Angle Recoding 2. Wu
2. AR (Angle Recode)

20180825 Sat

① AR [Hu]

skip certain micro rotations.

the rotation sequence $\mu(i) = \{-1, 0, +1\}$

$$\mu(i) = 0 \rightarrow \text{skip}$$

desire to minimize

$$\boxed{\sum_{i=0}^{N} |\mu(i)|}$$

so that the total number of CORDIC iterations can be minimized

Angle Recoding ← Multiplier Recoding

angle recoding method for efficient
implementation of the CORDIC algorithm
Hu & Naganathan, ISCAS 89

Greedy algorithm

the angle quantization error

$$\boxed{\xi_{m,AR} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i)}$$

$\theta(0) = \theta, \ \{\mu(i) = 0, \ 0 \leqslant i \leqslant N-1\}, \ k = 0$.

repeat until $|\theta(k)| < a(N-1)$ Do

Choose i_k, $0 \leqslant i_k \leqslant N-1$

$$\big|\,|\theta(k)| - a(i_k)\big| = \min_{0 \leqslant i \leqslant N-1} \big|\,|\theta(k)| - a(i_k)\big|$$

$$\theta(k+1) = \theta(k) - \mu(i_k)\, a(i_k)$$

$$\mu(i_k) = \mathrm{Sign}(\theta(k))$$

AQ & AR

$$\xi_{m,AR} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i)$$

$$= \theta - \left[\sum_{i=0}^{N'-1} \tan^{-1}\left(\alpha(j) \cdot 2^{-s(j)}\right) \right]$$

$$= \theta - \left[\sum_{i=0}^{N'-1} \tilde{\theta}(j) \right]$$

$N' \triangleq \sum_{i=0}^{N-1} |\mu(i)|$ the effective transition number

$s(j) \in \{0, 1, \cdots, N-1\}$ the rotational sequence
determines the micro-rotation angle
in the j-th iteration

$\alpha(j) \in \{-1, 0, +1\}$ the directional sequence
controls the direction of
the j-th micro-rotation of $a(s(j))$

$\tilde{\theta}(j)$ the jth micro-rotation angle

$$\tilde{\theta}(j) = \tan^{-1}\left(\alpha(j) \cdot 2^{-s(j)}\right)$$

AR essentially tries to approximate θ
with the combination of selected angle elements
from a pre-defined elementary angle set (EAS).

the EAS consists of all possible values of $\tilde{\theta}(j)$'s

the EAS S_1 in AR

$$S_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\}, \ s^* \in \{0, 1, \cdots, N-1\} \}$$

$$\xi_{m, AR} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i)$$

$$= \theta - \left[\sum_{i=0}^{N'-1} \tan^{-1}(\alpha(j) \cdot 2^{-S(j)}) \right]$$

$$= \theta - \left[\sum_{i=0}^{N'-1} \tilde{\theta}(j) \right]$$

$$\xi_m \triangleq \theta - \sum_{i=0}^{N_A - 1} \theta_i$$

AR performs AQ of the target angle θ

the sub-angle θ_i becomes $\tilde{\theta}(i) = \tan^{-1}(\alpha(i) \cdot 2^{-S(i)})$

N_A N'

EAS S, AR

elementary angle	value
r(1) = atan(-2^{-0})	
r(2) = atan(-2^{-1})	
r(3) = atan(-2^{-2})	
r(4) = atan(-2^{-3})	
r(5) = atan(-2^{-4})	
r(6) = atan(-2^{-5})	
r(7) = atan(-2^{-6})	
r(8) = atan(-2^{-7})	
r(9) = atan(0)	
r(10) = atan(2^{-7})	
r(11) = atan(2^{-6})	
r(12) = atan(2^{-5})	
r(13) = atan(2^{-4})	
r(14) = atan(2^{-3})	
r(15) = atan(2^{-2})	
r(16) = atan(2^{-1})	
r(17) = atan(2^{-0})	

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>> mu = [1, 0, 0, -1, 0, 0, -1, -1, 0, 0, 0, 1, 0, 0, 0, 1]
>> length(mu)
ans =  16
>> s =[ 0: 15]
>> atan(1)
ans =  0.78540
>> pi/4
ans =  0.78540
>> sum(atan(2.^(-s)) .* mu)
ans =  0.63813
>> 13 * pi / 32
ans =  1.2763
```