Applications of Pointers (1A)

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n-d access of a **1-d** array

- **2-d** array access of a **1-d** array
- **3-d** array access of a **1-d** array
- Accessing a **contiguous 1-d** array
- Accessing a **non-contiguous 1-d** arrays

- Accessing **static** allocated arrays
- Accessing **dynamically** allocated arrays

2-d array access of a **1-d** array

Array of Pointers

```
int     a [4] ;
int *   b [3] ;
```

```
int     a     [4]
```

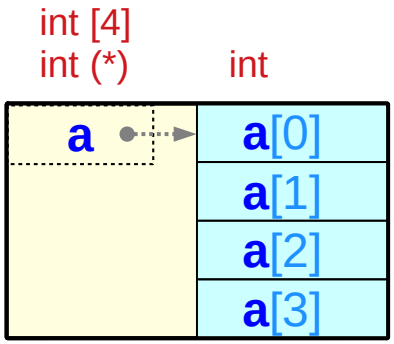
there are 4 elements

the type of each element:
an integer

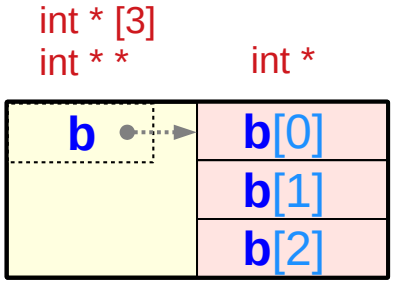
```
int *   b     [3]
```

there are 3 elements

the type of each element:
a pointer an integer



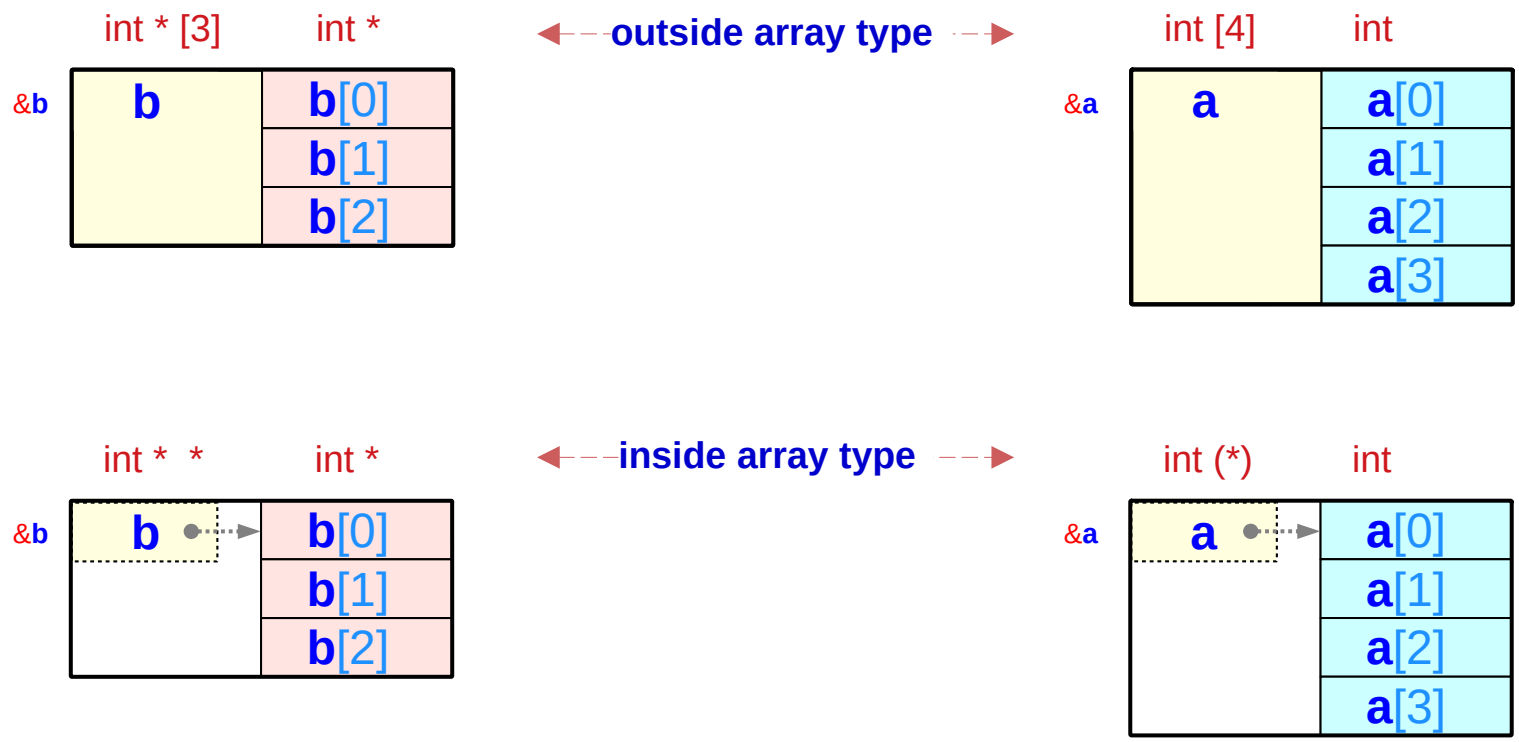
Integers



Integer pointers

Array of Pointers – a type view

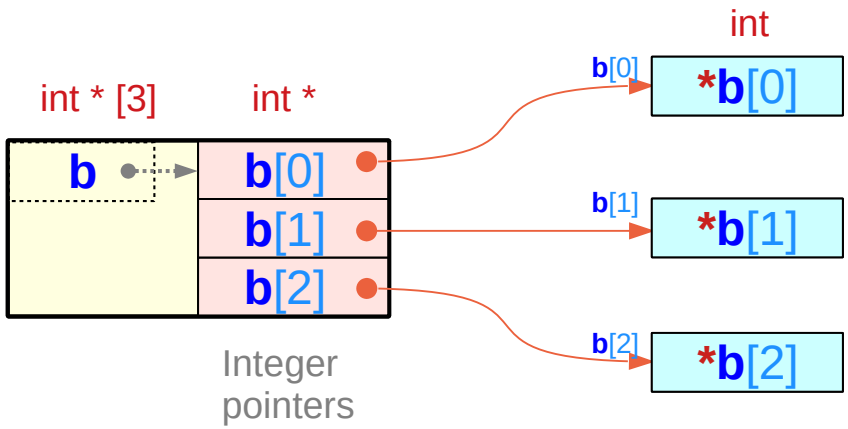
int * b [3] ; Pointer Array int a [4] ;

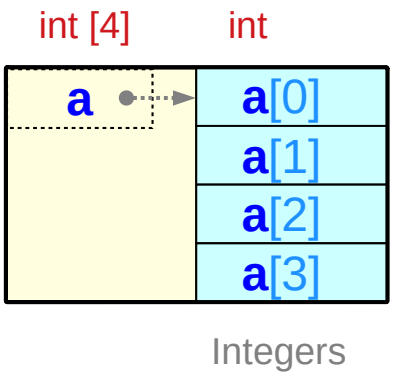


Array of Pointers – a variable view

int * b [3] ; Pointer Array

int a [4] ;





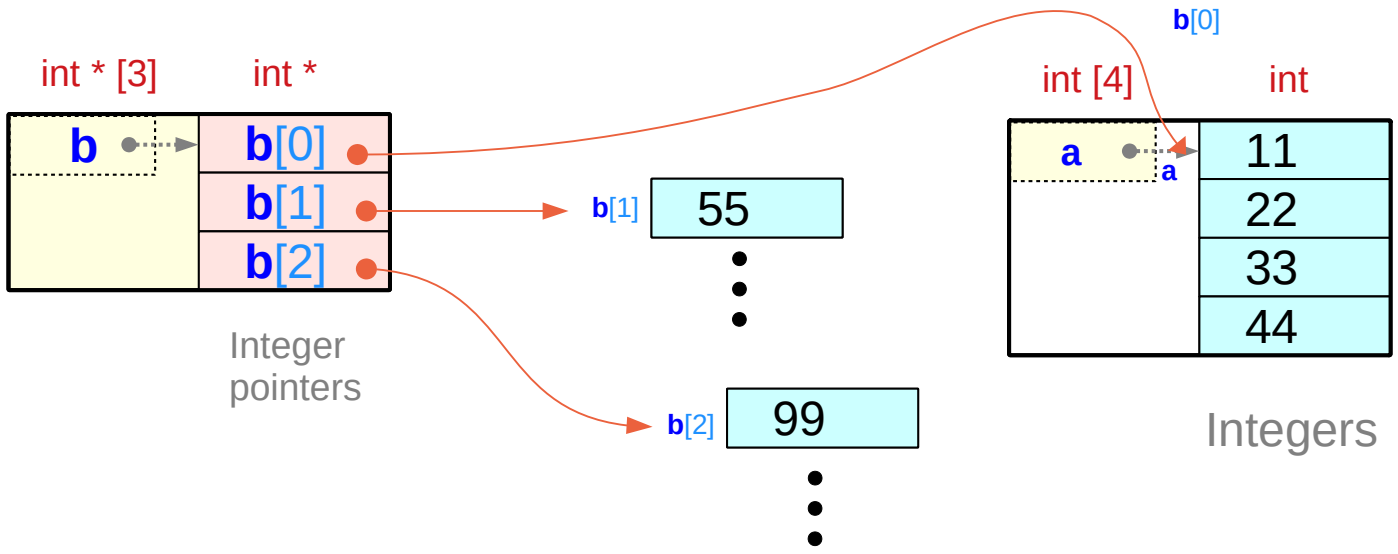
Assigning a **1-d** array name

int * b [3] ; Pointer Array

int a [4] ;

assignment

b[0] = **a** (= &**a**[0])



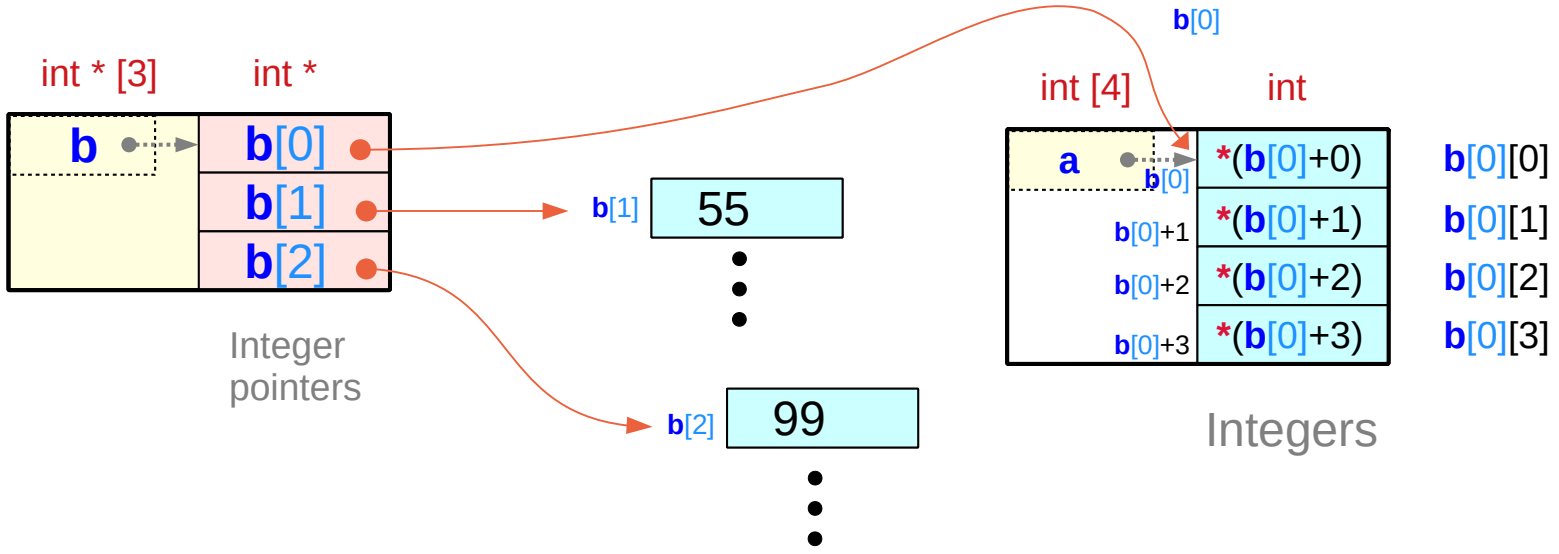
Assigning a **1-d** array name – equivalence

int * b [3] ; Pointer Array

int a [4] ;

assignment

b[0] = **a** (= &**a**[0])

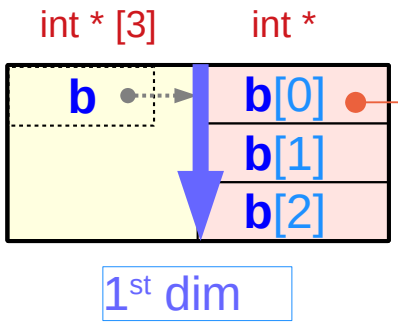


Array of Pointers – extended dimension

int * b [3] ; Pointer Array

int a [4] ;

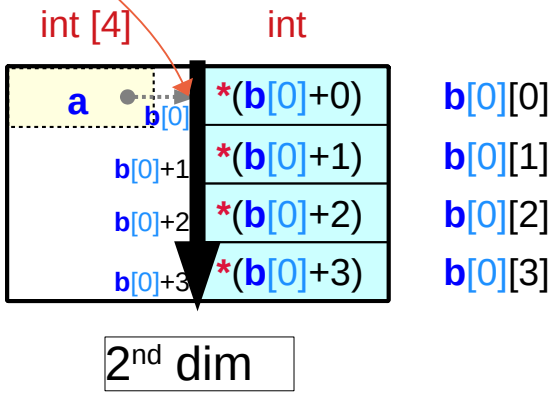
array name b



assignment

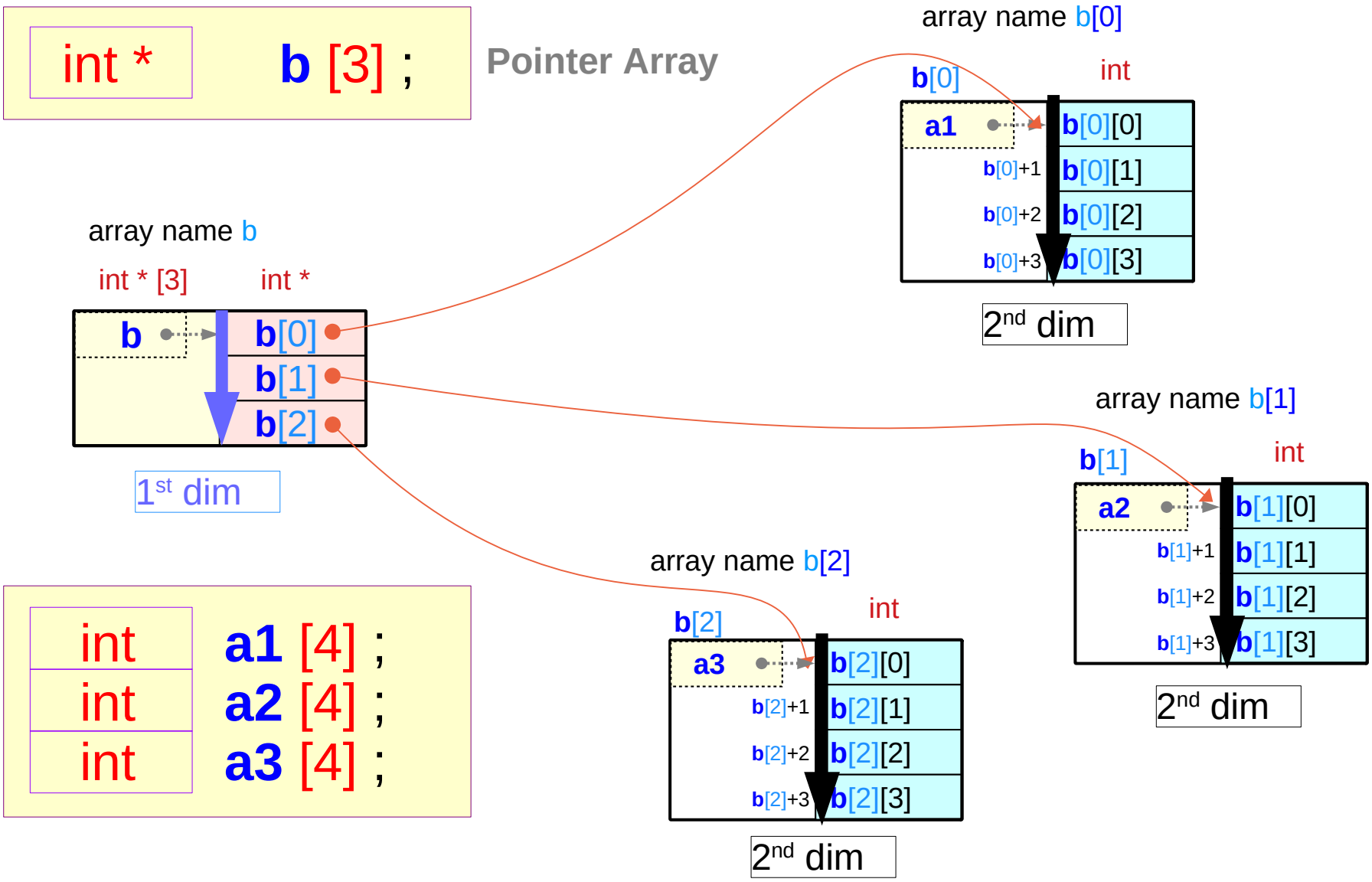
b[0] = **a** (= &**a**[0])

array name b[0]



a[0] ≡ b[0][0] ≡ *(*(b+0)+0)
a[1] ≡ b[0][1] ≡ *(*(b+0)+1)
a[2] ≡ b[0][2] ≡ *(*(b+0)+2)
a[3] ≡ b[0][3] ≡ *(*(b+0)+3)

