

Angle Recoding 2. Wu

2. AR (Angle Recode)

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① AR [Hu]

skip certain micro rotations

the rotation sequence $\mu(i) = \{-1, 0, +1\}$

$\mu(i) = 0 \rightarrow$ skip

desire to minimize

$$\sum_{i=0}^N |\mu(i)|$$

so that the total number of CORDIC iterations can be minimized

Angle Recoding \leftarrow Multiplier Recoding

angle recoding method for efficient implementation of the CORDIC algorithm
Hu & Naganathan, ISCAS 89

Greedy algorithm

the angle quantization error

$$\xi_{m,AR} \equiv \theta - \sum_{i=0}^M \mu(i) a(i)$$

$$\theta(0) = \theta, \{ \mu(i) = 0, 0 \leq i \leq N-1 \}, k=0$$

repeat until $|\theta(k)| < a(N-1) \rho_0$

Choose $i_k, 0 \leq i_k \leq N-1$

$$| |\theta(k)| - a(i_k) | = \underset{0 \leq i \leq N-1}{\text{Min}} | |\theta(k)| - a(i) |$$

$$\theta(k+1) = \theta(k) - \mu(i_k) a(i_k)$$

$$\mu(i_k) = \text{Sign}(\theta(k))$$

AQ & AR

$$\begin{aligned}\xi_{m,AR} &\equiv \theta - \sum_{i=0}^{N-1} \mu(i) a(i) \\ &= \theta - \left[\sum_{i=0}^{N-1} \tan^{-1}(\alpha(i) \cdot 2^{-s(i)}) \right] \\ &= \theta - \left[\sum_{i=0}^{N-1} \tilde{\theta}(i) \right]\end{aligned}$$

$$N' \triangleq \sum_{i=0}^{N-1} |\mu(i)| \quad \text{the effective transition number}$$

$s(j) \in \{0, 1, \dots, N-1\}$ the rotational sequence
determines the micro-rotation angle
in the j -th iteration

$\alpha(j) \in \{-1, 0, +1\}$ the directional sequence
controls the direction of
the j -th micro-rotation of $a(s(j))$

$\tilde{\theta}(j)$ the j th micro-rotation angle

$$\tilde{\theta}(j) = \tan^{-1}(\alpha(j) \cdot 2^{-s(j)})$$

AR essentially tries to approximate θ
with the combination of selected angle elements
from a pre-defined elementary angle set (EAS).

the EAS consists of all possible values of $\tilde{\theta}(j)$'s

the EAS S_1 in AR

$$S_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\}, s^* \in \{0, 1, \dots, N-1\} \}$$

$$\xi_{m, AR} \equiv \theta - \sum_{i=0}^{N-1} \mu(i) a(i)$$

$$= \theta - \left[\sum_{i=0}^{N-1} \tan^{-1}(\alpha(i) \cdot 2^{-s(i)}) \right]$$

$$= \theta - \left[\sum_{i=0}^{N-1} \tilde{\theta}(i) \right]$$

$$\xi_m \triangleq \theta - \sum_{i=0}^{N_A-1} \theta_i$$

AR performs AQ of the target angle θ

the sub-angle θ_i becomes $\tilde{\theta}(i) = \tan^{-1}(\alpha(i) \cdot 2^{-s(i)})$

N_A

N'

EAS S_1 AR

elementary angle	value
$r(1) = \text{atan}(-2^{\{-0\}})$	
$r(2) = \text{atan}(-2^{\{-1\}})$	
$r(3) = \text{atan}(-2^{\{-2\}})$	
$r(4) = \text{atan}(-2^{\{-3\}})$	
$r(5) = \text{atan}(-2^{\{-4\}})$	
$r(6) = \text{atan}(-2^{\{-5\}})$	
$r(7) = \text{atan}(-2^{\{-6\}})$	
$r(8) = \text{atan}(-2^{\{-7\}})$	
$r(9) = \text{atan}(0)$	
$r(10) = \text{atan}(2^{\{-7\}})$	
$r(11) = \text{atan}(2^{\{-6\}})$	
$r(12) = \text{atan}(2^{\{-5\}})$	
$r(13) = \text{atan}(2^{\{-4\}})$	
$r(14) = \text{atan}(2^{\{-3\}})$	
$r(15) = \text{atan}(2^{\{-2\}})$	
$r(16) = \text{atan}(2^{\{-1\}})$	
$r(17) = \text{atan}(2^{\{-0\}})$	

```
>> mu = [1, 0, 0, -1, 0, 0, -1, -1, 0, 0, 0, 1, 0, 0, 0, 1]
>> length(mu)
ans = 16
>> s = [ 0: 15]
>> atan(1)
ans = 0.78540
>> pi/4
ans = 0.78540
>> sum(atan(2.^(-s)) .* mu)
ans = 0.63813
>> 13 * pi / 32
ans = 1.2763
```