

Binary Numbers (1A)

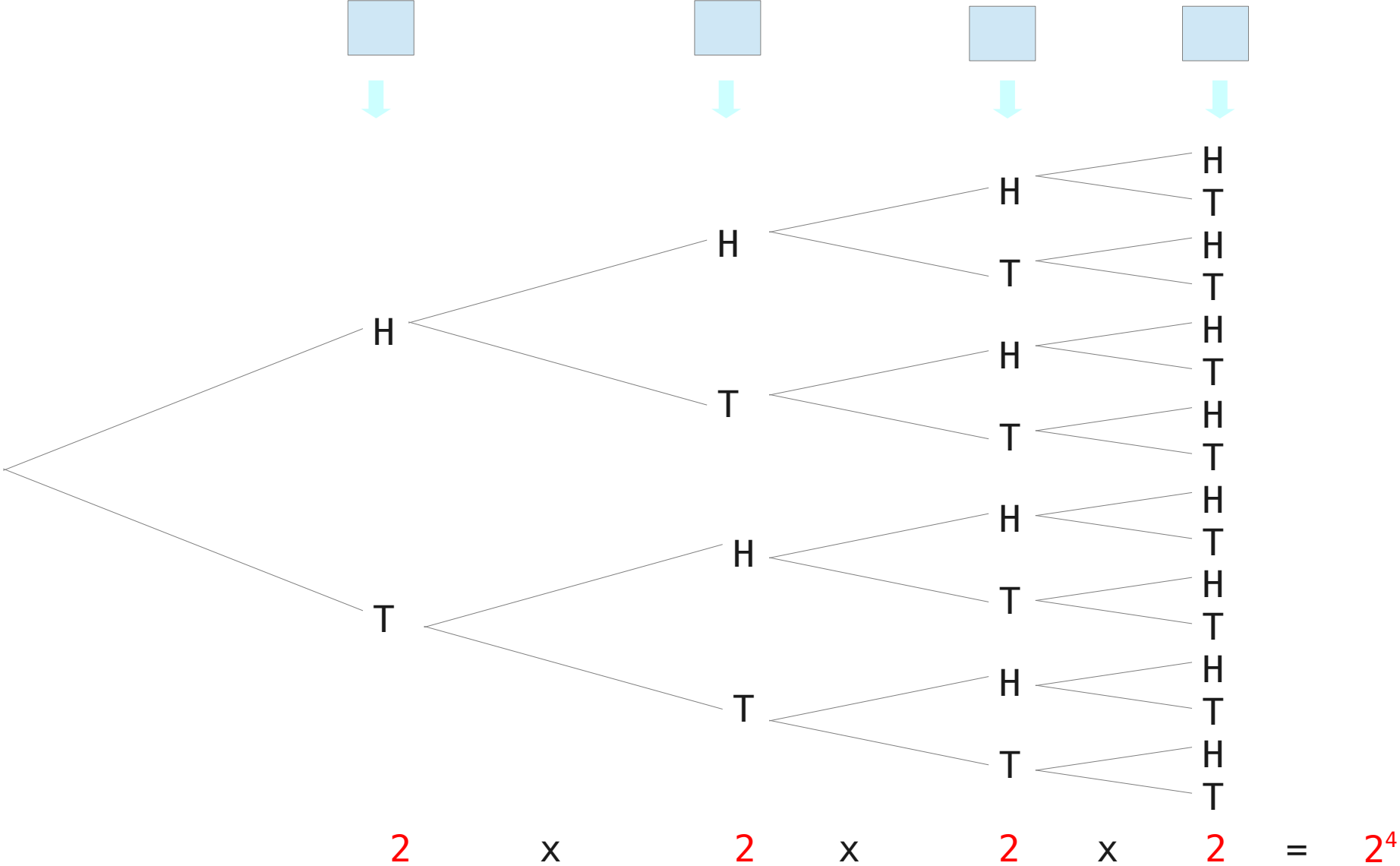
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