2-d access of 1-d arrays

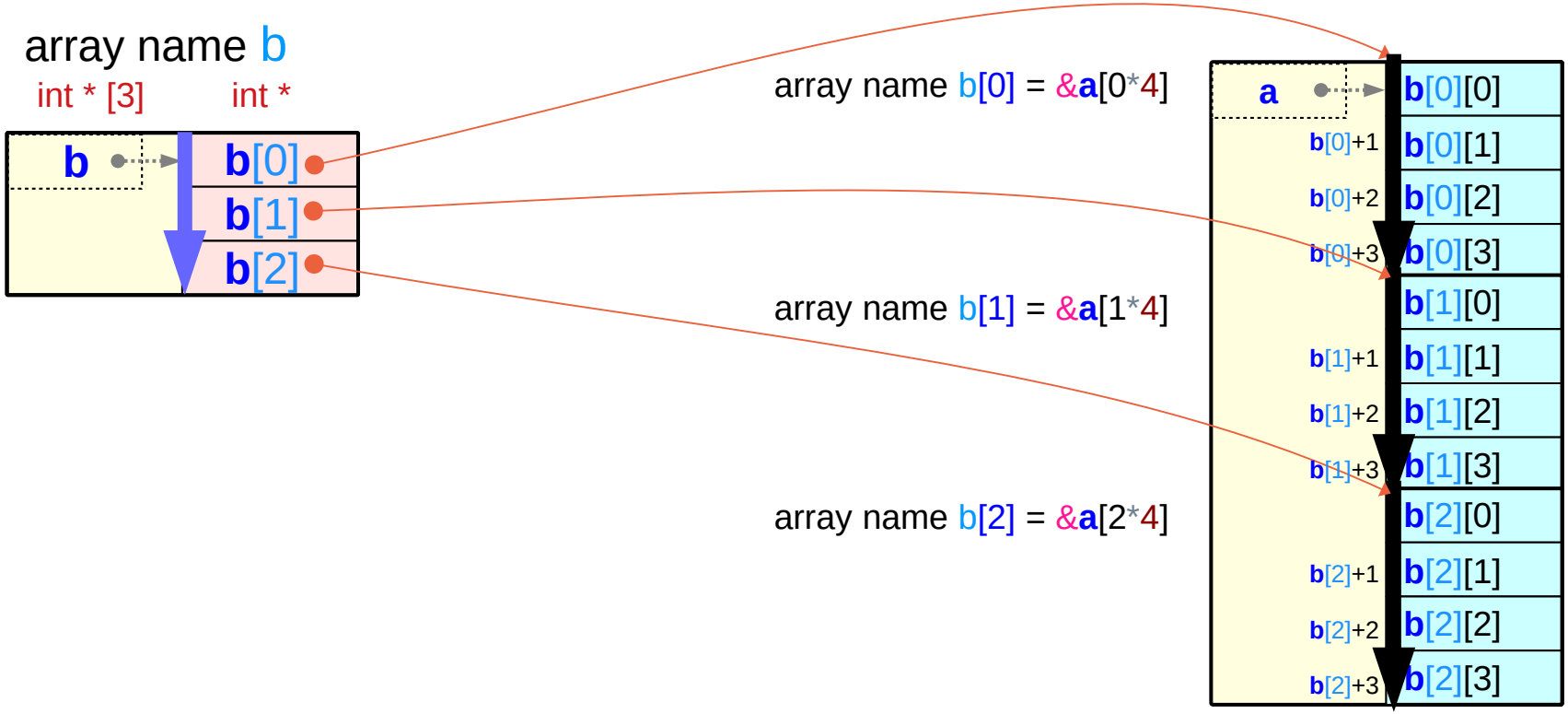


2-d access of a 1-d array

int * b [3] ;

Pointer Array

int * a [3*4] ;



2-d access of a 1-d array – pointer array assignment

int *	b [2*3] ;
int	a [2*3*4] ;

b[j] = &a[j*4] (= a+j*4)

b[j] + k = a+j*4 + k

*(b[j] + k) = *(a+j*4 + k)

b[j][k] ≡ a[j*4 +k]

j = [0:5] k = [0:3]

j*4+k = [0:23]

constraint : contiguous b[i][j] over j

2-d access of a **1-d** array

b[i][j] ≡ *(b[i] + j)

⬍ ⬍

a[i*4+j] ≡ *(a+i*4 + j)

1-d access of a **1-d** array

constraint : contiguous a[i*4+j] over j

3-d array access of a **1-d** array

3-d access of a 1-d array (1)

int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

```
int *    b [2*3] ;
int      a [2*3*4] ;
```

b[j] = &a[j*4] (= a+j*4)

b[j] + k = a+j*4 + k
*(b[j] + k) = *(a+j*4 + k)

b[j][k] ≡ a[j*4 +k]

j = [0:5] k = [0:3]
j*4+k = [0:23]

```
int **   c [2] ;
int *    b [2*3] ;
```

c[i] = &b[i*3] (= b+i*3)

c[i] +j = b+i*3 +j
*(c[i] +j) = *(b+i*3 +j)

c[i][j] = b[i*3 +j]

c[i][j] +k = b[i*3 +j] +k
*(c[i][j] +k) = b[i*3 +j][k]

c[i][j][k] = a[(i*3 +j)*4 +k]

c[i][j][k] ≡ a[(i*3+j)*4+k]

i = [0:1] j = [0:2] k = [0:3]
(i*3+j)*4+k = [0:23]

3-d access of a 1-d array (2)

int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

```
a[k]        ≡ *(a+k)
b[j][k]     ≡ *(*(b+j)+k)
c[i][j][k]  ≡ *(*(*(c+i)+j)+k)
```

constraint : contiguous a[i], b[i], c[i]

Assignments

```
c[i] = &b[i*3]  (= b+i*3)
b[j] = &a[j*4]  (= a+j*4)
```

Initializing pointer arrays **b** and **c**

3-d access of a **1-d** array

```
c[i][j][k]        ≡ *( c[i][j] +k)
b[i*3+j][k]       ≡ *( b[i*3+j] +k)
a[(i*3+j)*4 +k]   ≡ *(a+(i*3+j)*4 +k)
```

1-d access of a **1-d** array

3-d access of a 1-d array (3)

int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

```
a[k]        ≡ *(a+k)
b[j][k]     ≡ *(*(b+j)+k)
c[i][j][k]  ≡ *(*(*(c+i)+j)+k)
```

```
(((c[i])[j])[k])
≡ (((b+i*3)[j])[k])
≡ ((b[i*3+j])[k])
≡ ((a+(i*3+j)*4)[k])
≡ a[(i*3+j)*4+k]
```

c[i] = &b[i*3] = b+i*3

b[j] = &a[j*4] = a+j*4

```
*(*(*(c+i)+j)+k)
≡ *(*(b+i*3+j)+k)
≡ *(b[i*3+j]+k)
≡ *(a+(i*3+j)*4+k)
≡ a[(i*3+j)*4+k]
```

Equivalence relations in pointer array assignments

```
c[i] = &b[i*3] = b+i*3
b[j] = &a[j*4] = a+j*4
```

substitute c[i] in *(c[i]+j)

substitute b[m] in *(b[m]+k)

m = i*3+j

substitute c[i]

```
c[i][j]      = *(c[i]+j)
             = *(b+i*3+j) = b[i*3+j]
```

substitute b[m]

```
b[m][k]      = *(b[m]+k)
             = *(a+m*4+k) = a[m*4+k]

c[i][j][k]   = b[i*3+j][k] = a[(i*3+j)*4+k]
```

```
c[i] = &b[i*3] = b+i*3
b[j] = &a[j*4] = a+j*4
```

substitute c[i] in (c[i])[j]

substitute b[m] in (b[m])[k]

m = i*3+j

substitute c[i]

```
c[i][j]      = (b+i*3)[j]
             = *(b+i*3+j) = b[i*3+j]
```

substitute b[m]

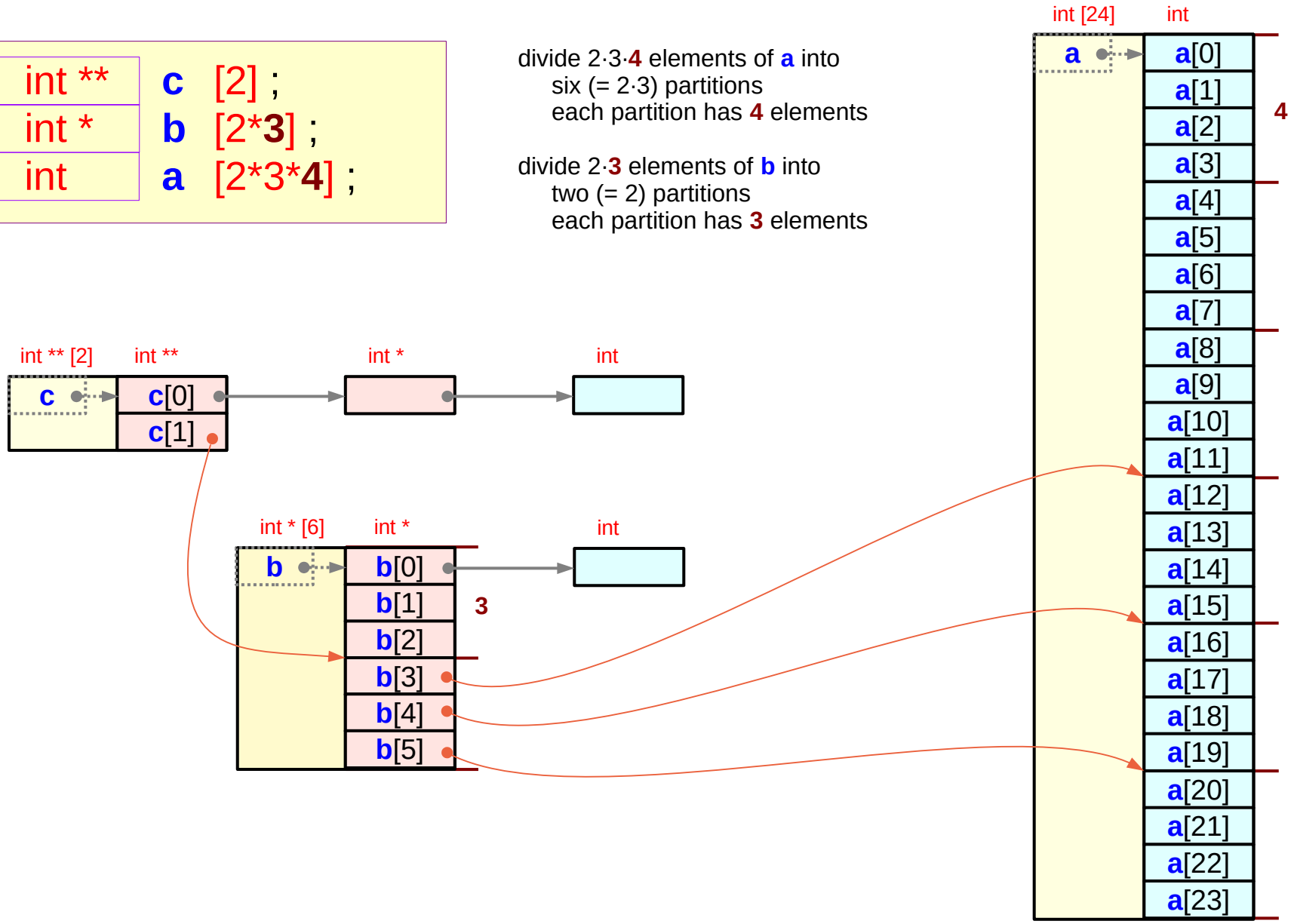
```
b[m][k]      = (a+m*4)[k]
             = *(a+m*4+k) = a[m*4+k]

c[i][j][k]   = b[i*3+j][k] = a[(i*3+j)*4+k]
```

Integer array **a** and pointer arrays **b**, **c**

int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

divide 2·3·**4** elements of **a** into
six (= 2·3) partitions
each partition has **4** elements

divide 2·**3** elements of **b** into
two (= 2) partitions
each partition has **3** elements



Pointer array initializations

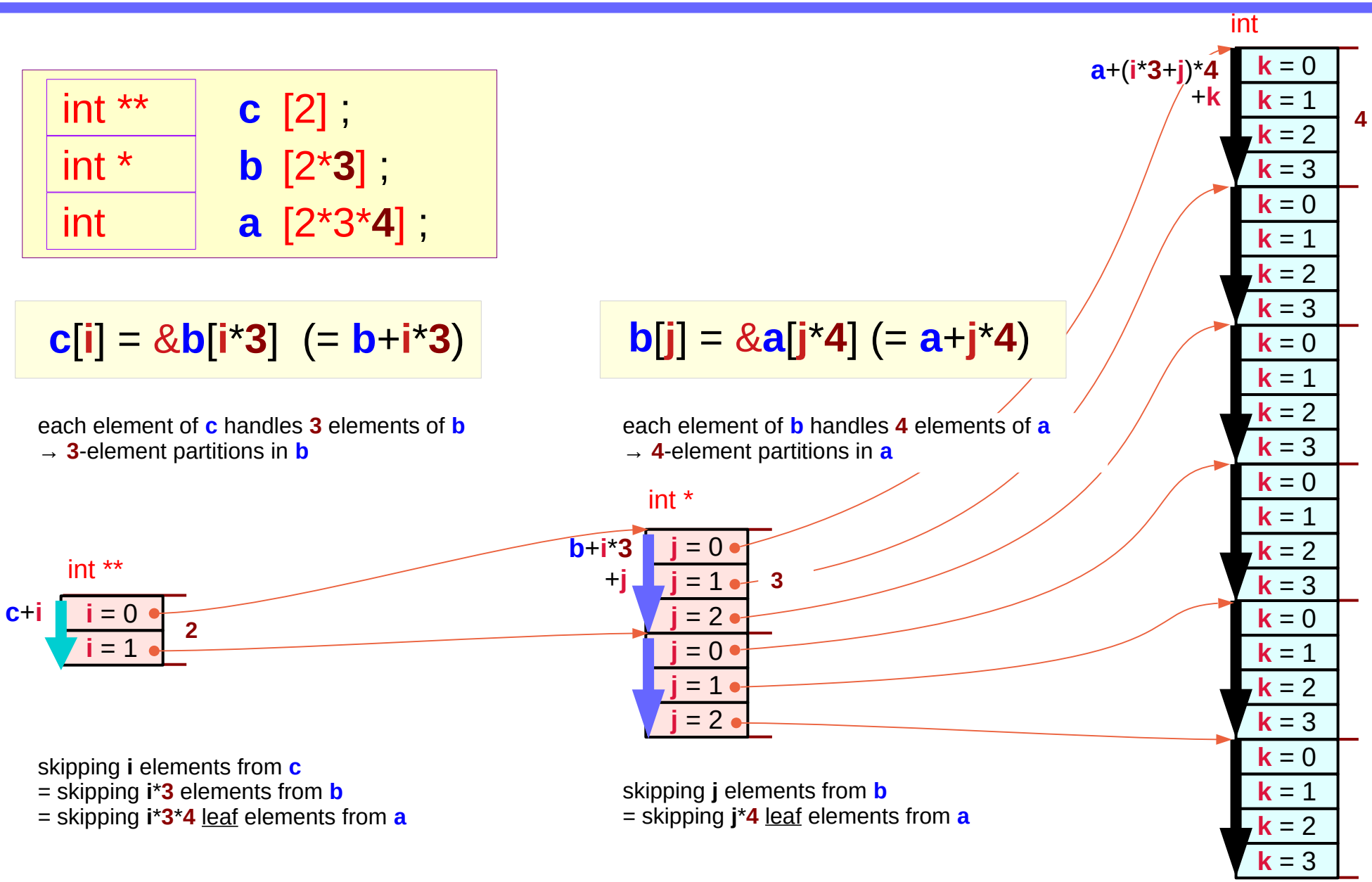
int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

c[i] = &b[i*3] (= b+i*3)

each element of **c** handles **3** elements of **b**
→ **3**-element partitions in **b**

b[j] = &a[j*4] (= a+j*4)

each element of **b** handles **4** elements of **a**
→ **4**-element partitions in **a**



skipping **i** elements from **c**
= skipping **i*3** elements from **b**
= skipping **i*3*4** leaf elements from **a**

