +j)

c[0][0] = *(c[0] + 0)
c[0][1] = *(c[0] + 1)
c[0][2] = *(c[0] + 2)
c[1][0] = *(c[1] + 0)
c[1][1] = *(c[2] + 1)
c[1][2] = *(c[3] + 2)

c[i]
int [3][4] **3** contiguous 1-d arrays
int (*) [4] points to the 1st 1-d array
int [4] 1-d array

[j] [k]
sizeof(**c[i]**)
sizeof(**c[i][j][k]**) * 3 * 4
sizeof(**int**) * 3 * 4



int c[2][3][4];

Address Value
c[i] + j
&c[i][0][0] + j * sizeof(*c[i])
&c[i][0][0] + j * sizeof(c[i][0])
&c[i][0][0] + j * 4 * 4

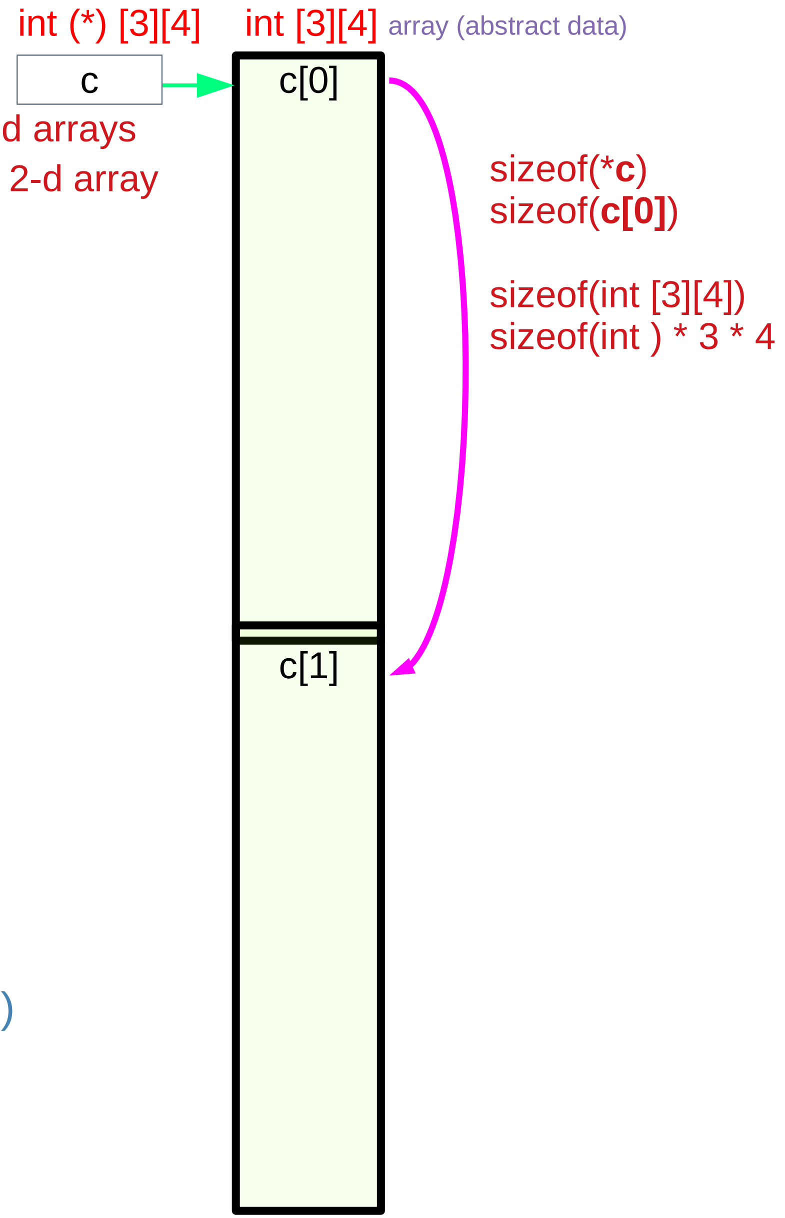
Contiguous array pointers c[i] ≡ *(c +i)

c[0] = *(c + 0)
c[1] = *(c + 1)

c

int [2][3][4]	**2** contiguous 2-d arrays
Int (*) [3][4]	points to the 1st 2-d array
int [3][4]	2-d array

sizeof(**c**)
sizeof(**c[i][j][k]**) * 2 * 3 * 4
sizeof(**int**) * 2 * 3 * 4



int c[2][3][4];