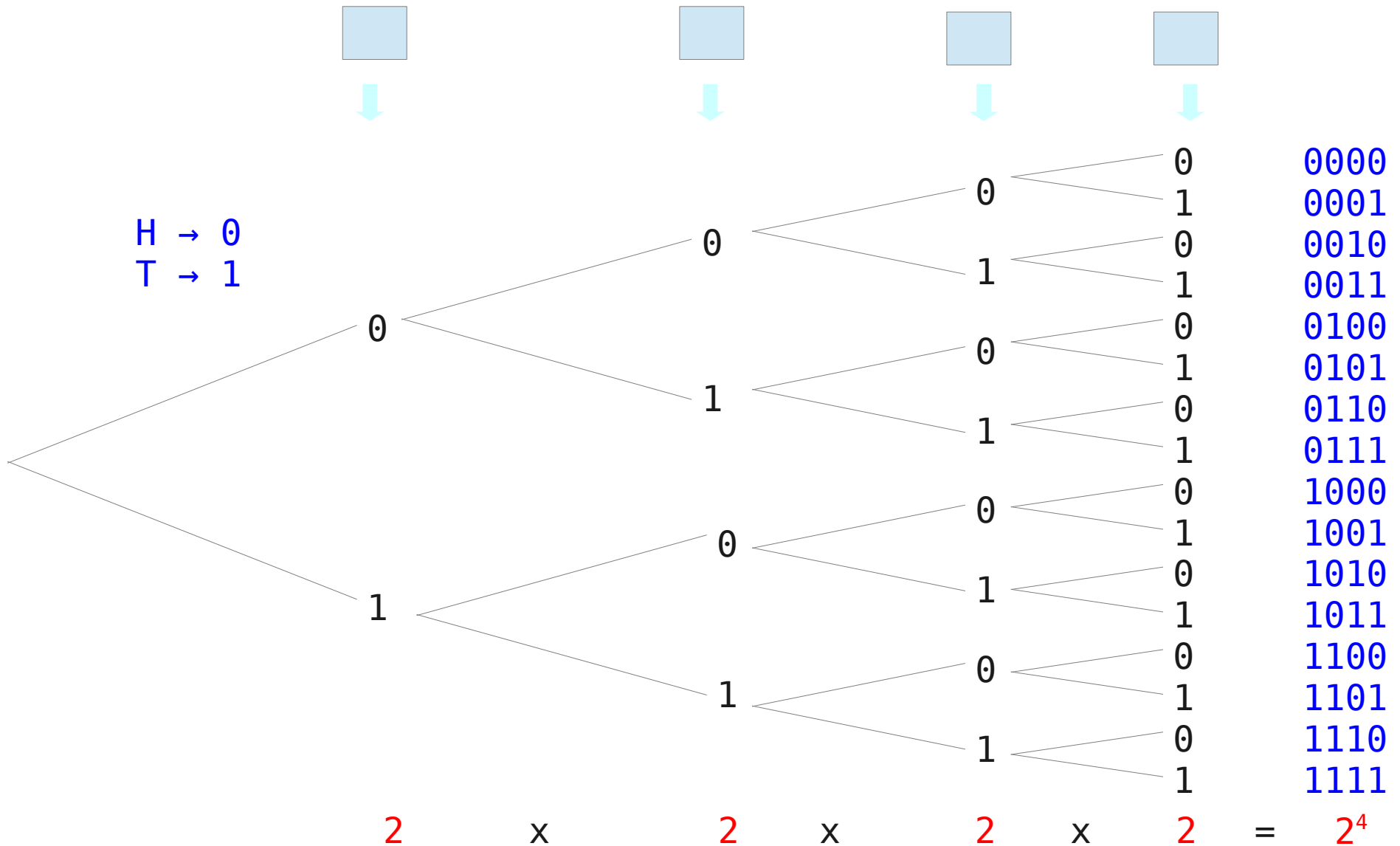
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Coin Tossing



