Propositional Logic– Arguments (5A)

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Based on

Contemporary Artificial Intelligence,
R.E. Neapolitan & X. Jiang

Logic and Its Applications,
Burkey & Foxley

Arguments

An **argument** consists of a set of propositions :

The **premises** propositions

The **conclusion** proposition

List of **premises** followed by the **conclusion**

A_1

A_2

…

A_n

\-\-\-\-\-\-\-\-

B

Entail

The **premises** is said to **entail** the **conclusion**
If in **every model** in which **all the premises** are **true**,
the **conclusion** is also **true**

List of **premises** followed by the **conclusion**

A_1
A_2
…
A_n

B

whenever
all the premises are **true**

the **conclusion** must be **true**
for the entailment

A Model

A mode or possible world:

Every atomic proposition is assigned a value T or F

The set of all these assignments constitutes
A model or a possible world

All possile worlds (assignments) are permissiable

Entailment Notation

Suppose we have *an argument*
whose premises are $A_1, A_2, \ldots, A_n$
whose conclusion is B

Then

$A_1, A_2, \ldots, A_n \models B$ if and only if
$A_1 \land A_2 \land \ldots \land A_n \Rightarrow B$ (logical implication)

logical implication: if $A_1 \land A_2 \land \ldots \land A_n \Rightarrow B$ is tautology (*always true*)

The **premises** is said to **entail** the **conclusion**
If in every model in which
all the premises are **true**,
the **conclusion** is also **true**

Entailment and Logical Implication

$A_1, A_2, \ldots, A_n \models B$

$\Leftrightarrow$ $A_1 \wedge A_2 \wedge \ldots \wedge A_n \Rightarrow B$

$\Leftrightarrow$ $A_1 \wedge A_2 \wedge \ldots \wedge A_n \Rightarrow B$ is a tautology

(logical implication)

If all the premises are **true**,
then the **conclusion** must be **true**

$T \wedge T \wedge \ldots \wedge T \Rightarrow T$

$T \wedge T \wedge \ldots \wedge T \Rightarrow$ ~~F~~

$F \wedge X \wedge \ldots \wedge X \Rightarrow X$

Sound Argument and Fallacy

A **sound** **argument**

$A_1, A_2, \ldots, A_n \models B$

If the **premises** **entails** the **conclusion**

A **fallacy**

$A_1, A_2, \ldots, A_n \not\models B$

If the **premises** does **not** **entail** the **conclusion**

Entailment Examples

$A, B \models A$

$A, B \Rrightarrow A$

$A \wedge B \Rrightarrow A$

$A, (A \Rightarrow B) \models B$

$A, (A \Rightarrow B) \Rrightarrow B$

$A \wedge (A \Rightarrow B) \Rrightarrow B$

Entailment Examples and Truth Tables

A	B	A∧B	A∧B ⇒ A
T	T	**T**	T
T	F	F	T
F	T	F	T
F	F	F	T

A∧B ⇒ A

The **premises** is said to **entail** the **conclusion**
If in every model in which
all the premises are **true**,
the **conclusion** is also **true**

any of the premises are **false**,
still **premises ⇒ conclusion** is **true**
(F⇒T and F⇒F always T)

Tautology

A	B	A⇒B	A∧(A⇒B)	A∧(A⇒B)⇒B
T	T	T	**T**	**T**
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

A∧(A⇒B) ⇒ B

Deduction System

Propositional logic

Given propositions (statements) : T or F
Deductive inference of T or F of *other propositions*

Deductive Inference
A process by which the truth of the conclusion
is shown to necessarily follow
from the truth of the premises

Deduction System

Deduction System : a set of inference rules

Inference rules are used to reason deductively

Sound Deduction System :

if it derives only sound arguments

Each of the inference rules is sound

Complete Deduction System :

