# Applications of Pointers (1A)

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3/27/24

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## **n-d** access of a **1-d** array

- **2-d** array access of a **1-d** array
- **3-d** array access of a **1-d** array
- Accessing a **contiguous 1-d** array
- Accessing a **non-contiguous 1-d** arrays

- Accessing **static** allocated arrays
- Accessing **dynamically** allocated arrays

## **2-d** array access of a **1-d** array

# Array of Pointers

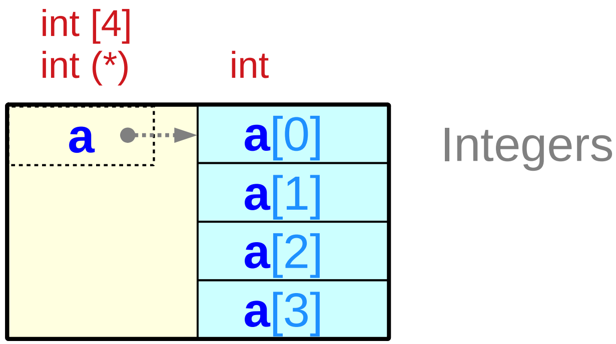
int a [4] ;

int * b [3] ;

int a [4]

there are 4 elements

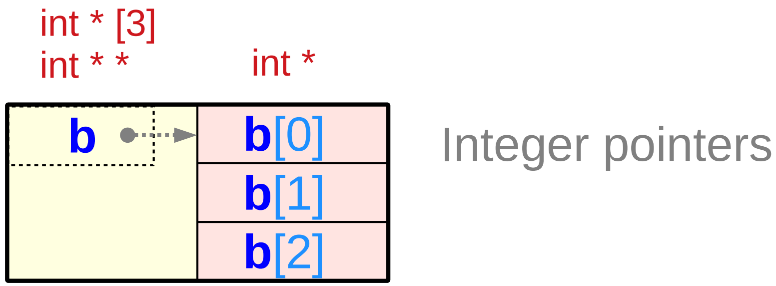
the type of each element:
an integer




int * b [3]

there are 3 elements

the type of each element:
a pointer an integer

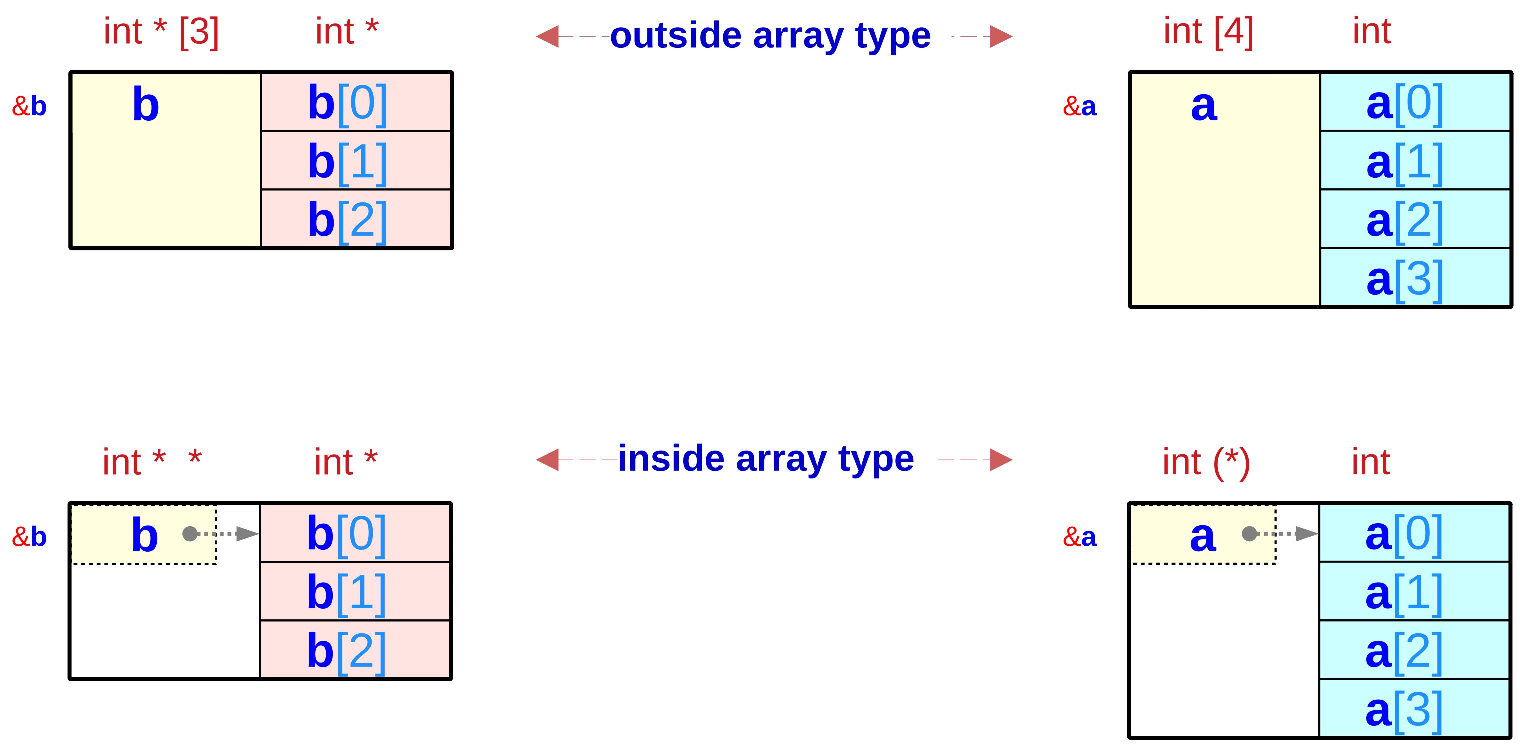


# Array of Pointers – a type view

int * b [3] ;

**Pointer Array**

int a [4] ;

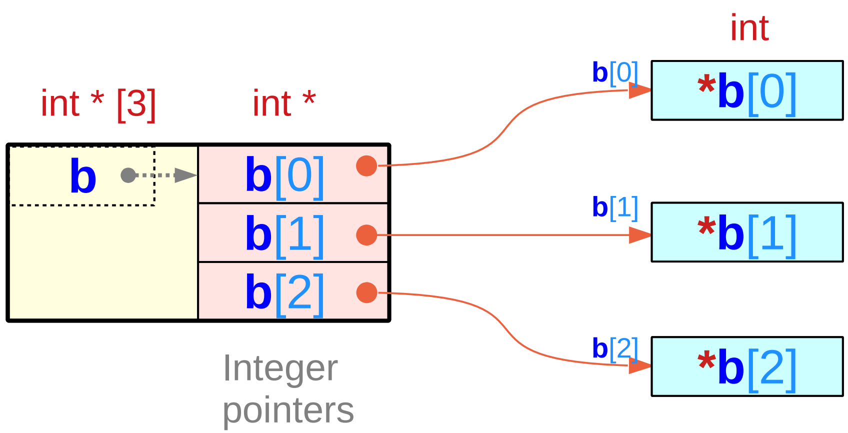


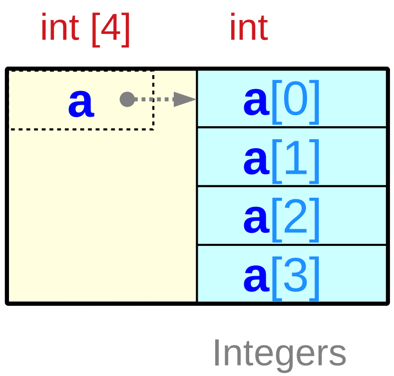
# Array of Pointers – a variable view

int * b [3] ;

**Pointer Array**

int a [4] ;



# Assigning a **1-d** array name

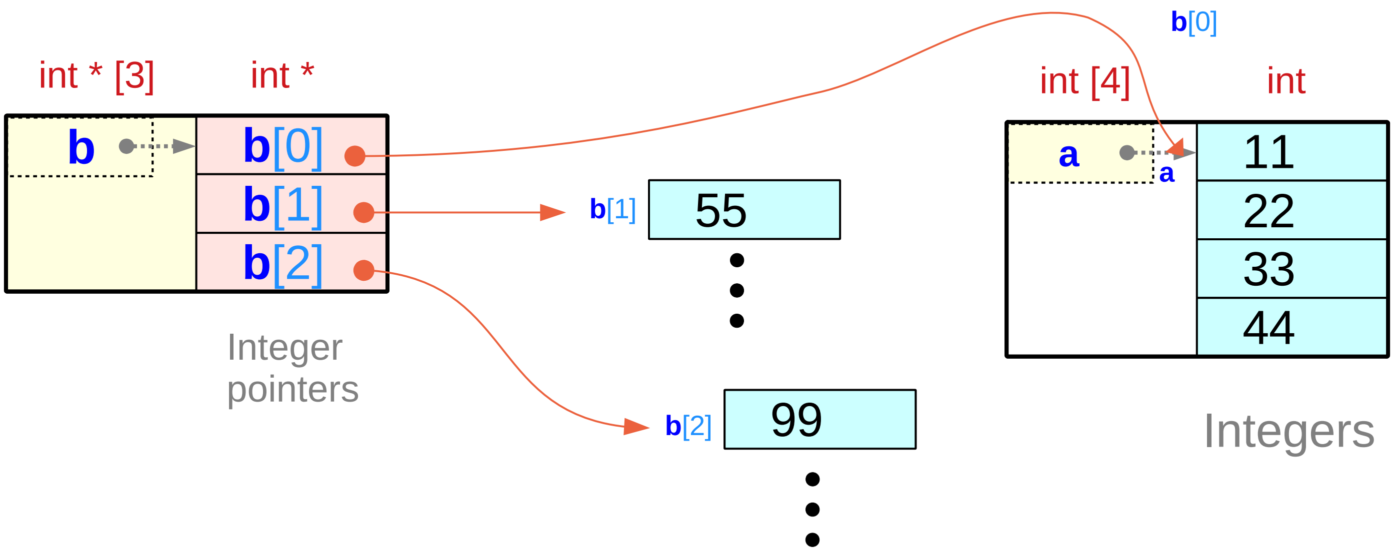
int * **b** [3] ;

**Pointer Array**

int **a** [4] ;

**assignment**

**b**[0] = **a** (= &**a**[0])



# Assigning a **1-d** array name – equivalence

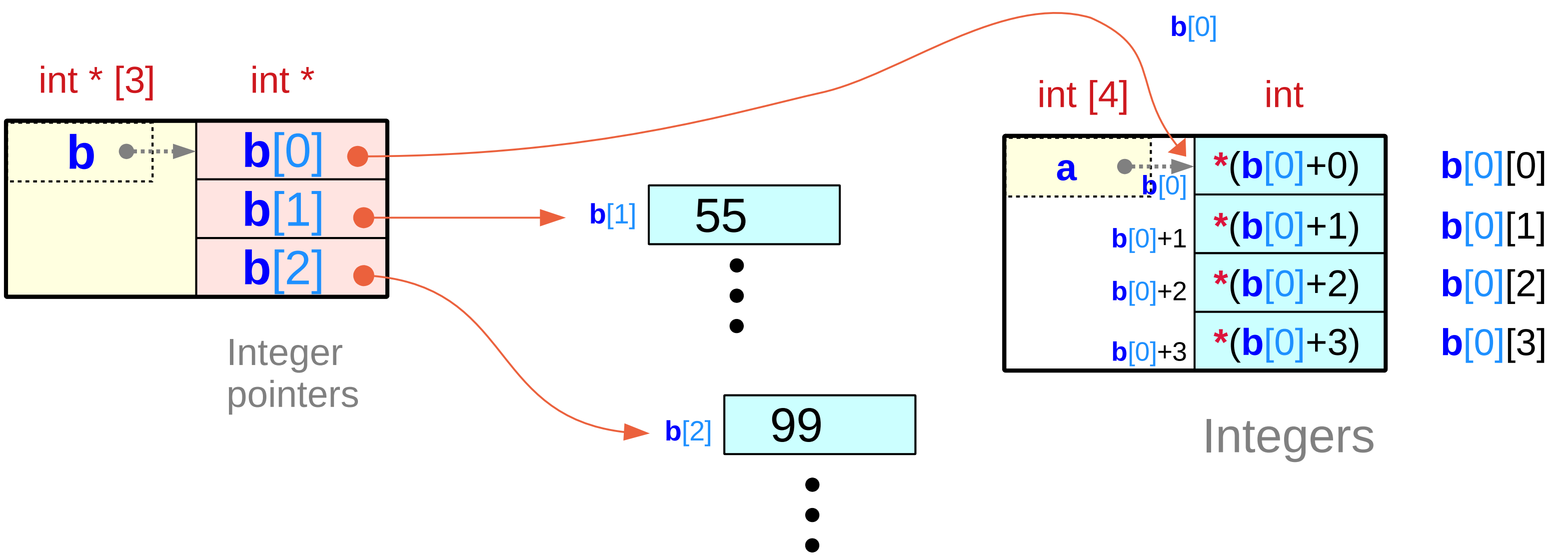
| int * | **b** [3] ; |
|---|---|

**Pointer Array**

| int | **a** [4] ; |
|---|---|

**assignment**

**b**[0] = **a** (= &**a**[0])



# Array of Pointers – extended dimension

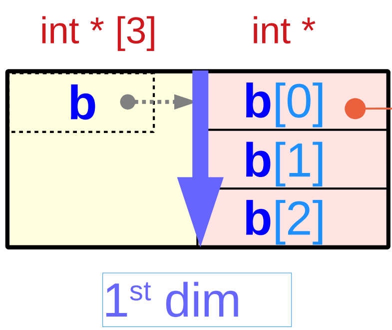
int * b [3] ; **Pointer Array**

int a [4] ;

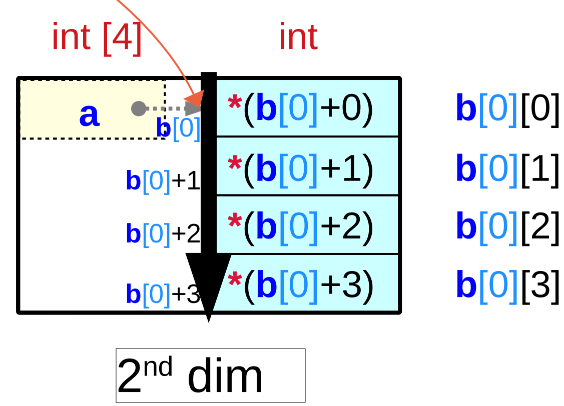
array name b

**assignment**

**b**[0] = **a** (= &**a**[0])

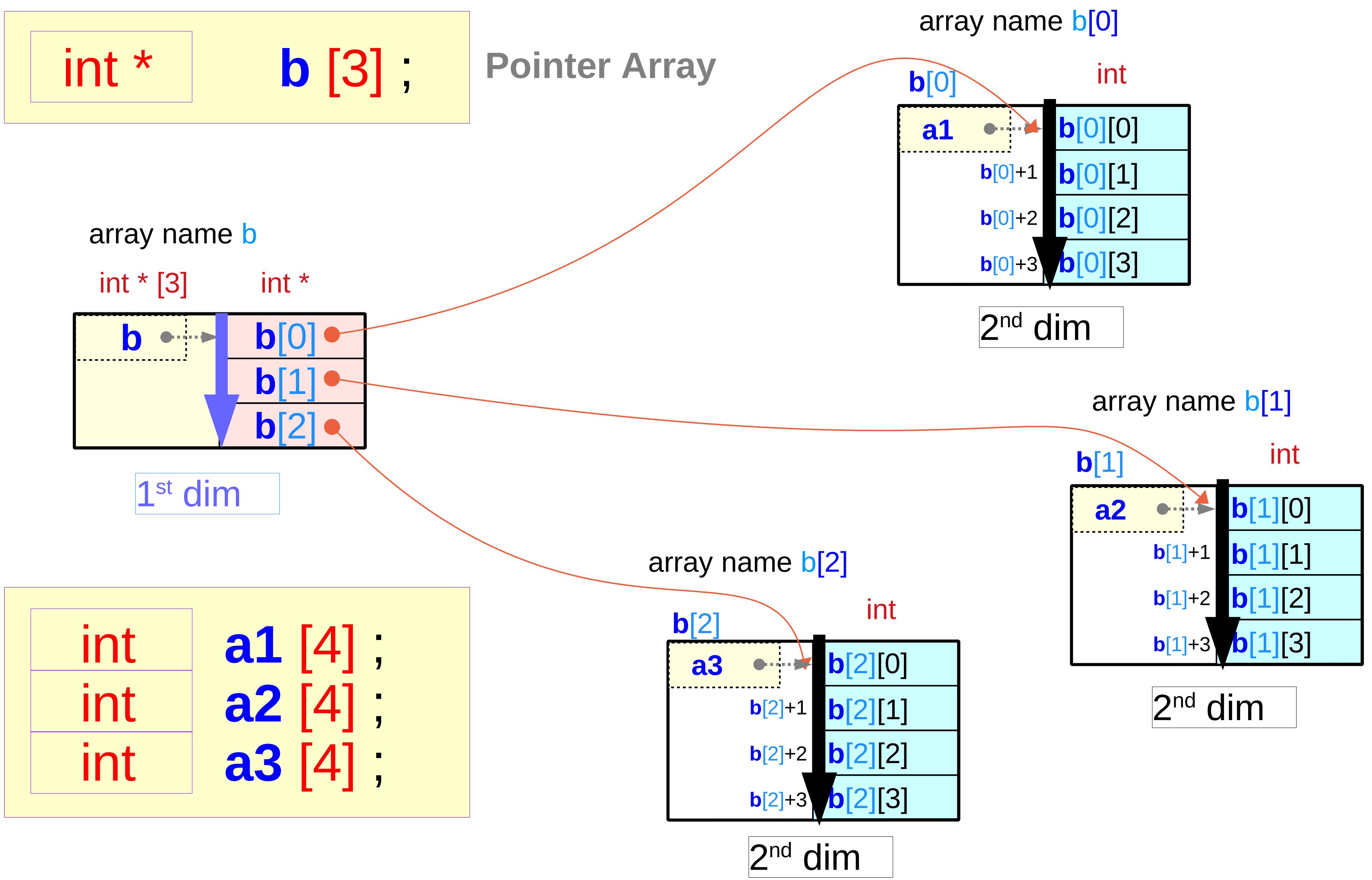



array name b[0]




a[0] ≡ b[0][0] ≡ *(*(b+0)+0)
a[1] ≡ b[0][1] ≡ *(*(b+0)+1)
a[2] ≡ b[0][2] ≡ *(*(b+0)+2)
a[3] ≡ b[0][3] ≡ *(*(b+0)+3)

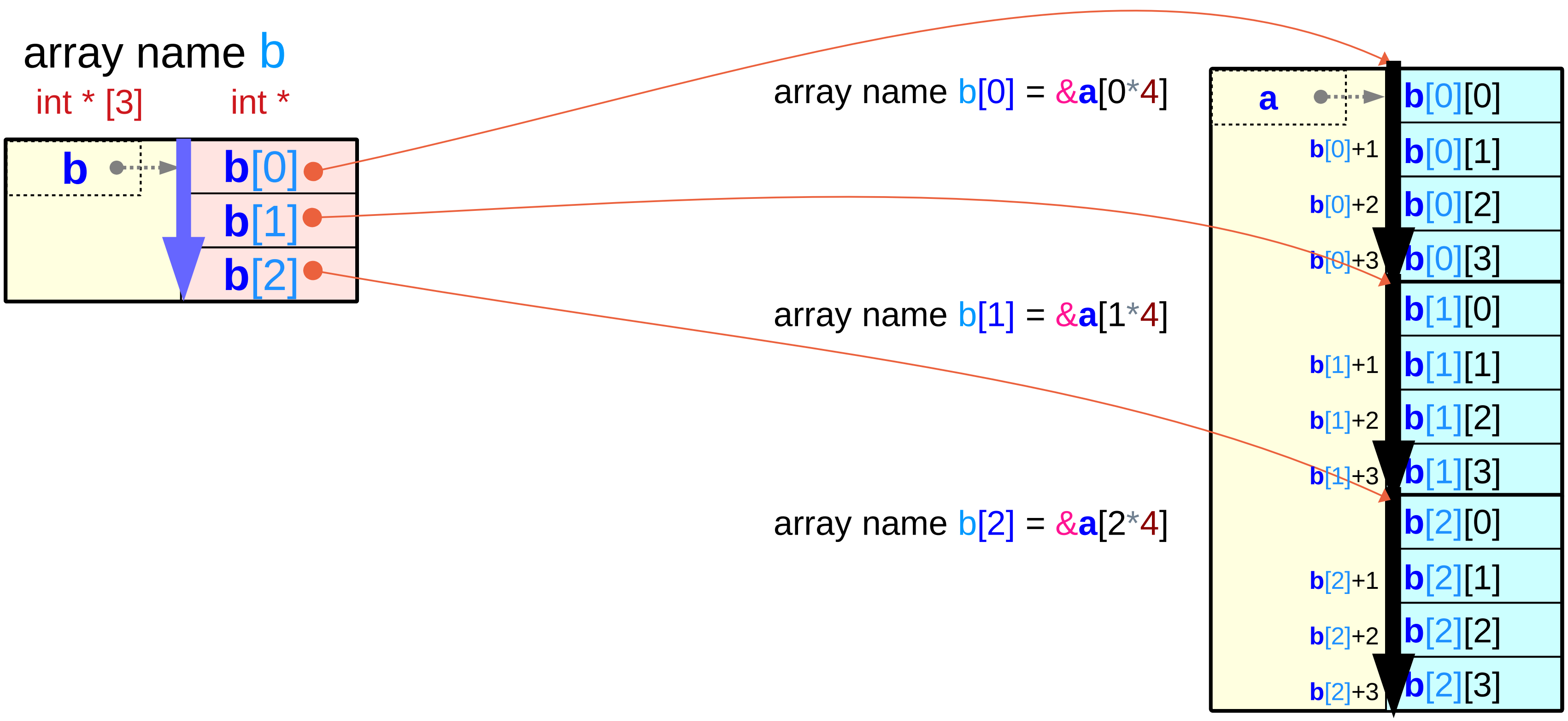
# 2-d access of 1-d arrays



# 2-d access of a 1-d array

int * b [3] ; Pointer Array

int * a [3*4] ;



# **2-d** access of a **1-d** array – pointer array assignment

| int * | b [2*3] ; |
|---|---|
| int | a [2*3*4] ; |

b[j] = &a[j*4] (= a+j*4)

b[j] + k = a+j*4 + k
*(b[j] + k) = *(a+j*4 + k)

b[j][k] ≡ a[j*4 +k]

j = [0:5] k = [0:3]

j*4+k = [0:23]

constraint : contiguous b[i][j] over j

**2-d** access of a **1-d** array

b[i][j] ≡ *( b[i] + j)

↕ ↕

a[i*4+j] ≡ *( a+i*4 + j)

**1-d** access of a **1-d** array

constraint : contiguous a[i*4+j] over j

# **3-d** array access of a **1-d** array

# 3-d access of a 1-d array (1)

```
int **   c  [2] ;
int *    b  [2*3] ;
int      a  [2*3*4] ;
```

```
int *    b  [2*3] ;
int      a  [2*3*4] ;
```

b[j] = &a[j*4]  (= a+j*4)

b[j] + k  =  a+j*4 + k
*(b[j] + k) = *(a+j*4 + k)

b[j][k] ≡ a[j*4 +k]

j = [0:5]  k = [0:3]

j*4+k = [0:23]

```
int **   c  [2] ;
int *    b  [2*3] ;
```

c[i] = &b[i*3]  (= b+i*3)

c[i] +j  =  b+i*3 +j
*(c[i] +j) = *(b+i*3 +j)

c[i][j] = b[i*3 +j]

c[i][j] +k  = b[i*3 +j] +k
*(c[i][j] +k)  = b[i*3 +j][k]

c[i][j][k]  = a[(i*3 +j)*4 +k]

c[i][j][k] ≡ a[(i*3+j)*4+k]

i = [0:1]  j = [0:2]  k = [0:3]

(i*3+j)*4+k = [0:23]

# 3-d access of a 1-d array (2)

| | |
|---|---|
| int ** | c [2] ; |
| int * | b [2*3] ; |
| int | a [2*3*4] ; |

a[k] ≡ *(a+k)
b[j][k] ≡ *(*(b+j)+k)
c[i][j][k] ≡ *(*(*(c+i)+j)+k)

constraint : contiguous a[i], b[i], c[i]

**Assignments**

c[i] = &b[i*3] (= b+i*3)
b[j] = &a[j*4] (= a+j*4)