Coin Tossing & Binary Numbers



Number Systems

radix=2

	2^3	2^2	2^1	2^0
	0	0	0	0
	0	0	0	1
	0	0	1	0
	0	0	1	1
	0	1	0	0
	0	1	0	1
	0	1	1	0
	0	1	1	1
	1	0	0	0
	1	0	0	1
	1	0	1	0
	1	0	1	1
	1	1	0	0
	1	1	0	1
	1	1	1	0
	1	1	1	1

Binary

radix=16

16^0
0
1
2
3
4
5
6
7
8
9
A
B
C
D
E
F

Hexadecimal

radix=8

8^1	8^0
0	0
0	1
0	2
0	3
0	4
0	5
0	6
0	7
1	0
1	1
1	2
1	3
1	4
1	5
1	6
1	7

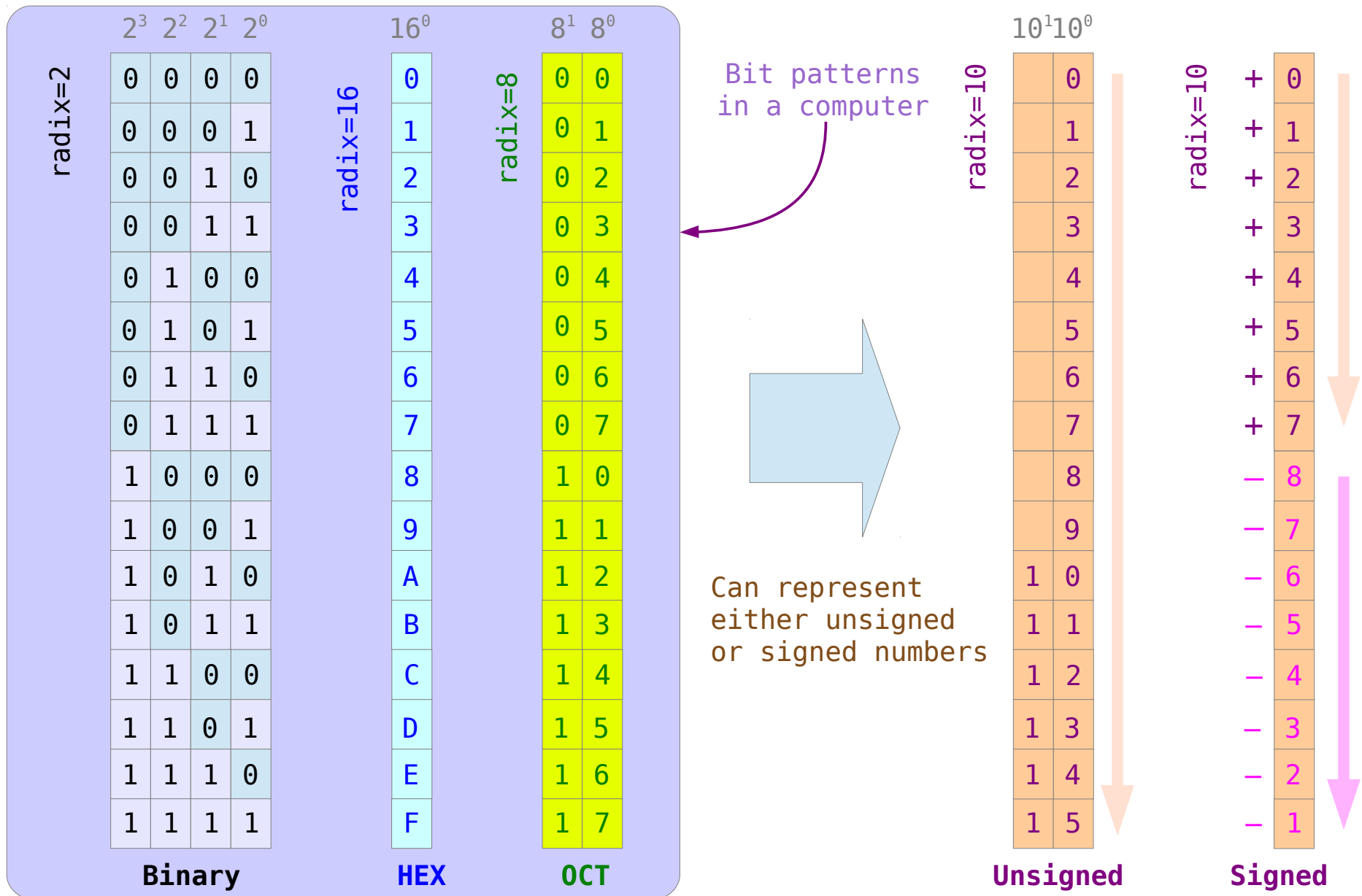
Octal

radix=10

10^1	10^0
	0
	1
	2
	3
	4
	5
	6
	7
	8
	9
1	0
1	1
1	2
1	3
1	4
1	5

Decimal

Bit Patterns



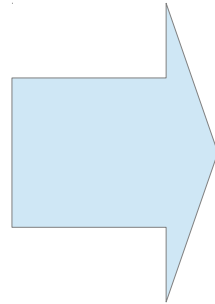
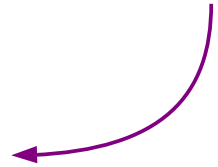
4-Bit Signed Numbers

radix=2

	2^3	2^2	2^1	2^0
0	0	0	0	0
0	0	0	0	1
0	0	0	1	0
0	0	0	1	1
0	0	1	0	0
0	0	1	0	1
0	0	1	1	0
0	0	1	1	1
1	0	0	0	0
1	0	0	0	1
1	0	0	1	0
1	0	0	1	1
1	0	1	0	0
1	0	1	0	1
1	0	1	1	0
1	0	1	1	1
1	1	0	0	0
1	1	0	0	1
1	1	0	1	0
1	1	0	1	1
1	1	1	0	0
1	1	1	0	1
1	1	1	1	0
1	1	1	1	1