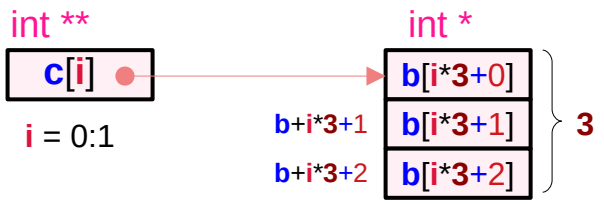
skipping **j** elements from **b**
= skipping **j*4** leaf elements from **a**

Partitioning arrays **a** and **b**

int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

Assigning pointer array

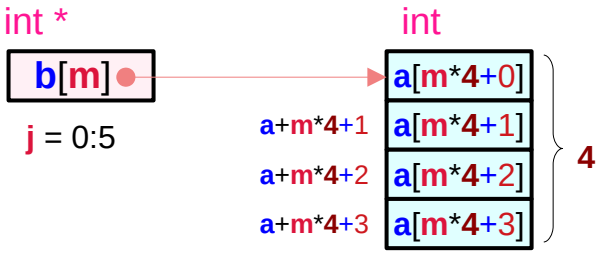
b[j] = &a[j*4] (= a+j*4)
c[i] = &b[i*3] (= b+i*3)



divide 2·3 elements of b
into two 3-element partitions
partition size = 3

c[i] contiguous c over i = 0:1
b[i*3+j] contiguous b over j = 0:2

c[0] = &b[0*3]; (= b + 0*3)
c[1] = &b[1*3]; (= b + 1*3)



divide 2·3·4 elements of a
into six 4-element partitions
partition size = 4

b[m] contiguous b over m = 0:5
a[m*4+k] contiguous a over k = 0:3

b[0] = &a[0*4]; (= a + 0*4)
b[1] = &a[1*4]; (= a + 1*4)
b[2] = &a[2*4]; (= a + 2*4)
b[3] = &a[3*4]; (= a + 3*4)
b[4] = &a[4*4]; (= a + 4*4)
b[5] = &a[5*4]; (= a + 5*4)

Skipping leaf elements

int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

b[j] = &a[j*4] (= a+j*4)

skipping 1 element in **b**
= skipping **4** leaf elements in **a**

c[i] = &b[i*3] (= b+i*3)

skipping 1 element in **c**
= skipping **3** elements in **b**
= skipping **3*4** leaf elements in **a**

c[i][j] ≡ b[i*3 +j]

skipping **i** elements from **c**
+ skipping **j** elements from **b**

= skipping **i*3+j** elements from **b**

c[i][j][k] ≡ a[(i*3 +j)*4+k]

skipping **i*3+j** elements from **b**
+ skipping **k** leaf elements from **a**

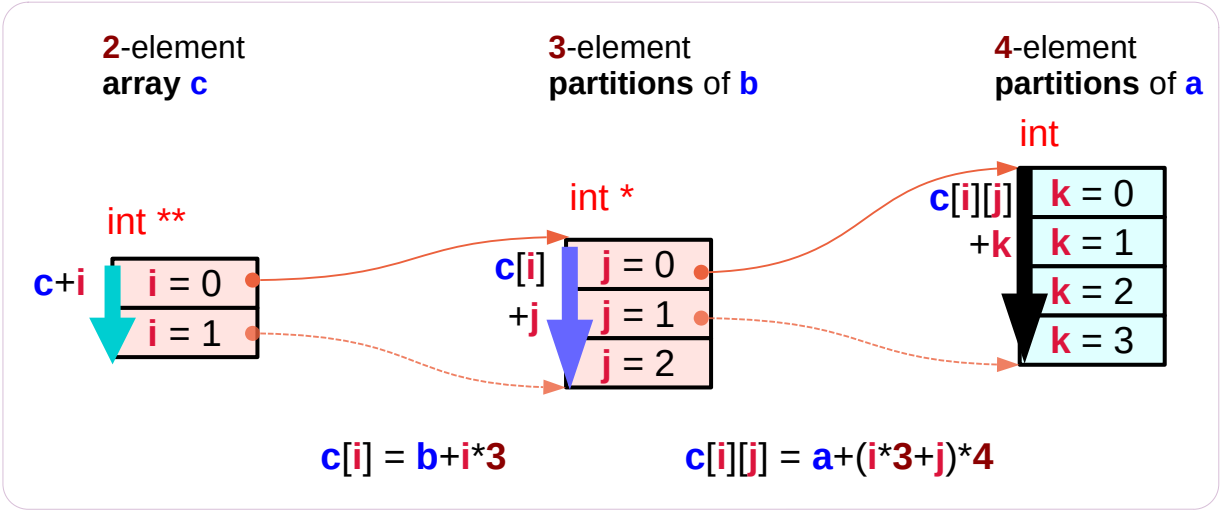
= skipping (**i*3+j**)***4+k** leaf elements from **a**

Contiguous constraints for **c[i][j][k]**

int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

b[j] = &a[j*4] (= a+j*4)
c[i] = &b[i*3] (= b+i*3)

c[i][j][k] ≡
a[(i*3 +j)*4+k]



contiguous 2-elements of array c, i = 0,1 ⟺ **c[i] ≡ *(c+i)**

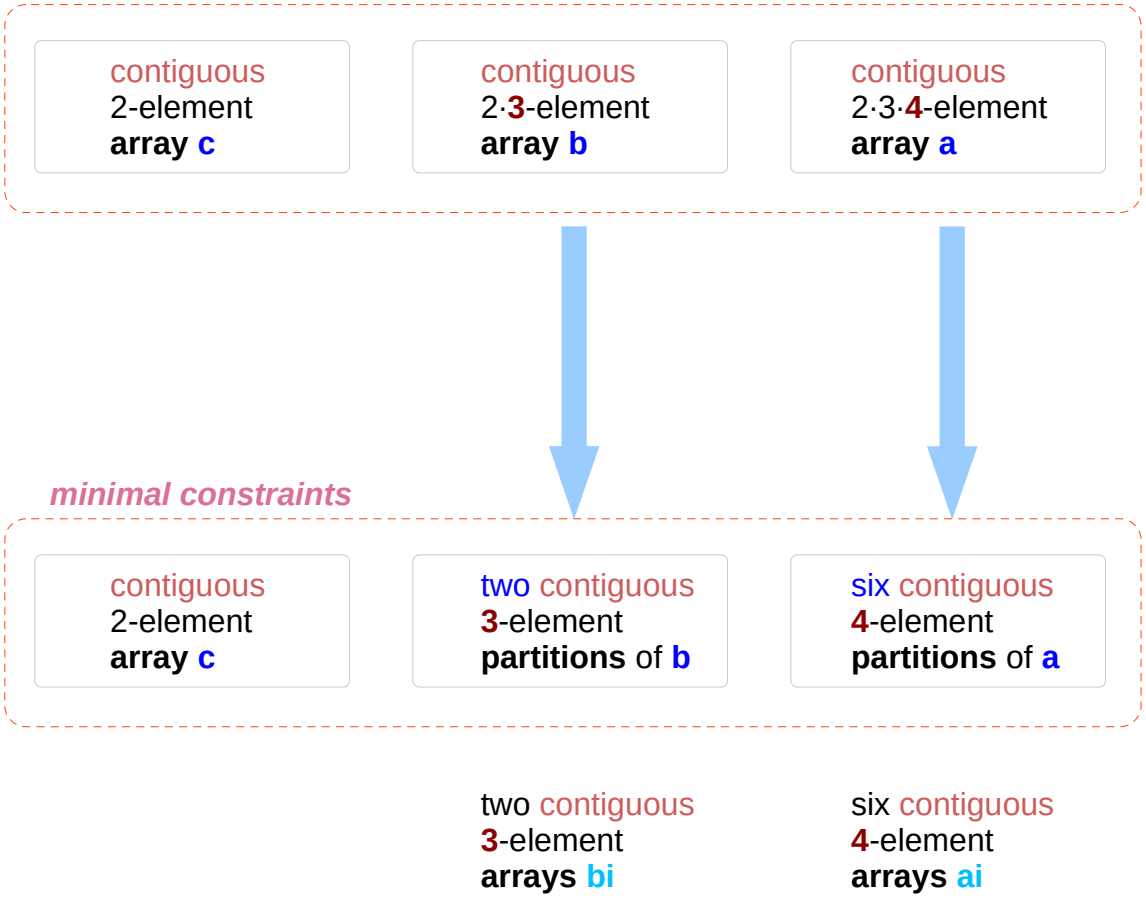
contiguous 3-element partitions of array b, j = 0,1,2 ⟺ **c[i][j] ≡ *(c[i]+j)**
c[i][j] ≡ *(*(c+i)+j)

contiguous 4-element partitions of array a, k = 0, 1,2,3 ⟺ **c[i][j][k] ≡ *(c[i][j]+k)**
c[i][j][k] ≡ *(*(*(c+i)+j)+k)

Minimal constraints and implementations

```
int c [2];        int b [2*3];      int c [2*3*4];

    c[0] = &b[0*3];   b[0] = &a[0*4];
    c[1] = &b[1*3];   b[1] = &a[1*4];
                      b[2] = &a[2*4];
                      b[3] = &a[3*4];
                      b[4] = &a[4*4];
                      b[5] = &a[5*4];
```

```
int c [2];        int b1 [3];       int a1 [4];
                  int b2 [3];       int a2 [4];
                                    int a3 [4];
                                    int a4 [4];
                                    int a5 [4];
                                    int a6 [4];

    c[0] = &b1[0];    b1[0] = &a1[0];
    c[1] = &b2[0];    b1[1] = &a2[0];
                      b1[2] = &a3[0];
                      b2[0] = &a4[0];
                      b2[1] = &a5[0];
                      b2[2] = &a6[0];
```



Accessing a **contiguous 1-d** array

- **1-d** array access
- **2-d** array access
- **3-d** array access

Accessing an int array **a** as a **1-d** array

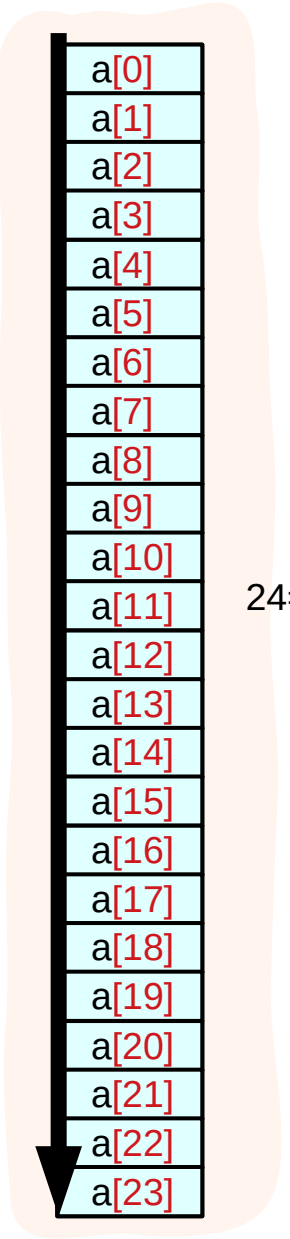
```
int        a [2*3*4] ;
```

```
a [k]
```

k = 0,1, …,23

```
c[i][j][k]  ≡ *(*(*(c+i)+j)+k)     int ** c[2] ;
b[j][k]     ≡ *(*(b+j)+k)          int * b[2*3] ;
a[k]        ≡ *(a+k)               int a[2*3*4] ;
```

int **a** [2*3*4] ;



24=2*3*4

Accessing an int array **a** as a **2-d** array using **b**

```
int      a [2*3*4] ;
int *    b [2*3] ;
```

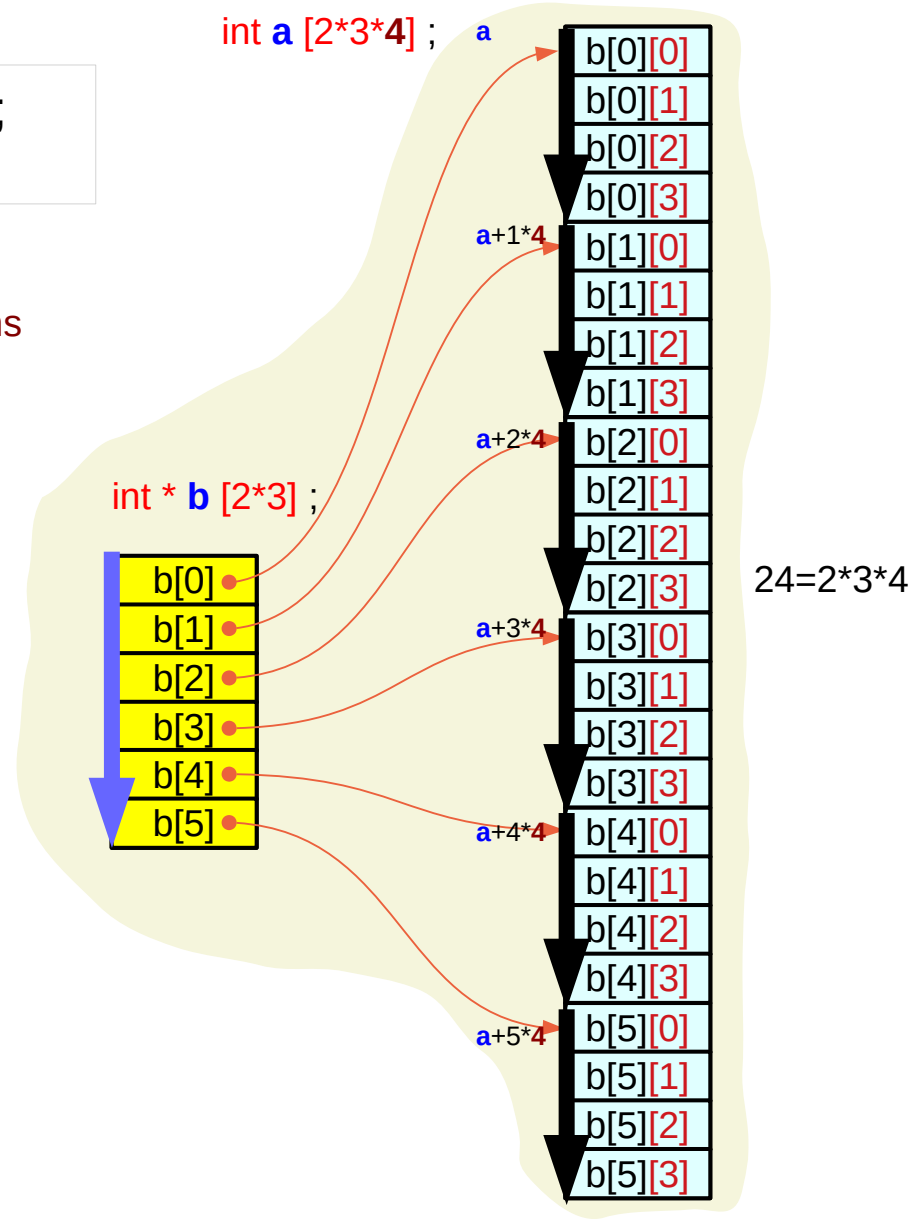
```
b[j] = &a[j*4];
```

a, **b** take actual memory locations

b [j][k] ≡ a [j*4 +k]

j = 0,1,2,3,4
k = 0,1,2,3

```
c[i][j][k]  ≡ *(*(*(c+i)+j)+k)     int ** c[2] ;
b[j][k]     ≡ *(*(b+j)+k)          int * b[2*3] ;
a[k]        ≡ *(a+k)               int a[2*3*4] ;
```



