

Propositional Logic– Arguments (5A)

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Based on

Contemporary Artificial Intelligence,
R.E. Neapolitan & X. Jiang

Logic and Its Applications,
Burkey & Foxley

Arguments

An **argument** consists of a set of propositions :

The **premises** propositions

The **conclusion** proposition

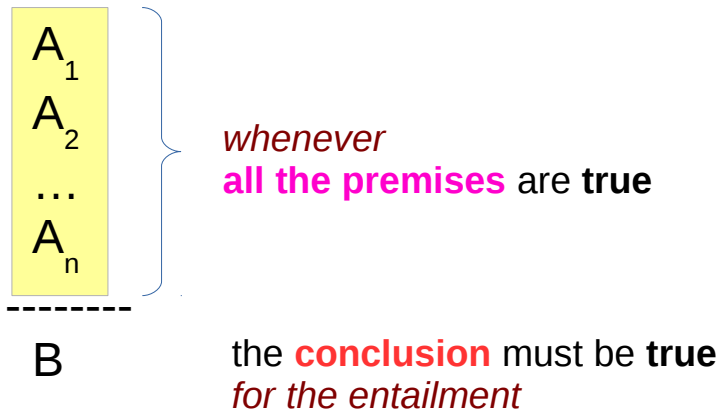
List of **premises** followed by the **conclusion**

$$\begin{array}{c} A_1 \\ A_2 \\ \dots \\ A_n \\ \hline B \end{array}$$

Entail

The **premises** is said to **entail** the **conclusion**
If in **every model** in which **all the premises** are **true**,
the **conclusion** is also **true**

List of **premises** followed by the **conclusion**



A Model

A model or possible world:

Every atomic proposition is assigned a value T or F

The set of all these assignments constitutes
A model or a possible world

All possible worlds (assignments) are permissible

Entailment Notation

Suppose we have *an argument*
whose premises are A_1, A_2, \dots, A_n
whose conclusion is B

Then

$$A_1, A_2, \dots, A_n \models B \quad \text{if and only if}$$
$$A_1 \wedge A_2 \wedge \dots \wedge A_n \Rightarrow B \quad (\text{logical implication})$$

logical implication: if $A_1 \wedge A_2 \wedge \dots \wedge A_n \Rightarrow B$ is **tautology** (*always true*)

The **premises** is said to **entail** the **conclusion**
If in every model in which
all the premises are **true**,
the **conclusion** is also **true**

Entailment and Logical Implication

$$A_1, A_2, \dots, A_n \models B$$

$$\iff A_1 \wedge A_2 \wedge \dots \wedge A_n \Rightarrow B$$

$$\iff A_1 \wedge A_2 \wedge \dots \wedge A_n \Rightarrow B \text{ is a tautology}$$

(logical implication)

If all the premises are true,
then the conclusion must be true

$$T \wedge T \wedge \dots \wedge T \Rightarrow T$$

$$T \wedge T \wedge \dots \wedge T \Rightarrow \text{X}$$

$$F \wedge X \wedge \dots \wedge X \Rightarrow X$$

Sound Argument and Fallacy

A **sound** argument

$A_1, A_2, \dots, A_n \models B$

If the **premises** entails the **conclusion**

A **fallacy**

$A_1, A_2, \dots, A_n \not\models B$

If the **premises** does not entail the **conclusion**

Entailment Examples

$A, B \models A$

$A, B \Rightarrow A$

$A \wedge B \Rightarrow A$

$A, (A \Rightarrow B) \models B$

$A, (A \Rightarrow B) \Rightarrow B$

$A \wedge (A \Rightarrow B) \Rightarrow B$

Entailment Examples and Truth Tables

A	B	$A \wedge B$	$A \wedge B \Rightarrow A$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

$$A \wedge B \Rightarrow A$$

The **premises** is said to **entail** the **conclusion**
 If in every model in which

all the premises are **true**,
 the **conclusion** is also **true**

any of the premises are **false**,
 still **premises** \Rightarrow **conclusion** is **true**
 ($F \Rightarrow T$ and $F \Rightarrow F$ always T)

Tautology

A	B	$A \Rightarrow B$	$A \wedge (A \Rightarrow B)$	$A \wedge (A \Rightarrow B) \Rightarrow B$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$$A \wedge (A \Rightarrow B) \Rightarrow B$$

Deduction System

Propositional logic

Given propositions (statements) : T or F

Deductive inference of T or F of *other propositions*

Deductive Inference

A process by which the truth of the conclusion
is shown to necessarily follow
from the truth of the premises

Deduction System

Deduction System : a set of inference rules

Inference rules are used to reason deductively

Sound Deduction System :

if it derives **only** sound arguments

Each of the inference rules is sound

Complete Deduction System :