Binary

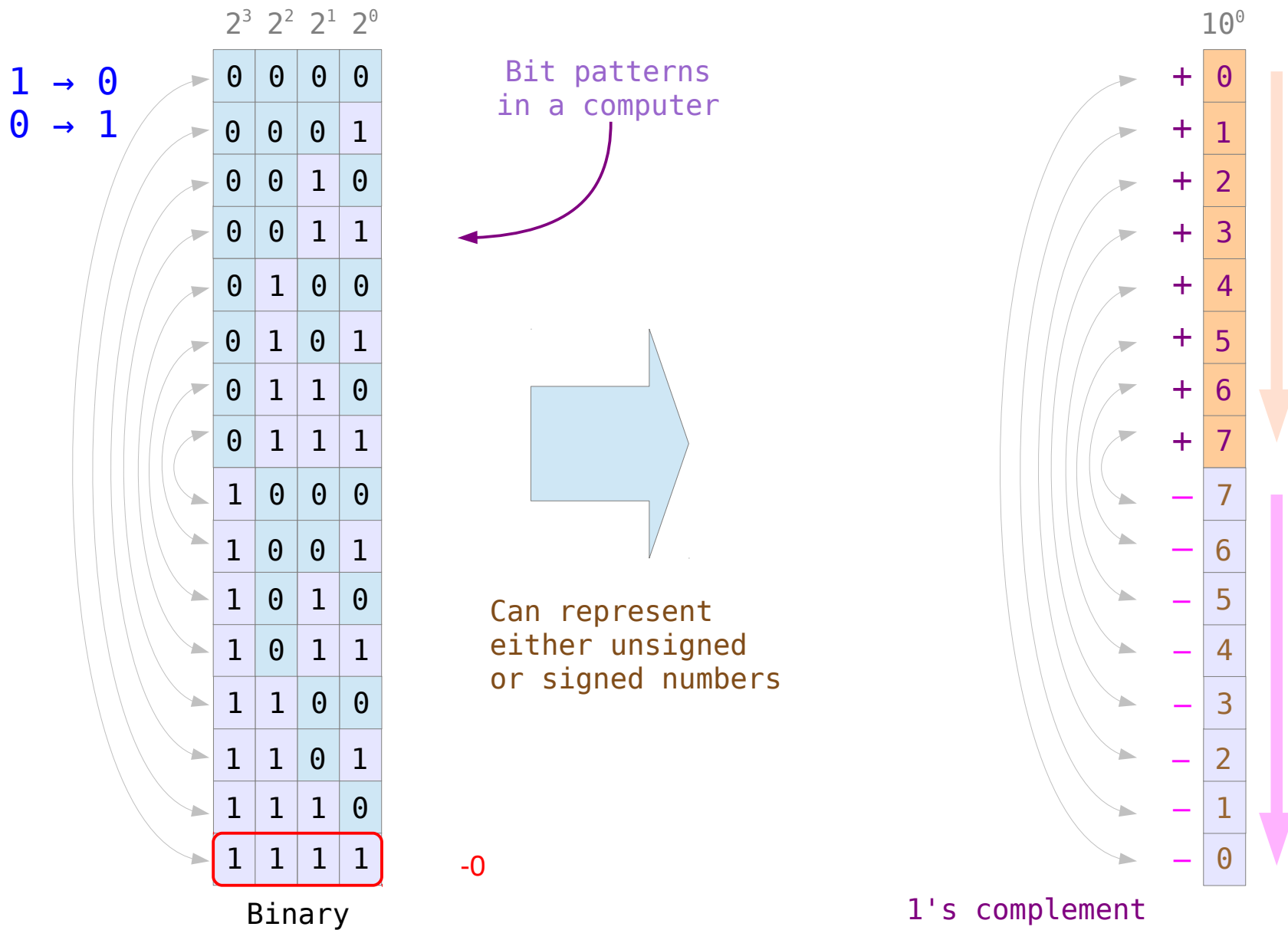
Bit patterns
in a computer



Can represent
either unsigned
or signed numbers

	10^0
radix=10	+ 0
radix=10	+ 1
radix=10	+ 2
radix=10	+ 3
radix=10	+ 4
radix=10	+ 5
radix=10	+ 6
radix=10	+ 7
2's complement	- 8
2's complement	- 7
2's complement	- 6
2's complement	- 5
2's complement	- 4
2's complement	- 3
2's complement	- 2
2's complement	- 1
radix=10	+ 0
radix=10	+ 1
radix=10	+ 2
radix=10	+ 3
radix=10	+ 4
radix=10	+ 5
radix=10	+ 6
radix=10	+ 7
1's complement	- 7
1's complement	- 6
1's complement	- 5
1's complement	- 4
1's complement	- 3
1's complement	- 2
1's complement	- 1
1's complement	0
sign magnitude	+ 0
sign magnitude	+ 1
sign magnitude	+ 2
sign magnitude	+ 3
sign magnitude	+ 4
sign magnitude	+ 5
sign magnitude	+ 6
sign magnitude	+ 7
sign magnitude	- 0
sign magnitude	- 1
sign magnitude	- 2
sign magnitude	- 3
sign magnitude	- 4
sign magnitude	- 5
sign magnitude	- 6
sign magnitude	- 7