Accessing an int array **a** as a **3-d** array

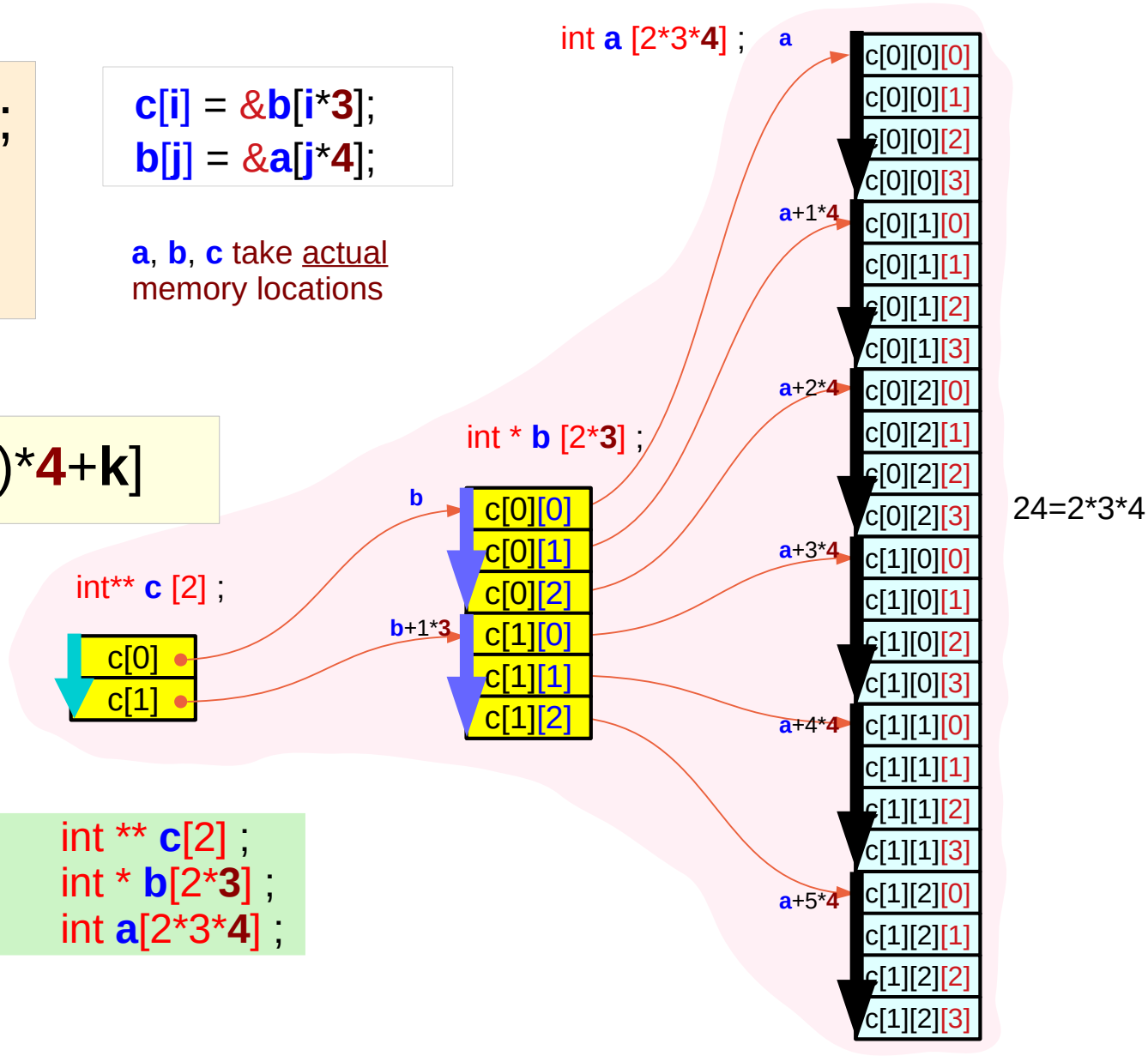
```
int      a [2*3*4] ;
int *    b [2*3] ;
int **   c [2] ;
```

```
c[i] = &b[i*3];
b[j] = &a[j*4];
```

a, **b**, **c** take actual memory locations

```
c [i][j][k] ≡ a [(i*3+j)*4+k]
```

i = 0, 1
j = 0, 1, 2
k = 0, 1, 2, 3




```
c[i][j][k]  ≡ *(*(*(c+i)+j)+k)      int ** c[2] ;
b[j][k]     ≡ *(*(b+j)+k)           int * b[2*3] ;
a[k]        ≡ *(a+k)                int a[2*3*4] ;
```

Accessing a **non-contiguous 1-d** arrays

- **3-d** array access

Accessing **non-contiguous 1-d** arrays as a **3-d** array (1)

```
int      a [2*3*4] ;
int *    b [2*3] ;
int **   c [2] ;
```

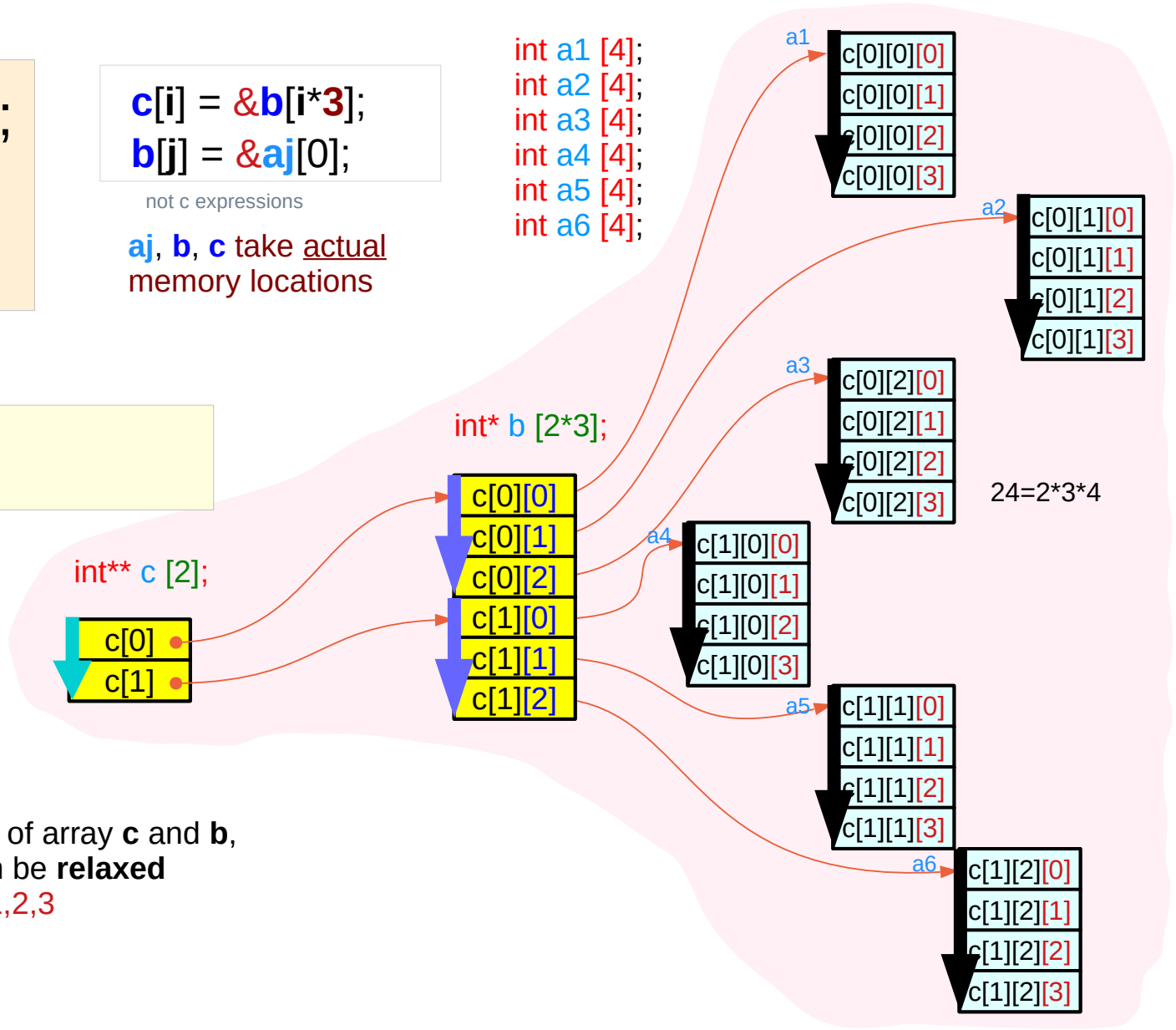
```
c [i][j][k]
```

i = 0, 1
j = 0, 1, 2
k = 0, 1, 2, 3

```
c[i] = &b[i*3];
b[j] = &aj[0];
```

not c expressions

aj, **b**, **c** take actual memory locations



Because the physical **allocation** of array **c** and **b**, the **contiguous constraints** can be **relaxed**
contiguous c[i][j][k] only for k=0,1,2,3

Accessing **non-contiguous 1-d** arrays as a **3-d** array (2)

```
int      a [2*3*4] ;
int *    b [2*3] ;
int **   c [2] ;
```

```
c [i][j][k]
```

i = 0, 1
j = 0, 1, 2
k = 0, 1, 2, 3

not c expressions

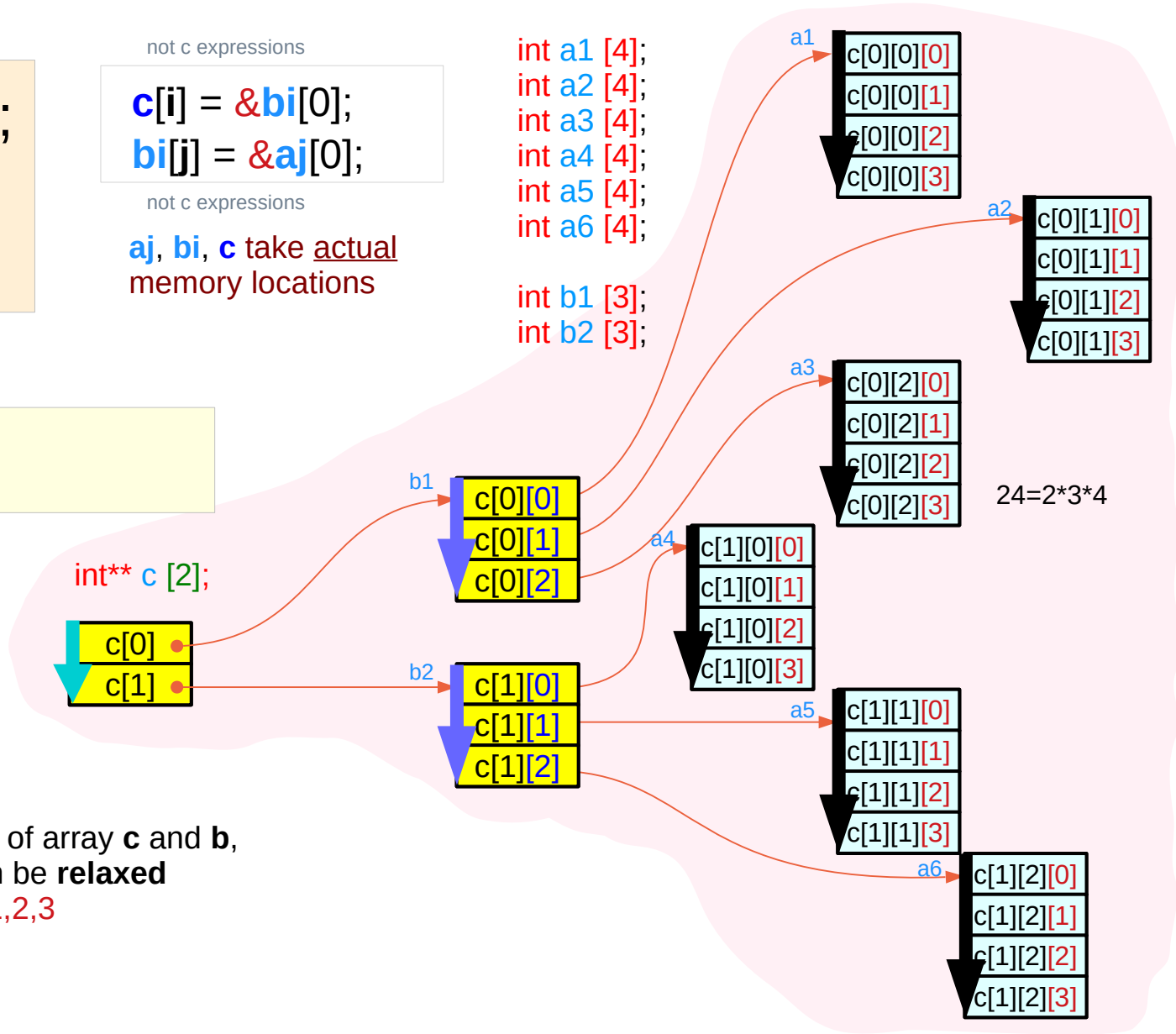
c[**i**] = &**bi**[0];
bi[**j**] = &**aj**[0];

not c expressions

aj, **bi**, **c** take actual memory locations

int a1 [4];
int a2 [4];
int a3 [4];
int a4 [4];
int a5 [4];
int a6 [4];

int b1 [3];
int b2 [3];



24=2*3*4

Because the physical **allocation** of array **c** and **b**, the **contiguous constraints** can be **relaxed**
contiguous c[i][j][k] only for k=0,1,2,3

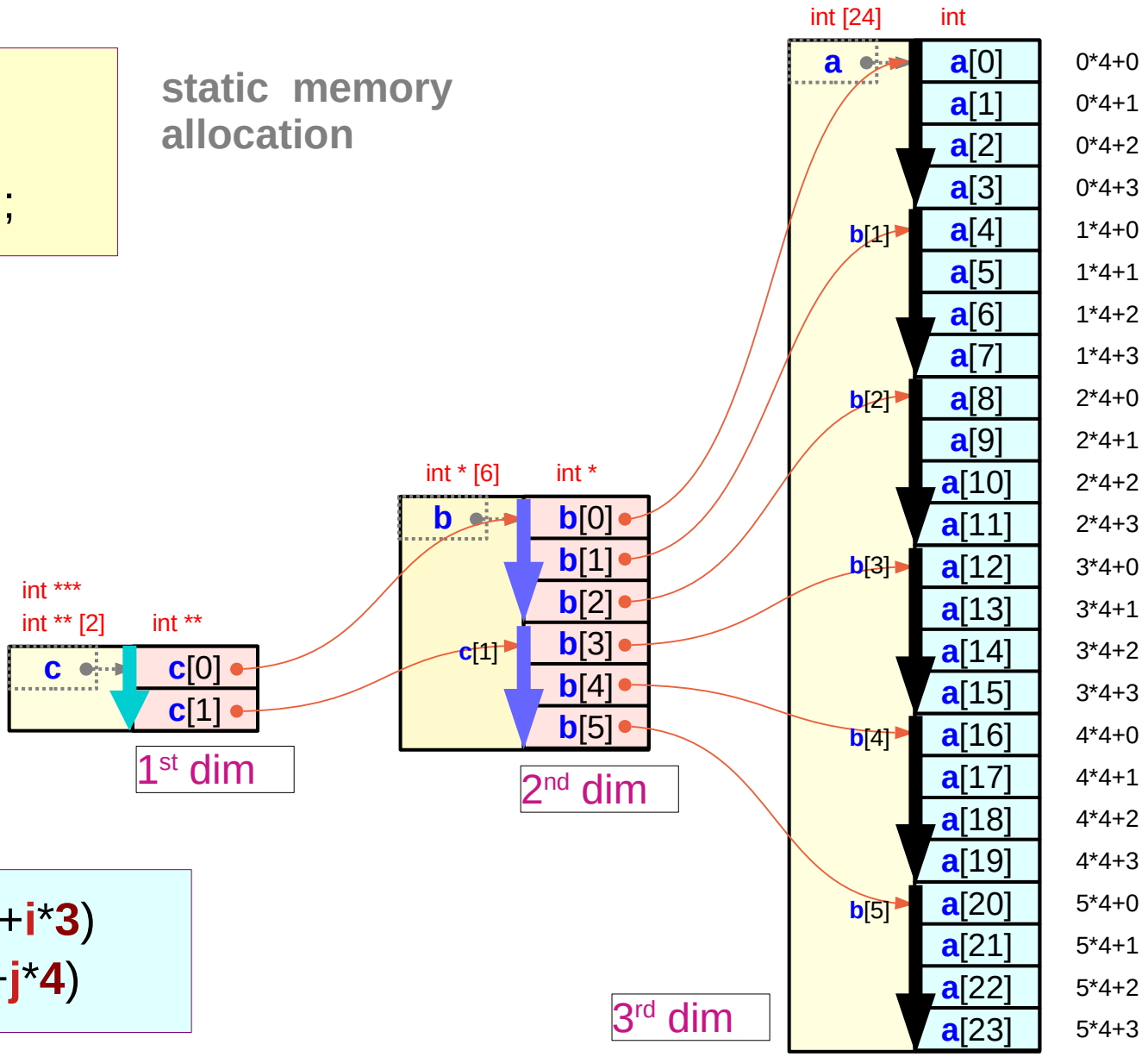
Accessing **statically** allocated arrays