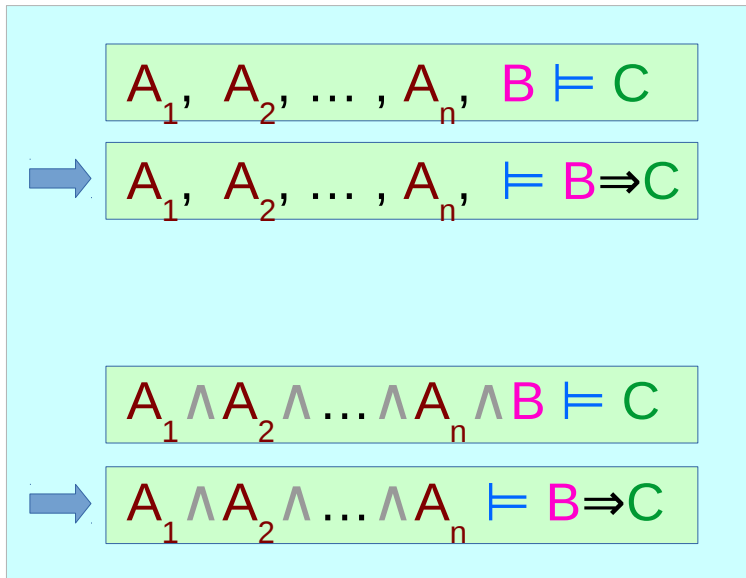
It can drive **every** sound argument

Must contain deduction theorem rule

Inference Rules

Combination Rule	$A, B \models A \wedge B$
Simplification Rule	$A \wedge B \models A$
Addition Rule	$A \models A \vee B$
Modus Ponens	$A, A \Rightarrow B \models B$
Modus Tolens	$\neg B, A \Rightarrow B \models \neg A$
Hypothetical Syllogism	$A \Rightarrow B, B \Rightarrow C \models A \Rightarrow C$
Disjunctive Syllogism	$A \vee B, \neg A \models B$
Rule of Cases	$A \Rightarrow B, \neg A \Rightarrow B \models B$
Equivalence Elimination	$A \Leftrightarrow B \models A \Rightarrow B$
Equivalence Introduction	$A \Rightarrow B, B \Rightarrow A \models A \Leftrightarrow B$
Inconsistency Rule	$A, \neg A \models B$
AND Commutivity Rule	$A \wedge B \models B \wedge A$
OR Commutivity Rule	$A \vee B \models B \vee A$
Deduction Theorem	If $A_1, A_2, \dots, A_n, B \models C$ then $A_1, A_2, \dots, A_n \models B \Rightarrow C$

Deduction Theorem



The **premises** is said to **entail** the **conclusion**
 If in every model in which
all the premises are **true**,
 the **conclusion** is also **true**

$A_1, A_2, \dots, A_n \models B$ if and only if
 $A_1 \wedge A_2 \wedge \dots \wedge A_n \Rightarrow B$
 ($A_1 \wedge A_2 \wedge \dots \wedge A_n \Rightarrow B$ is a tautology)

$$A, B \models C \iff A \models B \Rightarrow C$$

$$B \models C \iff \models B \Rightarrow C$$

If **A** is T, then **B ⇒ C** is always T (for the tautology)
 Even if **A, B** is T, then **B ⇒ C** is always T
 And if **A, B** is T, then **B** is T
 By modus ponens in the RHS, **A, B** is T, then **C** is true

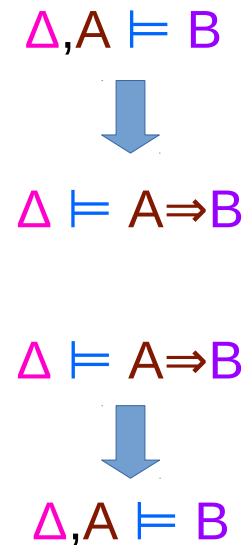
Deduction Theorem

$$\Delta, A \models B \quad \text{iff} \quad \Delta \models A \Rightarrow B$$

where Δ : a set of formulas,

if the formula B is deducible from a set Δ of assumptions, together with the assumption A , then the formula $A \Rightarrow B$ is deducible from Δ alone.

Conversely, if we can deduce $A \Rightarrow B$ from Δ , and if in addition we assume A , then B can be deduced.



<http://planetmath.org/deductiontheorem>

Deduction Theorem

The deduction theorem conforms with our intuitive understanding of how mathematical proofs work:

if we want to prove the statement “ A implies B ”,
then by assuming A , if we can prove B ,
we have established “ A implies B ”.