Initializing pointer arrays **b** and **c**

**3-d** access of a **1-d** array

c[i][j][k] ≡ *( c[i][j] +k)

b[i*3+j][k] ≡ *( b[i*3+j] +k)

a[(i*3+j)*4 +k] ≡ *(a+(i*3+j)*4 +k)

**1-d** access of a **1-d** array

# 3-d access of a 1-d array (3)

| | | |
|---|---|---|
| int ** | c | [2] ; |
| int * | b | [2*3] ; |
| int | a | [2*3*4] ; |

a[k] ≡ *(a+k)
b[j][k] ≡ *(*(b+j)+k)
c[i][j][k] ≡ *(*(*(c+i)+j)+k)

(((c[i])[j])[k])
≡ (((b+i*3)[j])[k])
≡ ((b[i*3+j])[k])
≡ ((a+(i*3+j)*4)[k])
≡ a[(i*3+j)*4+k]

c[i] = &b[i*3] = b+i*3

b[j] = &a[j*4] = a+j*4

*(*(*(c+i)+j)+k)
≡ *(*(b+i*3+j)+k)
≡ *(b[i*3+j]+k)
≡ *(a+(i*3+j)*4+k)
≡ a[(i*3+j)*4+k]

# Equivalence relations in pointer array assignments

c[i] = &b[i*3] = b+i*3
b[j] = &a[j*4] = a+j*4

substitute c[i] in *(c[i]+j)

substitute b[m] in *(b[m]+k)

m = i*3+j

c[i][j] = *(c[i]+j) (substitute c[i])
= *(b+i*3+j) = b[i*3+j]

b[m][k] = *(b[m]+k) (substitute b[m])
= *(a+m*4+k) = a[m*4+k]

c[i][j][k] = b[i*3+j][k] = a[(i*3+j)*4+k]

c[i] = &b[i*3] = b+i*3
b[j] = &a[j*4] = a+j*4

substitute c[i] in (c[i])[j]

substitute b[m] in (b[m])[k]

m = i*3+j

substitute c[i]
c[i][j] = (b+i*3)[j]
= *(b+i*3+j) = b[i*3+j]

substitute b[m]
b[m][k] = (a+m*4)[k]
= *(a+m*4+k) = a[m*4+k]

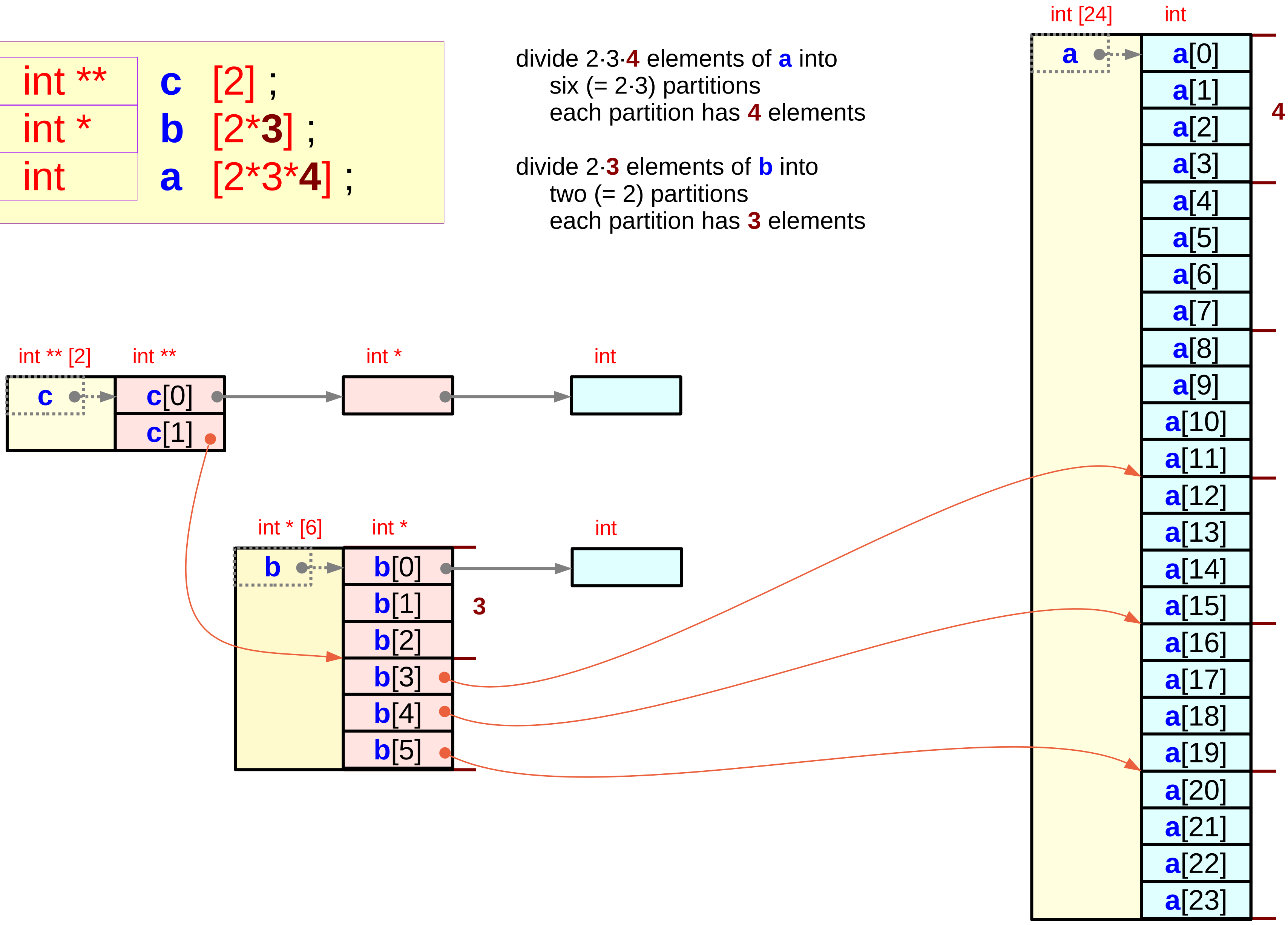
c[i][j][k] = b[i*3+j][k] = a[(i*3+j)*4+k]

# Integer array **a** and pointer arrays **b**, **c**

```
int **  c  [2] ;
int *   b  [2*3] ;
int     a  [2*3*4] ;
```

divide 2·3·**4** elements of **a** into
six (= 2·3) partitions
each partition has **4** elements

divide 2·**3** elements of **b** into
two (= 2) partitions
each partition has **3** elements



# Pointer array initializations

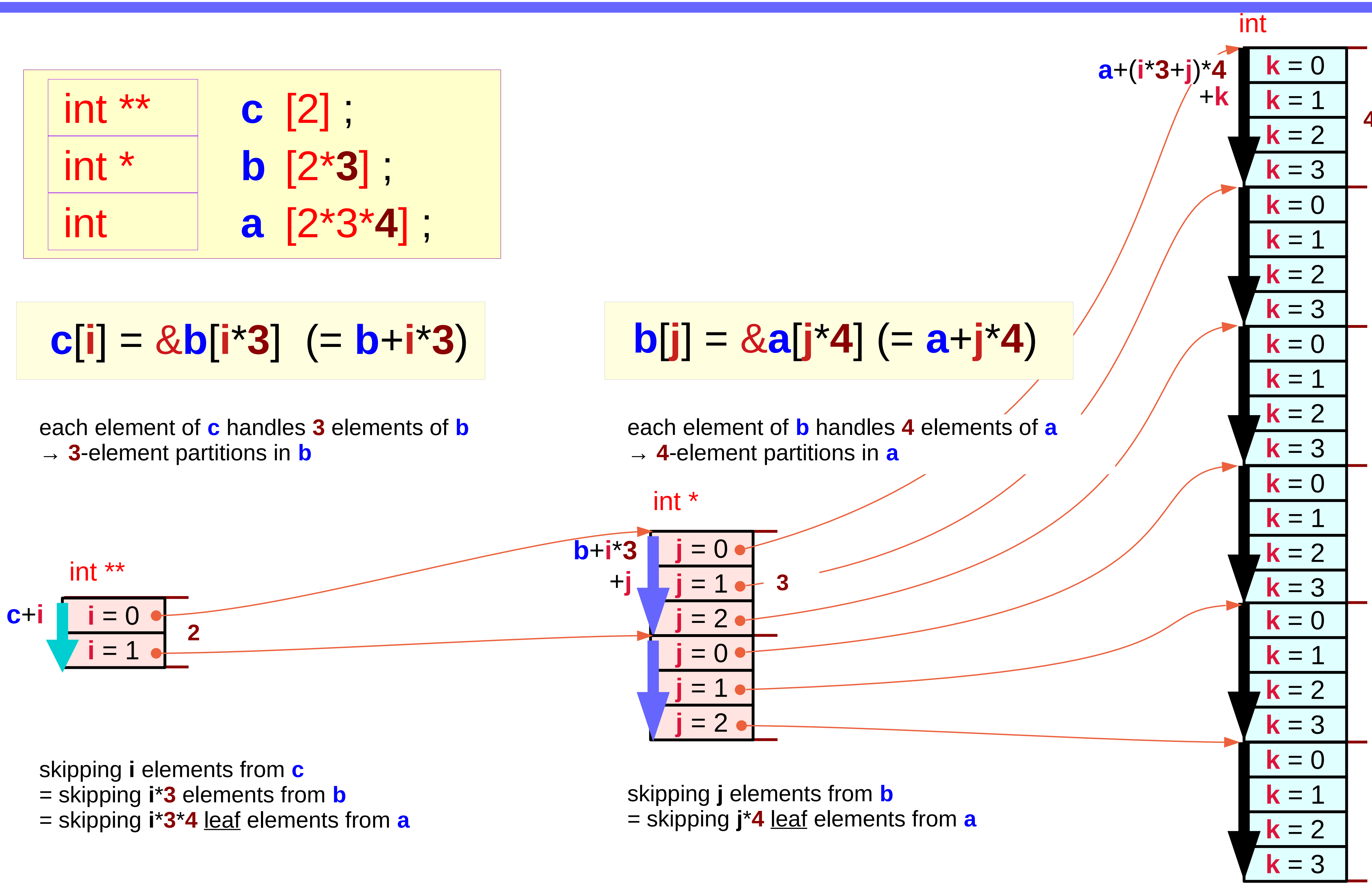
| | |
|---|---|
| int ** | c [2] ; |
| int * | b [2*3] ; |
| int | a [2*3*4] ; |

c[i] = &b[i*3] (= b+i*3)

b[j] = &a[j*4] (= a+j*4)

each element of c handles 3 elements of b
→ 3-element partitions in b

each element of b handles 4 elements of a
→ 4-element partitions in a




skipping i elements from c
= skipping i*3 elements from b
= skipping i*3*4 <u>leaf</u> elements from a

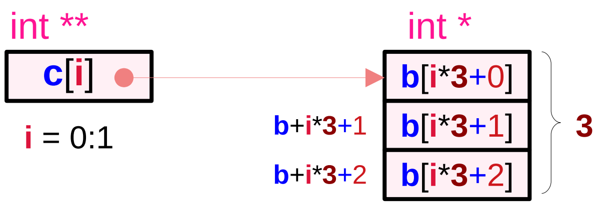
skipping j elements from b
= skipping j*4 <u>leaf</u> elements from a

# Partitioning arrays **a** and **b**

| | |
|---|---|
| int ** | c [2] ; |
| int * | b [2*3] ; |
| int | a [2*3*4] ; |

**Assigning pointer array**

b[j] = &a[j*4]  (= a+j*4)
c[i] = &b[i*3]  (= b+i*3)

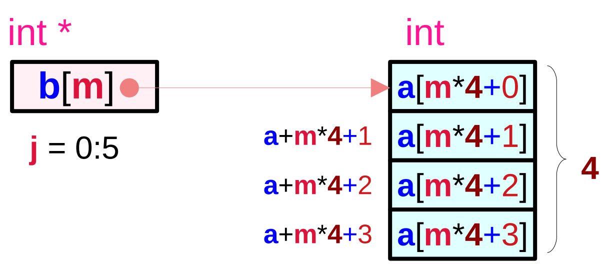



divide 2·3 elements of b
into two 3-element partitions
partition size = 3

c[i] contiguous c over i = 0:1

b[i*3+j] contiguous b over j = 0:2

c[0] = &b[0*3];  (= b + 0*3)
c[1] = &b[1*3];  (= b + 1*3)




divide 2·3·4 elements of a
into six 4-element partitions
partition size = 4

b[m] contiguous b over m = 0:5

a[m*4+k] contiguous a over k = 0:3

b[0] = &a[0*4];  (= a + 0*4)
b[1] = &a[1*4];  (= a + 1*4)
b[2] = &a[2*4];  (= a + 2*4)
b[3] = &a[3*4];  (= a + 3*4)
b[4] = &a[4*4];  (= a + 4*4)
b[5] = &a[5*4];  (= a + 5*4)

# Skipping leaf elements

| | |
|---|---|
| int ** | c [2] ; |
| int * | b [2*3] ; |
| int | a [2*3*4] ; |

b[j] = &a[j*4] (= a+j*4)

skipping 1 element in b
= skipping 4 leaf elements in a

c[i] = &b[i*3] (= b+i*3)

skipping 1 element in c
= skipping 3 elements in b
= skipping 3*4 leaf elements in a

c[i][j] ≡ b[i*3 +j]

skipping i elements from c
+ skipping j elements from b

= skipping i*3+j elements from b

c[i][j][k] ≡ a[(i*3 +j)*4+k]

skipping i*3+j elements from b
+ skipping k leaf elements from a

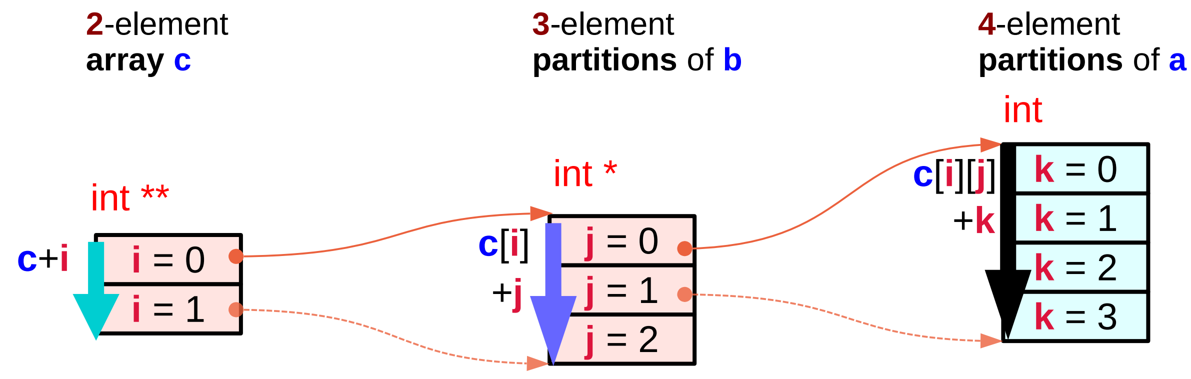
= skipping (i*3+j)*4+k leaf elements from a

# Contiguous constraints for c[i][j][k]

| | |
|---|---|
| int ** | c [2] ; |
| int * | b [2*3] ; |
| int | a [2*3*4] ; |

b[j] = &a[j*4] (= a+j*4)
c[i] = &b[i*3] (= b+i*3)

c[i][j][k] ≡
a[(i*3 +j)*4+k]




c[i] = b+i*3

c[i][j] = a+(i*3+j)*4

| | | |
|---|---|---|
| contiguous 2-elements of array c, i = 0,1 | ⟺ | c[i] ≡ *(c+i) |
| contiguous 3-element partitions of array b, j = 0,1,2 | ⟺ | c[i][j] ≡ *(c[i]+j)<br>c[i][j] ≡ *(*(c+i)+j) |
| contiguous 4-element partitions of array a, k = 0, 1,2,3 | ⟺ | c[i][j][k] ≡ *(c[i][j]+k)<br>c[i][j][k] ≡ *(*(*(c+i)+j)+k) |

# Minimal constraints and implementations

```
int c [2];        int b [2*3];      int c [2*3*4];

    c[0] = &b[0*3];     b[0] = &a[0*4];
    c[1] = &b[1*3];     b[1] = &a[1*4];
                        b[2] = &a[2*4];
                        b[3] = &a[3*4];
                        b[4] = &a[4*4];
                        b[5] = &a[5*4];
```

contiguous 2-element **array c**

contiguous 2·3-element **array b**

contiguous 2·3·4-element **array a**

*minimal constraints*

contiguous 2-element **array c**

two contiguous 3-element **partitions** of **b**

six contiguous 4-element **partitions** of **a**

```
int c [2];        int b1 [3];       int a1 [4];
                  int b2 [3];       int a2 [4];
                                    int a3 [4];
                                    int a4 [4];
                                    int a5 [4];
                                    int a6 [4];

    c[0] = &b1[0];      b1[0] = &a1[0];
    c[1] = &b2[0];      b1[1] = &a2[0];
                        b1[2] = &a3[0];
                        b2[0] = &a4[0];
                        b2[1] = &a5[0];
                        b2[2] = &a6[0];
```

two contiguous 3-element **arrays bi**

six contiguous 4-element **arrays ai**

## Accessing a **contiguous 1-d** array

- **1-d** array access
- **2-d** array access
- **3-d** array access

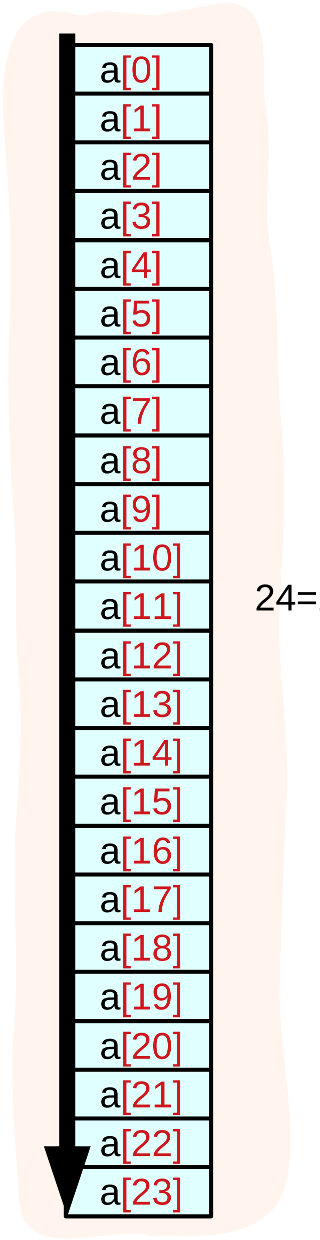
# Accessing an int array **a** as a **1-d** array

int **a** [2*3*4] ;

**a** [**k**]

**k** = 0,1, …,23

int a [2*3*4] ;




24=2*3*4

| | | |
|---|---|---|
| **c**[**i**][**j**][**k**] | ≡ *(*(*(**c**+**i**)+**j**)+**k**) | int ** **c**[2] ; |
| **b**[**j**][**k**] | ≡ *(*(**b**+**j**)+**k**) | int * **b**[2*3] ; |
| **a**[**k**] | ≡ *****(**a**+**k**) | int **a**[2*3*4] ; |

# Accessing an int array **a** as a **2-d** array using **b**

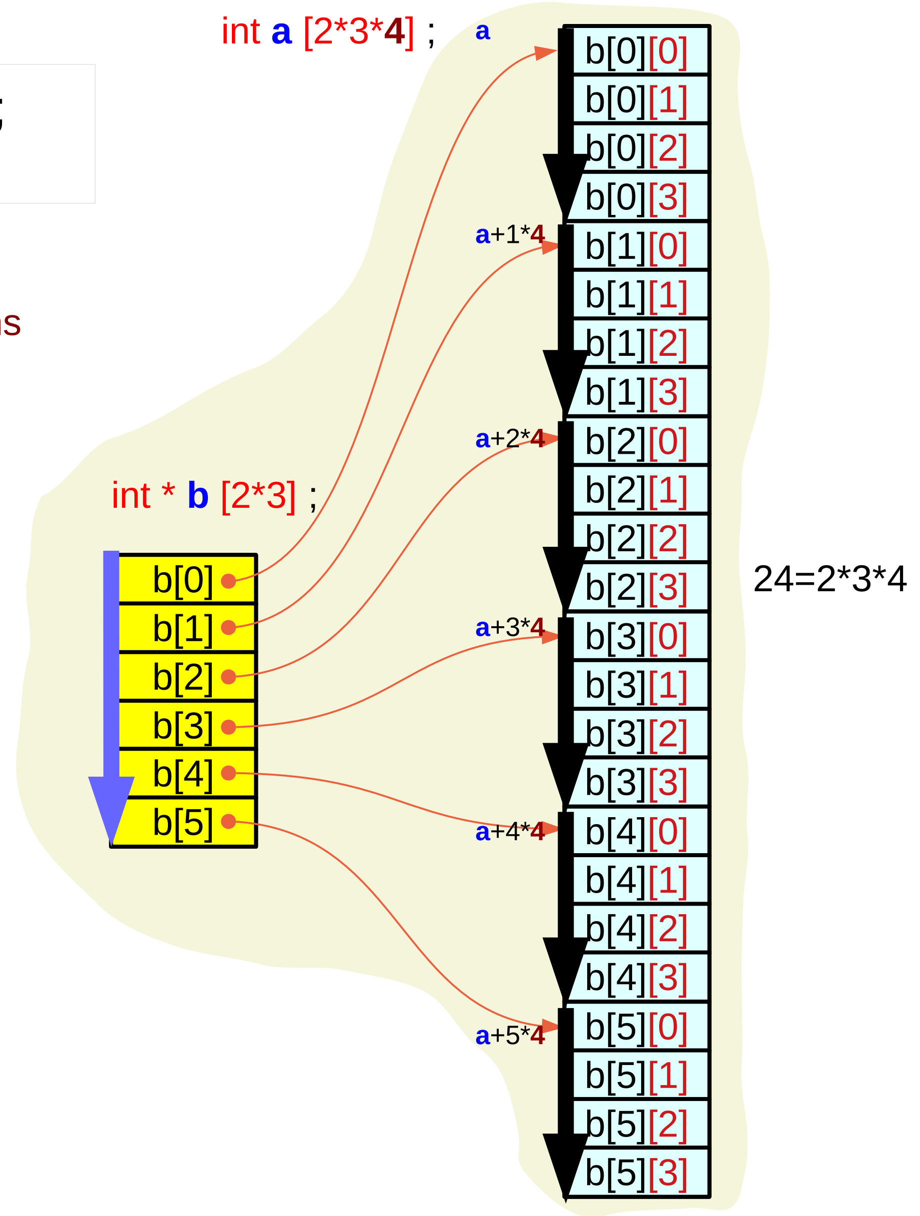
```
int     a [2*3*4] ;
int *   b [2*3] ;
```

b [j][k] ≡ a [j*4 +k]

j = 0,1,2,3,4
k = 0,1,2,3

b[j] = &a[j*4];

**a**, **b** take actual memory locations




| | | |
|---|---|---|
| c[i][j][k] | ≡ *(*(*(c+i)+j)+k) | int ** c[2] ; |
| b[j][k] | ≡ *(*(b+j)+k) | int * b[2*3] ; |
| a[k] | ≡ *(a+k) | int a[2*3*4] ; |