Accessing **dynamically** allocated arrays

Using arrays a, b, c – statically allocated

int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

static memory allocation

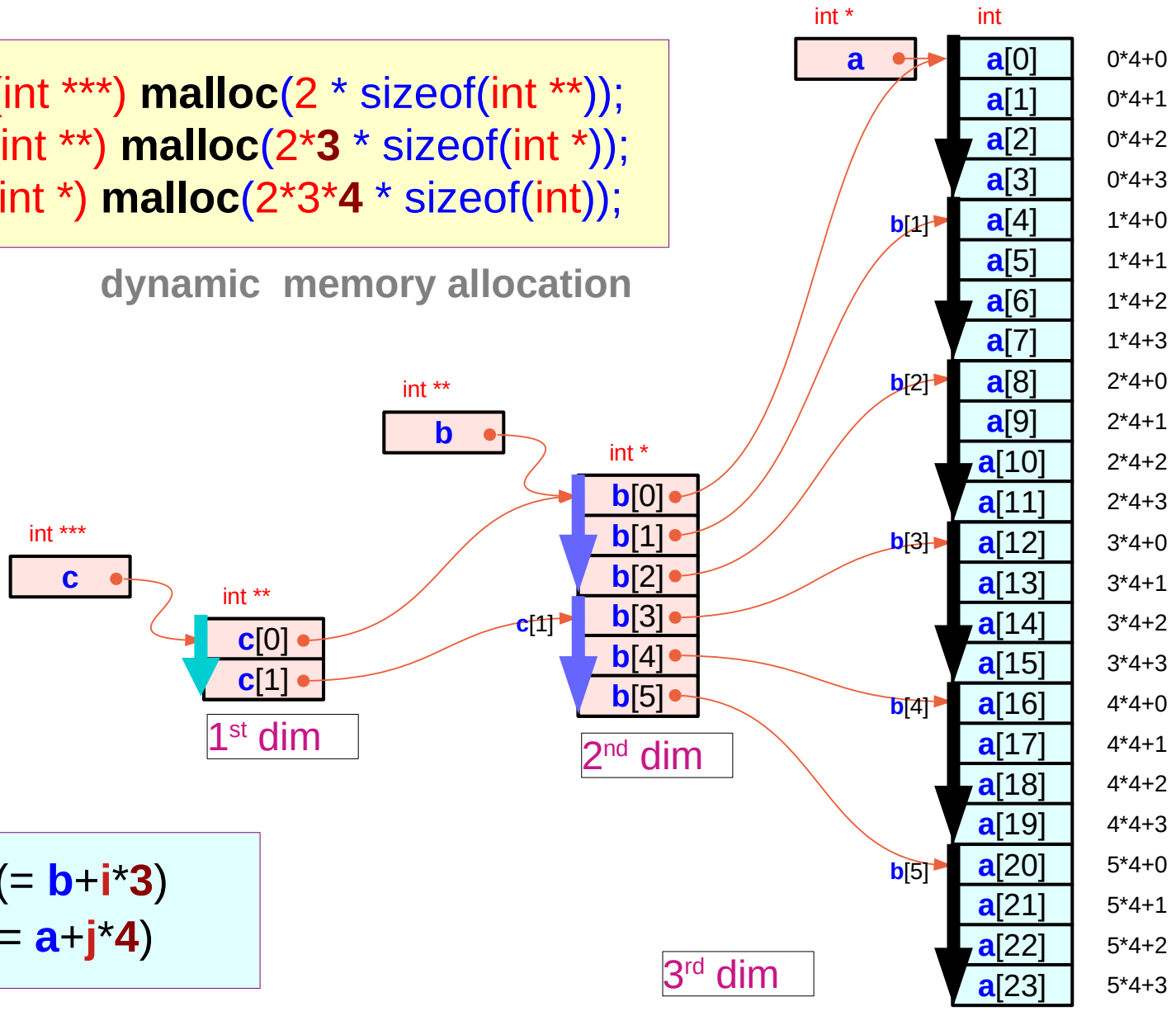


c[i] = &b[i*3] (= b+i*3)
b[j] = &a[j*4] (= a+j*4)

Using pointer **a**, **b**, **c** – dynamically allocated

int ***	**c** = (int ***) **malloc**(2 * sizeof(int **));
int **	**b** = (int **) **malloc**(2*3 * sizeof(int *));
int *	**a** = (int *) **malloc**(2*3*4 * sizeof(int));

dynamic memory allocation



c[**i**] = &**b**[**i***3] (= **b**+**i***3)
b[**j**] = &**a**[**j***4] (= **a**+**j***4)

Static v.s. dynamic allocation (1)

int **	c [2] ;
int *	b [2*3] ;
int	a [2*3*4] ;

int ***	c = (int ***) malloc(2 * sizeof(int **));
int **	b = (int **) malloc(2*3 * sizeof(int *));
int *	a = (int *) malloc(2*3*4 * sizeof(int));

static memory allocations

type(**c**) = int ** [2] → int ***
type(**b**) = int * [2*3] → int **
type(**a**) = int [2*3*4] → int *

sizeof(**c**) = 2 * sizeof(int **)
sizeof(**b**) = 2*3 * sizeof(int *)
sizeof(**a**) = 2*3*4 * sizeof(int)

value(**c[i]**) = **b** + 3***i**
value(**b[j]**) = **a** + 4***j**

dynamic memory allocations

type(**c**) = int ***
type(**b**) = int **
type(**a**) = int *

sizeof(**c**) = 4 bytes on 32-bit system
sizeof(**b**) = 4 bytes on 32-bit system
sizeof(**a**) = 4 bytes on 32-bit system

value(**c[i]**) = **b** + 3***i**
value(**b[j]**) = **a** + 4***j**

c[**i**] = &**b**[**i***3] (= **b**+**i***3)
b[**j**] = &**a**[**j***4] (= **a**+**j***4)

Static v.s. dynamic allocation (2)

- **static allocation**

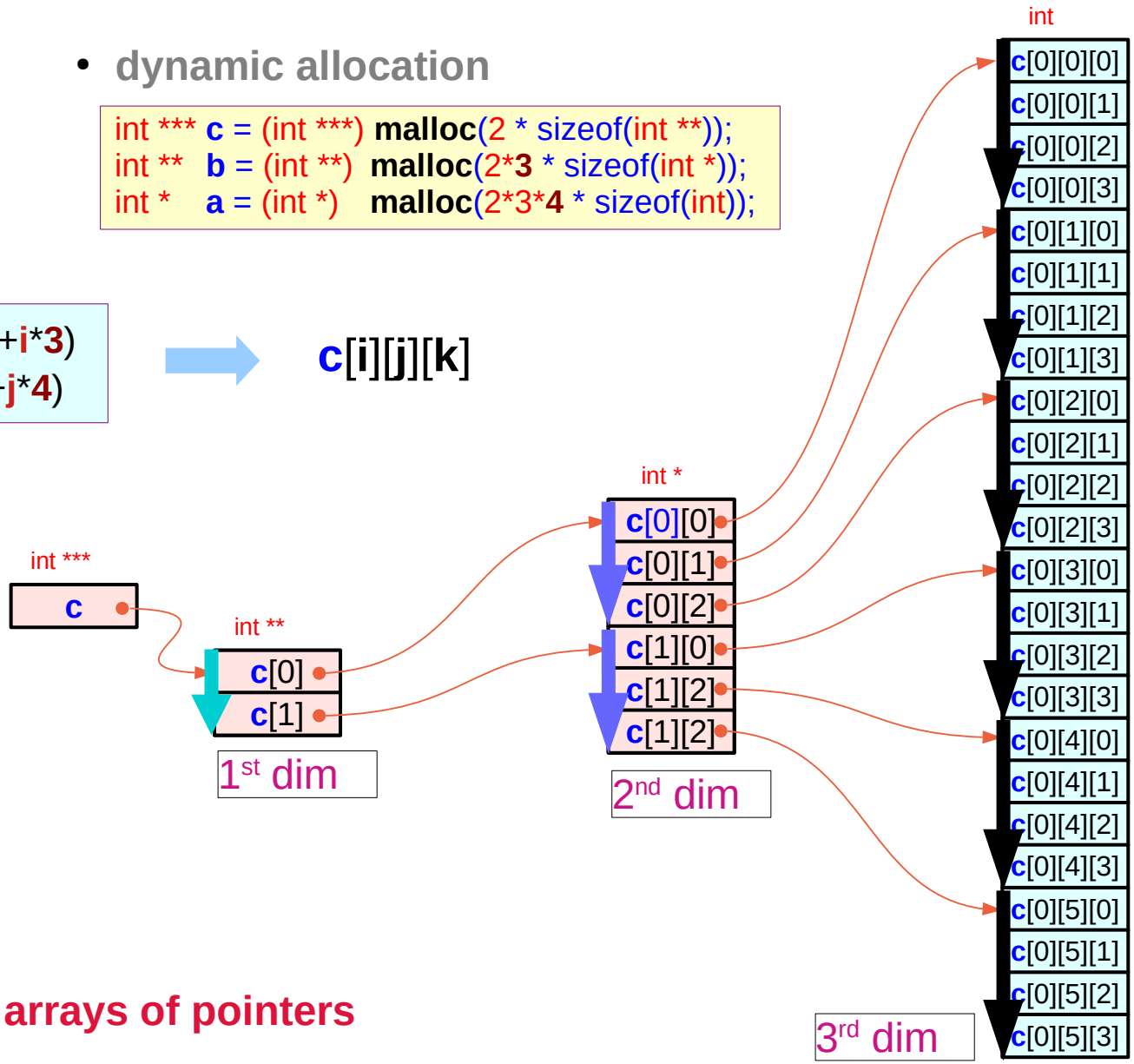
```
int **   c  [2] ;
int *    b  [2*3] ;
int      a  [2*3*4] ;
```

- **dynamic allocation**

```
int *** c = (int ***) malloc(2 * sizeof(int **));
int **  b = (int **)  malloc(2*3 * sizeof(int *));
int *   a = (int *)   malloc(2*3*4 * sizeof(int));
```

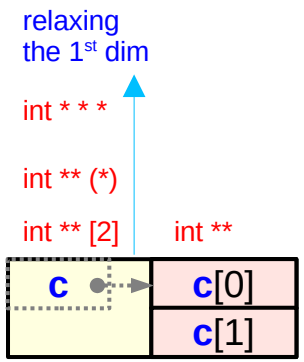
```
c[i] = &b[i*3]  (= b+i*3)
b[j] = &a[j*4] (= a+j*4)
```

→ **c[i][j][k]**



arrays of pointers

Static v.s. dynamic allocation (3)

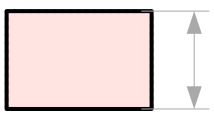


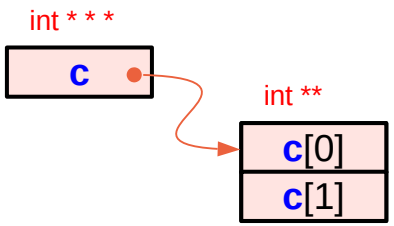
int ** c [2] ;

static memory allocation

int *** c = (int ***) **malloc**(2 * sizeof(int **));

malloc(2 * sizeof(int **));



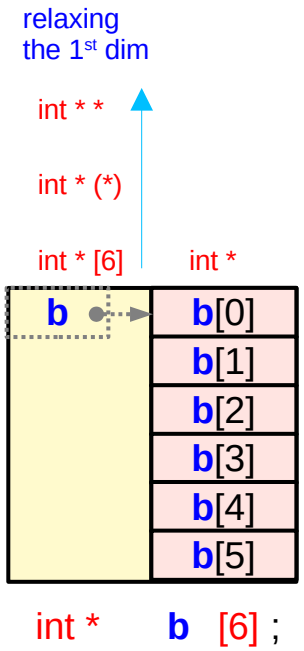


int *** c = (int ***) **malloc**(2 * sizeof(int **));

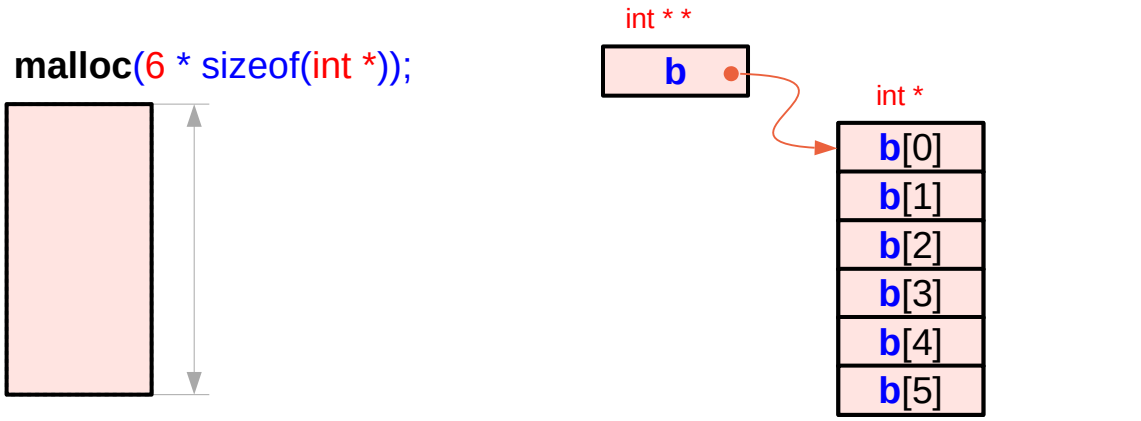
dynamic memory allocation

Static v.s. dynamic allocation (4)

```
int **   b = (int ***)  malloc(6 * sizeof(int *));
```



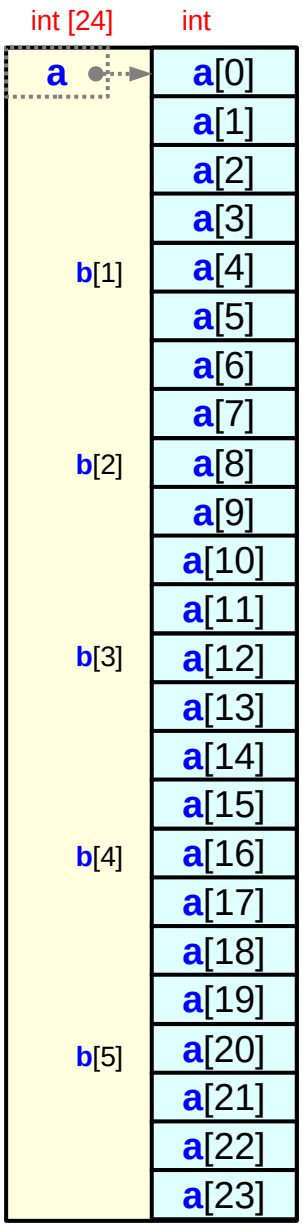
static memory allocation



int ** **b** = (int **) **malloc**(6 * sizeof(int *));

dynamic memory allocation

Static v.s. dynamic allocation (5)



relaxing
the 1st dim

int *

int (*)

int [24]

int * **a** [24] ;

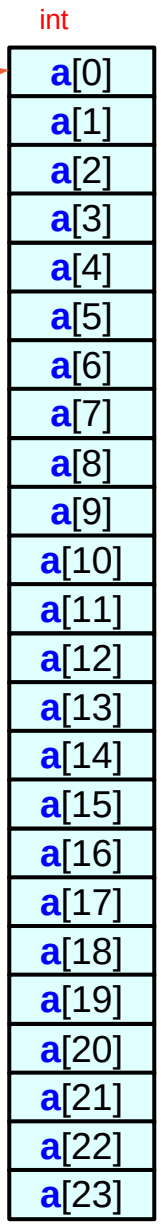
static memory allocation

int * **a** = (int *) **malloc**(24 * sizeof(int));

int *

a

int



int * **a** = (int *) **malloc**(24 * sizeof(int));

dynamic memory allocation

& address-of operator

* dereference operator

Address-of operator and dereferencing operator

the address of a variable :
address-of operator ***&***

the content at an address :
dereferencing operator *******

& variable *:*
returns the address of a variable

variable *has memory locations whose value can be changed by an assignment*

(***variable*** *must be an lvalue*)

**** address*** *:*
returns the value at the address

**** address*** *has memory locations whose value can be changed by an assignment*

(**** address*** *is an lvalue*)

lvalue and rvalue in assignments

Left Hand Side *Right Hand Side*

LHS = ***RHS***

```
int    a, b = 10 ;
int  * p, q = &a ;
```

lvalue	=	*lvalue*
lvalue	=	*rvalue*
~~*rvalue*~~	=	*lvalue*
~~*rvalue*~~	=	*rvalue*

p	=	**q** ;
p	=	&**a** ;
~~&**a**~~	=	**p** ;
~~&**a**~~	=	&**b** ;

*in the **LHS,** only lvalue can exist*
*rvalue can exist only in the **RHS***

a, **b**, **p**, **q**	: lvalues	… variables	… RW
***p**, ***q**	: lvalues	… variables	… RW
&**a**, &**b**	: rvalues	… constants	… RO

lvalue and rvalue with * and & operators

lvalue	←	* lvalue	lvalue	←	* p
~~rvalue~~	←	* lvalue	~~rvalue~~	←	* p
lvalue	←	* rvalue	lvalue	←	* (&a)
~~rvalue~~	←	* rvalue	~~rvalue~~	←	* (&a)
~~lvalue~~	←	& lvalue	~~lvalue~~	←	& p
rvalue	←	& lvalue	rvalue	←	& p
~~lvalue~~	←	& ~~rvalue~~	lvalue	←	~~& (&a)~~
~~rvalue~~	←	& ~~rvalue~~	rvalue	←	~~& (&a)~~

```
int    a = 10 ;
int  *  p = &a ;
```

* can be applied
to either an **lvalue** variable
or a **rvalue** address

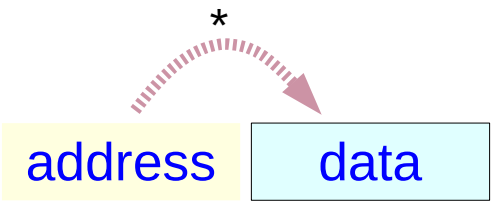
* **operand** becomes
an **lvalue** variable
thus can be applied
successively.