<http://planetmath.org/deductiontheorem>

Deduction Theorem

The converse statement of the deduction theorem

turns out to be a trivial consequence of **modus ponens**: $A, A \Rightarrow B \models B$

if $\Delta \models A \Rightarrow B$,

then certainly $\Delta, A \models A \Rightarrow B$

Since $\Delta, A \models A$, we get,

via **modus ponens**, $\Delta, A \models B$ as a result.

$$\Delta, A \models A \Rightarrow B$$

$$\Delta, A \models A$$

$$\Delta, A \models B$$

<http://planetmath.org/deductiontheorem>

Deduction Theorem

Deduction theorem is needed to derive arguments that has no premises

An argument without premises is simply a tautology

$\models A \vee \neg A$

no premises appear before the \models symbol

an argument without premises

Tautology if it is sound

Argument without premises

A

$A \vee \neg A$

$A \Rightarrow A \vee \neg A$

$\neg A$

$\neg A \vee A$

$A \vee \neg A$

$\neg A \Rightarrow A \vee \neg A$

$A \vee \neg A$

assume A

$X \models X \vee Y$

$A \models A \vee \neg A \Rightarrow \models A \Rightarrow A \vee \neg A$

discharge A

assume $\neg A$

$X \models X \vee Y$

$X \vee Y \models Y \vee X$

$\neg A \models A \vee \neg A \Rightarrow \models \neg A \Rightarrow A \vee \neg A$

discharge A

$X \Rightarrow Y, \neg X \Rightarrow Y \models Y$

$A \Rightarrow A \vee \neg A, \neg A \Rightarrow A \vee \neg A \models A \vee \neg A$

$\models A \vee \neg A$

Argument without premises

A

$A \vee \neg A$

$A \Rightarrow A \vee \neg A$

A

assume A

$A \models A \vee \neg A$

$\models A \Rightarrow A \vee \neg A$

discharge A

$\neg A$

$\neg A \vee A$

$A \vee \neg A$

$\neg A \Rightarrow A \vee \neg A$

$\neg A$

assume $\neg A$

$\neg A \models \neg A \vee A$

$\neg A \models A \vee \neg A$

$\models \neg A \Rightarrow A \vee \neg A$

discharge A

$A \vee \neg A$

$\models A \vee \neg A$

To prove a sound argument

Prove using truth tables

Whether an argument is sound or fallacy

1. time complexity (2^n)
2. not the way which humans do

Prove using inference rules

To reason deductively

Double Turnstile

1. semantic consequence: with a set of sentences on the left and a single sentence on the right, to denote that if every sentence on the left is true, the sentence on the right must be true, e.g. $\Gamma \models \varphi$. This usage is closely related to the single-barred turnstile symbol which denotes syntactic consequence.

2. satisfaction: with a **model** (or truth-structure) on the left and **a set of sentences** on the right, to denote that **the structure is a model for (or satisfies) the set of sentences**, e.g. $\mathbf{A} \models \Gamma$.

3. a tautology: $\models \varphi$. which is to say that the expression φ is a **semantic consequence of the empty set**.

https://en.wikipedia.org/wiki/Double_turnstile

Syntactic Consequences

A formula **A** is a syntactic consequence within some formal system **FS** of a set Γ of formulas if there is a **formal proof** in **FS** of **A** from the set Γ .

$$\Gamma \vdash_{\text{FS}} \mathbf{A}$$

Syntactic consequence does **not depend** on any **interpretation** of the formal system.

A formal proof or derivation is a finite sequence of sentences (called well-formed formulas in the case of a formal language), each of which is an axiom, an assumption, or follows from the preceding sentences in the sequence by a **rule of inference**. The last sentence in the sequence is a **theorem** of a formal system.

https://en.wikipedia.org/wiki/Double_turnstile

Semantic Consequences

A formula **A** is a semantic consequence within some formal system **FS** of a set of statements Γ

$$\Gamma \models_{\text{FS}} \mathbf{A}$$

if and only if there is **no model I** in which all members of Γ are true and **A is false**. Or, in other words, the set of the **interpretations** that make all members of Γ true is a **subset** of the set of the interpretations that make **A** true.

https://en.wikipedia.org/wiki/Double_turnstile

Semantic Consequences

$$A \vdash_S B$$

means **there is a derivation**, in the proof-system S , from the premise A to the conclusion B . [If context fixes the relevant system S , we suppress the subscript.]

$$A \models_L B$$

means that **on every possible interpretation** of the non-logical vocabulary of language L , if A comes out true, so does B . [If context fixes the relevant language L we suppress the subscript.]

$$A \rightarrow B$$

on the truth-functional interpretation, if the atomic wff p happens to be false and the atomic wff q happens to be false too, then $p \rightarrow q$ evaluates as true. But of course we don't have $p \models q$ (q isn't true on every valuation which makes p true).

<http://math.stackexchange.com/questions/365569/whats-the-difference-between-syntactic-consequence-%E2%8A%A2-and-semantic-consequence-%E2%8A%A8>

Logical Equivalences

\neg, \wedge, \vee

\neg, \wedge, \vee

$\wedge \vee \neg \neg$
 $\Rightarrow \Leftrightarrow \equiv \equiv$
 \vDash

$\wedge \vee \neg \neg$
 $\Rightarrow \Leftrightarrow \equiv \equiv$
 \vDash

\Rightarrow
 \Leftrightarrow
 \equiv

\Rightarrow
 \Leftrightarrow
 \equiv

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