It can drive every sound argument

Must contain deduction theorem rule

Inference Rules

Combination Rule	$A, B \models A \wedge B$
Simplification Rule	$A \wedge B \models A$
Addition Rule	$A \models A \vee B$
Modus Pones	$A, A \Rightarrow B \models B$
Modus Tolens	$\neg B, A \Rightarrow B \models \neg A$
Hypothetical Syllogism	$A \Rightarrow B,\ B \Rightarrow C \models A \Rightarrow C$
Disjunctive Syllogism	$A \vee B, \neg A \models B$
Rule of Cases	$A \Rightarrow B, \neg A \Rightarrow B \models B$
Equivalence Elimination	$A \Leftrightarrow B \models A \Rightarrow B$
Equivalence Introduction	$A \Rightarrow B,\ B \Rightarrow A \models A \Leftrightarrow B$
Inconsistency Rule	$A, \neg A \models B$
AND Commutivity Rule	$A \wedge B \models B \wedge A$
OR Commutivity Rule	$A \vee B \models B \vee A$
Deduction Theorem	If $A_1, A_2, \ldots, A_n, B \models C$ then $A_1, A_2, \ldots, A_n, \models B \Rightarrow C$

Deduction Theorem

$A_1,\ A_2,\ \ldots,\ A_n,\ B \vDash C$

$\Rightarrow$ $A_1,\ A_2,\ \ldots,\ A_n,\ \vDash B \Rightarrow C$

$A_1 \wedge A_2 \wedge \ldots \wedge A_n \wedge B \vDash C$

$\Rightarrow$ $A_1 \wedge A_2 \wedge \ldots \wedge A_n \vDash B \Rightarrow C$

The **premises** is said to **entail** the **conclusion**
If in every model in which
all the premises are **true**,
the **conclusion** is also **true**

$A_1, A_2, \ldots, A_n \vDash B$ if and only if
$A_1 \wedge A_2 \wedge \ldots \wedge A_n \Rightarrow B$
($A_1 \wedge A_2 \wedge \ldots \wedge A_n \Rightarrow B$ is a tautology)

$A, B \vDash C \rightleftarrows A \vDash B \Rightarrow C$

$B \vDash C \rightleftarrows \ \vDash B \Rightarrow C$

If A is T, then B⇒C is always T (for the tautology)
Even if A,B is T, then B⇒C is always T
And if A,B is T, then B is T
By modus ponens in the RHS, A,B is T, then C is true

Deduction Theorem

$\Delta, A \models B$ iff $\Delta \models A \Rightarrow B$

where Δ : a set of formulas,

if the formula B is deducible
from a set Δ of assumptions,
together with the assumption A,
then the formula $A \Rightarrow B$ is deducible from Δ alone.

$\Delta, A \models B$

↓

$\Delta \models A \Rightarrow B$

Conversely, if we can deduce $A \Rightarrow B$ from Δ,
and if in addition we assume A,
then B can be deduced.

$\Delta \models A \Rightarrow B$

↓

$\Delta, A \models B$

http://planetmath.org/deductiontheorem

Deduction Theorem

The deduction theorem conforms with
our intuitive understanding of how mathematical proofs work:

if we want to prove the statement "A implies B",
then by assuming A, if we can prove B,
we have established "A implies B".

http://planetmath.org/deductiontheorem

Deduction Theorem

The converse statement of the deduction theorem
turns out to be a trivial consequence of **modus ponens**: $A, A \Rightarrow B \vDash B$

if $\Delta \vDash A \Rightarrow B$,
then certainly $\Delta, A \vDash A \Rightarrow B$
Since $\Delta, A \vDash A$, we get,
via **modus ponens**, $\Delta, A \vDash B$ as a result.

$\Delta, A \vDash A \Rightarrow B$

$\Delta, A \vDash A$

$\Delta, A \vDash B$

http://planetmath.org/deductiontheorem

Deduction Theorem

Deduction theorem is needed to derive arguments that has no premises

An argument without premises is simply a tautology

$\models A \lor \neg A$

no premises appear before the $\models$ symbol

an argument without premises

Tautology if it is sound

Argument without premises

A

$A \vee \neg A$

$A \Rightarrow A \vee \neg A$

$\neg A$

$\neg A \vee A$

$A \vee \neg A$

$\neg A \Rightarrow A \vee \neg A$

$A \vee \neg A$

assume A

$X \models X \vee Y$

$A \models A \vee \neg A \quad \Rightarrow \quad \models A \Rightarrow A \vee \neg A$

discharge A

assume ¬A

$X \models X \vee Y$

$X \vee Y \models Y \vee X$

$\neg A \models A \vee \neg A \quad \Rightarrow \quad \models \neg A \Rightarrow A \vee \neg A$

discharge A

$X \Rightarrow Y, \neg X \Rightarrow Y \models Y$

$A \Rightarrow A \vee \neg A, \ \neg A \Rightarrow A \vee \neg A \models A \vee \neg A$