# Accessing an int array **a** as a **3-d** array

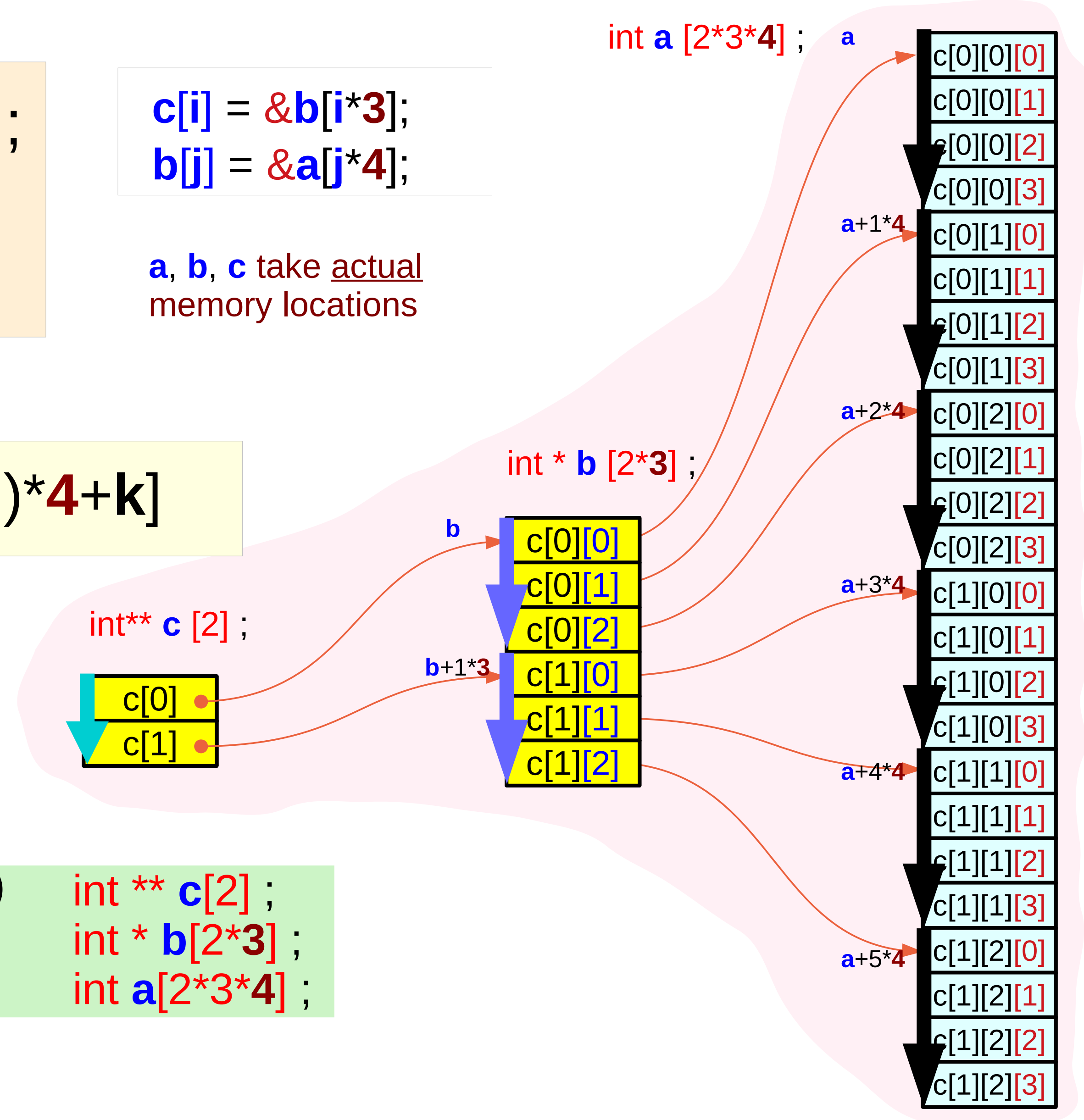
```
int      a [2*3*4] ;
int *    b [2*3] ;
int **   c [2] ;
```

c[i] = &b[i*3];
b[j] = &a[j*4];

**a**, **b**, **c** take actual memory locations

**c [i][j][k] ≡ a [(i*3+j)*4+k]**

i = 0, 1
j = 0, 1, 2
k = 0, 1, 2, 3




| | | |
|---|---|---|
| c[i][j][k] | ≡ *(*(*(c+i)+j)+k) | int ** c[2] ; |
| b[j][k] | ≡ *(*(b+j)+k) | int * b[2*3] ; |
| a[k] | ≡ *(a+k) | int a[2*3*4] ; |

Accessing a **non-contiguous 1-d** arrays

- **3-d** array access

# Accessing **non-contiguous 1-d** arrays as a **3-d** array (1)

```
int      a [2*3*4] ;
int *    b [2*3] ;
int **   c [2] ;
```

**c** [**i**][**j**][**k**]

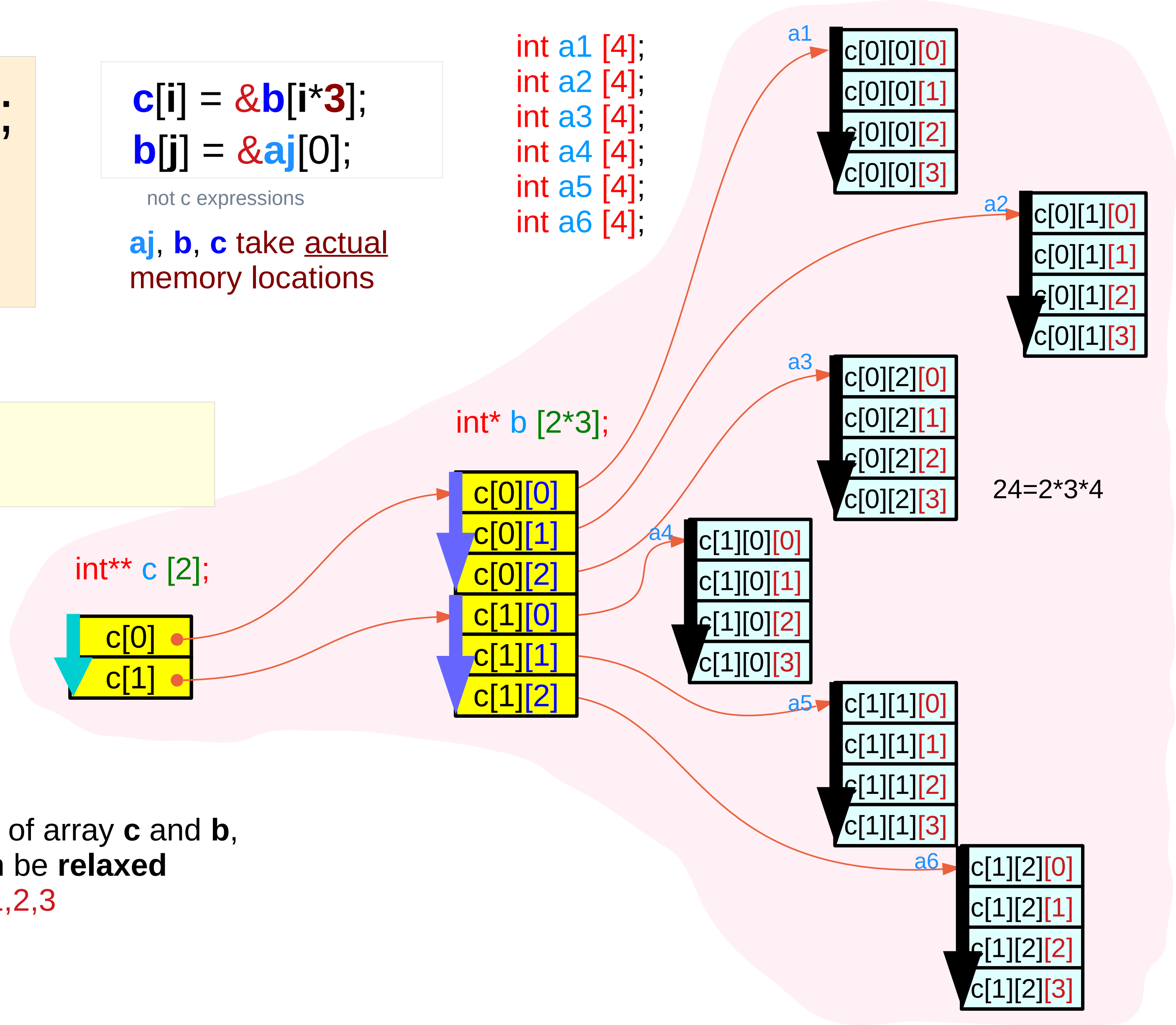
**i** = 0, 1
**j** = 0, 1, 2
**k** = 0, 1, 2, 3

**c**[**i**] = &**b**[**i***3];
**b**[**j**] = &**aj**[0];

not c expressions

**aj**, **b**, **c** take actual memory locations

int a1 [4];
int a2 [4];
int a3 [4];
int a4 [4];
int a5 [4];
int a6 [4];




Because the physical **allocation** of array **c** and **b**,
the **contiguous constraints** can be **relaxed**
contiguous c[i][j][k] only for k=0,1,2,3

# Accessing **non-contiguous 1-d** arrays as a **3-d** array (2)

```
int     a [2*3*4] ;
int *   b [2*3] ;
int **  c [2] ;
```

**c** [i][j][k]

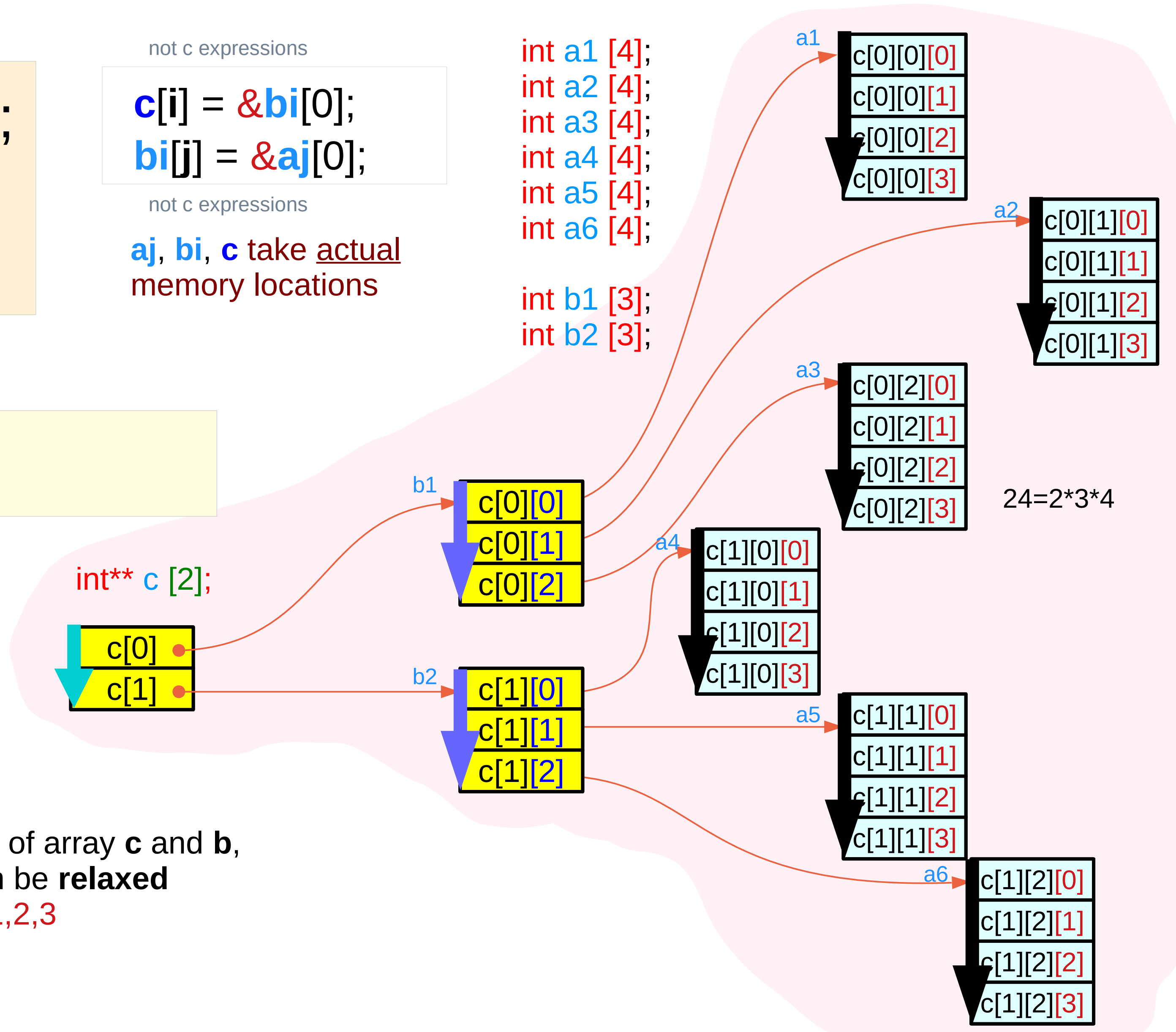
**i** = 0, 1
**j** = 0, 1, 2
**k** = 0, 1, 2, 3

not c expressions

**c**[**i**] = &**bi**[0];
**bi**[**j**] = &**aj**[0];

not c expressions

**aj**, **bi**, **c** take actual memory locations




Because the physical **allocation** of array **c** and **b**,
the **contiguous constraints** can be **relaxed**
contiguous c[i][j][k] only for k=0,1,2,3

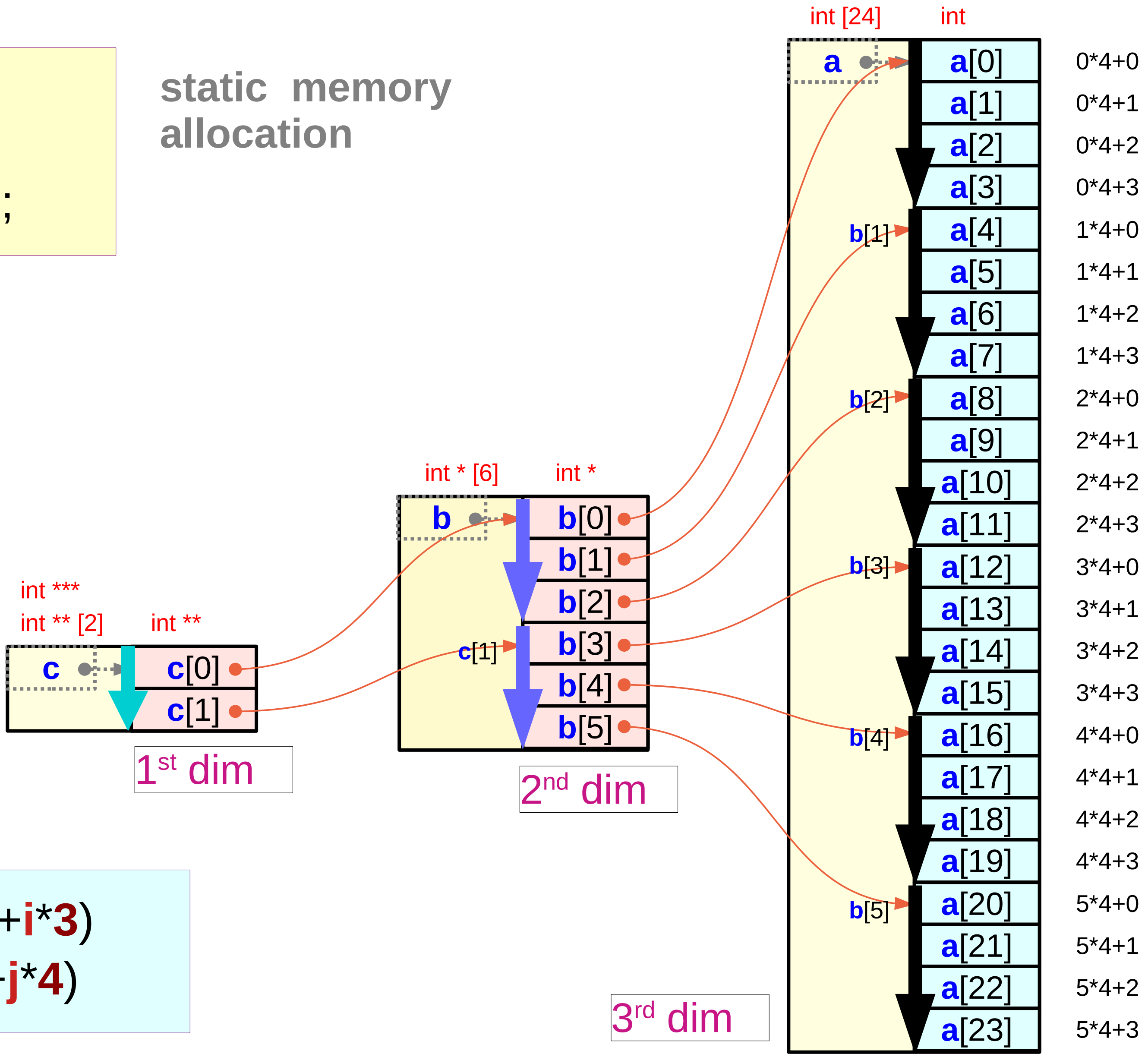
Accessing **statically** allocated arrays

Accessing **dynamically** allocated arrays

# Using arrays **a**, **b**, **c** – statically allocated

| | | |
|---|---|---|
| int ** | c | [2] ; |
| int * | b | [2*3] ; |
| int | a | [2*3*4] ; |

**static memory allocation**



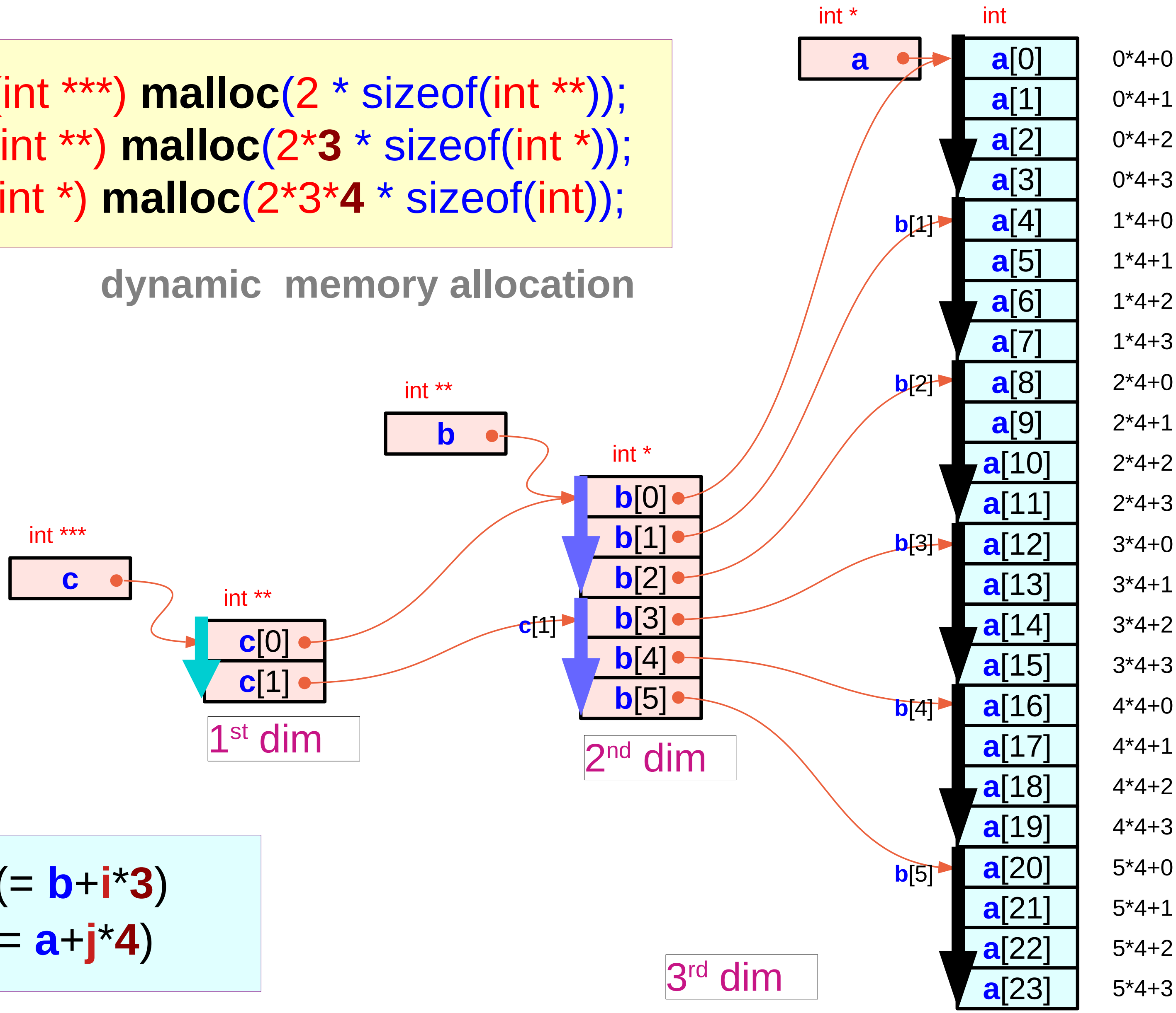

c[i] = &b[i*3] (= b+i*3)

b[j] = &a[j*4] (= a+j*4)

# Using <u>pointer</u> **a**, **b**, **c** – dynamically allocated

| | | |
|---|---|---|
| int *** | **c** | = (int ***) **malloc**(2 * sizeof(int **)); |
| int ** | **b** | = (int **) **malloc**(2*3 * sizeof(int *)); |
| int * | **a** | = (int *) **malloc**(2*3*4 * sizeof(int)); |

**dynamic memory allocation**




**c**[**i**] = &**b**[**i***3] (= **b**+**i***3)

**b**[**j**] = &**a**[**j***4] (= **a**+**j***4)

# Static v.s. dynamic allocation (1)

```
int **  c  [2] ;
int *   b  [2*3] ;
int     a  [2*3*4] ;
```

```
int ***  c  = (int ***) malloc(2 * sizeof(int **));
int **   b  = (int **)  malloc(2*3 * sizeof(int *));
int *    a  = (int *)   malloc(2*3*4 * sizeof(int));
```

## static memory allocations

type(**c**) = int ** [2] → int ***
type(**b**) = int * [2*3] → int **
type(**a**) = int [2*3*4] → int *

sizeof(**c**) = 2 * sizeof(int **)
sizeof(**b**) = 2*3 * sizeof(int *)
sizeof(**a**) = 2*3*4 * sizeof(int)

value(**c[i]**) = **b** + 3***i**
value(**b[j]**) = **a** + 4***j**

## dynamic memory allocations

type(**c**) = int ***
type(**b**) = int **
type(**a**) = int *

sizeof(**c**) = 4 bytes on 32-bit system
sizeof(**b**) = 4 bytes on 32-bit system
sizeof(**a**) = 4 bytes on 32-bit system

value(**c[i]**) = **b** + 3***i**
value(**b[j]**) = **a** + 4***j**

**c**[**i**] = &**b**[**i***3] (= **b**+**i***3)
**b**[**j**] = &**a**[**j***4] (= **a**+**j***4)

# Static v.s. dynamic allocation (2)

- **static allocation**

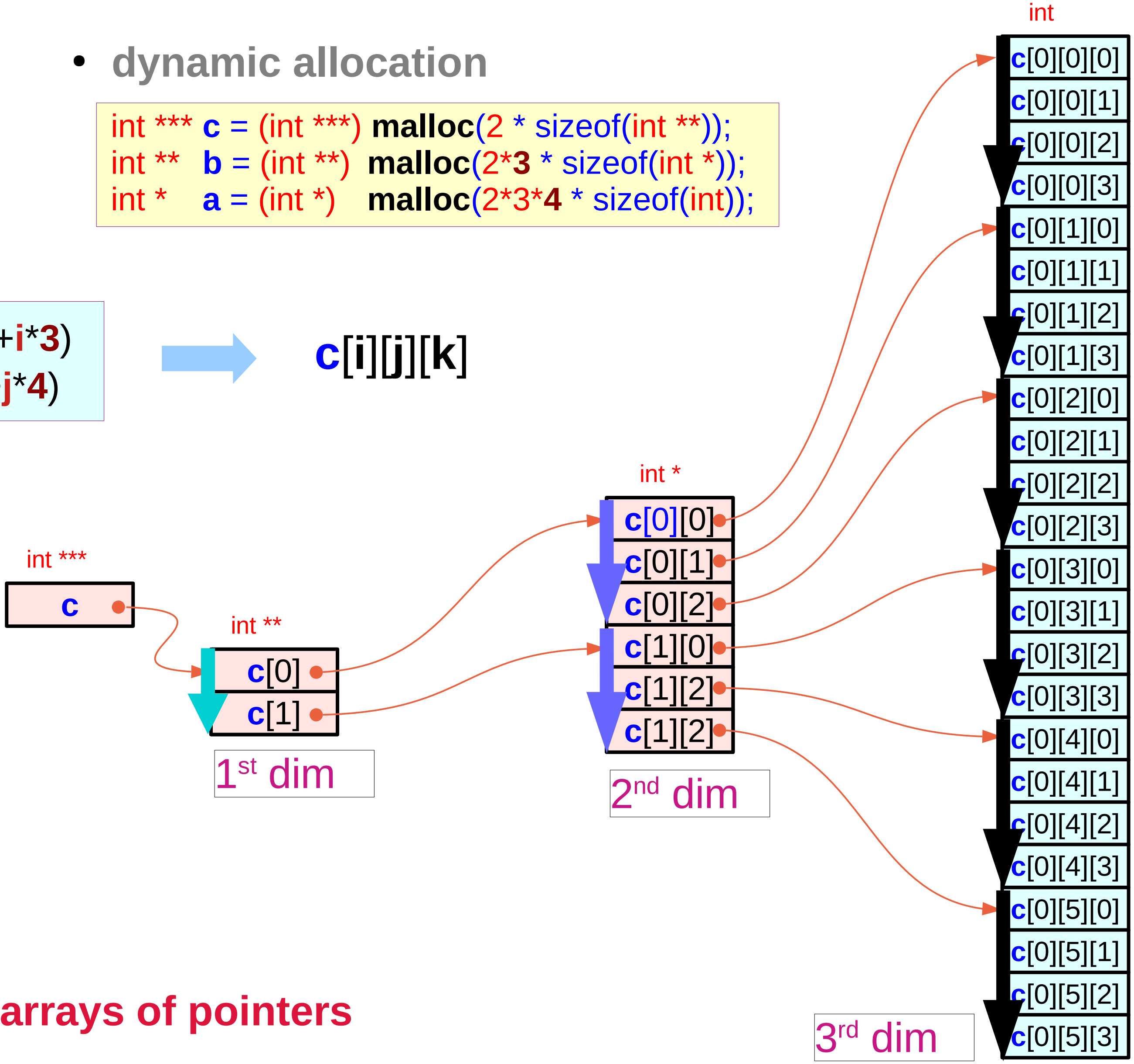
```
int **   c  [2] ;
int *    b  [2*3] ;
int      a  [2*3*4] ;
```