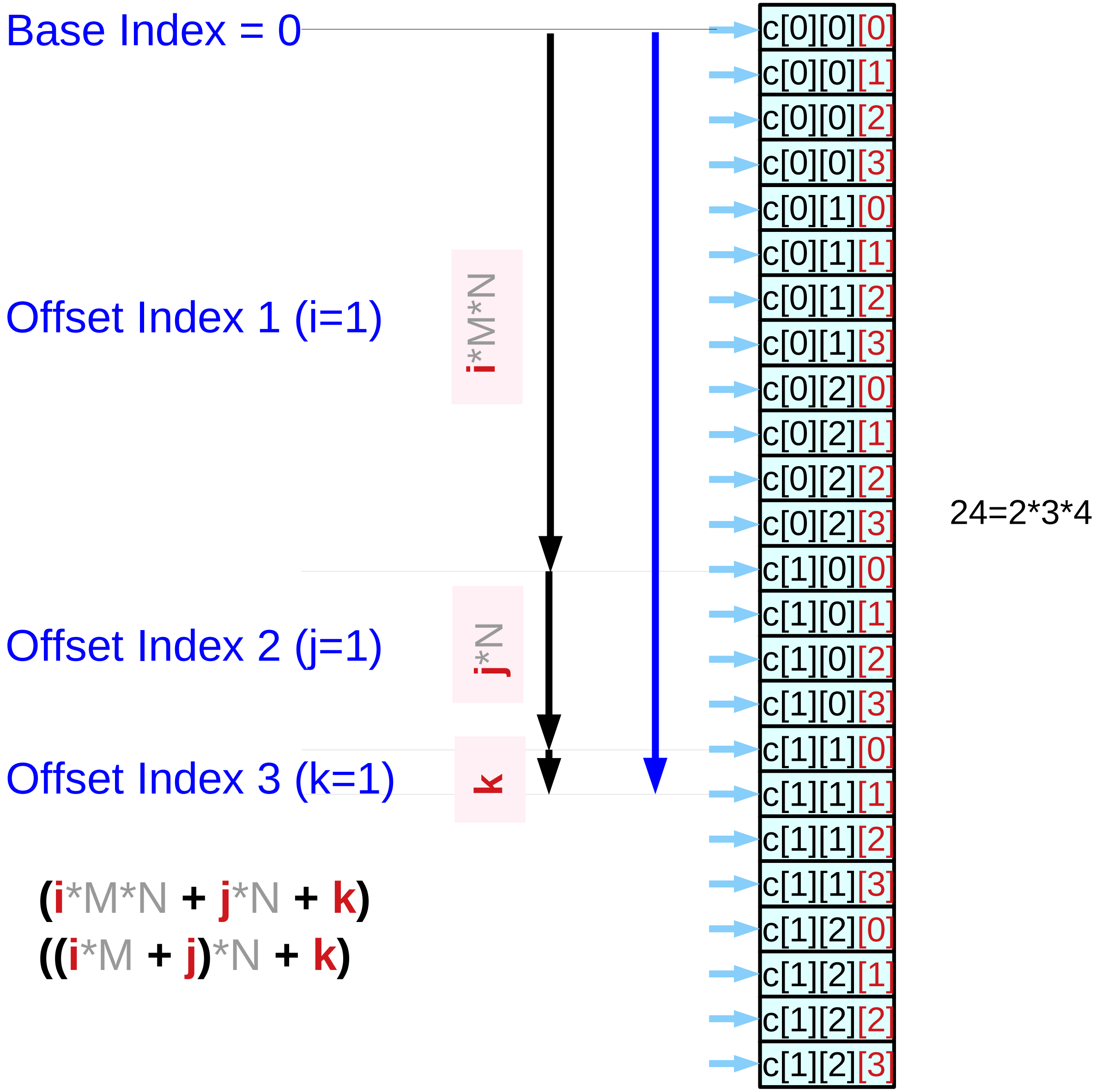
Address Value
c + i
&c[0][0][0] + i * sizeof(*c)
&c[0][0][0] + i * sizeof(c[0])
&c[0][0][0] + i * 4 * 3 * 4