$\models A \vee \neg A$

Argument without premises

A	A assume A
A ∨ ¬A	A ⊨ A ∨ ¬A
A ⇒ A ∨ ¬A	⊨ A ⇒ A ∨ ¬A
	discharge A
¬A	¬A assume ¬A
¬A ∨ A	¬A ⊨ ¬A ∨ A
A ∨ ¬A	¬A ⊨ A ∨ ¬A
¬A ⇒ A ∨ ¬A	⊨ ¬A ⇒ A ∨ ¬A
	discharge A
A ∨ ¬A	⊨ A∨¬A

To prove a sound argument

Prove using truth tables

Whether an argument is sound or fallacy

1. time complexity (2^n)
2. not the way which humans do

Prove using inference rules

To reason deductively

Double Turnstile

1. semantic consequence: with a set of sentences on the left and a single sentence on the right, to denote that if every sentence on the left is true, the sentence on the right must be true, e.g. $\boldsymbol{\Gamma \vDash \varphi}$. This usage is closely related to the single-barred turnstile symbol which denotes syntactic consequence.

2. satisfaction: with a model (or truth-structure) on the left and a set of sentences on the right, to denote that the structure is a model for (or satisfies) the set of sentences, e.g. $\boldsymbol{A \vDash \Gamma}$.

3. a tautology: $\boldsymbol{\vDash \varphi}$. which is to say that the expression φ is a semantic consequence of the empty set.

https://en.wikipedia.org/wiki/Double_turnstile

Syntactic Consequences

A formula **A** is a syntactic consequence within some formal system **FS** of a set **Γ** of formulas if there is a formal proof in **FS** of **A** from the set **Γ**.

$$\mathbf{\Gamma} \vdash_{FS} \mathbf{A}$$

Syntactic consequence does not depend on any interpretation of the formal system.

A formal proof or derivation is a finite sequence of sentences (called well-formed formulas in the case of a formal language), each of which is an axiom, an assumption, or follows from the preceding sentences in the sequence by a **rule of inference**. The last sentence in the sequence is a **theorem** of a formal system.

https://en.wikipedia.org/wiki/Double_turnstile

Semantic Consequences

A formula **A** is a semantic consequence within some formal system **FS** of a set of statements **Γ**

$$\mathbf{\Gamma} \models_{FS} \mathbf{A}$$

if and only if there is no model **I** in which all members of **Γ** are true and **A** is false. Or, in other words, the set of the interpretations that make all members of **Γ** true is a **subset** of the set of the interpretations that make **A** true.

https://en.wikipedia.org/wiki/Double_turnstile

Semantic Consequences

$A \vdash_S B$

means there is a derivation, in the proof-system S, from the premise A to the conclusion B. [If context fixes the relevant system S, we suppress the subscript.]

$A \vDash_L B$

means that on every possible interpretation of the non-logical vocabulary of language L, if A comes out true, so does B. [If context fixes the relevant language L we suppress the subscript.]

$A \rightarrow B$

on the truth-functional interpretation, if the atomic wff p happens to be false and the atomic wff q happens to the false too, then $p \rightarrow q$ evaluates as true. But of course we don't have $p \vDash q$ (q isn't true on every valuation which makes p true).

http://math.stackexchange.com/questions/365569/whats-the-difference-between-syntactic-consequence-%E2%8A%A2-and-semantic-consequence-%E2%8A%A8

Logical Equivalences

¬, ∧, ∨

¬, ∧, ∨

∧∨⌐¬
⇒⇔≡⇛
⊨

∧∨⌐¬
⇒⇔≡⇛
⊨

⇒ ⇒
⇔ ⇔
≡ ≡

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