- **dynamic allocation**

```
int *** c = (int ***) malloc(2 * sizeof(int **));
int **  b = (int **)  malloc(2*3 * sizeof(int *));
int *   a = (int *)   malloc(2*3*4 * sizeof(int));
```

**c**[**i**] = &**b**[**i***3] (= **b**+**i***3)
**b**[**j**] = &**a**[**j***4] (= **a**+**j***4)

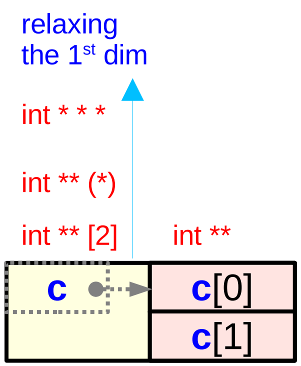
→ **c[i][j][k]**




**arrays of pointers**

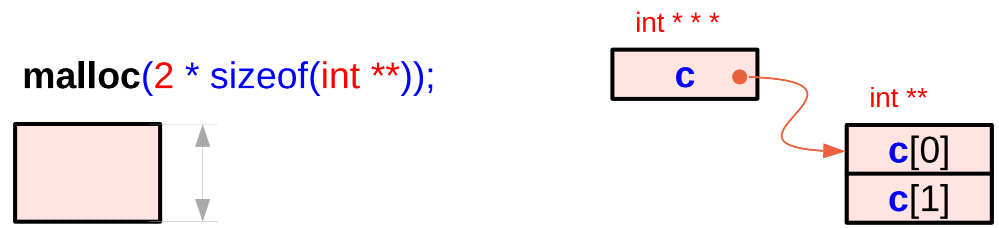
# Static v.s. dynamic allocation (3)

```
int ***   c = (int ***)  malloc(2 * sizeof(int **));
```




```
int **   c   [2] ;
```

**static memory allocation**

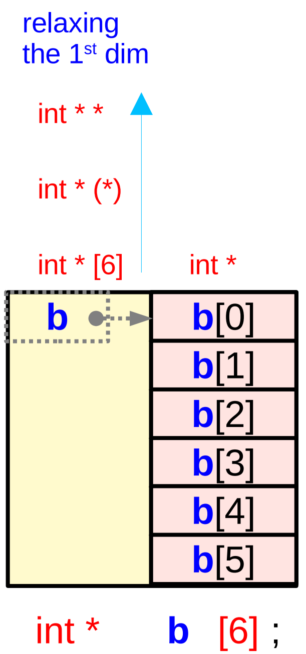


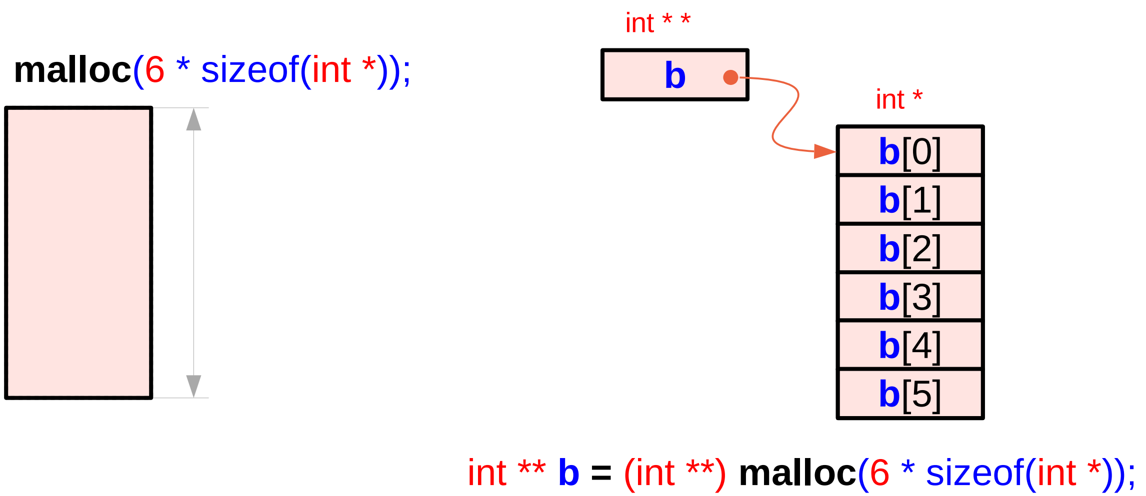

```
int *** c = (int ***) malloc(2 * sizeof(int **));
```

**dynamic memory allocation**

# Static v.s. dynamic allocation (4)

```
int **   b = (int ***)  malloc(6 * sizeof(int *));
```

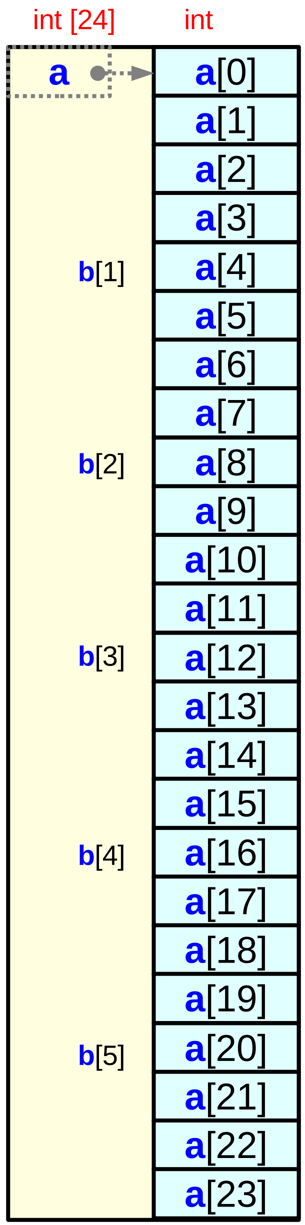



**static memory allocation**

**dynamic memory allocation**

# Static v.s. dynamic allocation (5)




relaxing
the 1st dim

int *

int (*)

int [24]

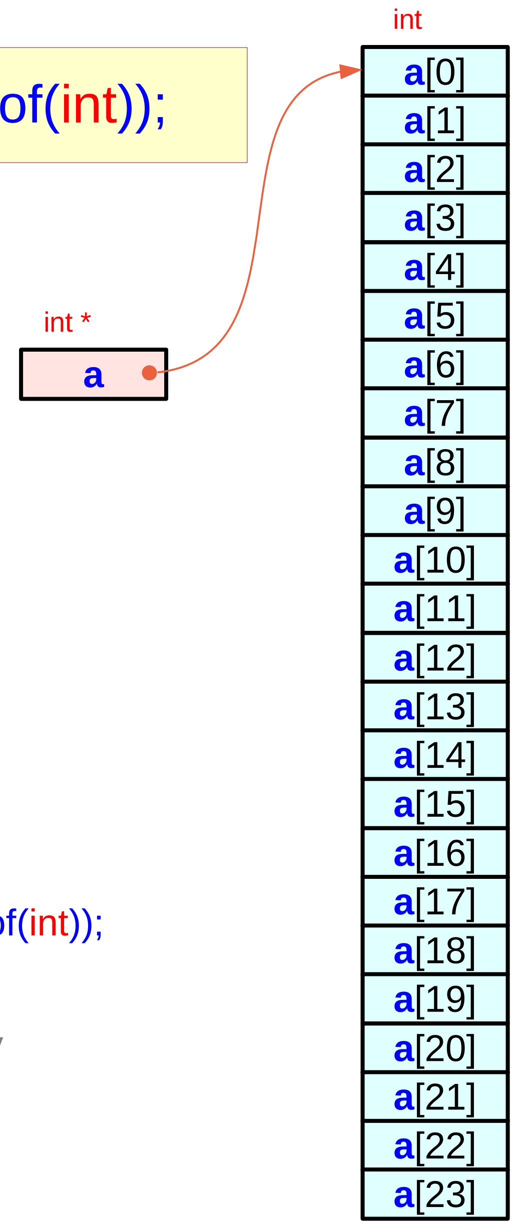
int * a [24] ;

**static memory allocation**

int * a = (int *) **malloc**(24 * sizeof(int));

int * a = (int *) **malloc**(24 * sizeof(int));

**dynamic memory allocation**



**&** address-of operator

***** dereference operator

# Address-of operator and dereferencing operator

*the address of a variable :*
*address-of operator* ***&***

***& variable** :*
*returns the address of a variable*

***variable** has memory locations*
*whose value can be changed*
*by an assignment*

(***variable** must be an lvalue*)

*the content at an address :*
*dereferencing operator* *******

**** address** :*
*returns the value at the address*

**** address** has memory locations*
*whose value can be changed*
*by an assignment*

(**** address** is an lvalue*)

# lvalue and rvalue in assignments

*Left Hand Side* *Right Hand Side*
***LHS*** **=** ***RHS***

```
int   a,  b = 10 ;
int * p, q = &a ;
```

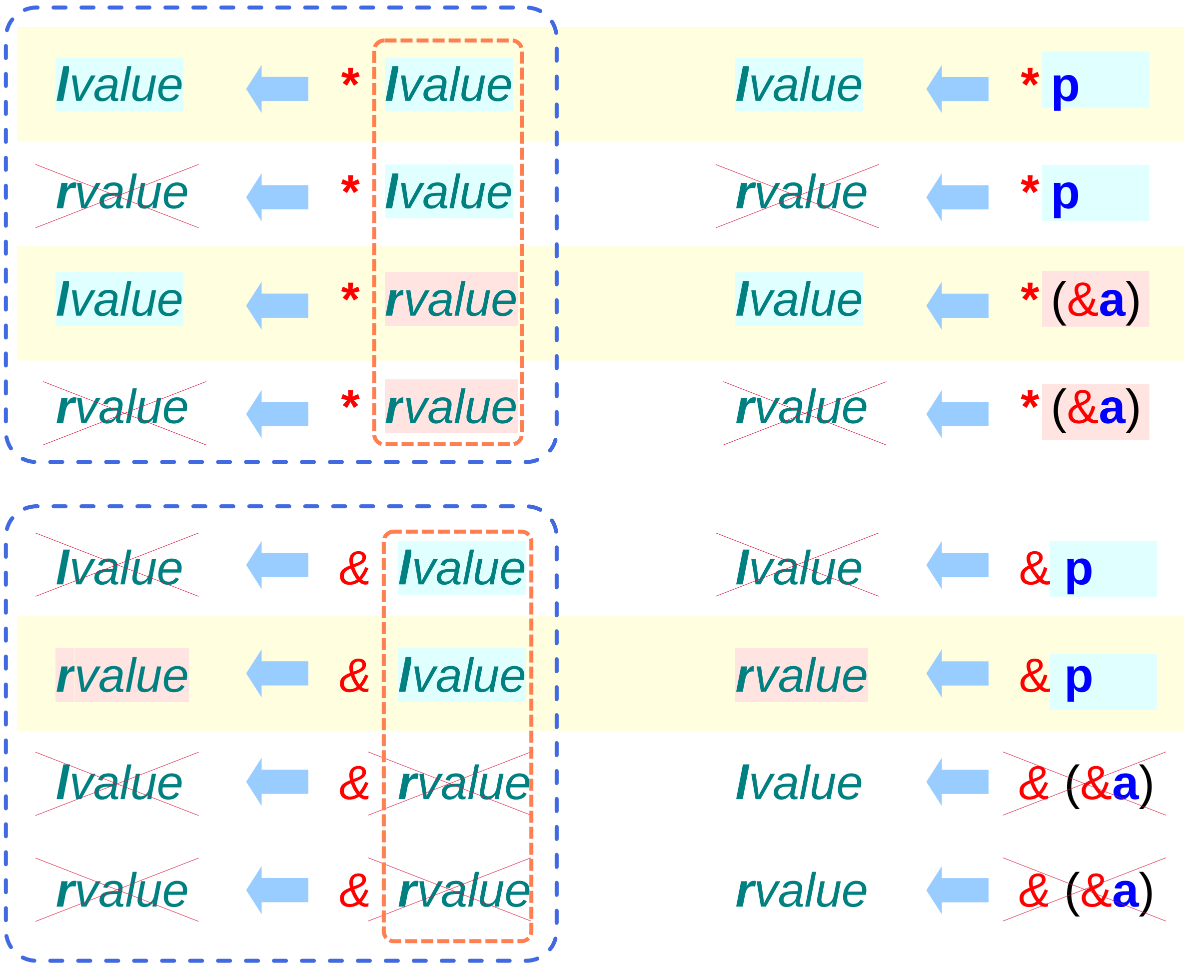
| LHS | | RHS |
|---|---|---|
| lvalue | = | lvalue |
| lvalue | = | rvalue |
| ~~rvalue~~ | = | lvalue |
| ~~rvalue~~ | = | rvalue |

| LHS | | RHS |
|---|---|---|
| p | = | q ; |
| p | = | &a ; |
| ~~&a~~ | = | p ; |
| ~~&a~~ | = | &b ; |

*in the **LHS,** only **l**value can exist*
***r**value can exist only in the **RHS***

| | | | |
|---|---|---|---|
| a, b, p, q | : lvalues | … variables | … RW |
| *p, *q | : lvalues | … variables | … RW |
| &a, &b | : rvalues | … constants | … RO |

# lvalue and rvalue with * and & operators




```
int    a = 10 ;
int *  p = &a ;
```

***** can be applied
to either an **lvalue** variable
or a **rvalue** address

*** operand** becomes
an **lvalue** variable
thus can be applied
successively.

**&** can be applied
to only an **lvalue** variable and
returns only an **rvalue** address

| | | | |
|---|---|---|---|
| **a**, **p** | : lvalues | … variables | … RW |
| *p | : lvalues | … variables | … RW |
| &a | : rvalues | … constants | … RO |

# Address-of and dereference operators

## Primitive Data Type

*

address | data

*lvalue* p
*pointer*

*lvalue* *p
*variable*

*rvalue* &a
*constant*

*lvalue* a
*variable*

&

address | data

*rvalue* &a
*constant*

*lvalue* a
*variable*

## Pointer Data Type

*

address | address

*lvalue* q
*d pointer*

*lvalue* *q
*pointer*

*rvalue* &q
*constant*

*lvalue* q
*d pointer*

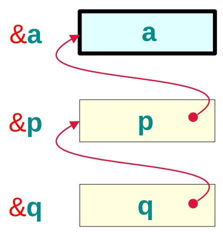
&

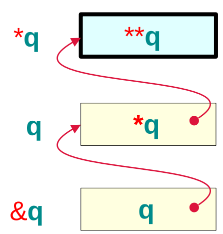
address | address

*rvalue* &q
*constant*

*lvalue* q
*d pointer*

```
int    a ;
int *  p ;
int ** q ;
```



# Byte addresses of sub-arrays in an array

~~&(&(&(c[i])[j])[k])~~

**& C operator**

can be applied to only **lvalue** variable
returns address value
thus, the above expression is not possible

successive application of & is not possible

In constrast, ***p** becomes a lvalue variable
* operator can be applied successively.

# Finding sub-array sizes

int  **c** [2][3][4] ;

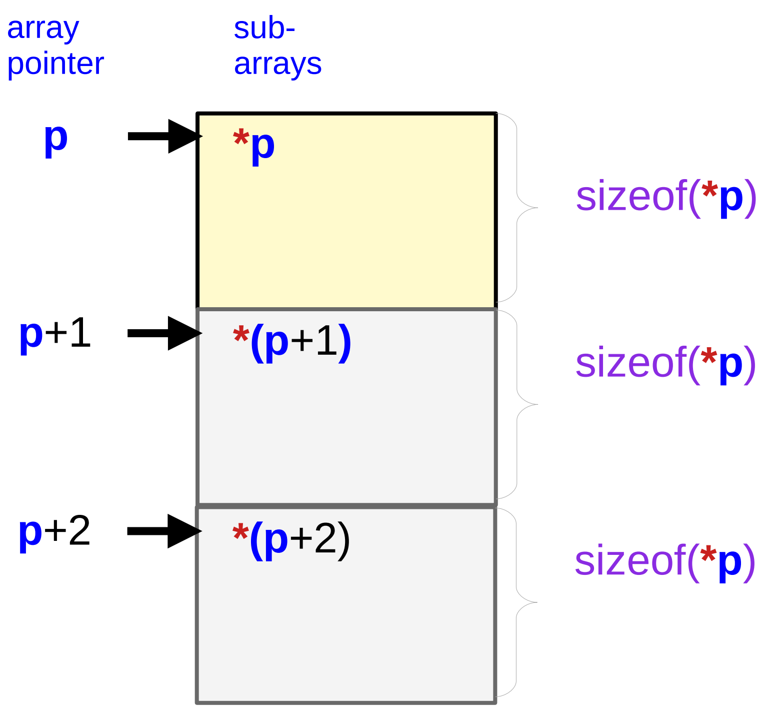
[2] [3] [4]
sizeof(**c**[**i**][**j**][0]) = sizeof(int)

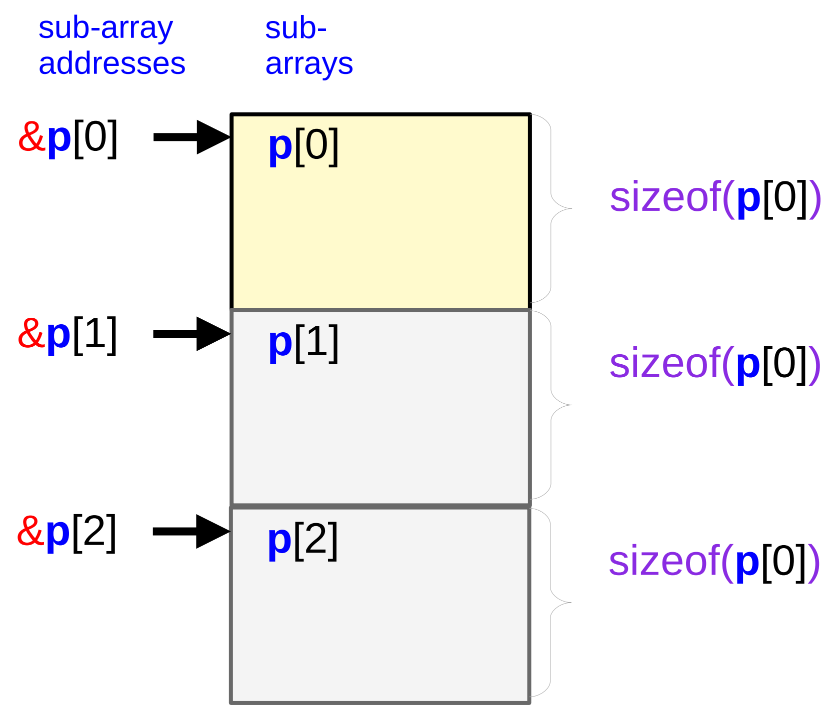
[2] [3] [4]
sizeof(**c**[**i**][0]) = 4 * sizeof(int)

[2] [3] [4]
sizeof(**c**[**i**]) = 3 * 4 * sizeof(int)

[2] [3] [4]
sizeof(**c**) = 2 * 3 * 4 * sizeof(int))

# Pointer increments and byte addresses




byte address    byte address    byte size

value(p+i) = value(p) + i * sizeof(*p)

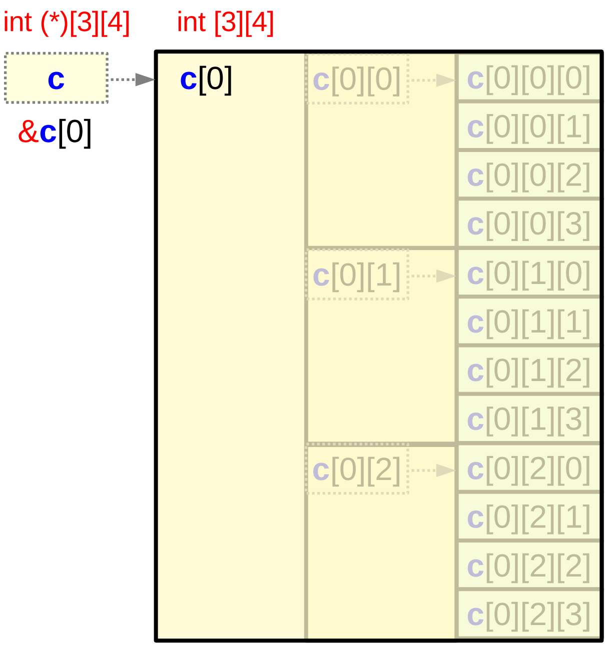
math expression with an explicit size information: $(p+i)_{sizeof(*p)}$

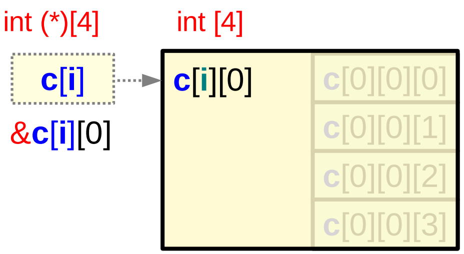
byte address    byte address    byte size

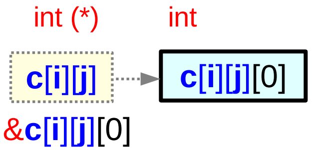
value(&p[i]) = value(p) + i * sizeof(p[0])

math expression with an explicit size information: $(\&p[i])_{sizeof(*p)}$

# Byte addresses of subarrays &c[i], &c[i][j], &c[i][j][k]




value(&c[i]) = value(c+i)
= value(c) + i * sizeof(*c)
= value(c) + i * sizeof(c[0])
= value(c) + i * sizeof(int) * 3 * 4

skip i elements of *c from c

$(c + i)_{3 \cdot 4 \cdot 4}$

value(&c[i][j]) = value(c[i]+j)
= value(c[i]) + j * sizeof(*c[i])
= value(c[i]) + j * sizeof(c[i][0])
= value(c[i]) + j * sizeof(int) * 4

skip j elements of *c[i] from c[i]

$(c[i] + j)_{4 \cdot 4}$

value(&c[i][j][k]) = value(c[i][j]+k)
= value(c[i][j]) + k * sizeof(*c[i][j])
= value(c[i][j]) + k * sizeof(c[i][j][0])
= value(c[i][j]) + k * sizeof(int)

skip k elements of *c[i][j] from c[i][j]

$(c[i][j] + k)_{1 \cdot 4}$

i = 0:1
j = 0:2
k = 0:3

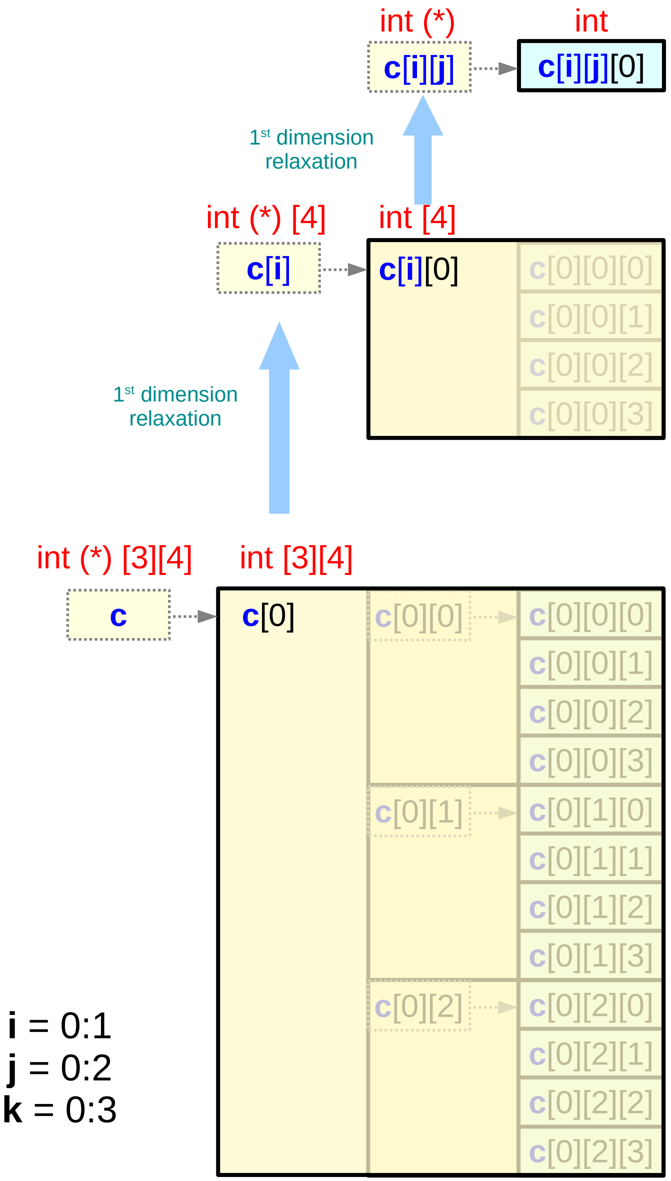
# Address replications and subarray addresses