Contiguous linear layout

```
int     c [L][M][N];

        c [i][j][k];
```

L	M	N
i	**j**	**k**
i*M*N	**j*N**	**k**

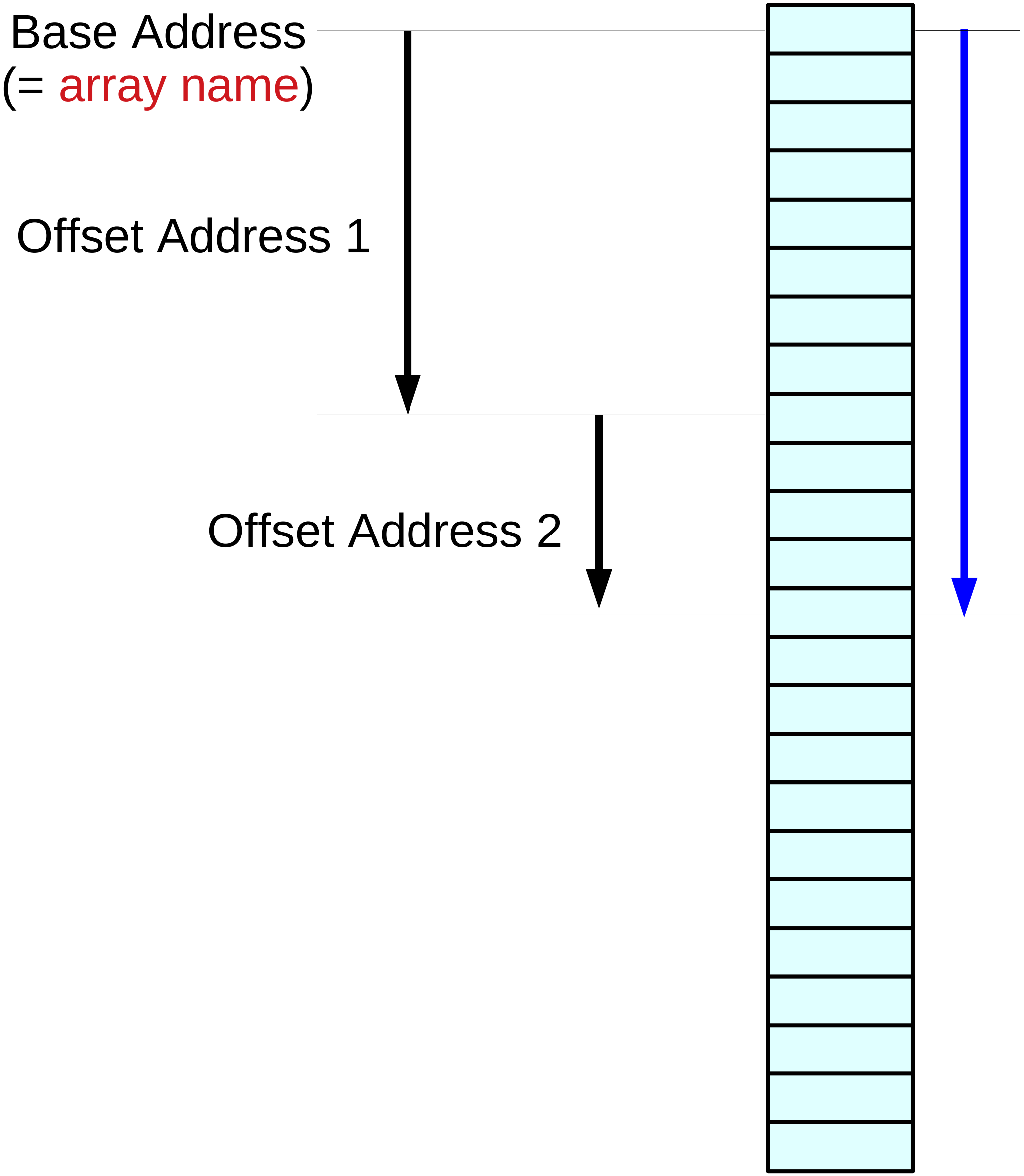


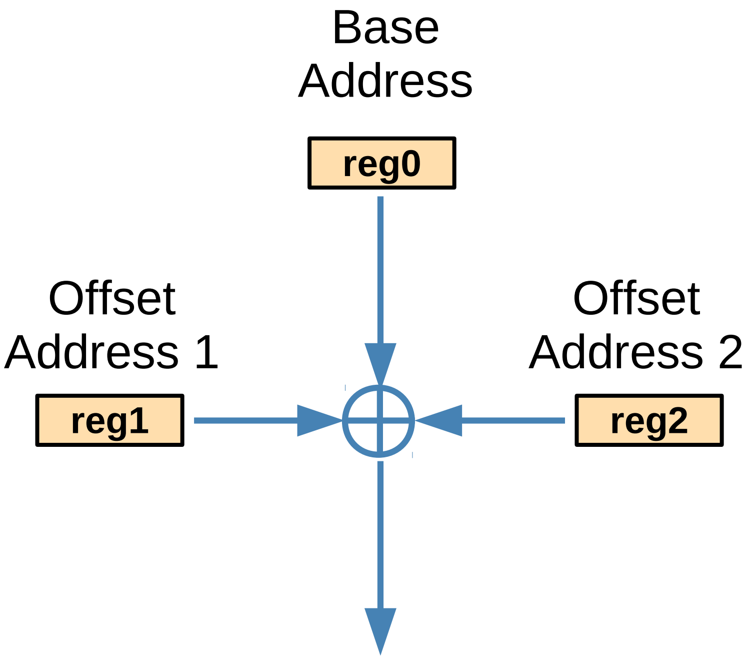
c[0][0][0]
c[0][0][1]
c[0][0][2]
c[0][0][3]
c[0][1][0]
c[0][1][1]
c[0][1][2]
c[0][1][3]
c[0][2][0]
c[0][2][1]
c[0][2][2]
c[0][2][3]
c[1][0][0]
c[1][0][1]
c[1][0][2]
c[1][0][3]
c[1][1][0]
c[1][1][1]
c[1][1][2]
c[1][1][3]
c[1][2][0]
c[1][2][1]
c[1][2][2]
c[1][2][3]