& can be applied
to only an **lvalue** variable and
returns only an **rvalue** address

a, **p**	: lvalues	… variables	… RW
***p**	: lvalues	… variables	… RW
&a	: rvalues	… constants	… RO

Address-of and dereference operators

Primitive Data Type

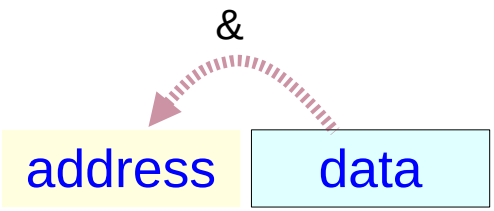


lvalue **p** — *pointer*

lvalue ***p** — *variable*

rvalue **&a** — *constant*

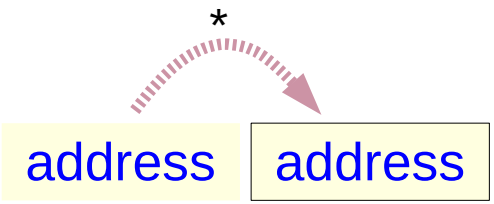
lvalue **a** — *variable*



rvalue **&a** — *constant*

lvalue **a** — *variable*

Pointer Data Type

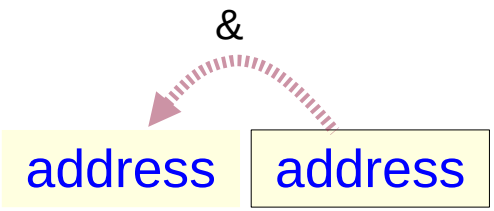


lvalue **q** — *d pointer*

lvalue ***q** — *pointer*

rvalue **&q** — *constant*

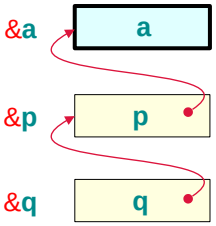
lvalue **q** — *d pointer*

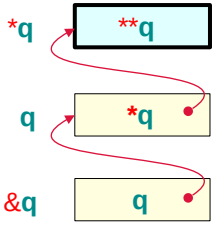


rvalue **&q** — *constant*

lvalue **q** — *d pointer*

```
int     a ;
int *   p ;
int **  q ;
```





Byte addresses of sub-arrays in an array

~~&(&(&(c[i])[j])[k])~~

& C operator

can be applied to only **lvalue** variable

returns address value

thus, the above expression is not possible

successive application of & is not possible

In constrast, ***p** becomes a lvalue variable

* operator can be applied successively.

Finding sub-array sizes

int c [2][3][4] ;

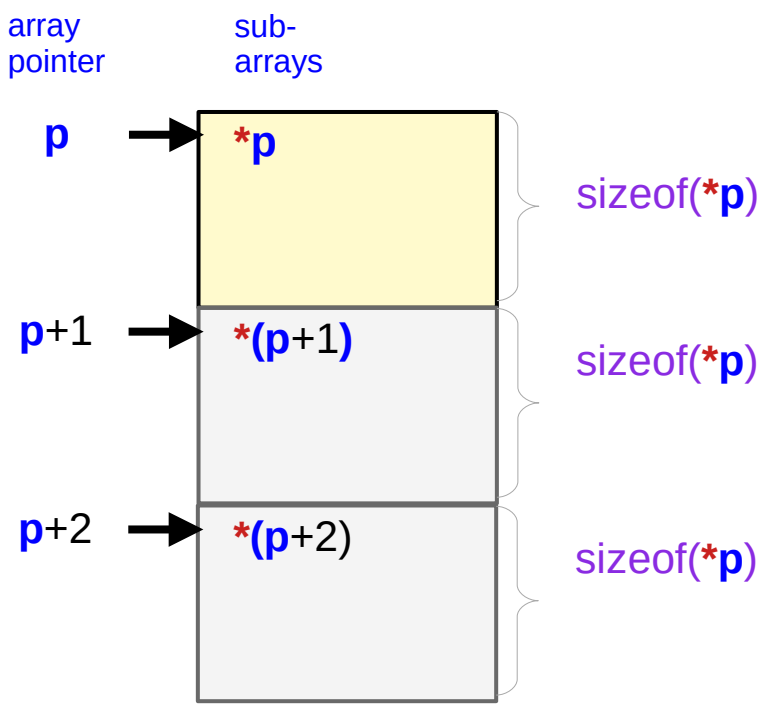
[2] [3] [4]
sizeof(c[i][j][0]) = sizeof(int)

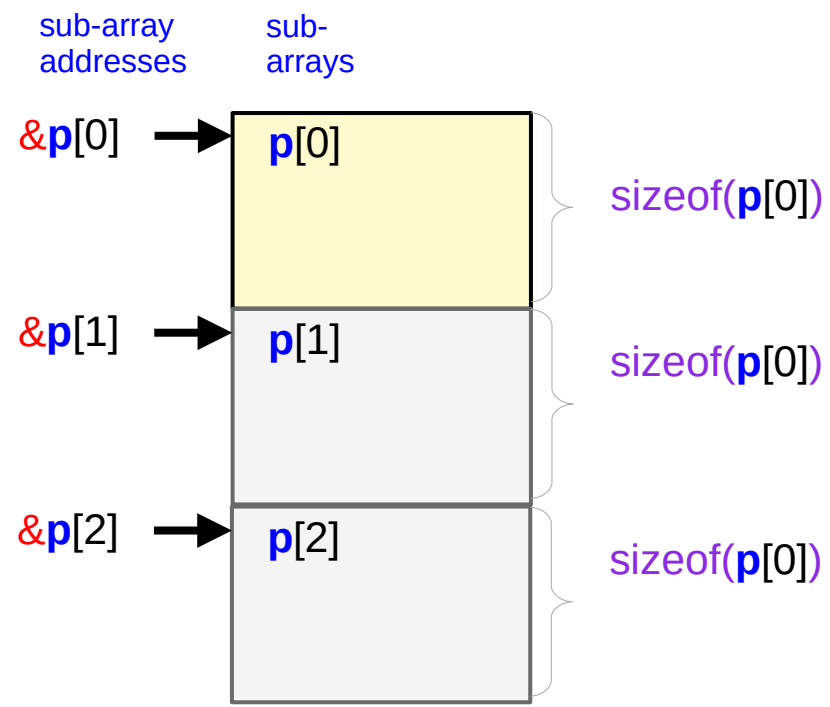
[2] [3] [4]
sizeof(c[i][0]) = 4 * sizeof(int)

[2] [3] [4]
sizeof(c[i]) = 3 * 4 * sizeof(int)

[2] [3] [4]
sizeof(c) = 2 * 3 * 4 * sizeof(int))

Pointer increments and byte addresses





byte address byte address byte size

value(**p**+**i**) = value(**p**) + **i** * sizeof(***p**)

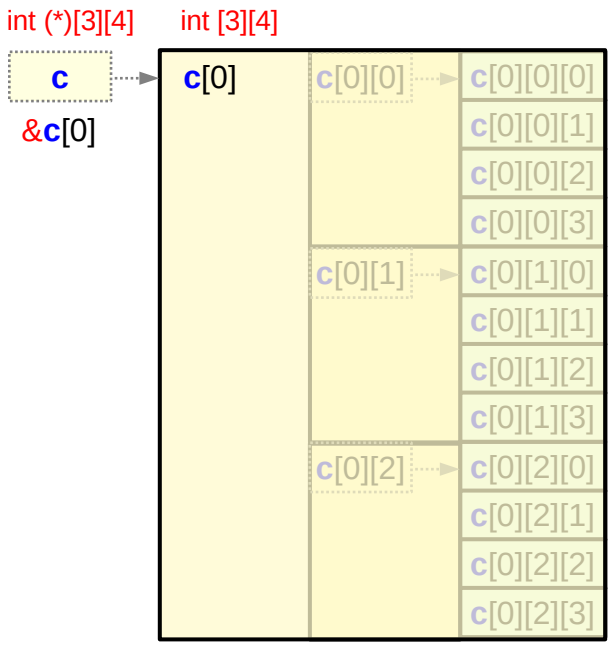
math expression with an explicit size information (**p**+**i**)sizeof(***p**)

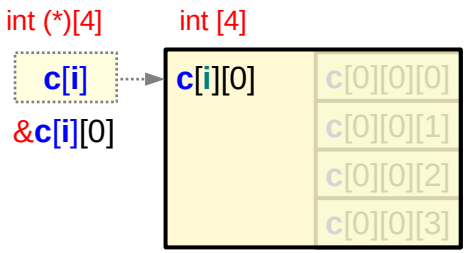
byte address byte address byte size

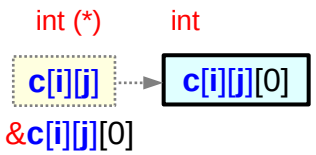
value(&**p**[**i**]) = value(**p**) + **i** * sizeof(**p**[0])

math expression with an explicit size information (&**p**[**i**])sizeof(***p**)

Byte addresses of subarrays &c[i], &c[i][j], &c[i][j][k]







value(&c[i]) = value(c+i)
= value(c) + i * sizeof(*c)
= value(c) + i * sizeof(c[0])
= value(c) + i * sizeof(int) * 3 * 4

skip i elements of *c from c

$(c + i)_{3 \cdot 4 \cdot 4}$

value(&c[i][j]) = value(c[i]+j)
= value(c[i]) + j * sizeof(*c[i])
= value(c[i]) + j * sizeof(c[i][0])
= value(c[i]) + j * sizeof(int) * 4

skip j elements of *c[i] from c[i]

$(c[i] + j)_{4 \cdot 4}$

value(&c[i][j][k]) = value(c[i][j]+k)
= value(c[i][j]) + k * sizeof(*c[i][j])
= value(c[i][j]) + k * sizeof(c[i][j][0])
= value(c[i][j]) + k * sizeof(int)

skip k elements of *c[i][j] from c[i][j]

$(c[i][j] + k)_{1 \cdot 4}$

i = 0:1
j = 0:2
k = 0:3

Address replications and subarray addresses

Address Replication

transferring pointing address
to the pointer that references itself

&X = X

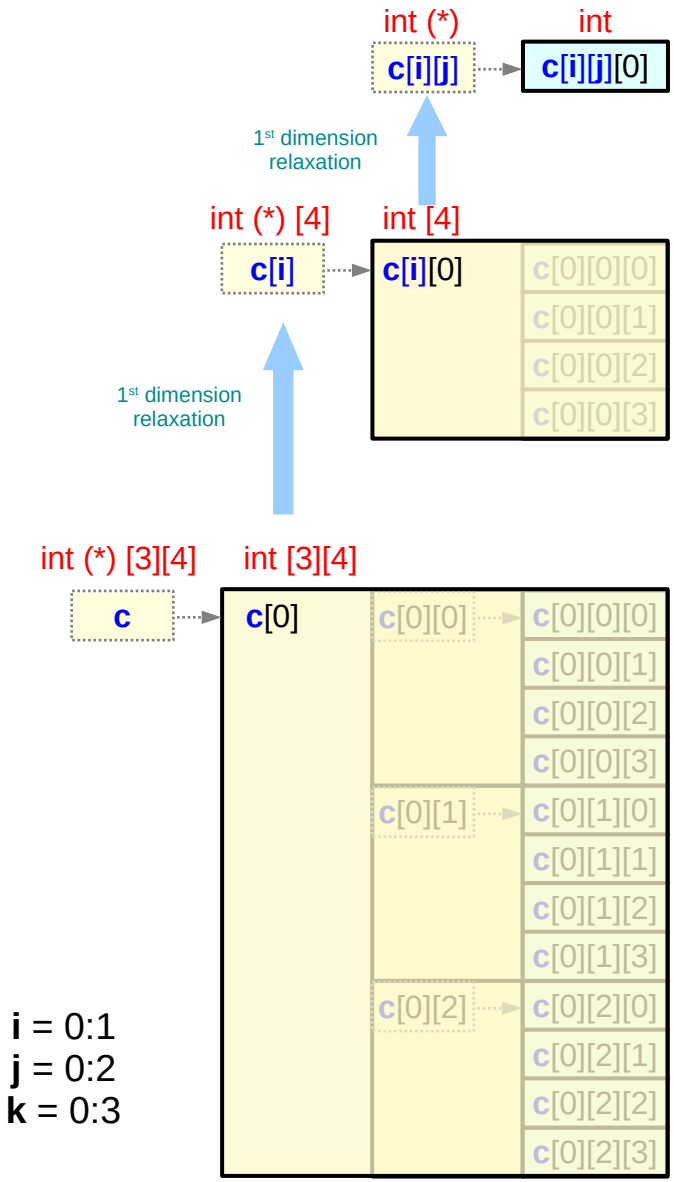
int (*) int
value(&**c**[**i**][**j**][**k**]) = value(**c**[**i**][**j**]) + **k** * sizeof(int)

int (*) [4] int [4]
value(&**c**[**i**][**j**]) = value(**c**[**i**]) + **j** * 4 * sizeof(int)

int (*) [3][4] int [3][4]
value(&**c**[**i**]) = value(**c**) + **i** * 3 * 4 * sizeof(int)

byte addresses
of abstract data

values of
virtual pointers



Address replications in a multi-dimensional array

int c [2][3][4] ;