1's Complement Bit Pattern



The Sum 1's Complement Numbers

$$\begin{array}{r}
 0000 + 0 \\
 1111 - 0 \\
 \hline
 1111 - 0
 \end{array}$$

$$\begin{array}{r}
 0100 + 4 \\
 1011 - 4 \\
 \hline
 1111 - 0
 \end{array}$$

$$\begin{array}{r}
 +X \\
 -X \\
 \hline
 (2^n - 1)
 \end{array}$$



$$\begin{array}{r}
 0001 + 1 \\
 1110 - 1 \\
 \hline
 1111 - 0
 \end{array}$$

$$\begin{array}{r}
 0101 + 5 \\
 1010 - 5 \\
 \hline
 1111 - 0
 \end{array}$$

n-bit
No carry

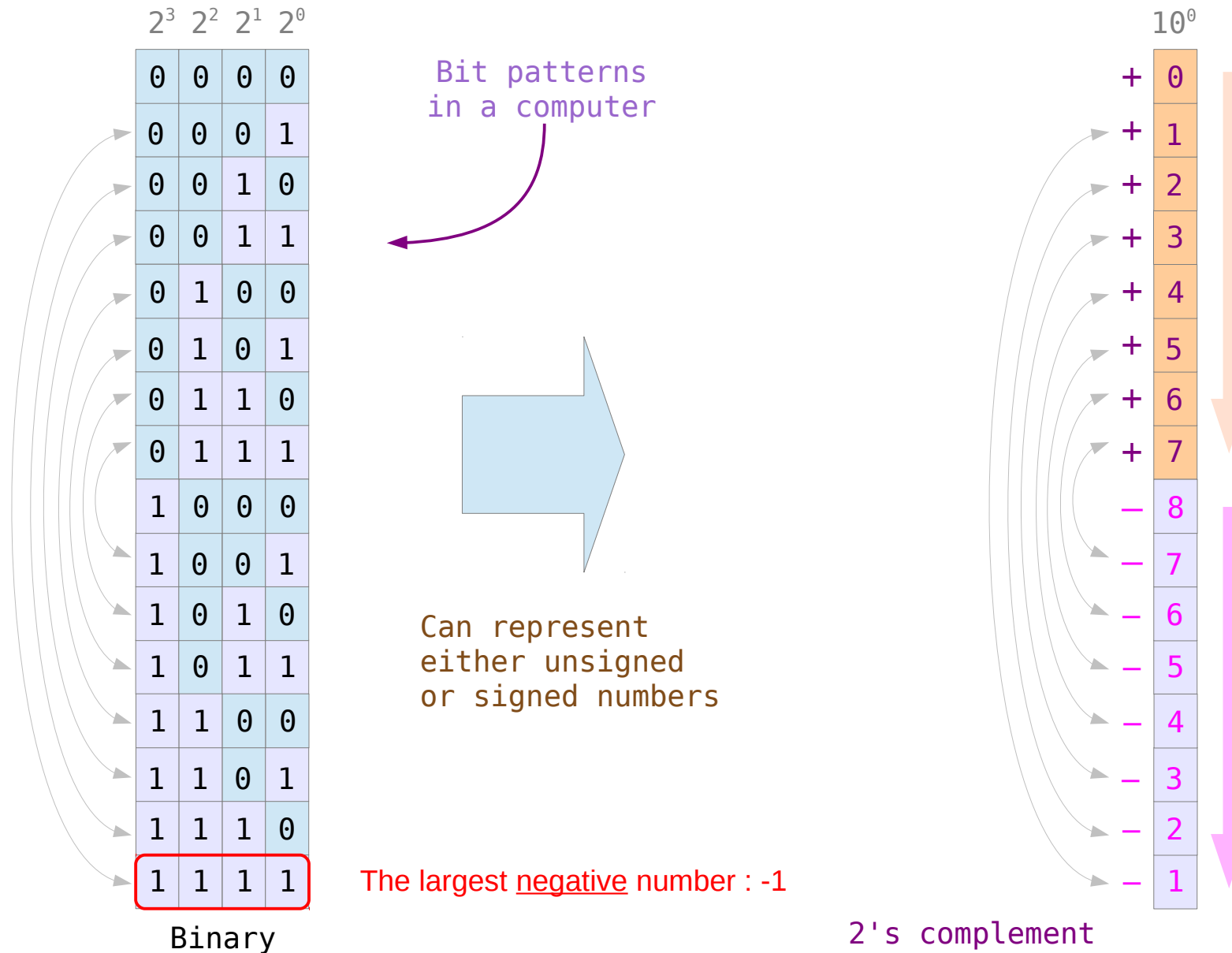
$$\begin{array}{r}
 0010 + 2 \\
 1101 - 2 \\
 \hline
 1111 - 0
 \end{array}$$

$$\begin{array}{r}
 0110 + 6 \\
 1001 - 6 \\
 \hline
 1111 - 0
 \end{array}$$

$$\begin{array}{r}
 0011 + 3 \\
 1100 - 3 \\
 \hline
 1111 - 0
 \end{array}$$

$$\begin{array}{r}
 0111 + 7 \\
 1000 - 7 \\
 \hline
 1111 - 0
 \end{array}$$

2's Complement Bit Pattern




The Sum of 2's Complement Numbers

$$\begin{array}{r}
 0000 \\
 1000 \\
 \hline
 11000
 \end{array}
 \begin{array}{r}
 0 \\
 -8 \\
 \hline
 8
 \end{array}$$