**Address Replication**

transferring pointing address
to the pointer that references itself

**&X = X**

| | | |
|---|---|---|
| int (*) int<br>value(&**c**[**i**][**j**][**k**]) | = | int (*)<br>value(**c**[**i**][**j**]) + **k** * sizeof(int) |
| int (*) [4] int [4]<br>value(&**c**[**i**][**j**]) | = | int (*) [4]<br>value(**c**[**i**]) + **j** * 4 * sizeof(int) |
| int (*) [3][4] int [3][4]<br>value(&**c**[**i**]) | = | int (*) [3][4]<br>value(**c**) + **i** * 3 * 4 * sizeof(int) |
| byte addresses<br>of abstract data | | values of<br>virtual pointers |




**i** = 0:1
**j** = 0:2
**k** = 0:3

# Address replications in a multi-dimensional array

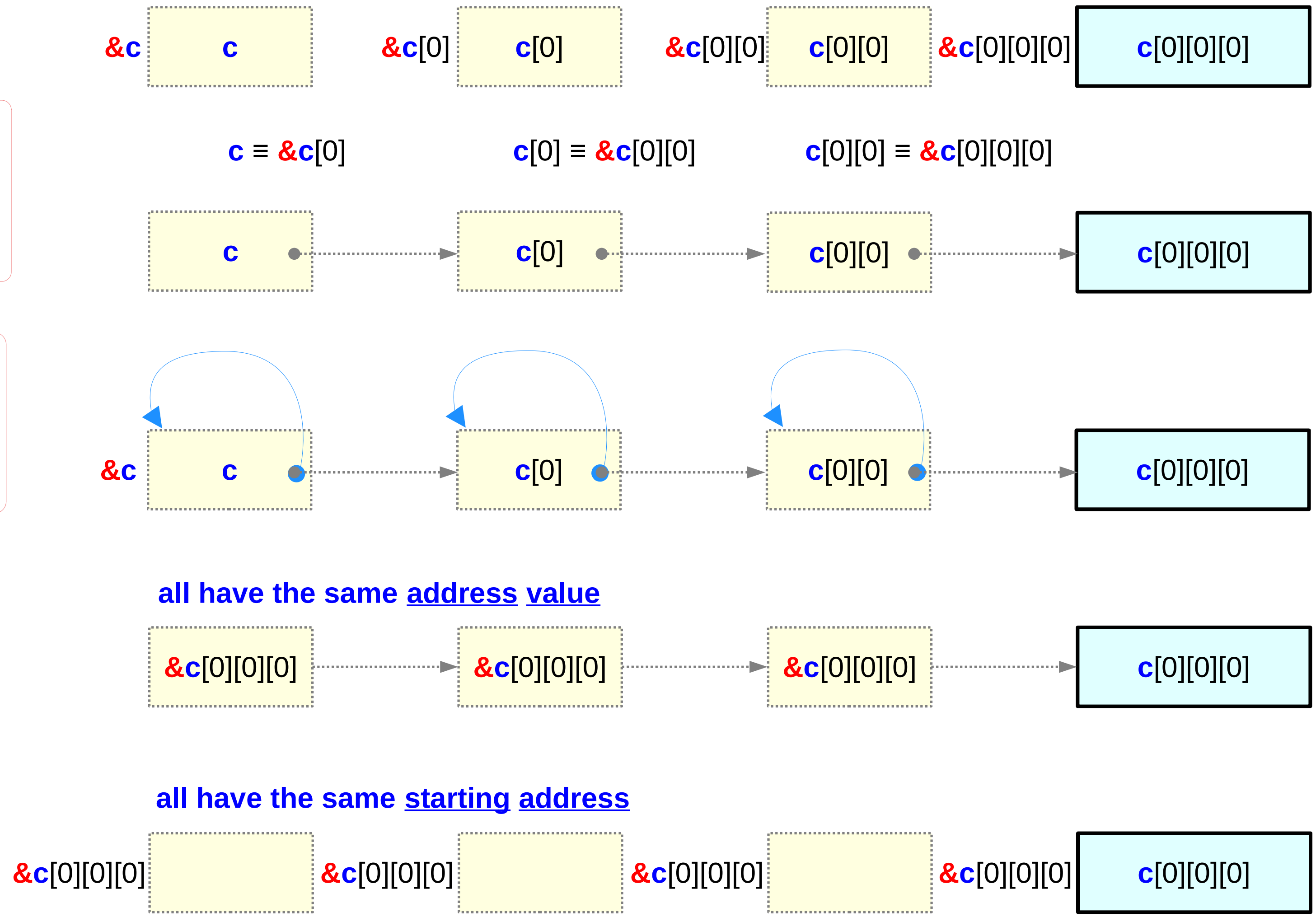
int **c** [2][3][4] ;

**equivalences**

c[i][j] ≡ &c[i][j][0]
c[i] ≡ &c[i][0]
c ≡ &c[0]

**address replication**

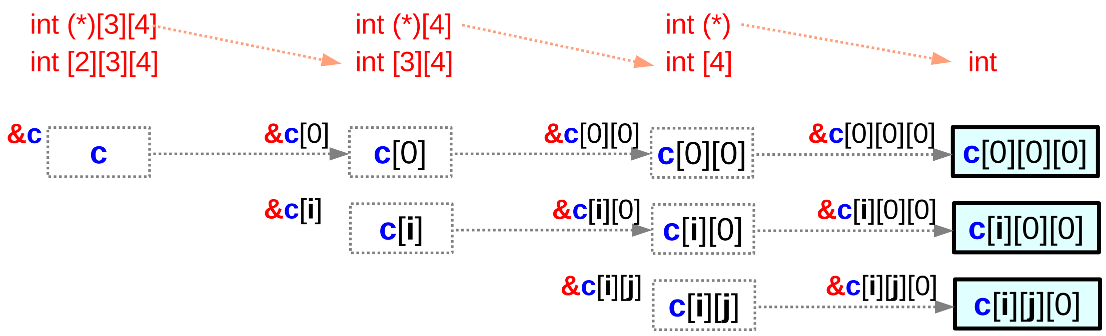
value(c[i][j]) = value(&c[i][j])
value(c[i]) = value(&c[i])
value(c) = value(&c)

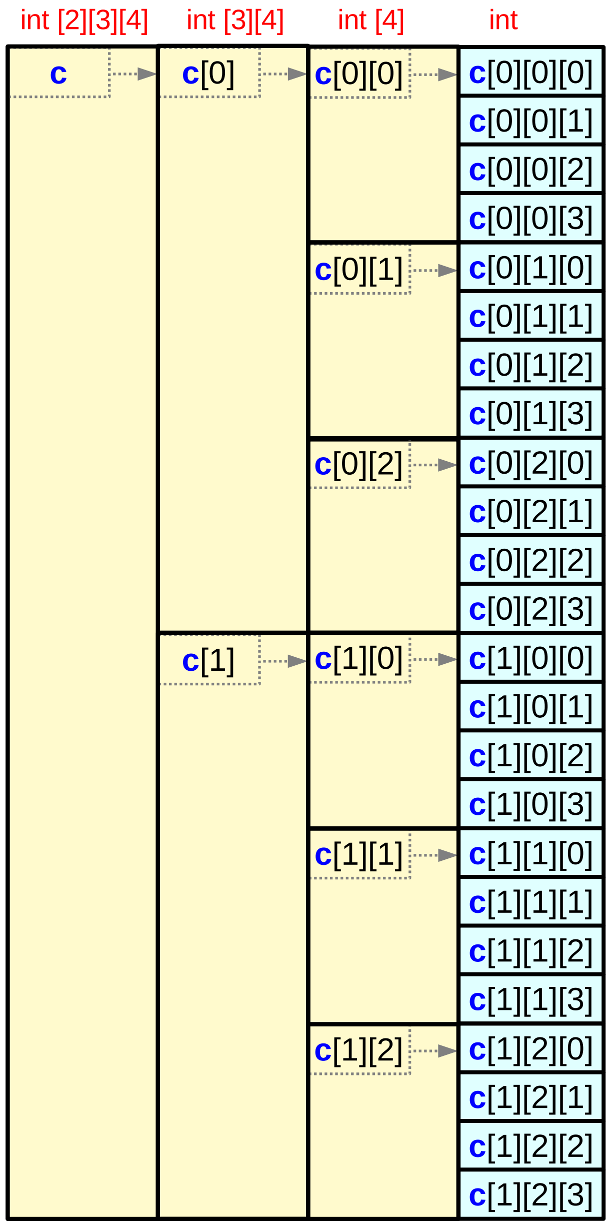



c, c[0], c[0][0] :
these virtual pointers have
the same address value

a physical location
has a unique address

# Referencing sub-arrays




**equivalence relations**

| | | |
|---|---|---|
| c[i][j] ≡ *(c[i]+j) | &c[i][j] ≡ (c[i]+j) | &c[i][0] ≡ c[i] |
| c[i] ≡ *(c+i) | &c[i] ≡ (c+i) | &c[0] ≡ c |

**address replication**

value(c[i][j]) = value(&c[i][j]) = value(c[i]+j) = *value(c[i]+j)
value(c[i]) = value(&c[i]) = value(c+i) = *value(c+i)

c[i], c[i][0] point to the same data c[i][0][0]
c, c[0], c[0][0] point to the same data c[0][0][0]

# Types, sizes, and values of sub-arrays

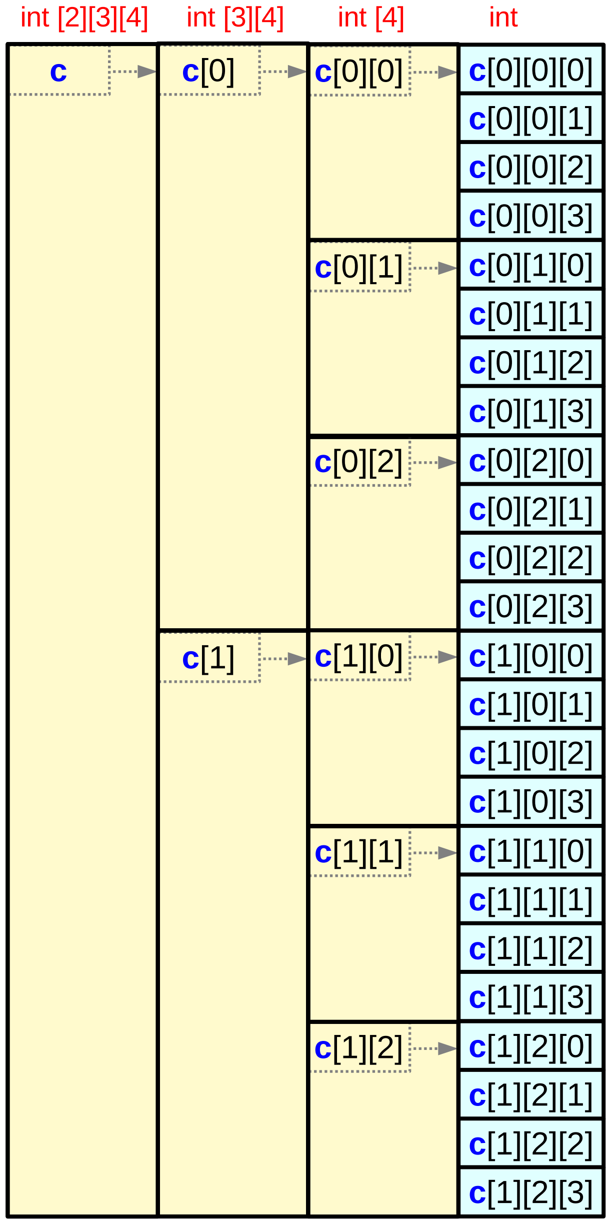
int c [2][3][4] ;        **static allocation**

value(**c**) = value(**c**[0]) = value(**c**[0][0]) = &**c**[0][0][0]
value(**c**[0][1]) = &**c**[0][1][0]
value(**c**[0][2]) = &**c**[0][2][0]
value(**c**[1]) = value(**c**[1][0]) = &**c**[1][0][0]
value(**c**[1][1]) = &**c**[1][1][0]
value(**c**[1][2]) = &**c**[1][2][0]

sizeof(**c**) = 2*3*4 * sizeof(int)
sizeof(**c**[**i**]) = 3*4 * sizeof(int)
sizeof(**c**[**i**][**j**]) = 4 * sizeof(int)

type(**c**) = int [2][3][4]
int (*)[3][4]
type(**c**[**i**]) = int [3][4]
int (*)[4]
type(**c**[**i**][**j**]) = int [4]
int (*)

**pointers to arrays**



# Using multi-dimensional arrays

**Pointer Array Approach**

– using <u>explicit</u> pointers

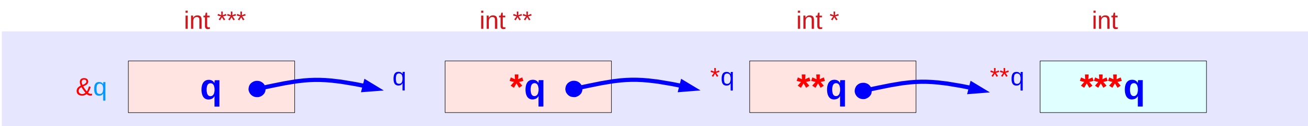
**Array Pointer Approach**

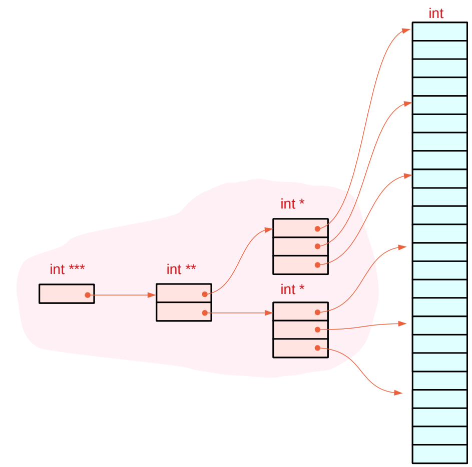
– using <u>implicit</u> pointers

# Two types of **3-d** array accesses

## Pointer Array Approach (arrays of pointers)

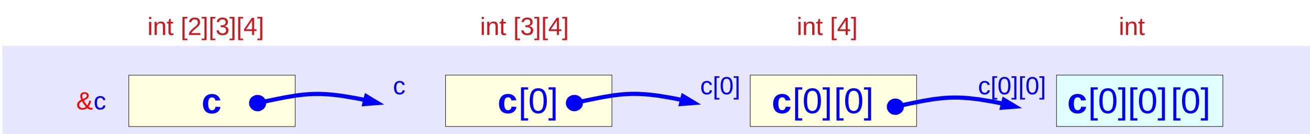
### Pointer Chain Type I

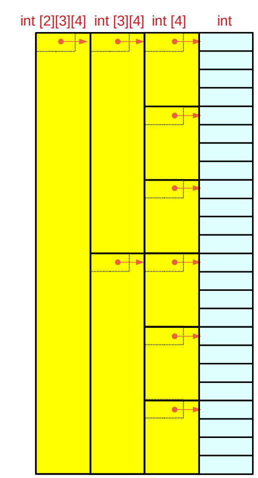



## Array Pointer Approach (pointers to arrays)

### Pointer Chain Type II



# Pointer addition – **math** and **c** expressions

**Accessing c[i][j][k]** – *unified **c** expressions*

skip **i** elements of **c**[**i**] from **c**

skip **j** elements of **c**[**i**][**j**] from **c**[**i**]

skip **k** elements of **c**[**i**][**j**][**k**] from **c**[**i**][**j**]

**(c + i)**

**(c[i] + j)**

**(c[i][j] + k)**

## Pointer Array Approach

$(c + i)_{1 \cdot 4}$

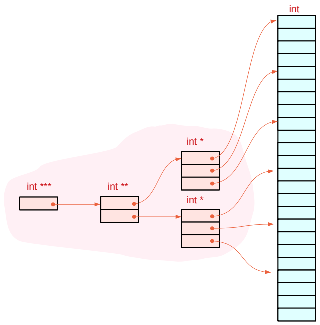
sizeof(*c) = 1*4

$(c[i] + j)_{1 \cdot 4}$

sizeof(*c[i]) = 1*4

$(c[i][j] + k)_{1 \cdot 4}$

sizeof(*c[i][j]) = 1*4




## Array Pointer Approach

$(c + i)_{3 \cdot 4 \cdot 4}$

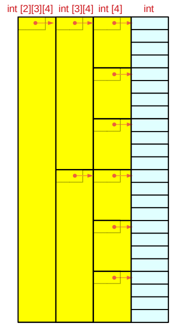
sizeof(*c) = 3*4*4

$(c[i] + j)_{4 \cdot 4}$

sizeof(*c[i]) = 4*4

$(c[i][j] + k)_{1 \cdot 4}$

sizeof(*c[i][j]) = 1*4



# Accessing c[i][j][k] element

## Accessing c[i][j][k]

skip **i** elements
of **c**[i] from **c**

$(c + i)$

skip **j** elements
of **c**[i][j] from **c**[i]

$(c[i] + j)$

skip **k** elements
of **c**[i][j][k] from **c**[i][j]

$(c[i][j] + k)$

## Pointer Array Approach

skip **i** * sizeof(int **)
(= **i***4) bytes from **c**

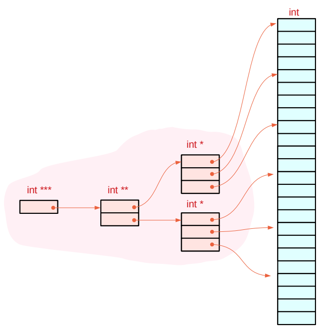
$(c + i)_{1 \cdot 4}$

skip **j** * sizeof(int *)
(= **j***4) bytes from **c**[i]

$(c[i] + j)_{1 \cdot 4}$

skip **k** * sizeof(int)
(= **k***4) bytes from **c**[i][j]

$(c[i][j] + k)_{1 \cdot 4}$




## Array Pointer Approach

skip **i** * sizeof(int [3][4])
(= **i***3*4*4) bytes from **c**

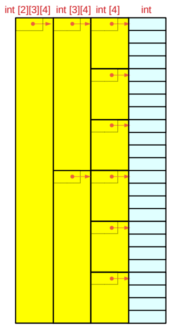
$(c + i)_{3 \cdot 4 \cdot 4}$

skip **j** * sizeof(int [4])
(= **j***4*4) bytes from **c**[i]

$(c[i] + j)_{4 \cdot 4}$

skip **k** * sizeof(int)
(= **k***4) bytes from **c**[i][j]

$(c[i][j] + k)_{1 \cdot 4}$



# Accessing c[i][j][k] – Pointer Array Approach

## Pointer Array Approach

skip **i** * sizeof(int **)
= **i** *4 bytes from **c**

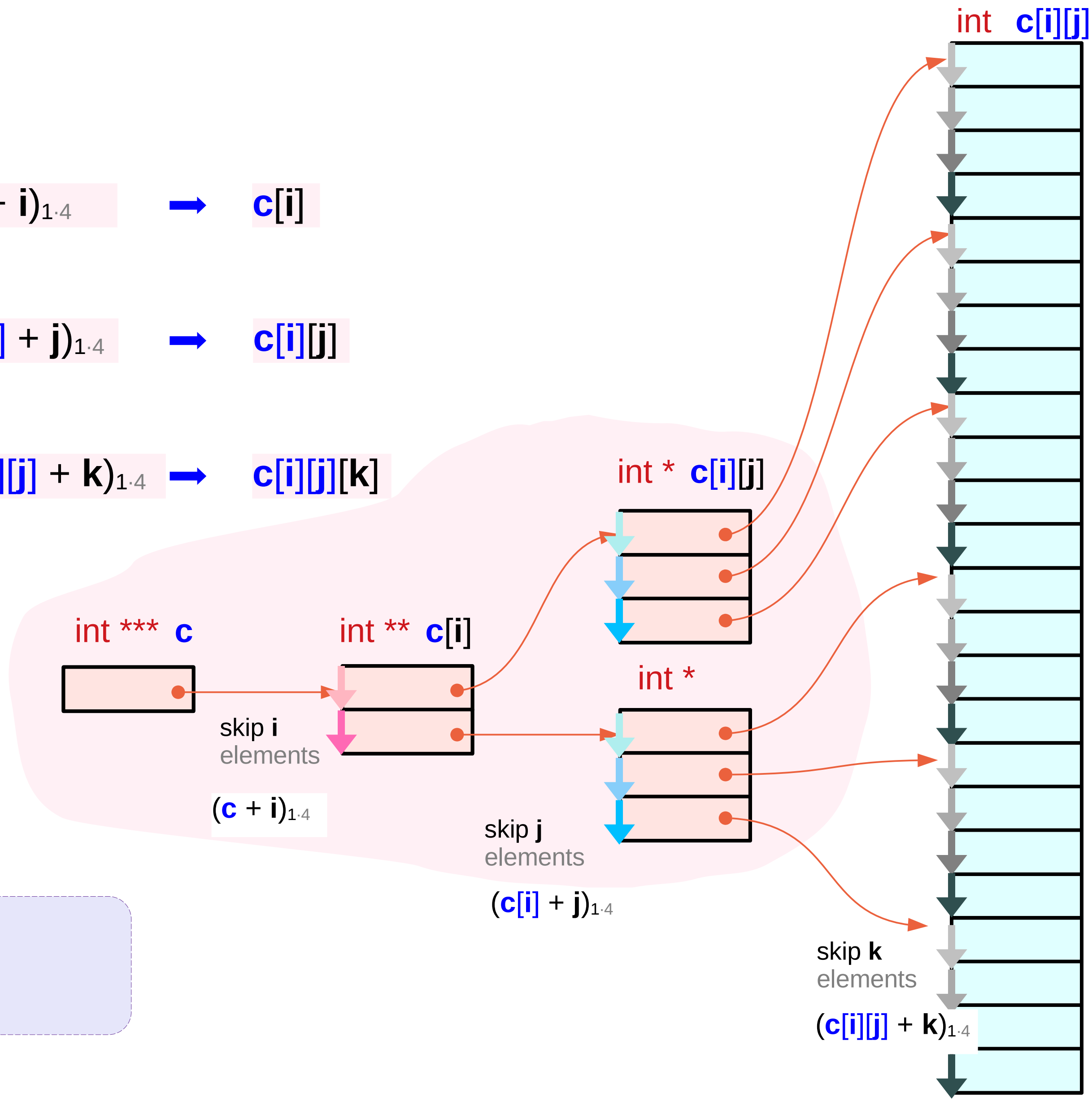
skip **j** * sizeof(int *)
= **j** *4 bytes from **c**[**i**]

skip **k** * sizeof(int)
= **k** *4 bytes from **c**[**i**][**j**]

$(\mathbf{c} + \mathbf{i})_{1\cdot 4}$ ➡ **c**[**i**]

$(\mathbf{c}[\mathbf{i}] + \mathbf{j})_{1\cdot 4}$ ➡ **c**[**i**][**j**]

$(\mathbf{c}[\mathbf{i}][\mathbf{j}] + \mathbf{k})_{1\cdot 4}$ ➡ **c**[**i**][**j**][**k**]




sizeof(**c**[**i**][**j**][**k**]) = sizeof(int) = 4
sizeof(**c**[**i**][**j**]) = sizeof(int *) = 4
sizeof(**c**[**i**]) = sizeof(int **) = 4

