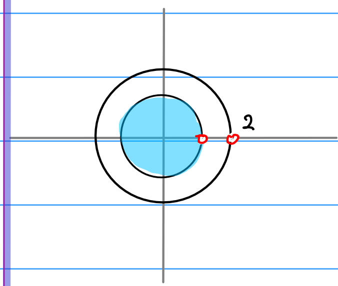
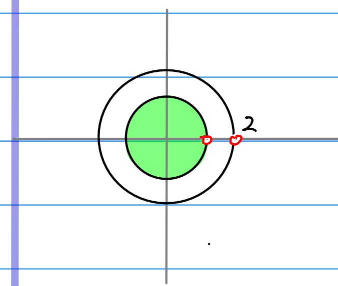
Laurent Series and z-Transform Examples case 4.A

20171016

I





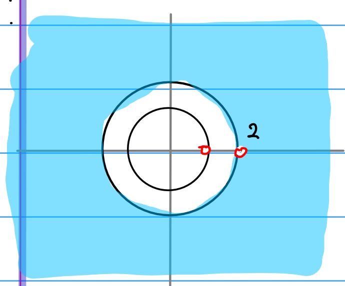
$$a_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

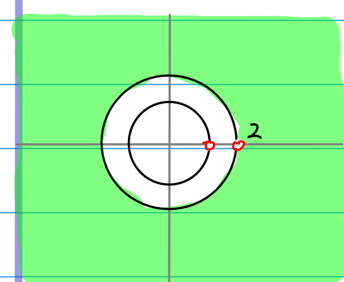
$$x_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=1}^{\infty} \left[1 - 2^{n-1} \right] z^n$$

$$X(z) = \sum_{n=-1}^{\infty} \left[1 - \left(\frac{1}{2}\right)^{n+1} \right] z^{-n}$$

II





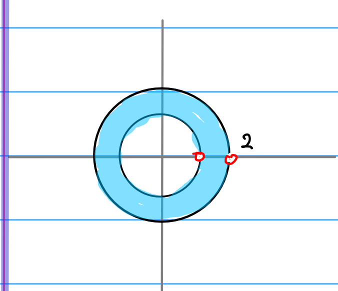
$$a_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

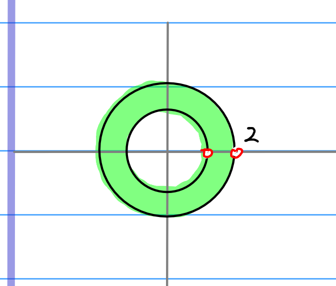
$$x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} \left[2^{n-1} - 1 \right] z^n$$

$$X(z) = \sum_{n=1}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1 \right] z^{-n}$$

III





$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

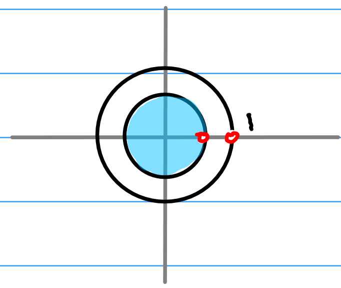
$$x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

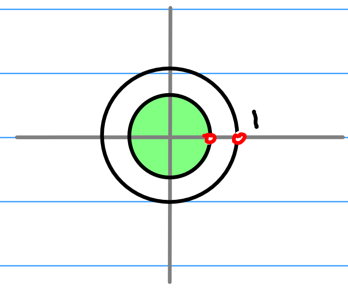
$$f(z) = + \sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n$$

$$X(z) = + \sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n}$$

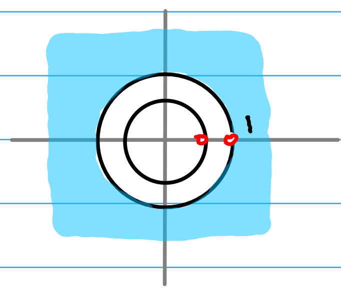
4.A

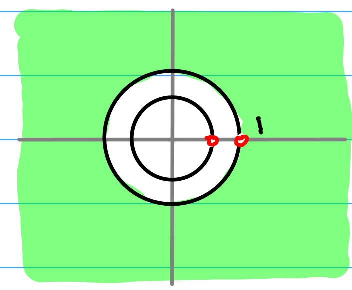
$$f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)} = X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$



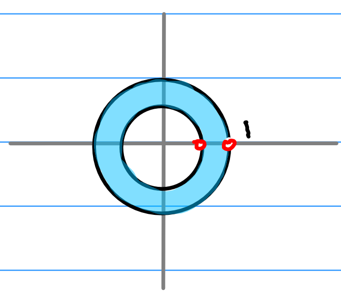


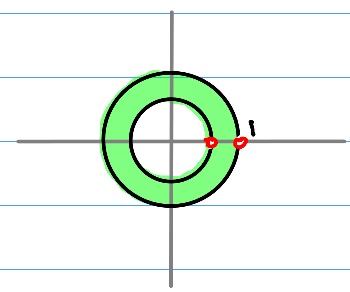
$$\sum_{n=1}^{\infty}\left[1-2^{n-1}\right]z^{n} \xrightarrow{-n} \sum_{n=-1}^{\infty}\left[1-\left(\tfrac{1}{2}\right)^{n+1}\right]z^{-n}$$