$$\begin{array}{r}
 0100 \\
 1100 \\
 \hline
 10000
 \end{array}
 \begin{array}{r}
 +4 \\
 -4 \\
 \hline
 0
 \end{array}$$

$$\begin{array}{r}
 +X \\
 -X \\
 \hline
 2^n
 \end{array}$$


 (n+1)-bit

$$\begin{array}{r}
 0001 \\
 1111 \\
 \hline
 10000
 \end{array}
 \begin{array}{r}
 +1 \\
 -1 \\
 \hline
 0
 \end{array}$$

$$\begin{array}{r}
 0101 \\
 1011 \\
 \hline
 10000
 \end{array}
 \begin{array}{r}
 +5 \\
 -5 \\
 \hline
 0
 \end{array}$$

$$\begin{array}{r}
 0010 \\
 1110 \\
 \hline
 10000
 \end{array}
 \begin{array}{r}
 +2 \\
 -2 \\
 \hline
 0
 \end{array}$$

$$\begin{array}{r}
 0110 \\
 1010 \\
 \hline
 10000
 \end{array}
 \begin{array}{r}
 +6 \\
 -6 \\
 \hline
 0
 \end{array}$$

$$\begin{array}{r}
 0011 \\
 1101 \\
 \hline
 10000
 \end{array}
 \begin{array}{r}
 +3 \\
 -3 \\
 \hline
 0
 \end{array}$$

$$\begin{array}{r}
 0111 \\
 1001 \\
 \hline
 10000
 \end{array}
 \begin{array}{r}
 +7 \\
 -7 \\
 \hline
 0
 \end{array}$$

Decimal to Binary (1)

$$2 \) \ \underline{14}$$

$$2 \) \ \underline{7} \ \dots \ 0$$

$$2 \) \ \underline{3} \ \dots \ 1$$

$$1 \ \dots \ 1$$

$$14 = 2 \cdot 7 + 0$$

$$7 = 2 \cdot 3 + 1$$

$$3 = 2 \cdot 1 + 1$$

$$14 = 2 \cdot 7 + 0$$

$$14 = 2 \cdot (2 \cdot 3 + 1) + 0$$

$$14 = 2 \cdot (2 \cdot (2 \cdot 1 + 1) + 1) + 0$$

$$14 = 2 \cdot (2 \cdot (2 \cdot 1 + 1) + 1) + 0 \quad 1 \cdot 2^3$$

$$14 = 2 \cdot (2 \cdot (2 \cdot 1 + 1) + 1) + 0 \quad 1 \cdot 2^2$$

$$14 = 2 \cdot (2 \cdot (2 \cdot 1 + 1) + 1) + 0 \quad 1 \cdot 2^1$$

Decimal to Binary (2)

$$14 = 2 \cdot 7 + 0$$

$$14 = 2 \cdot (2 \cdot 3 + 1) + 0$$

$$14 = 2 \cdot (2 \cdot (2 \cdot 1 + 1) + 1) + 0$$

$$14 = 7 \cdot 2 + 0$$

$$14 = (3 \cdot 2 + 1) \cdot 2 + 0$$

$$14 = ((1 \cdot 2 + 1) \cdot 2 + 1) \cdot 2 + 0$$

$$\left(\cdots \left(\left(A_{n-1} r + A_{n-2} \right) r + A_{n-3} \right) r + \cdots + A_1 \right) r + A_0$$

A_{n-1}	A_{n-2}	A_{n-3}		...		A_1	A_0
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References

- [1] <http://en.wikipedia.org/>
- [2] M. M. Mano, C. R. Kime, "Logic and Computer Design Fundamentals", 4th ed.
- [3] M. M. Mano, M. D. Ciletti, "Digital Design", 5th ed.
- [4] D. M. Harris, S. L. Harris, "Digital Design and Computer Architecture"