# Accessing **c[i][j][k]** – Array Pointer Approach

## Array Pointer Approach

skip **i** * sizeof(int [3][4])
= **i** *3*4*4 bytes from **c**

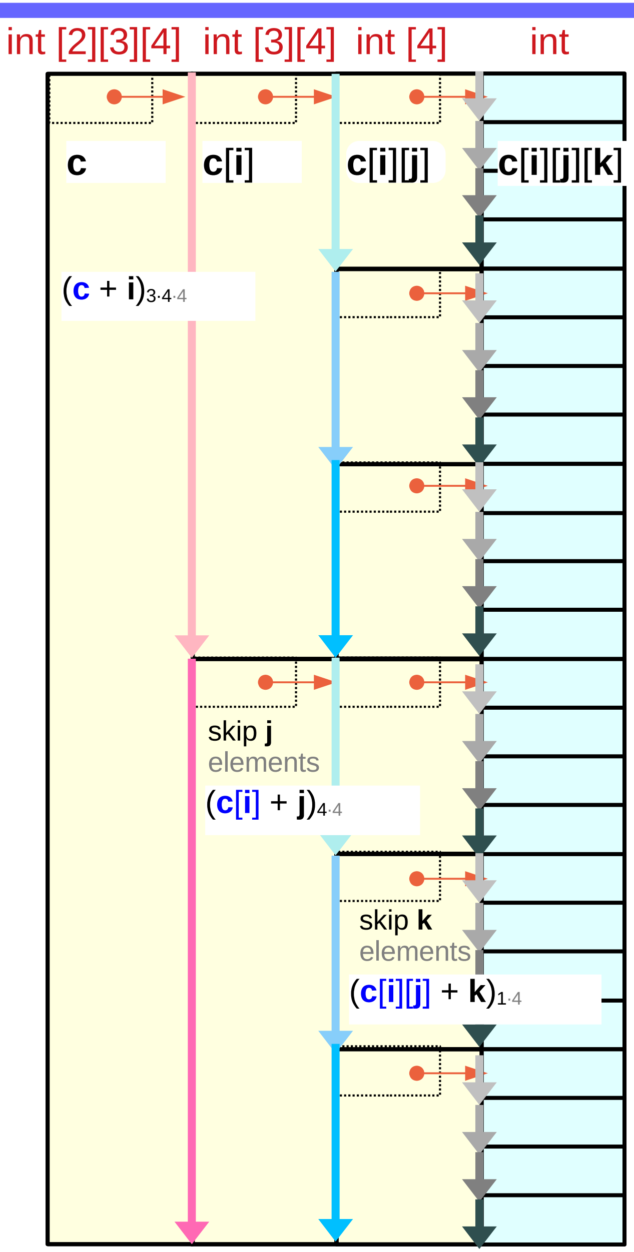
$(\mathbf{c} + \mathbf{i})_{3 \cdot 4 \cdot 4}$ ➡ **c**[**i**]

skip **j** * sizeof(int [4])
= **j** *4*4 bytes from **c**[**i**]

$(\mathbf{c}[\mathbf{i}] + \mathbf{j})_{4 \cdot 4}$ ➡ **c**[**i**][**j**]

skip **k** * sizeof(int)
= **k** *4 bytes from **c**[**i**][**j**]

$(\mathbf{c}[\mathbf{i}][\mathbf{j}] + \mathbf{k})_{1 \cdot 4}$ ➡ **c**[**i**][**j**][**k**]




- ***subarray partitioning***
- ***address replication***

***size information***

sizeof(**c**[**i**][**j**][**k**]) = sizeof(int) = 4
sizeof(**c**[**i**][**j**]) = sizeof(int [4]) = 4*4
sizeof(**c**[**i**]) = sizeof(int [3][4]) = 3*4*4

# Array element address – Pointer Array Approach

*equivalence relations – c expressions*

```
&c[i][j][k]  = (c[i][j] + k)
&c[i][j]     = (c[i] + j)
&c[i]        = (c + i)
```

```
c[i][j][k]   = *(c[i][j] + k)
c[i][j]      = *(c[i] + j)
c[i]         = *(c + i)
```

*size information*

```
sizeof(c[i][j][k]) = sizeof(int) = 4
sizeof(c[i][j]) = sizeof(int *) = 4
sizeof(c[i]) = sizeof(int **) = 4
```

*address fetch – math expressions*

value(c[i][j]) ⇒ *value( $(c[i] + j)_{1\cdot 4}$ ) = *value(c[i] + j * 4) ⇐ sizeof(*c[i])

value(c[i]) ⇒ *value( $(c + i)_{1\cdot 4}$ ) = *value(c + i * 4) ⇐ sizeof(*c)

*address of c[i][j][k] – math expressions*

&c[i][j][k] = value( $(c[i][j] + k)_{1\cdot 4}$ )

= value( *value( $(c[i] + j)_{1\cdot 4}$ ) + k * 4 )

= value( *value( *value( $(c + i)_{1\cdot 4}$ ) + j * 4 ) + k * 4 )

= value( *value( *value( c + i * 4 ) + j * 4 ) + k * 4 )

⇐ &c[i][j][k] ≡ (c[i][j] + k)

⇐ c[i][j] ≡ *(c[i] + j)

⇐ c[i] ≡ *(c + i)

# Array element address – Array Pointer Approach

*equivalence relations – c expressions*

&c[i][j][k] = (c[i][j] + k)
&c[i][j] = (c[i] + j)
&c[i] = (c + i)

c[i][j][k] = *(c[i][j] + k)
c[i][j] = *(c[i] + j)
c[i] = *(c + i)

*size information*

sizeof(c[i][j][k]) = sizeof(int) = 4
sizeof(c[i][j]) = sizeof(int [4]) = 4*4
sizeof(c[i]) = sizeof(int [3][4]) = 3*4*4

*address replication – math expressions*

value(c[i][j]) = value(&c[i][j]) ⇒ value( $(c[i] + j)_{4\cdot 4}$ ) = value(c[i]) + j * 4*4 ⇐ sizeof(*c[i])
value(c[i]) = value(&c[i]) ⇒ value( $(c + i)_{3\cdot 4\cdot 4}$ ) = value(c) + i * 3*4*4 ⇐ sizeof(*c)

*address of c[i][j][k] – math expressions*

&c[i][j][k] = value( $(c[i][j] + k)_{1\cdot 4}$ )
= value( $(c[i] + j)_{4\cdot 4}$ ) + k * 4
= value( $(c + i)_{3\cdot 4\cdot 4}$ ) + j * 4*4 + k * 4
= value( c ) + i * 3*4*4 + j * 4*4 + k * 4

⇐ &c[i][j][k] ≡ c[i][j] + k
⇐ &c[i][j] ≡ c[i] + j ⇐ c[i][j] address replication
⇐ &c[i] ≡ c + i ⇐ c[i] address replication

- ***address replication***
- ***combining size and address information***

# Accessing **c[i][j][k]** via byte addresses

## Pointer Array Approach

&c[i][j][k] = value( (c[i][j] + k)$_{1\cdot 4}$ ) = value( c[i][j] + k * 4 )
&c[i][j] = value( (c[i] + j)$_{1\cdot 4}$ ) = value( c[i] + j * 4 )
&c[i] = value( (c + i)$_{1\cdot 4}$ ) = value( c + i * 4 )

c[i][j][k] = *value( c[i][j] + k * 4 )
c[i][j] = *value( c[i] + j * 4 )
c[i] = *value( c + i * 4 )

c[i][j][k] = *value( c[i][j] + k * 4 )
= *value( *value( c[i] + j * 4 ) + k * 4 )
= *value( *value( *value( c + i * 4 ) + j * 4 ) + k * 4 )

three memory accesses for **c[i][j][k]**

## Array Pointer Approach

&c[i][j][k] = value( (c[i][j] + k)$_{1\cdot 4}$ ) = value( c[i][j] + k * 4 )
&c[i][j] = value( (c[i] + j)$_{4\cdot 4}$ ) = value( c[i] + j * 4*4 )
&c[i] = value( (c + i)$_{3\cdot 4\cdot 4}$ ) = value( c + i * 3*4*4 )

c[i][j][k] = *value( c[i][j] + k * 4 )
c[i][j] = *value( c[i] + j * 4*4 )
c[i] = *value( c + i * 3*4*4 )

c[i][j][k] = *value( c[i][j] + k * 4 )
= *value( *value( c[i] + j * 4*4 ) + k * 4 )
= *value( *value( *value( c + i * 3*4*4 ) + j * 4*4 ) + k * 4 )
= *value( value( value( c + i * 3*4*4 ) + j * 4*4 ) + k * 4 )
= *value( c + i * 3*4*4 + j * 4*4 + k * 4 )

address replication
single memory access for **c[i][j][k]**

# Equivalence relations in **c[i][j][k]**

## Pointer Array Approach

&c[i][j][k] = value( c[i][j] + k * 4 ) = value( c[i][j] ) + k * 4
&c[i][j] = value( c[i] + j * 4 ) = value( c[i] ) + j * 4
&c[i] = value( c + i * 4 ) = value( c ) + i * 4

c[i][j][k] ≠ *value( c[i][j] ) + k * 4
c[i][j] ≠ *value( c[i] ) + j * 4
c[i] ≠ *value( c ) + i * 4

c[i][j][k] = *value( c[i][j] + k * 4 )
c[i][j] = *value( c[i] + j * 4 )
c[i] = *value( c + i * 4 )

different semantics !
be careful in mixed c and math expressions

## Array Pointer Approach

&c[i][j][k] = value( $(c[i][j] + k)_{1 \cdot 4}$ ) = value( c[i][j] ) + k * 4
&c[i][j] = value( $(c[i] + j)_{4 \cdot 4}$ ) = value( c[i] ) + j * 4*4
&c[i] = value( $(c + i)_{3 \cdot 4 \cdot 4}$ ) = value( c ) + i * 3*4*4

c[i][j][k] ≠ *value( c[i][j] ) + k * 4
c[i][j] ≠ *value( c[i] ) + j * 4*4
c[i] ≠ *value( c ) + i * 3*4*4

c[i][j][k] = *value( c[i][j] + k * 4 )
c[i][j] = *value( c[i] + j * 4*4 )
c[i] = *value( c + i * 3*4*4 )

different semantics !
be careful in mixed c and math expressions

# Accessing c[i][j][k]

int c [L][M][N] ;

| | | |
|---|---|---|
| c[i] | ≡ | *(c + i) |
| c[i][j] | ≡ | *(c[i] + j) |
| c[i][j][k] | ≡ | *(c[i][j] + k) |

| | | |
|---|---|---|
| &c[i] | ≡ | (c + i) |
| &c[i][j] | ≡ | (c[i] + j) |
| &c[i][j][k] | ≡ | (c[i][j] + k) |

| equivalence relations | | | multiple indirections | address replications |
|---|---|---|---|---|
| c[i] | ≡ | *(c+i) | ≡ *(c+i) | ≡ (c+i) |
| c[i][j] | ≡ | *(c[i]+j) | ≡ *(*(c+i)+j) | ≡ ((c+i)+j) |
| c[i][j][k] | ≡ | *(c[i][j]+k) | ≡ *(*(*(c+i)+j)+k) | ≡ (((c+i)+j)+k) |
| | | | | ➡ *(c +i +j +k) |

Pointer Array Approach →

Array Pointer Approach →

# Conditions for c[i][j][k]

## Equivalence relations in c[i][j][k]

| | | |
|---|---|---|
| c[i][j][k] | ≡ | *(c[i][j] +k) |
| *(c[i][j] +k) | ≡ | *(*(c[i] +j) +k) |
| *(*(c[i] +j) +k) | ≡ | *(*(*(c +i) +j) +k) |

| | | |
|---|---|---|
| c[i][j][k] | ≡ | (c[i][j] +k) |
| c[i][j] | ≡ | (c[i] +j) |
| c[i] | ≡ | (c +i) |

## Pointer Array Approach

| | | |
|---|---|---|
| c[i][j][k] | ≡ | (c[i][j] +k) |
| c[i][j] | ≡ | (c[i] +j) |
| c[i] | ≡ | (c +i) |

⟺

| | | |
|---|---|---|
| contiguous 4 c[i][j][k]'s | 4 * (int | 4 bytes) |
| contiguous 3 c[i][j]'s | 3 * (int * | 4 or 8 bytes) |
| contiguous 2 c[i]'s | 2 * (int ** | 4 or 8 bytes) |

## Array Pointer Approach

| | | |
|---|---|---|
| c[i][j][k] | ≡ | (c[i][j] +k) |
| c[i][j] | ≡ | (c[i] +j) |
| c[i] | ≡ | (c +i) |

⟺

| | | |
|---|---|---|
| contiguous 4 c[i][j][k]'s | 4 * (int | 4 bytes) |
| contiguous 3 c[i][j]'s | 3 * (int [4] | 4*4 bytes) |
| contiguous 2 c[i]'s | 2 * (int [3][4] | 3*4*4 bytes) |

# Skipping leaf elements

## Continuity Constraints

| | | |
|---|---|---|
| c[i][j][k] | ≡ | (c[i][j] +k) |
| c[i][j] | ≡ | (c[i] +j) |
| c[i] | ≡ | (c +i) |

contiguous **c[i][j][k]** over **k**=0:3
contiguous **c[i][j]** over **j**=0:2
contiguous **c[i]** over **i**=0:1

## Pointer Array Approach

$(c[i][j] + k)_{1 \cdot 4}$ skip **k** * 4 bytes from **c[i][j]**
$(c[i] + j)_{1 \cdot 4}$ skip **j** * 4 bytes from **c[i]**
$(c + i)_{1 \cdot 4}$ skip **i** * 4 bytes from **c**

**k** leaf elements — **k** * 4 bytes
**j** * **4** leaf elements — **j** * **4** * 4 bytes
**i** * **3** * **4** leaf elements — **i** * **3** * **4** * 4 bytes

## Array Pointer Approach

$(c[i][j] + k)_{1 \cdot 4}$ skip **k** * 4 bytes from **c[i][j]**
$(c[i] + j)_{4 \cdot 4}$ skip **j** * 4*4 bytes from **c[i]**
$(c + i)_{3 \cdot 4 \cdot 4}$ skip **i** * 3*4*4 bytes from **c**

**k** leaf elements — **k** * 4 bytes
**j** * **4** leaf elements — **j** * **4** * 4 bytes
**i** * **3** * **4** leaf elements — **i** * **3** * **4** * 4 bytes

# **3-d** Access **c**[**i**][**j**][**k**]

# Accessing c[i][j][k] – Conditions

## General requirements

```
c[i][j][k] = *(c[i][j]+k)
c[i][j]    = *(c[i]+j)
c[i]       = *(c+i)
```

```
&c[i][j][k] = c[i][j]+k
&c[i][j]    = c[i]+j
&c[i]       = c+i
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]       = c
```

## Pointer array approach

```
int** c[2];
int*  b[2*3];
int   c[2*3*4];
```

| | |
|---|---|
| c[i][j][k] | :: int |
| c[i][j] | :: int * |
| c[i] | :: int ** |

| | | |
|---|---|---|
| c[i] | ⟸ | &b[i*3] |
| b[j] | ⟸ | &a[j*4] |

**Hierarchical Pointer Arrays**

## Array pointer approach

```
int c[2][3][4];
```

| | |
|---|---|
| c[i][j][k] | :: int |
| c[i][j] | :: int [4] |
| c[i] | :: int [3][4] |

| | | |
|---|---|---|
| c | ⟸ | &c[0][0][0] |
| c[i] | ⟸ | &c[i][0][0] |
| c[i][j] | ⟸ | &c[i][j][0] |

**Virtual Array Pointers**

# Accessing c[i][j][k] – Pointer Array Approach (1)

c[i] ⇐ &b[i*3]
b[j] ⇐ &a[j*4]

**[2][3][4]**

c[i] ≡ *(c + i)
c[i][j] ≡ *(c[i] + j)
c[i][j][k] ≡ *(c[i][j] + k)

&c[i] ≡ (c + i)
&c[i][j] ≡ (c[i] + j)
&c[i][j][k] ≡ (c[i][j] + k)

b[j] ≡ (a+j*4)

*(b[j]+k) = *(a+j*4+k);

b[j][k] ≡ a[j*4+k]

c[i] ≡ (b+i*3)

*(c[i]+j) = *(b+i*3+j);

c[i][j] ≡ b[i*3+j] ⇒ c[i][j] ≡ (a+(i*3+j)*4)

*(c[i][j]+k) = *(b[i*3+j]+k);

*(c[i][j]+k) = *(a+(i*3+j)*4+k);

c[i][j][k] ≡ a[(i*3+j)*4+k]

```
int** c[2];
int*  b[2*3];
int   a[2*3*4];
```

# Accessing c[i][j][k] – Pointer Array Approach (2)

c[i] ⇐ &b[i*3]
b[j] ⇐ &a[j*4]

[2][3][4]

$(c + i)_{1\cdot4}$ — skip i * sizeof(int **) — i * 4 bytes from c

$(c[i] + j)_{1\cdot4}$ — skip j * sizeof(int *) — j * 4 bytes from c[i]

$(c[i][j] + k)_{1\cdot4}$ — skip k * sizeof(int) — k * 4 bytes from c[i][j]

b[j] ≡ (a+j*4)

skip j elements of b — skip j*4 elements of a

c[i] ≡ (b+i*3)

skip i elements of c — skip i*3 elements of b

b[j][k] ≡ a[j*4+k]

skip j elements of b +
skip k elements of a

c[i][j] ≡ b[i*3+j]

skip i elements of c +
skip j elements of b

skip i*3 elements of b +
skip j elements of b

⇒ c[i][j] ≡ (a+(i*3+j)*4)

skip i*3*4 elements of a +
skip j*4 elements of a +

```
int** c[2];
int*  b[2*3];
int   a[2*3*4];
```

c[i][j][j] ≡ a[(i*3+j)*4+k]

skip i*3*4 elements of a +
skip j*4 elements of a +
skip k elements of a

# Accessing c[i][j][k] – Array Pointer Approach (1)

| | | |
|---|---|---|
| c | ⇐ | &c[0][0][0] |
| c[i] | ⇐ | &c[i][0][0] |
| c[i][j] | ⇐ | &c[i][j][0] |

➡

| | |
|---|---|
| c[i] | ≡ *(c + i) |
| c[i][j] | ≡ *(c[i] + j) |
| c[i][j][k] | ≡ *(c[i][j] + k) |

| | |
|---|---|
| &c[i] | ≡ (c + i) |
| &c[i][j] | ≡ (c[i] + j) |
| &c[i][j][k] | ≡ (c[i][j] + k) |

**[2][3][4]**

value(c) = &c[0][0][0]
value(c[i]) = &c[i][0][0]
value(c[i][j]) = &c[i][j][0]
value(c[i][j][k]) = &c[i][j][k]

➡

sizeof(c) = 2*3*4*sizeof(int)
sizeof(c[i]) = 3*4*sizeof(int)
sizeof(c[i][j]) = 4*sizeof(int)
sizeof(c[i][j][k]) = sizeof(int)

➡

value(c[i]) = &c[0][0][0] + i * 3*4*sizeof(int)
value(c[i][j]) = &c[i][0][0] + j * 4*sizeof(int)
value(c[i][j][k]) = &c[i][j][0] + k * sizeof(int)

➡

&c[i] = value(c) + i * sizeof(*c)
&c[i][j] = value(c[i]) + j * sizeof(*c[i])
&c[i][j][k] = value(c[i][j]) + k * sizeof(*c[i][j])

➡

| | |
|---|---|
| c[i] | ≡ *(c + i) |
| c[i][j] | ≡ *(c[i] + j) |
| c[i][j][k] | ≡ *(c[i][j] + k) |

# Accessing c[i][j][k] – Array Pointer Approach (2)

| | | |
|---|---|---|
| c | ⟸ | &c[0][0][0] |
| c[i] | ⟸ | &c[i][0][0] |
| c[i][j] | ⟸ | &c[i][j][0] |

**[2][3][4]**

$(c + i)_{3\cdot4\cdot4}$

skip
i * sizeof(int [3][4])
i * 3*4*4 bytes
from c

$(c[i] + j)_{4\cdot4}$

skip
j * sizeof(int [4])
j *4*4 bytes
from c[i]

$(c[i][j] + k)_{1\cdot4}$

skip
k * sizeof(int)
k * 4 bytes
from c[i][j]

value(c[i]) = &c[i][0][0]

| skip **i** elements of c[i] | c[i] | skip **i*3*4** elements of c[i][0][0] |
|---|---|---|

value(c[i][j]) = &c[i][j][0]

| skip **i** elements of c[i] | c[i] | skip **i*3*4** elements of c[i][0][0] |
|---|---|---|
| + | | + |
| skip **j** elements of c[i][j] | c[i][j] | skip **j*4** elements of c[i][j][0] |

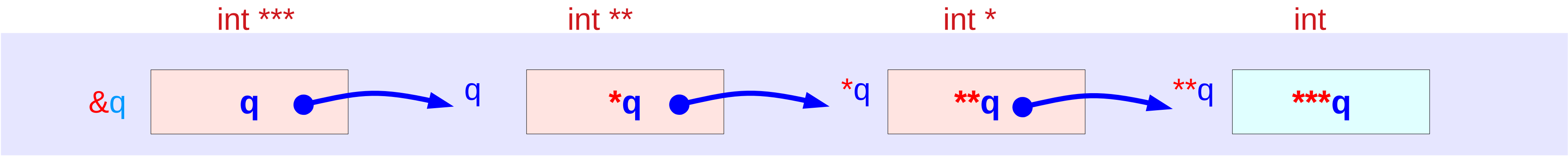
value(c[i][j][k]) = &c[i][j][k]

| skip **i** elements of c[i] | c[i] | skip **i*3*4** elements of c[i][0][0] |
|---|---|---|
| + | | + |
| skip **j** elements of c[i][j] | c[i][j] | skip **j*4** elements of c[i][j][0] |
| + | | + |
| skip **k** elements of c[i][j][k] | c[i][j][k] | skip **k** elements of c[i][j][k] |

# Pointer Chain Types

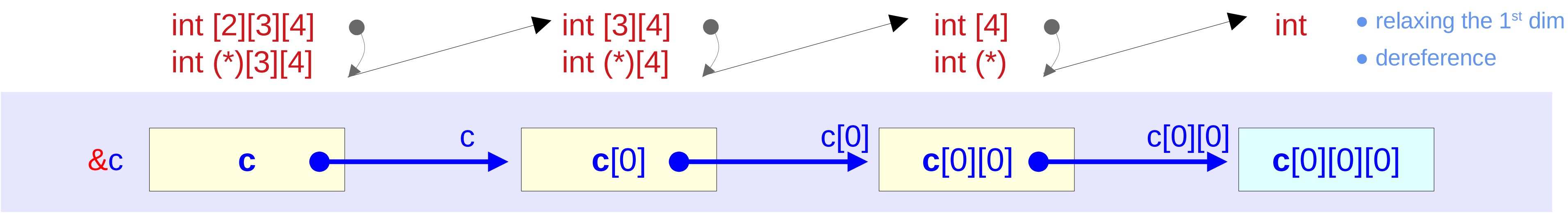
# Pointer Chain Types

## Pointer Chain Type I



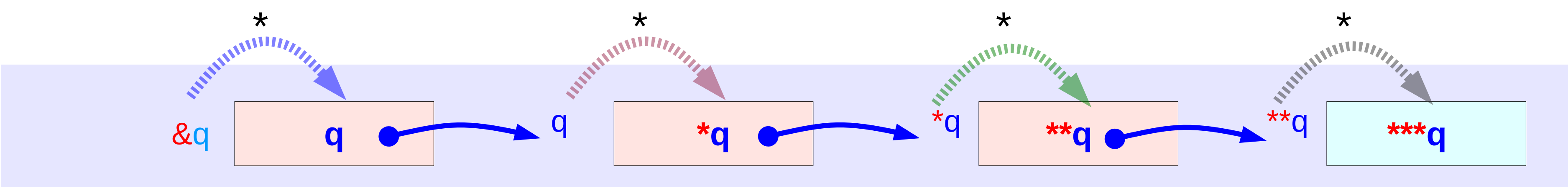

- **Pointer Array Approach** (index operations are handled by a user)

## Pointer Chain Type II




- **Array Pointer Approach** (index operations are handled by a compiler)

# Pointer Chain Type I – * and & operators




*(&q) ≡ q

*(q) ≡ *q

*(*q) ≡ **q

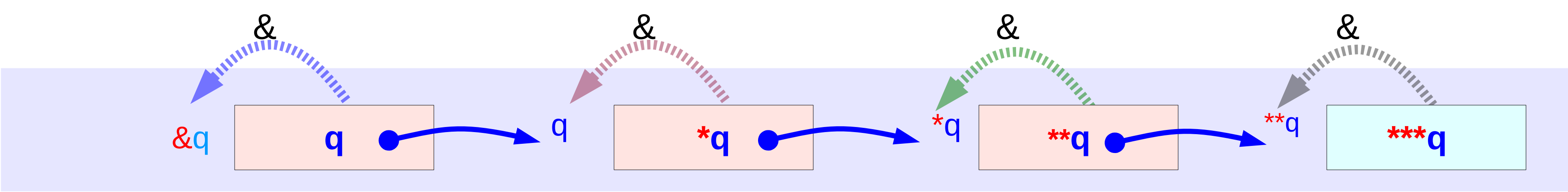
*(**q) ≡ ***q

C expression *(&q) equals to the variable q

C expression *(q) equals to the variable q

C expression *(*q) equals to the variable **q

C expression *(**q) equals to the variable ***q




&q

&(*q) ≡ value(q)