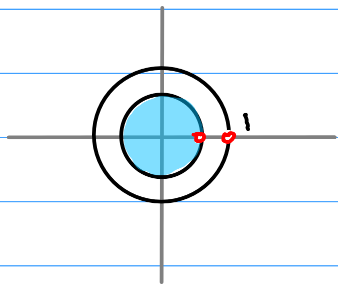
$$\sum_{n=-1}^{-\infty}\left[2^{n-1}-1\right]z^{n} \xrightarrow{-n} \sum_{n=1}^{\infty}\left[\left(\tfrac{1}{2}\right)^{n+1}-1\right]z^{-n}$$





$$+\sum_{n=1}^{\infty}z^{n} + \sum_{n=0}^{-\infty}2^{n-1}z^{n} \xrightarrow{-n} +\sum_{n=-1}^{-\infty}z^{-n} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1}z^{-n}$$

$$f(z) = \frac{-0.5\, z^2}{(z-1)(z-0.5)} = \frac{-z}{z-1} + \frac{0.5 z}{z-0.5}$$

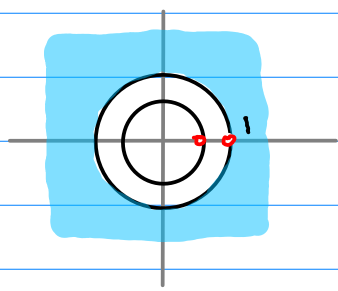


$$+\frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} - \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{2z}{1}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{z}{1}\right)^n - \sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{2z}{1}\right)^n$$

$$= +\sum_{n=0}^{\infty} z^{n+1} - \sum_{n=0}^{\infty} 2^n z^{n+1}$$

$$= \sum_{n=1}^{\infty}\left[1 - 2^{n-1}\right] z^n$$

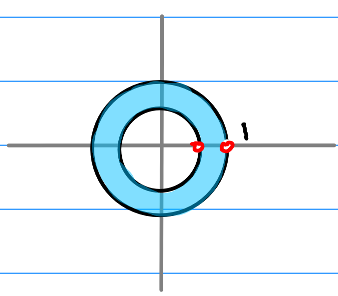


$$-\frac{(1)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$$

$$= -\sum_{n=0}^{\infty}(1)\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n$$

$$= \sum_{n=0}^{\infty}\left[2^{-n-1} - 1\right] z^{-n}$$

$$= \sum_{n=-1}^{-\infty}\left[2^{n-1} - 1\right] z^n$$



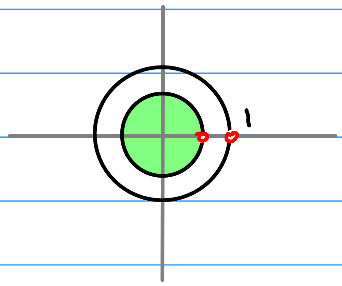
$$+\frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{z}{1}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n$$

$$= +\sum_{n=0}^{\infty} z^{n+1} + \sum_{n=0}^{\infty} 2^{-n-1} z^{-n}$$

$$= +\sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n$$

$$X(z) = \frac{-0.5\, z^2}{(z-1)(z-0.5)} = \frac{-z}{z-1} + \frac{0.5z}{z-0.5}$$



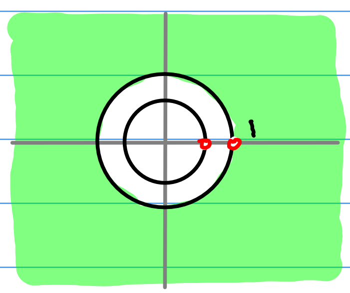
$$+\frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} - \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{2z}{1}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{z}{1}\right)^n - \sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{2z}{1}\right)^n$$

$$= +\sum_{n=0}^{\infty} z^{n+1} - \sum_{n=0}^{\infty} 2^n z^{n+1}$$

$$= \sum_{n=1}^{\infty}\left[1 - 2^{n-1}\right] z^n$$

$$\boxed{\sum_{n=-1}^{\infty}\left[1-\left(\tfrac{1}{2}\right)^{n+1}\right] z^{-n}}$$



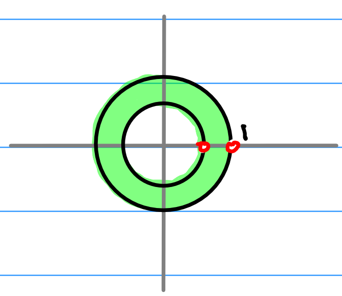
$$-\frac{(1)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$$

$$= -\sum_{n=0}^{\infty}(1)\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n$$

$$= \sum_{n=0}^{\infty}\left[2^{-n-1} - 1\right] z^{-n}$$

$$= \sum_{n=-1}^{-\infty}\left[2^{n-1} - 1\right] z^n$$

$$\boxed{\sum_{n=1}^{\infty}\left[\left(\tfrac{1}{2}\right)^{n+1} - 1\right] z^{-n}}$$



$$+\frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{z}{1}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n$$

$$= +\sum_{n=0}^{\infty} z^{n+1} + \sum_{n=0}^{\infty} 2^{-n-1} z^{-n}$$

$$= +\sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n$$

$$\boxed{+\sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty}\left(\tfrac{1}{2}\right)^{n+1} z^{-n}}$$