$(i*M*N + j*N + k)$

$((i*M + j)*N + k)$

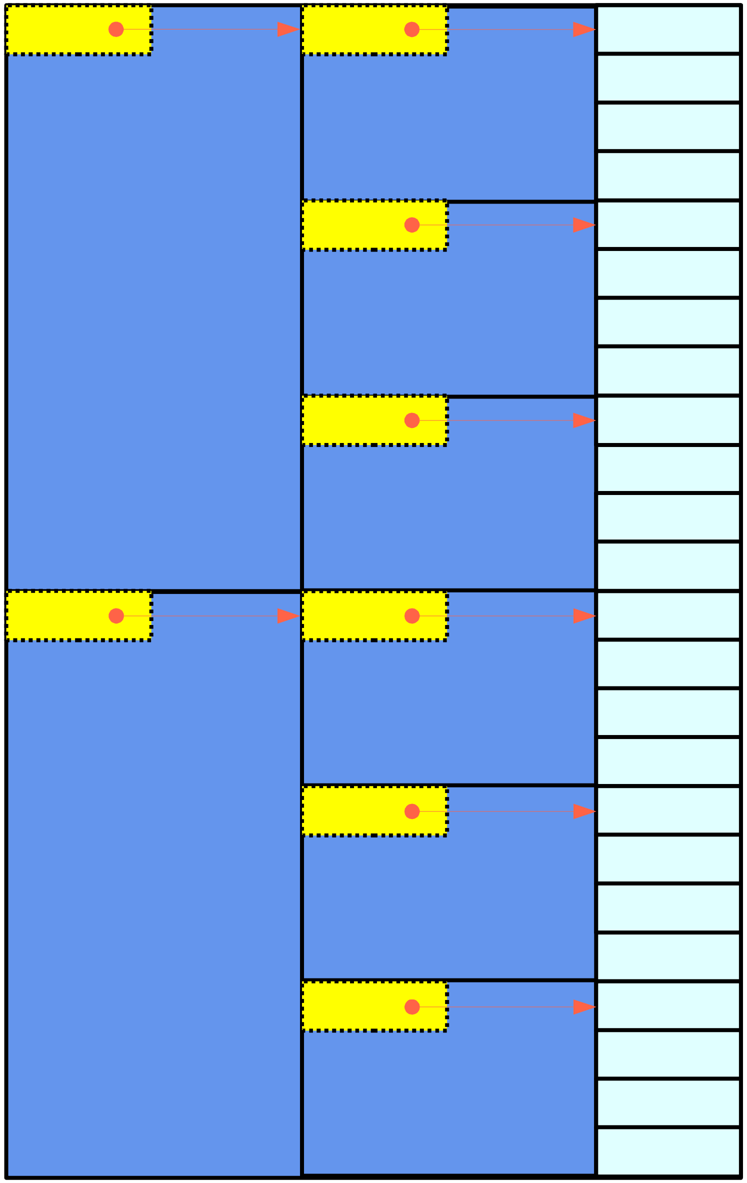
Base and Offset Addressing

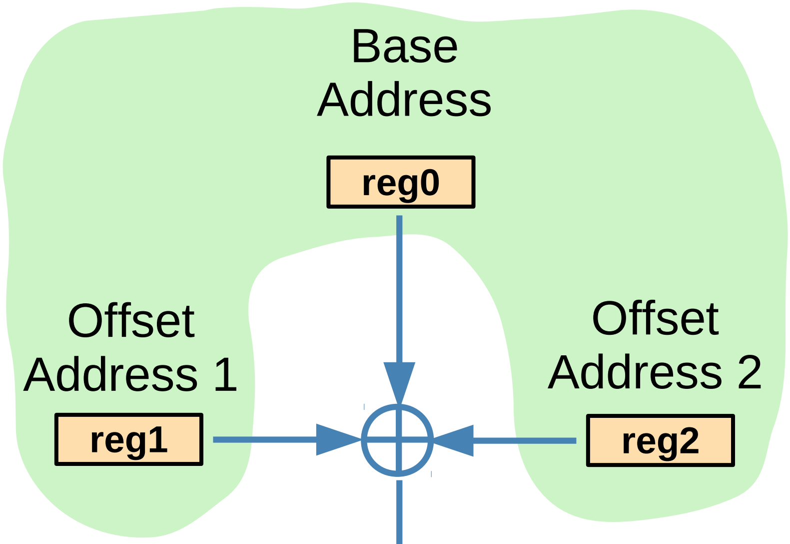




compiler
assembly instruction
registers in the CPU

Array Pointer Approach





register based address **computations**
eliminate the pointer arrays – by a compiler

Array Pointer Approach
(pointer to arrays)

References

[1] Essential C, Nick Parlante
[2] Efficient C Programming, Mark A. Weiss
[3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
[4] C Language Express, I. K. Chun