&(**q) ≡ value(*q)

&(***q) ≡ value(**q)

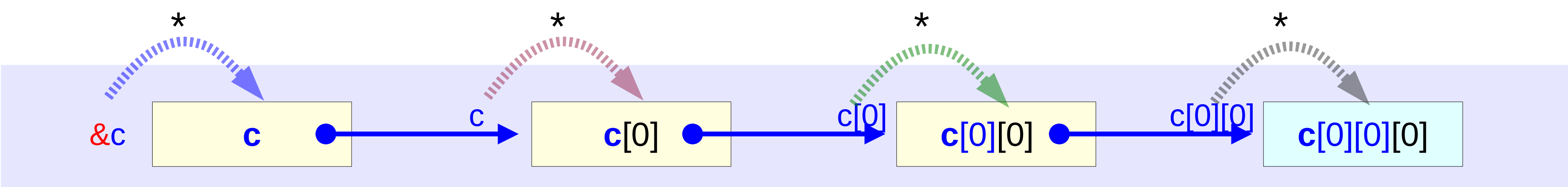
C expression &q equals to value(&q) which is the address value of a variable q

C expression &(*q) equals to value(q) which is the address value of a variable *q

C expression &(**q) equals to value(*q) which is the address value of a variable **q

C expression &(***q) equals to value(**q) which is the address value of a variable ***q

# Pointer Chain Type II – * and & operators




*(&c) ≡ c

*(c) ≡ c[0]

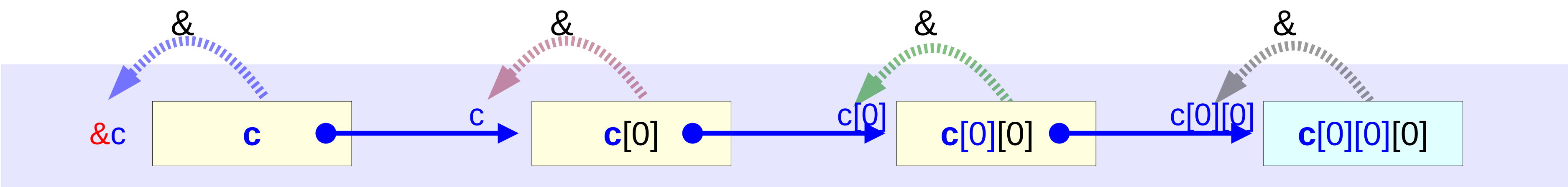
*(c[0]) ≡ c[0][0]

*(c[0][0]) ≡ c[0][0][0]

(int (*)[3][4]) c can be viewed as a pointer to (int [3][4]) c[0]

(int (*)[4]) c[0] can be viewed as a pointer to (int [4]) c[0][0]

(int (*)) c[0][0] can be viewed as a pointer to (int) c[0][0][0]




&c

&(c[0]) ≡ value(c)

&(c[0][0]) ≡ value(c[0])

&(c[0][0][0]) ≡ value(c[0][0])

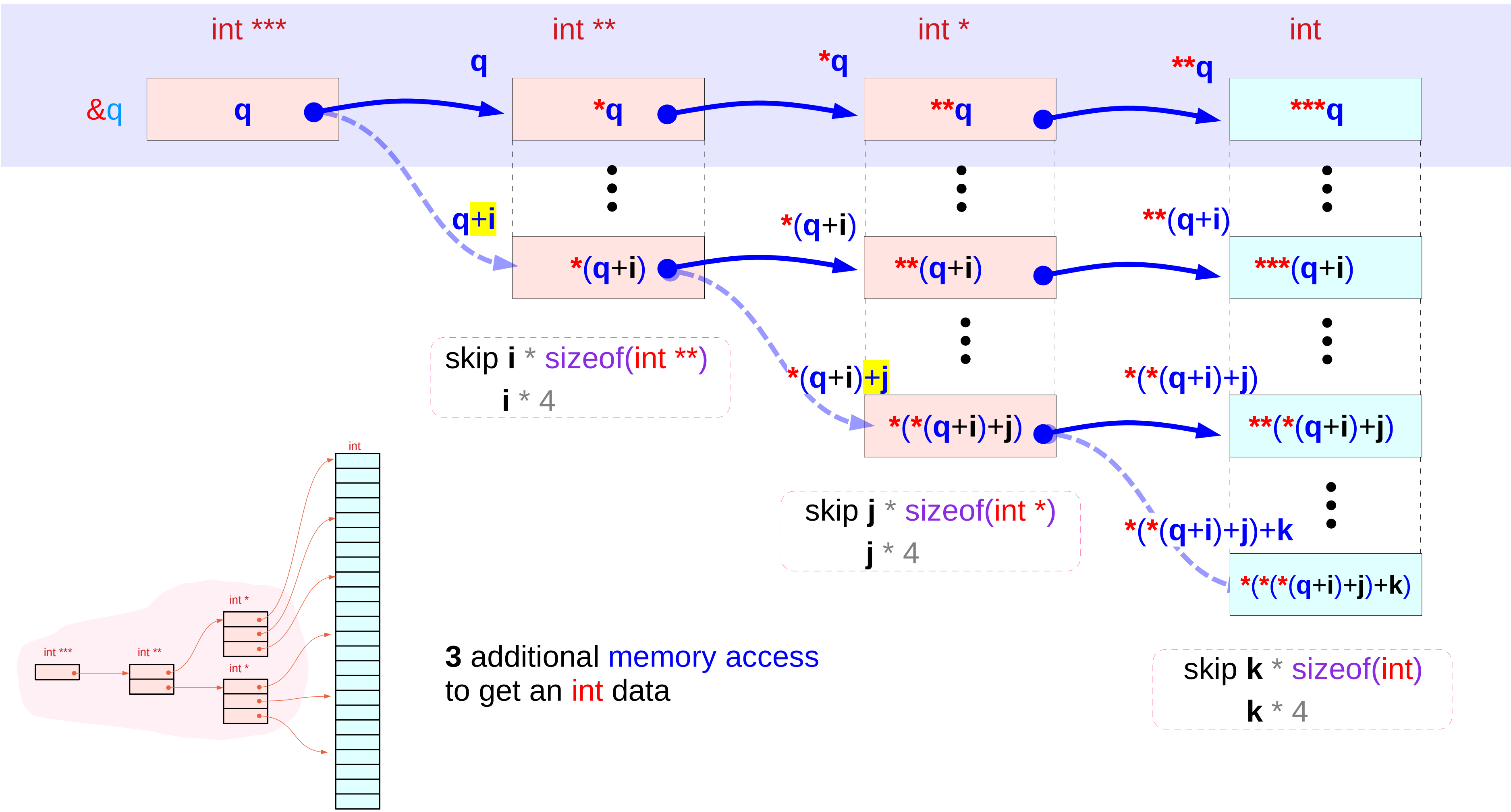
(int (*)[3][4]) c has the address value of (int [3][4]) c[0]

(int (*)[4]) c[0] has the address value of (int [4]) c[0][0]

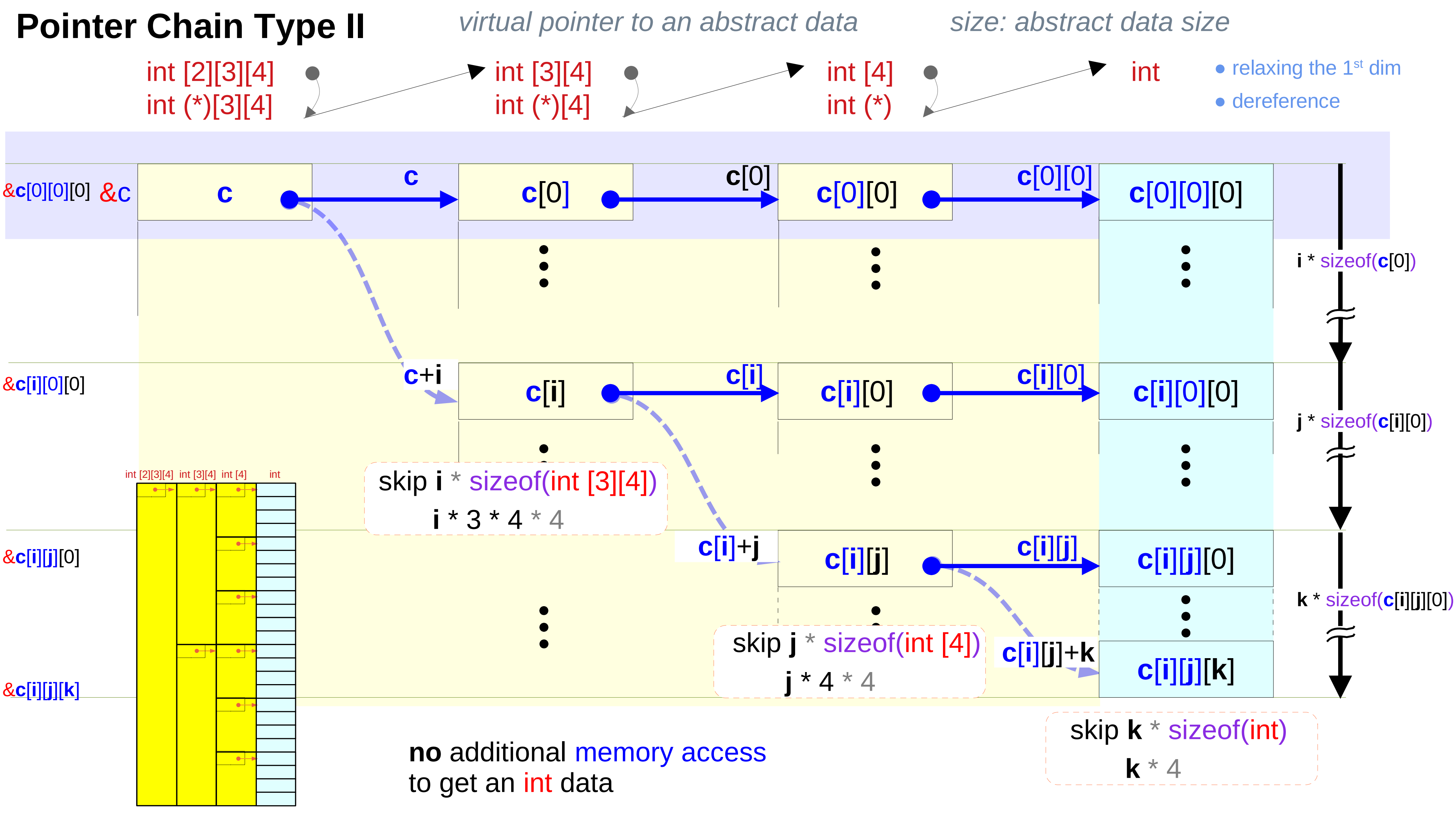
(int (*)) c[0][0] has the address value of (int) c[0][0][0]

# Pointer Chain Type II – skipping elements

**Pointer Chain Type I** *multiple indirection* *size: pointer size*

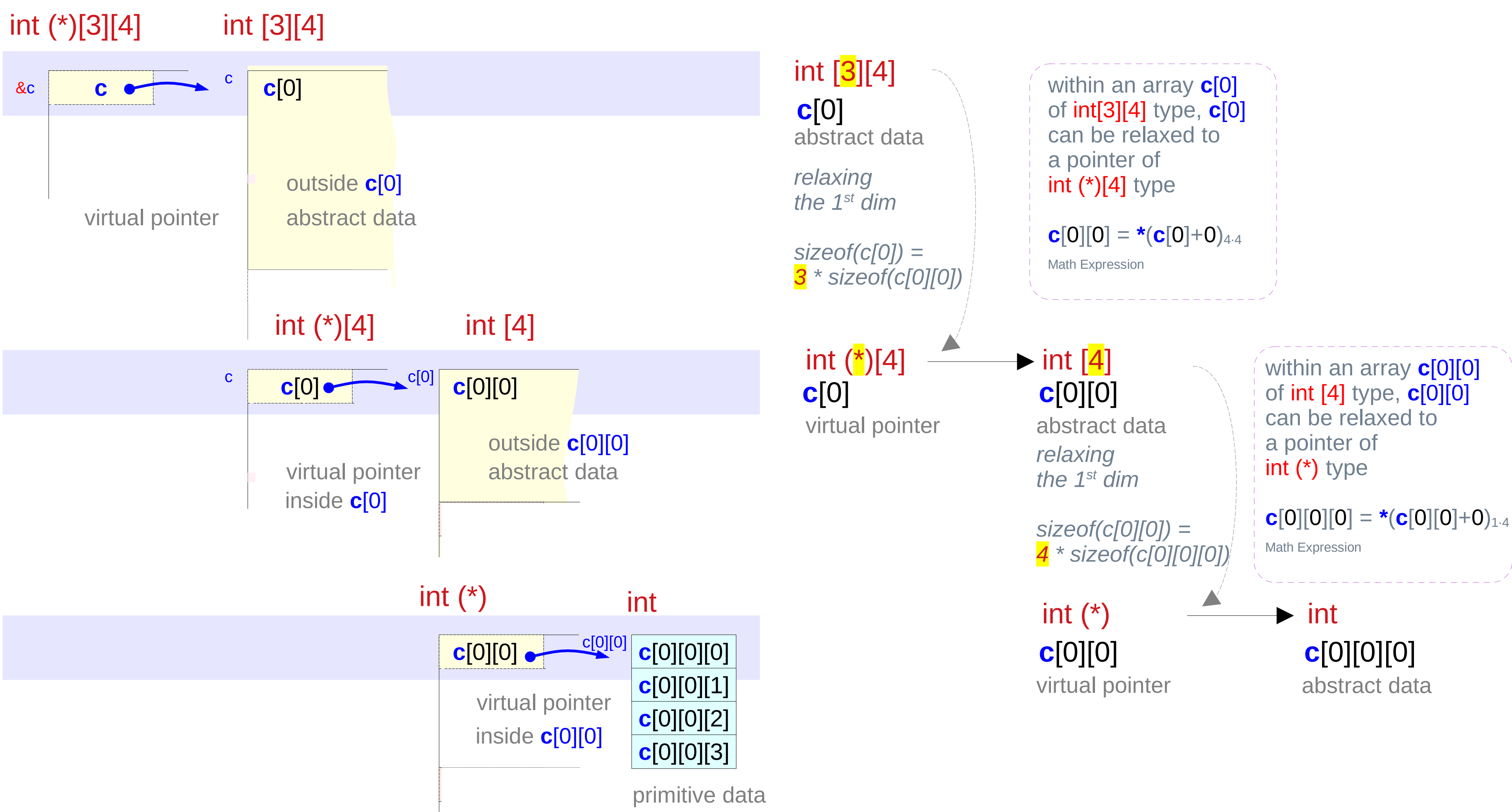


# Pointer Chain Type I – skipping elements

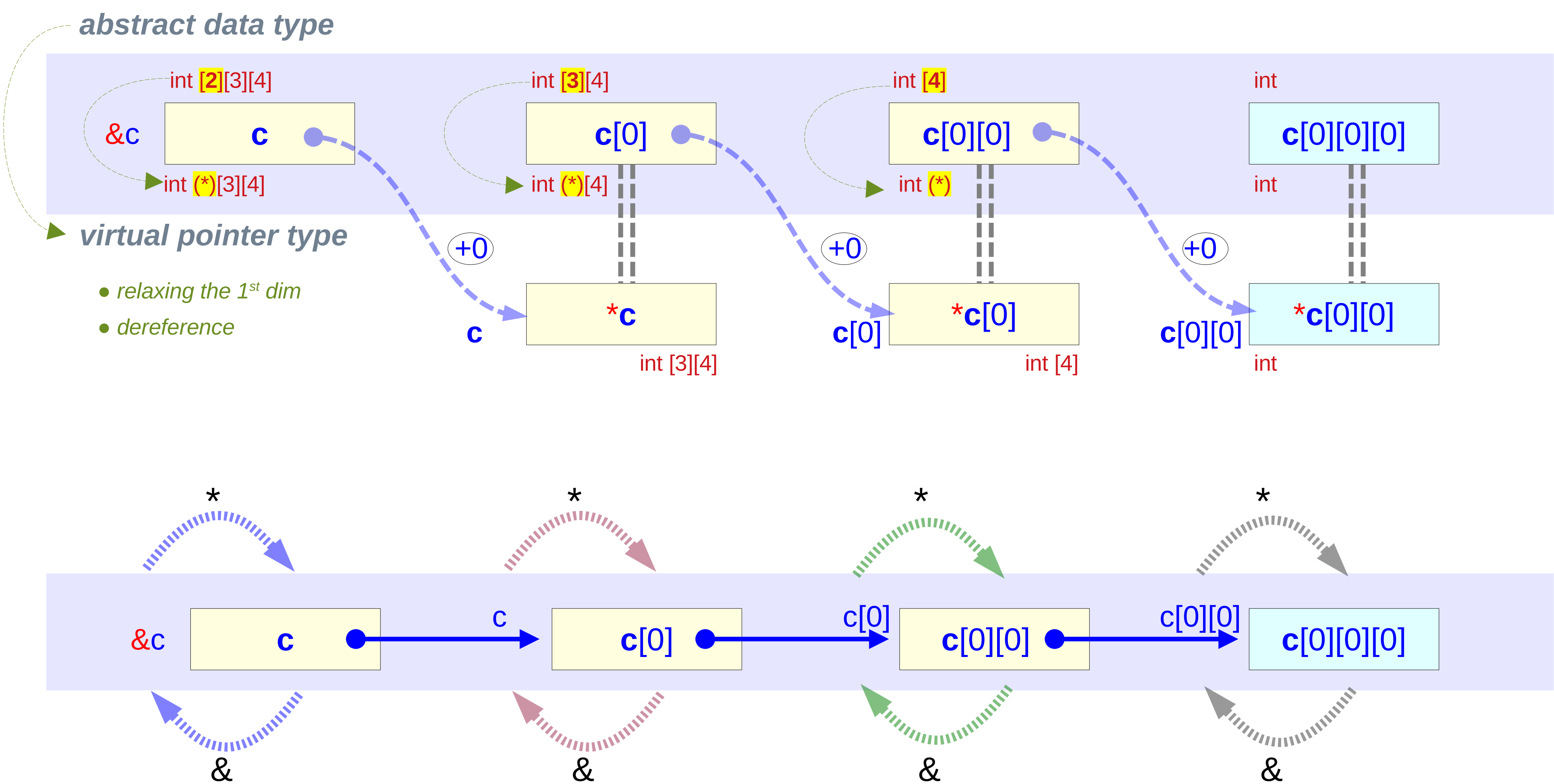



**no** additional memory access
to get an int data

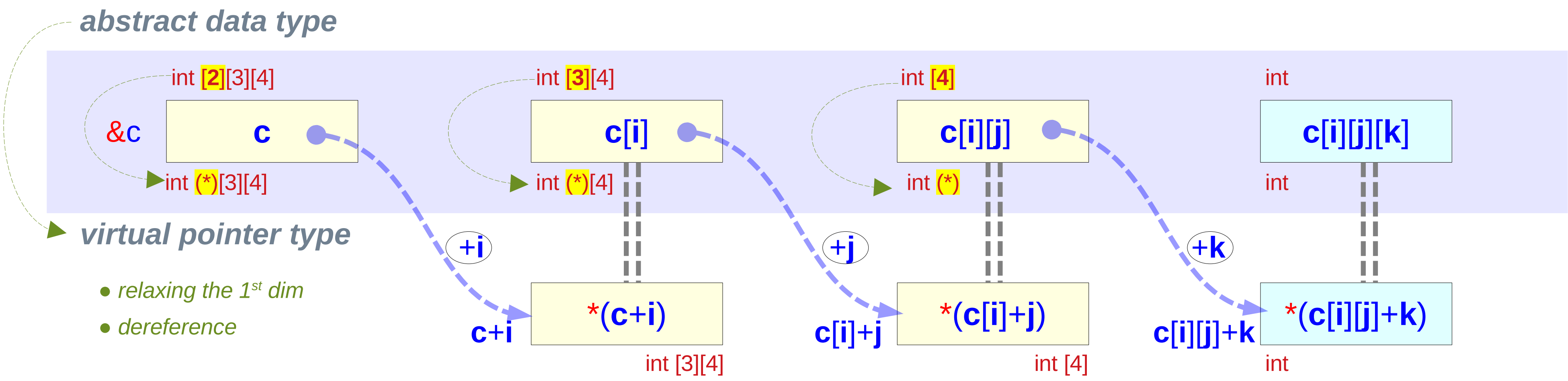
# Relaxing the 1st dimension and derferencing