equivalences

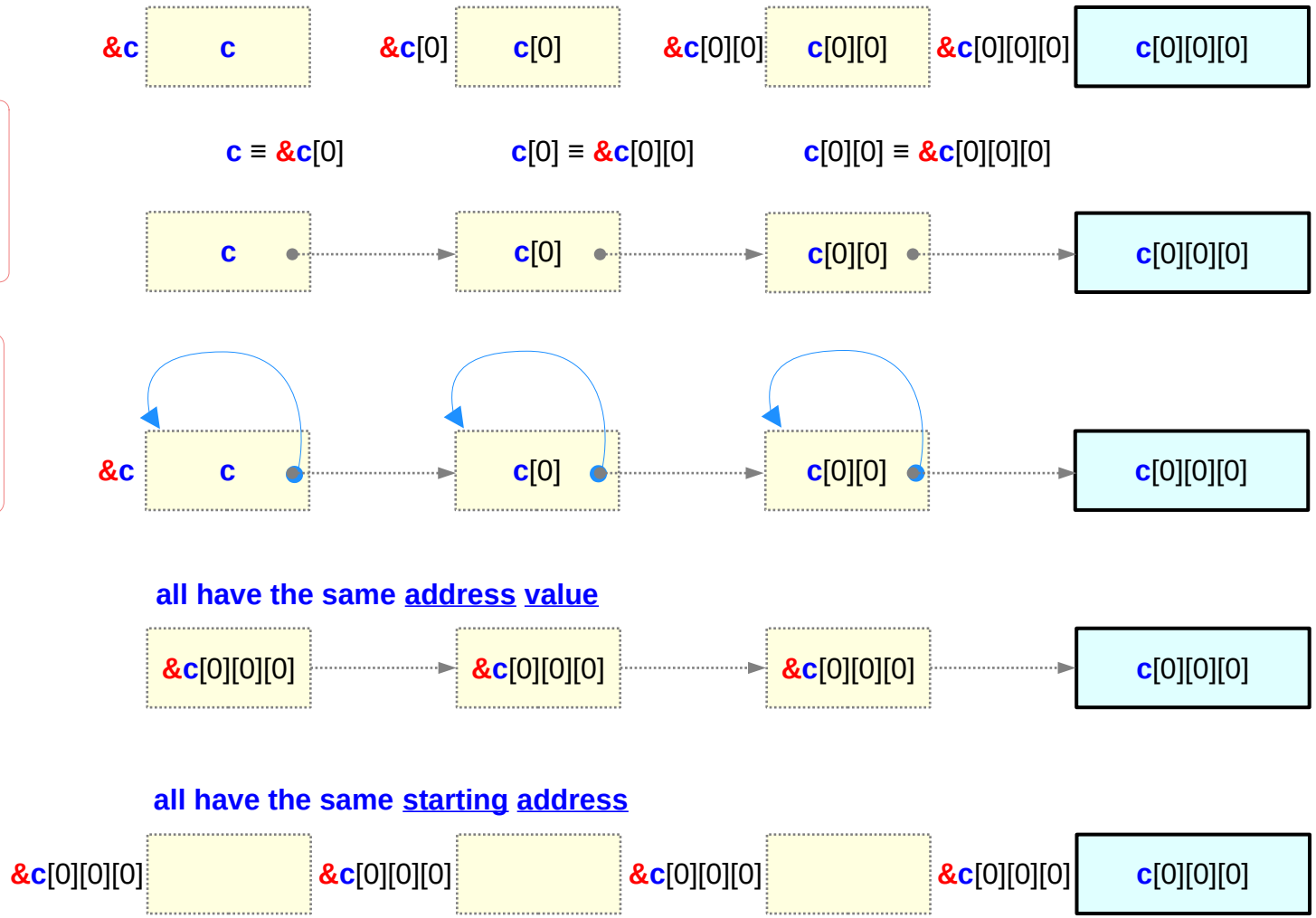
c[i][j] ≡ &c[i][j][0]
c[i] ≡ &c[i][0]
c ≡ &c[0]

address replication

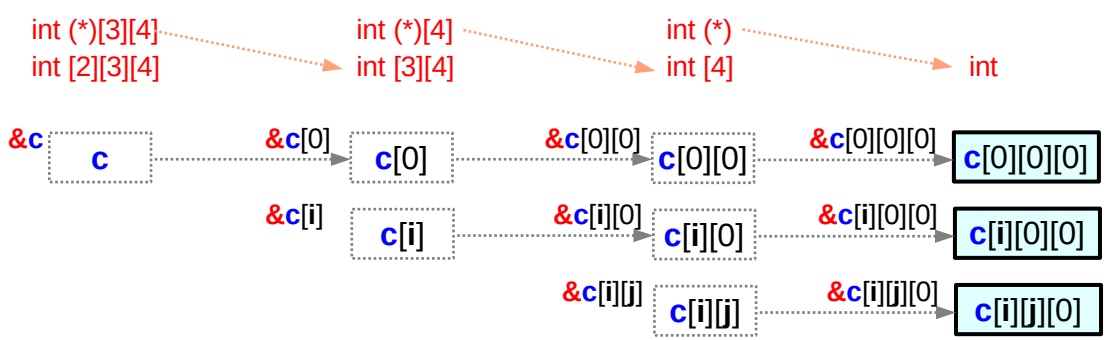
value(c[i][j]) = value(&c[i][j])
value(c[i]) = value(&c[i])
value(c) = value(&c)

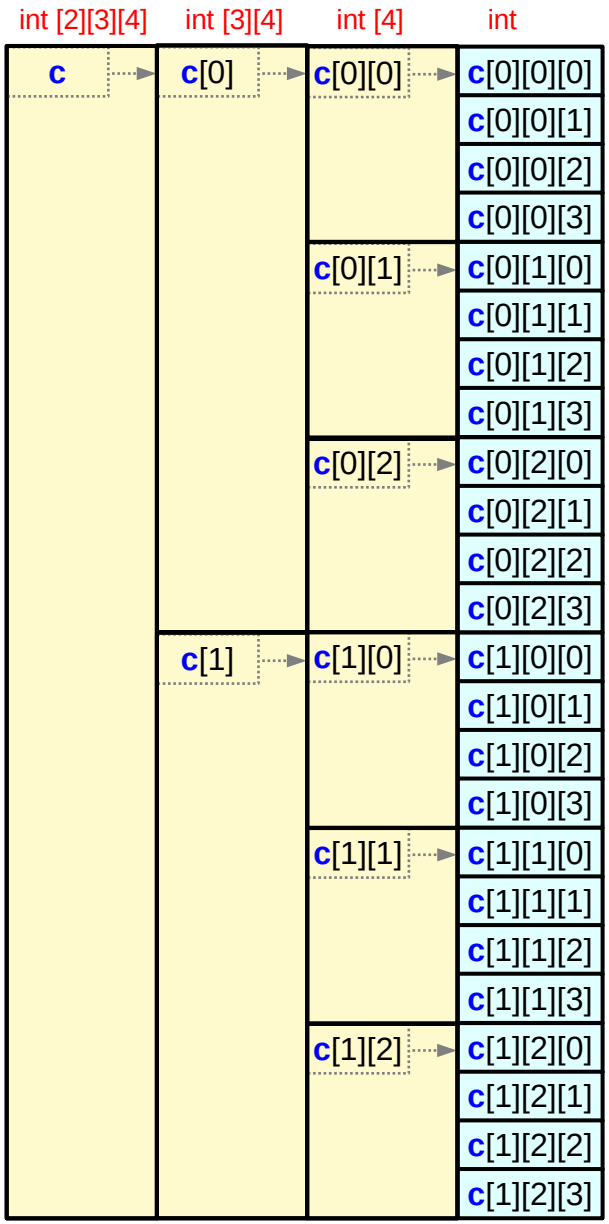
c, c[0], c[0][0] :
these virtual pointers have
the same address value

a physical location
has a unique address



Referencing sub-arrays





equivalence relations

c[i][j] ≡ *(c[i]+j)	&c[i][j] ≡ (c[i]+j)	&c[i][0] ≡ c[i]
c[i] ≡ *(c+i)	&c[i] ≡ (c+i)	&c[0] ≡ c

address replication

value(c[i][j]) = value(&c[i][j]) = value(c[i]+j) = *value(c[i]+j)
value(c[i]) = value(&c[i]) = value(c+i) = *value(c+i)

c[i], c[i][0] point to the same data c[i][0][0]
c, c[0], c[0][0] point to the same data c[0][0][0]

Types, sizes, and values of sub-arrays

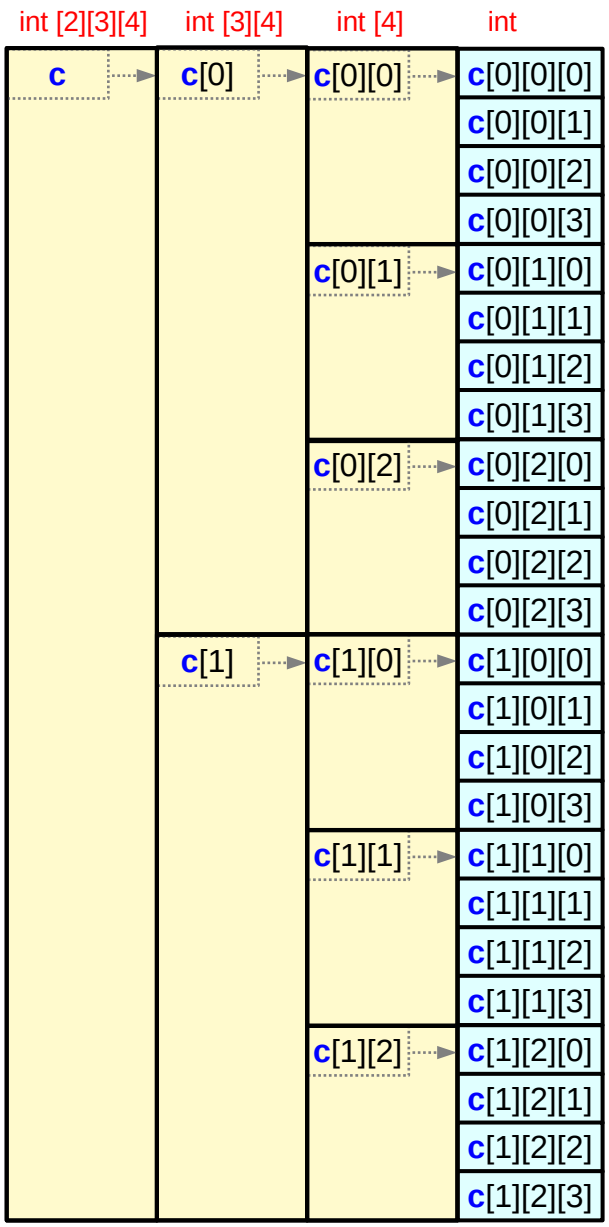
int **c** [2][3][4] ; **static allocation**

value(**c**) = value(**c**[0]) = value(**c**[0][0]) = &**c**[0][0][0]
value(**c**[0][1]) = &**c**[0][1][0]
value(**c**[0][2]) = &**c**[0][2][0]
value(**c**[1]) = value(**c**[1][0]) = &**c**[1][0][0]
value(**c**[1][1]) = &**c**[1][1][0]
value(**c**[1][2]) = &**c**[1][2][0]

sizeof(**c**) = 2*3*4 * sizeof(int)
sizeof(**c**[**i**]) = 3*4 * sizeof(int)
sizeof(**c**[**i**][**j**]) = 4 * sizeof(int)

type(**c**) = int [2][3][4]
int (*)[3][4]
type(**c**[**i**]) = int [3][4]
int (*)[4]
type(**c**[**i**][**j**]) = int [4]
int (*)

pointers to arrays



Using multi-dimensional arrays

Pointer Array Approach

– using explicit pointers

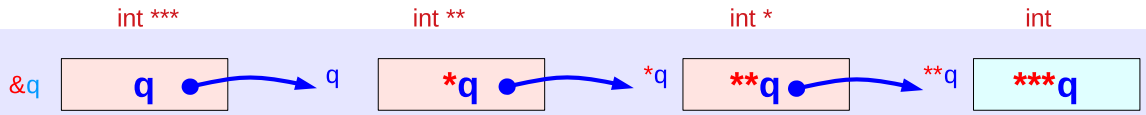
Array Pointer Approach

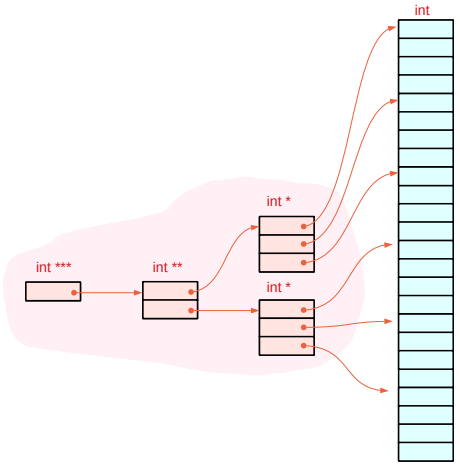
– using implicit pointers

Two types of **3-d** array accesses

Pointer Array Approach (arrays of pointers)

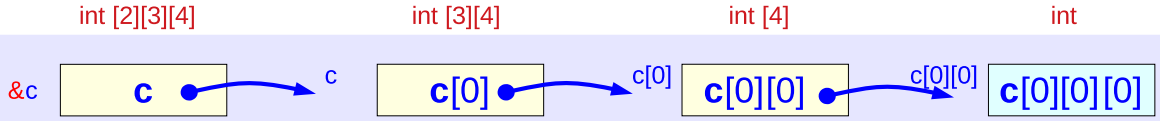
Pointer Chain Type I

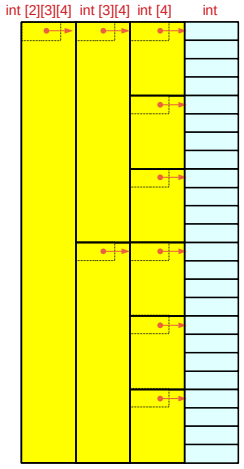




Array Pointer Approach (pointers to arrays)

Pointer Chain Type II





Pointer addition – **math** and **c** expressions

Accessing c[i][j][k] – *unified **c** expressions*

skip **i** elements of **c**[**i**] from **c**

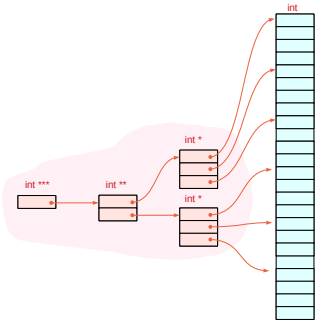
skip **j** elements of **c**[**i**][**j**] from **c**[**i**]

skip **k** elements of **c**[**i**][**j**][**k**] from **c**[**i**][**j**]

(c + i)

(c[i] + j)

(c[i][j] + k)



Pointer Array Approach

(c + i)$_{1 \cdot 4}$

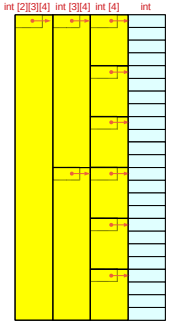
(c[i] + j)$_{1 \cdot 4}$

(c[i][j] + k)$_{1 \cdot 4}$

sizeof(***c**) = 1*4

sizeof(***c**[**i**]) = 1*4

sizeof(***c**[**i**][**j**]) = 1*4



Array Pointer Approach

(c + i)$_{3 \cdot 4 \cdot 4}$

(c[i] + j)$_{4 \cdot 4}$

(c[i][j] + k)$_{1 \cdot 4}$

sizeof(***c**) = 3*4*4

sizeof(***c**[**i**]) = 4*4

sizeof(***c**[**i**][**j**]) = 1*4

Accessing **c[i][j][k]** element

Accessing c[i][j][k]

skip **i** elements
of **c**[**i**] from **c**

(**c** + **i**)

skip **j** elements
of **c**[**i**][**j**] from **c**[**i**]

(**c**[**i**] + **j**)

skip **k** elements
of **c**[**i**][**j**][**k**] from **c**[**i**][**j**]

(**c**[**i**][**j**] + **k**)

Pointer Array Approach

skip **i** * sizeof(int **)
(= **i***4) bytes from **c**

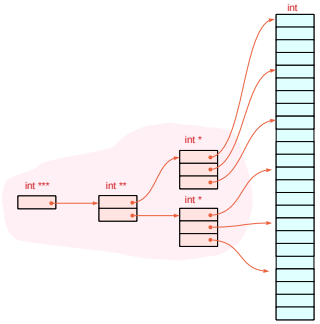
$(\mathbf{c} + \mathbf{i})_{1 \cdot 4}$

skip **j** * sizeof(int *)
(= **j***4) bytes from **c**[**i**]

$(\mathbf{c}[\mathbf{i}] + \mathbf{j})_{1 \cdot 4}$

skip **k** * sizeof(int)
(= **k***4) bytes from **c**[**i**][**j**]

$(\mathbf{c}[\mathbf{i}][\mathbf{j}] + \mathbf{k})_{1 \cdot 4}$



Array Pointer Approach

skip **i** * sizeof(int [3][4])
(= **i***3*4*4) bytes from **c**

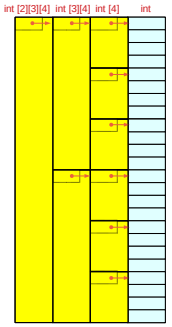
$(\mathbf{c} + \mathbf{i})_{3 \cdot 4 \cdot 4}$

skip **j** * sizeof(int [4])
(= **j***4*4) bytes from **c**[**i**]

$(\mathbf{c}[\mathbf{i}] + \mathbf{j})_{4 \cdot 4}$

skip **k** * sizeof(int)
(= **k***4) bytes from **c**[**i**][**j**]