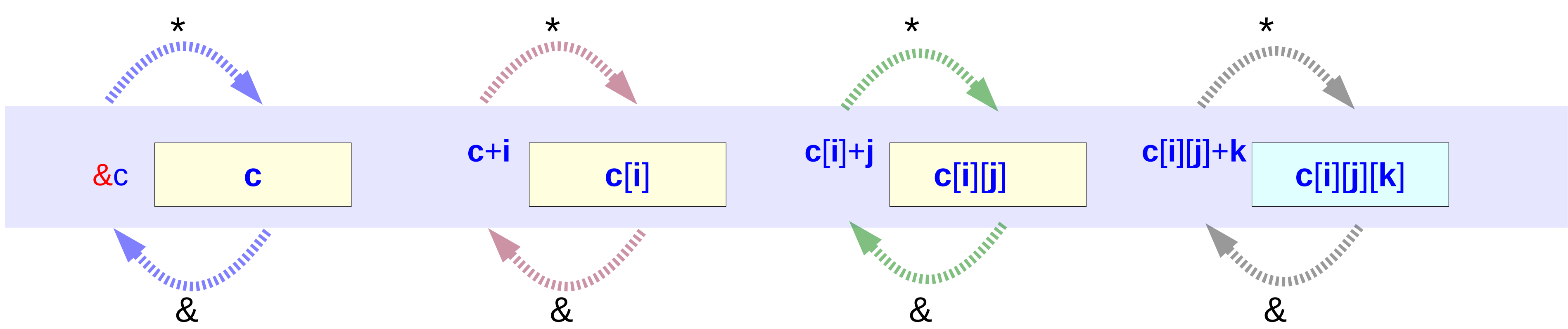


# Two step deferencing in type II (1) – without skipping

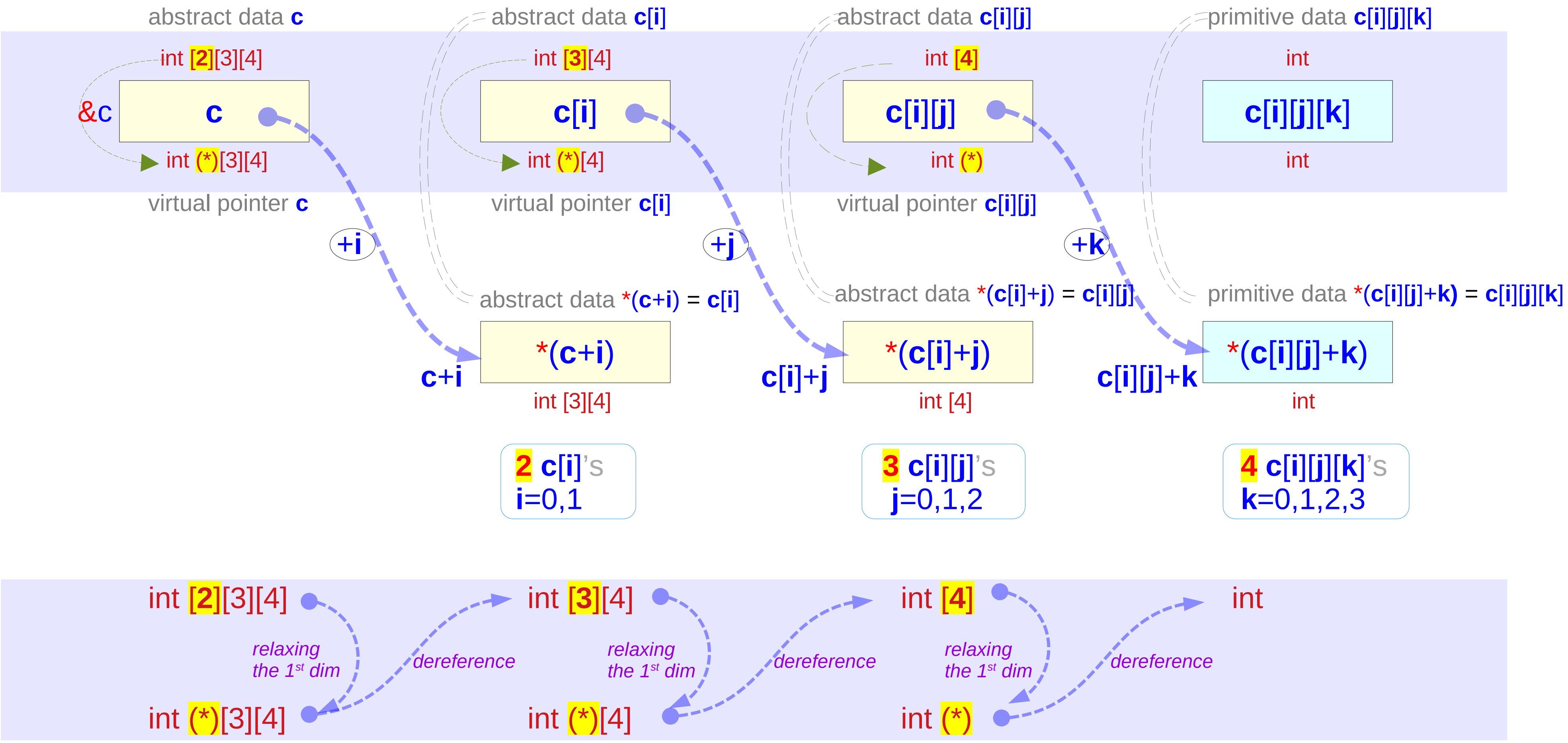


# Two step deferencing in type II (2) – with skipping

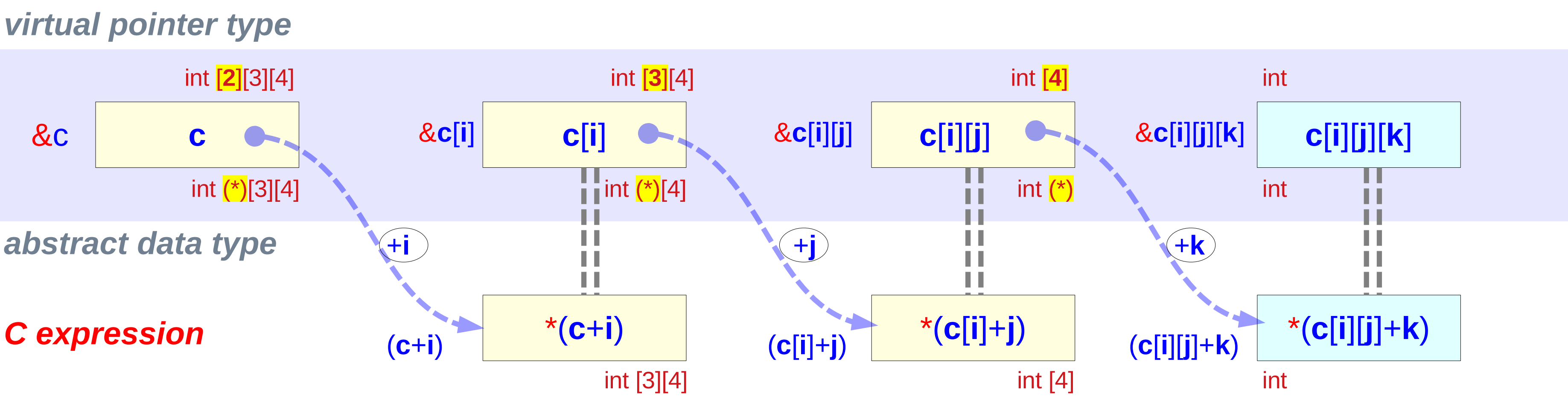


# Two step deferencing in type II (3)



# Skipping elements




***Math expression***

$(c+i)_{3\cdot4\cdot4}$ — skip $i\cdot3\cdot4$ integer elements from **c**

$(c[i]+j)_{4\cdot4}$ — skip $j\cdot4$ integer elements from **c[i]**

$(c[i][j]+k)_{1\cdot4}$ — skip $k\cdot1$ integer elements from **c[i][j]**

sizeof(*c) = 3*4*4

sizeof(*c[i]) = 4*4

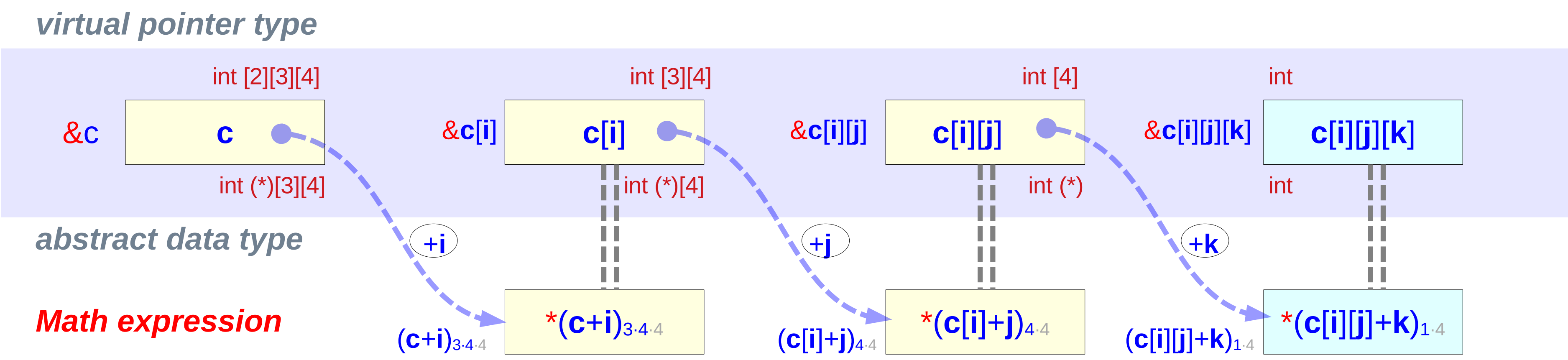
sizeof(*c[i][j]) = 1*4

value($(c+i)_{3\cdot4}$)
= value(c) + i *3*4*4

value($(c[i]+j)_{4}$)
= value(c[i]) + j *4*4

value($(c[i][j]+k)_{1}$)
= value(c[i][j]) + k *4

# Address replication




***equivalence relations – c expressions***

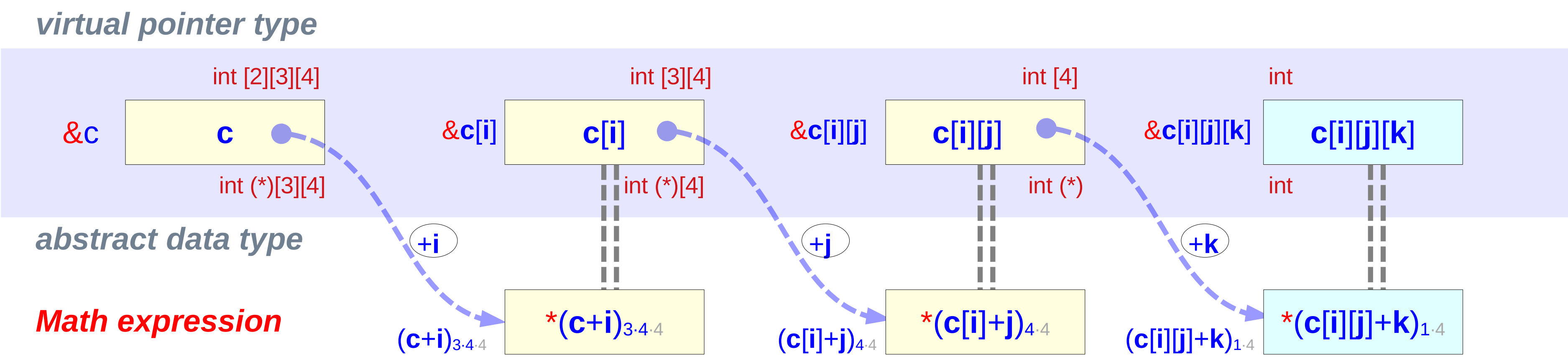
c[i][j][k] = *(c[i][j] + k)
c[i][j] = *(c[i] + j)
c[i] = *(c + i)

&c[i][j][k] = (c[i][j] + k)
&c[i][j] = (c[i] + j)
&c[i] = (c + i)

***address replication – math expressions***

value( c[i][j] ) = value( $(c[i] + j)_{4 \cdot 4}$ )
value( c[i] ) = value( $(c + i)_{3 \cdot 4 \cdot 4}$ )

# Applying address replication




element size
= sizeof(int)
= 4 bytes

*address replication*

value((c+i)$_{3 \cdot 4 \cdot 4}$)
= value(c)
+ i *3*4*4

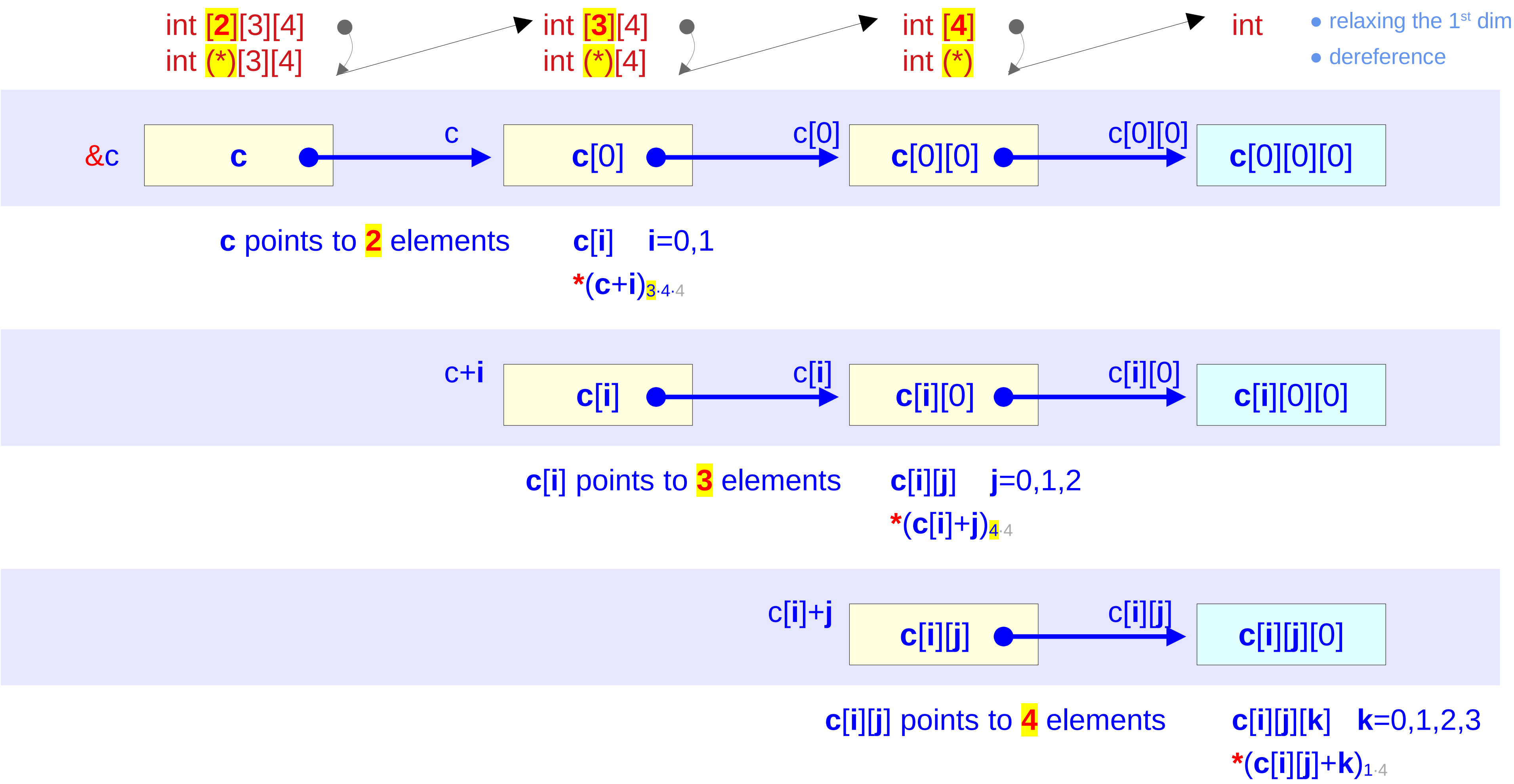
*address replication*

value((c[i]+j)$_{4 \cdot 4}$)
= value(c[i])
+ j *4*4

value((c[i][j]+k)$_{1 \cdot 4}$)
= value(c[i][j])
+ k *4

# Pointer Chain Types

## Pointer Chain Type II



# Pointer Chain Types

- relaxing the 1st dim
- dereference

**i** = 0,1

int [**2**][3][4] → int [3][4]

int (*)[3][4]

**c** points to **2** elements

**c**[**i**] **i**=0,1

***(c+i)** $_{3 \cdot 4 \cdot 4}$

**j** = 0,1,2

int [**3**][4] → int [4]

int (*)[4]

**c**[**i**] points to **3** elements

**c**[**i**][**j**] **j**=0,1,2

***(c[i]+j)** $_{4 \cdot 4}$

**k** = 0,1,2,3

int [**4**] → int

int (*)

**c**[**i**][**j**] points to **4** elements

**c**[**i**][**j**][**k**] **k**=0,1,2,3

***(c[i][j]+k)** $_{1 \cdot 4}$

# const pointers

# const type, const pointer type (1)

```
const int  *  p;          ⟺     int  *  p;

int * const q  ;          ⟺     int  *  q  ;

const int  *  const r  ;  ⟺     int  *   r  ;
```

**_constant_** *must not be changed*
*must not be updated*
*must not be written*
*must not be assigned*

# const type, const pointer type (2)

```
const int * p ;
```
constant integer

```
int * const q ;
```
constant pointer

```
const int * const r ;
```
constant integer

```
const int * const r ;
```
constant pointer

***p : constant integer value**

**q : constant (int *) pointer**

***r : constant integer value**

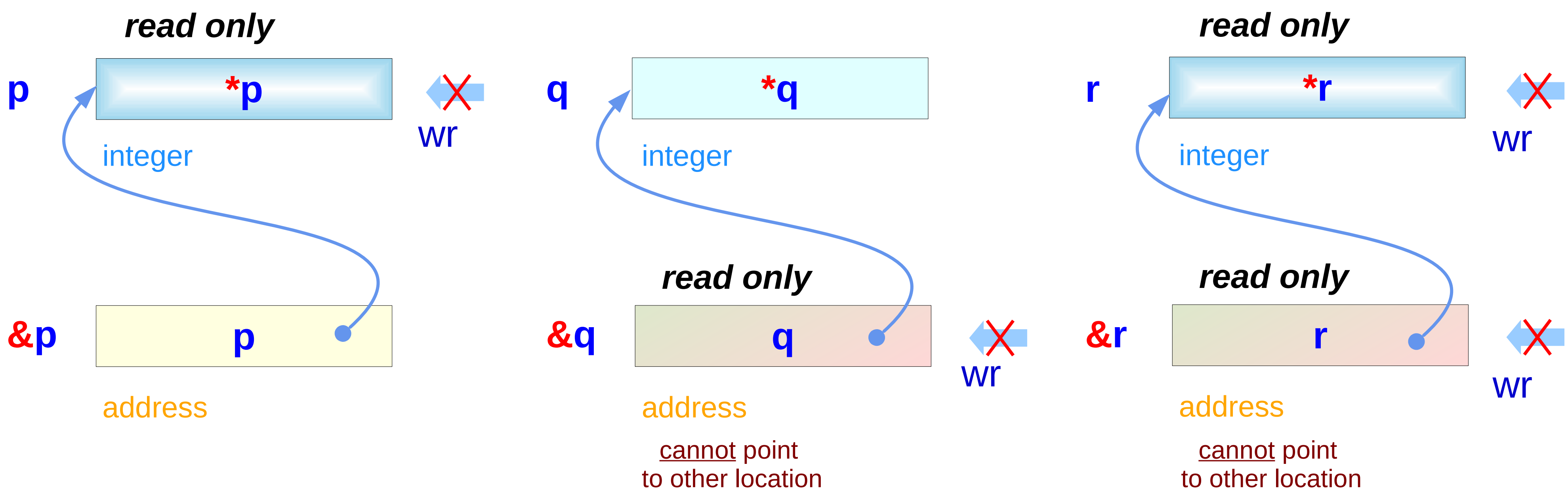
**r : constant (int *) pointer**

**const** [ ]

group with the following

# const type, const pointer type (3)

const int *p;    int *const q ;    const int *const r ;



# **const** examples (1)

```
const int * pc;
      int * p, i;
const int   ic;
```

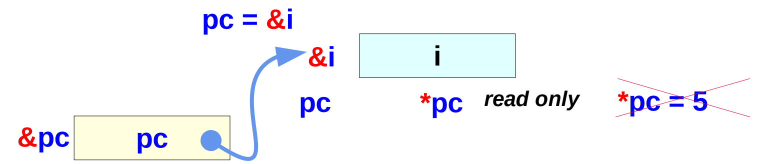
```
 pc = &i;     // (const int *) ← (int *)
*pc  = 5;     // (const int) error
```

Writing to the writable memory location (**i**)
is forbidden via **pc** … (no harm, OK)

```
 p  = &ic;    // (int *) ← (const int *) warning
*p  = 5;      // (int)
```

Writing to the read only memory location (**ic**)
is not forbidden via **p** … (hazardous, not OK)

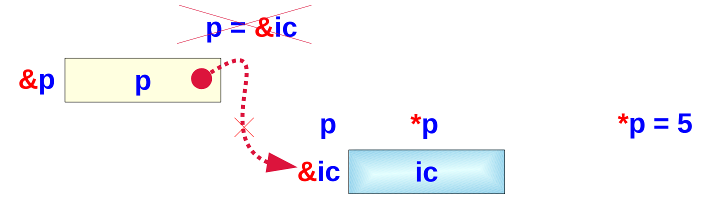
C A Reference Manual, Harbison & Steele Jr.




**pc** can point to **i**
***pc** must be **const**

the same memory location
that can be written via **i**
cannot be written via ***pc**

***pc** should not write
the writable memory location




Assume **p** points to **const ic**

the same memory location
that cannot be written via **ic,**
can be written via ***p**

thus ***p** can write
the **const** memory location

therefore, **p** should not point to **const ic**

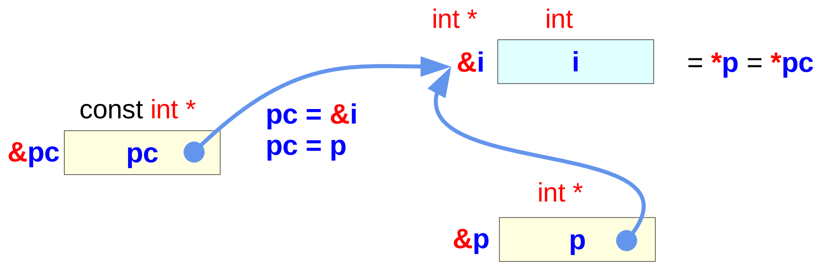
# const examples (2)

```
const int * pc;
      int * p, i = 5;
const int   ic = 7;
```

```
p   = &i;
pc  = &ic
```

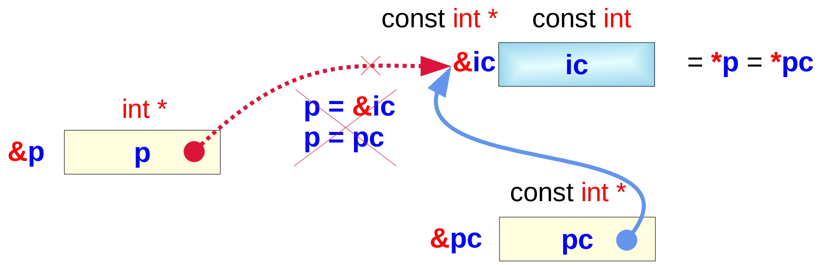
**// more constrained type ← general type (O)**

```
pc  = &i;    // (const int *  ←  int *)
pc  =  p;    // (const int *  ←  int *)
```




**// general type ← more constrained type (X)**

```
p   = &ic;   // (int *  ←  const int *)   warning
p   =  pc;   // (int *  ←  const int *)   warning
```

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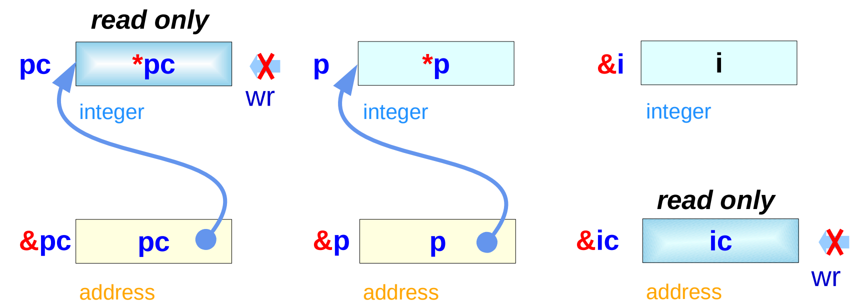
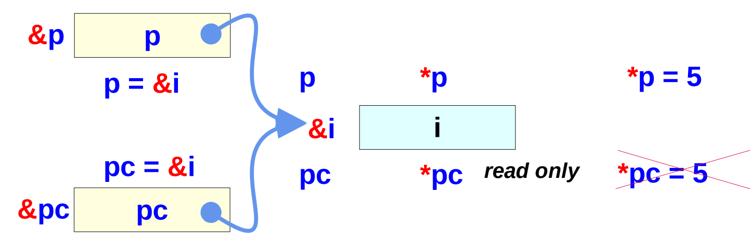
# const examples (3)

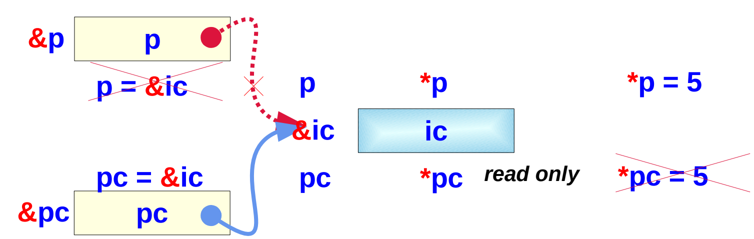
```
const int * pc;
      int * p, i;
const int   ic;
```




```
 p   = &i;    // (int *) ← (int *)
*p   = 5;     // (int)
 pc  = &i;    // (const int *) ← (int *)
*pc  = 5;     // (const int) error
```




```
 p   = &ic;   // (int *) ← (const int *) warning
*p   = 5;     // (int)
 pc  = &ic;   // (const int *) ← (const int *)
*pc  = 5;     // (const int) error
```

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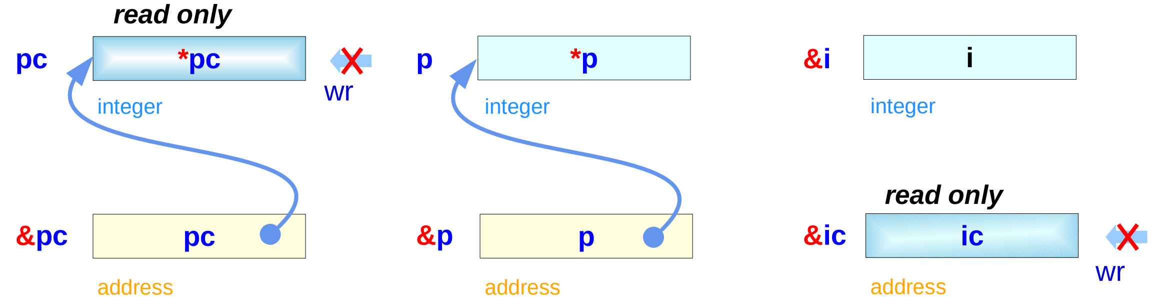
# const examples (4)

```
const int * pc;
          int * p, i;
const int   ic;
```




```
 pc  =   p = &i;
 pc  = &ic
*p   =   5;
*pc  =   5;              // invalid      *pc :: cons int
```

```
pc   = &i;               //              (const int *  ←   int *)
pc   =   p;              //              (const int *  ←   int *)
p    = &ic;              // invalid      (int * ← const int *)
p    =   pc;             // invalid      (int * ← const int *)
p    = (int *) &ic;      // type cast
p    = (int *)   pc;     // type cast
```

C A Reference Manual, Harbison & Steele Jr.

## References

[1] Essential C, Nick Parlante
[2] Efficient C Programming, Mark A. Weiss
[3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
[4] C Language Express, I. K. Chun