$(\mathbf{c}[\mathbf{i}][\mathbf{j}] + \mathbf{k})_{1 \cdot 4}$



Accessing **c[i][j][k]** – Pointer Array Approach

Pointer Array Approach

skip **i** * sizeof(int **)
= **i** *4 bytes from **c**

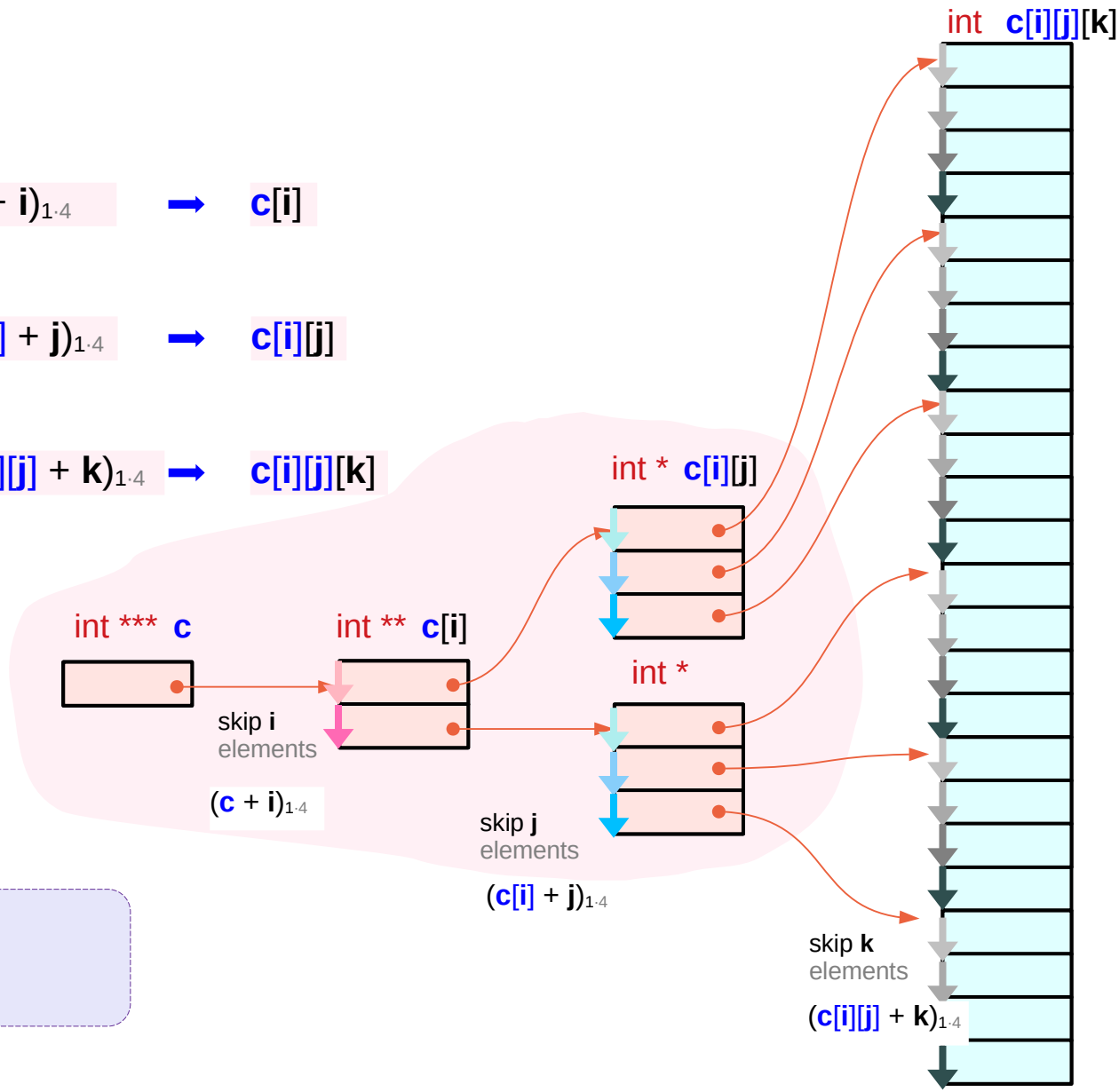
(**c** + **i**)$_{1\cdot 4}$ ➡ **c**[**i**]

skip **j** * sizeof(int *)
= **j** *4 bytes from **c**[**i**]

(**c**[**i**] + **j**)$_{1\cdot 4}$ ➡ **c**[**i**][**j**]

skip **k** * sizeof(int)
= **k** *4 bytes from **c**[**i**][**j**]

(**c**[**i**][**j**] + **k**)$_{1\cdot 4}$ ➡ **c**[**i**][**j**][**k**]



sizeof(**c**[**i**][**j**][**k**]) = sizeof(int) = 4
sizeof(**c**[**i**][**j**]) = sizeof(int *) = 4
sizeof(**c**[**i**]) = sizeof(int **) = 4

Accessing **c[i][j][k]** – Array Pointer Approach

Array Pointer Approach

skip **i** * sizeof(int [3][4])
= **i** *3*4*4 bytes from **c**

$(c + i)_{3\cdot4\cdot4}$ ➡ **c[i]**

skip **j** * sizeof(int [4])
= **j** *4*4 bytes from **c[i]**

$(c[i] + j)_{4\cdot4}$ ➡ **c[i][j]**

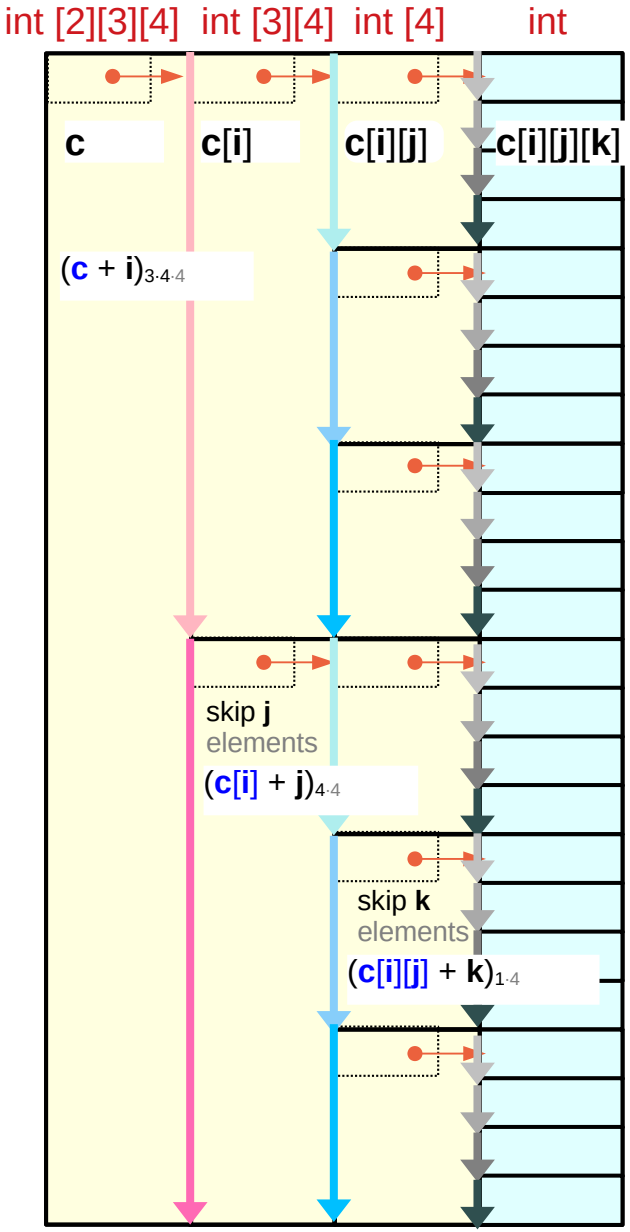
skip **k** * sizeof(int)
= **k** *4 bytes from **c[i][j]**

$(c[i][j] + k)_{1\cdot4}$ ➡ **c[i][j][k]**

- ***subarray partitioning***
- ***address replication***

size information

sizeof(**c[i][j][k]**) = sizeof(int) = 4
sizeof(**c[i][j]**) = sizeof(int [4]) = 4*4
sizeof(**c[i]**) = sizeof(int [3][4]) = 3*4*4



Array element address – Pointer Array Approach

equivalence relations – c expressions

```
&c[i][j][k]  = (c[i][j] + k)
&c[i][j]     = (c[i] + j)
&c[i]        = (c + i)
```

```
c[i][j][k]   = *(c[i][j] + k)
c[i][j]      = *(c[i] + j)
c[i]         = *(c + i)
```

size information

```
sizeof(c[i][j][k]) = sizeof(int) = 4
sizeof(c[i][j]) = sizeof(int *) = 4
sizeof(c[i]) = sizeof(int **) = 4
```

address fetch – math expressions

```
value(c[i][j])  ➡  *value( (c[i] + j)1·4 )  = *value(c[i] + j * 4)   ⬅  sizeof(*c[i])
value(c[i])     ➡  *value( (c + i)1·4 )     = *value(c + i * 4)      ⬅  sizeof(*c)
```

address of c[i][j][k] – math expressions

```
&c[i][j][k] = value( (c[i][j] + k)1·4 )                                  ⬅  &c[i][j][k]  ≡ (c[i][j] + k)
            = value( *value( (c[i] + j)1·4 ) + k * 4 )                   ⬅  c[i][j]      ≡ *(c[i] + j)
            = value( *value( *value( (c + i)1·4 ) + j * 4 ) + k * 4 )    ⬅  c[i]         ≡ *(c + i)
            = value( *value( *value( c + i * 4 ) + j * 4 ) + k * 4 )
```

Array element address – Array Pointer Approach

equivalence relations – c expressions

&c[i][j][k] = (c[i][j] + k)
&c[i][j] = (c[i] + j)
&c[i] = (c + i)

c[i][j][k] = *(c[i][j] + k)
c[i][j] = *(c[i] + j)
c[i] = *(c + i)

size information

sizeof(c[i][j][k]) = sizeof(int) = 4
sizeof(c[i][j]) = sizeof(int [4]) = 4*4
sizeof(c[i]) = sizeof(int [3][4]) = 3*4*4

address replication – math expressions

value(c[i][j]) = value(&c[i][j]) ➡ value($(c[i] + j)_{4\cdot 4}$) = value(c[i]) + j * 4*4 ⬅ sizeof(*c[i])
value(c[i]) = value(&c[i]) ➡ value($(c + i)_{3\cdot 4\cdot 4}$) = value(c) + i * 3*4*4 ⬅ sizeof(*c)

address of c[i][j][k] – math expressions

&c[i][j][k] = value($(c[i][j] + k)_{1\cdot 4}$) ⬅ &c[i][j][k] ≡ c[i][j] + k
= value($(c[i] + j)_{4\cdot 4}$) + k * 4 ⬅ &c[i][j] ≡ c[i] + j ⬅ c[i][j] address replication
= value($(c + i)_{3\cdot 4\cdot 4}$) + j * 4*4 + k * 4 ⬅ &c[i] ≡ c + i ⬅ c[i] address replication
= value(c) + i * 3*4*4 + j * 4*4 + k * 4

- *address replication*
- *combining size and address information*

Accessing **c[i][j][k]** via byte addresses

Pointer Array Approach

&c[i][j][k] = value((c[i][j] + k)$_{1\cdot 4}$) = value(c[i][j] + k * 4)
&c[i][j] = value((c[i] + j)$_{1\cdot 4}$) = value(c[i] + j * 4)
&c[i] = value((c + i)$_{1\cdot 4}$) = value(c + i * 4)

c[i][j][k] = *value(c[i][j] + k * 4)
c[i][j] = *value(c[i] + j * 4)
c[i] = *value(c + i * 4)

c[i][j][k] = *value(c[i][j] + k * 4)
= *value(*value(c[i] + j * 4) + k * 4)
= *value(*value(*value(c + i * 4) + j * 4) + k * 4)

three memory accesses for **c[i][j][k]**

Array Pointer Approach

&c[i][j][k] = value((c[i][j] + k)$_{1\cdot 4}$) = value(c[i][j] + k * 4)
&c[i][j] = value((c[i] + j)$_{4\cdot 4}$) = value(c[i] + j * 4*4)
&c[i] = value((c + i)$_{3\cdot 4\cdot 4}$) = value(c + i * 3*4*4)

c[i][j][k] = *value(c[i][j] + k * 4)
c[i][j] = *value(c[i] + j * 4*4)
c[i] = *value(c + i * 3*4*4)

c[i][j][k] = *value(c[i][j] + k * 4)
= *value(*value(c[i] + j * 4*4) + k * 4)
= *value(*value(*value(c + i * 3*4*4) + j * 4*4) + k * 4)
= *value(value(value(c + i * 3*4*4) + j * 4*4) + k * 4)
= *value(c + i * 3*4*4 + j * 4*4 + k * 4)

address replication

single memory access for **c[i][j][k]**

Equivalence relations in **c[i][j][k]**

Pointer Array Approach

&c[i][j][k] = value(c[i][j] + k * 4) = value(c[i][j]) + k * 4
&c[i][j] = value(c[i] + j * 4) = value(c[i]) + j * 4
&c[i] = value(c + i * 4) = value(c) + i * 4

c[i][j][k] ≠ *value(c[i][j]) + k * 4
c[i][j] ≠ *value(c[i]) + j * 4
c[i] ≠ *value(c) + i * 4

different semantics !

be careful in mixed c and math expressions

c[i][j][k] = *value(c[i][j] + k * 4)
c[i][j] = *value(c[i] + j * 4)
c[i] = *value(c + i * 4)

Array Pointer Approach

&c[i][j][k] = value($(c[i][j] + k)_{1 \cdot 4}$) = value(c[i][j]) + k * 4
&c[i][j] = value($(c[i] + j)_{4 \cdot 4}$) = value(c[i]) + j * 4*4
&c[i] = value($(c + i)_{3 \cdot 4 \cdot 4}$) = value(c) + i * 3*4*4

c[i][j][k] ≠ *value(c[i][j]) + k * 4
c[i][j] ≠ *value(c[i]) + j * 4*4
c[i] ≠ *value(c) + i * 3*4*4

different semantics !

be careful in mixed c and math expressions

c[i][j][k] = *value(c[i][j] + k * 4)
c[i][j] = *value(c[i] + j * 4*4)
c[i] = *value(c + i * 3*4*4)

Accessing c[i][j][k]

```
int    c [L][M][N] ;
```

c[i]	≡ *(c + i)
c[i][j]	≡ *(c[i] + j)
c[i][j][k]	≡ *(c[i][j] + k)

&c[i]	≡ (c + i)
&c[i][j]	≡ (c[i] + j)
&c[i][j][k]	≡ (c[i][j] + k)

equivalence relations		multiple indirections	address replications
c[i]	≡ *(c+i)	≡ *(c+i)	≡ (c+i)
c[i][j]	≡ *(c[i]+j)	≡ *(*(c+i)+j)	≡ ((c+i)+j)
c[i][j][k]	≡ *(c[i][j]+k)	≡ *(*(*(c+i)+j)+k)	≡ (((c+i)+j)+k)
			→ *(c +i +j +k)

Pointer Array Approach →

Array Pointer Approach →

Conditions for **c[i][j][k]**

Equivalence relations in c[i][j][k]

c[i][j][k]	≡	*(c[i][j] +k)
*(c[i][j] +k)	≡	*(*(c[i] +j) +k)
((c[i] +j) +k)	≡	*(*(*(c +i) +j) +k)

c[i][j][k]	≡	(c[i][j] +k)
c[i][j]	≡	(c[i] +j)
c[i]	≡	(c +i)

Pointer Array Approach

c[i][j][k]	≡	(c[i][j] +k)
c[i][j]	≡	(c[i] +j)
c[i]	≡	(c +i)

⟺

contiguous **4** c[i][j][k]'s	**4** * (int	4 bytes)
contiguous **3** c[i][j]'s	**3** * (int *	4 or 8 bytes)
contiguous **2** c[i]'s	**2** * (int **	4 or 8 bytes)

Array Pointer Approach

c[i][j][k]	≡	(c[i][j] +k)
c[i][j]	≡	(c[i] +j)
c[i]	≡	(c +i)

⟺

contiguous **4** c[i][j][k]'s	**4** * (int	4 bytes)
contiguous **3** c[i][j]'s	**3** * (int [4]	4*4 bytes)
contiguous **2** c[i]'s	**2** * (int [3][4]	3*4*4 bytes)

Skipping leaf elements

Continuity Constraints

c[i][j][k]	≡	(c[i][j] +k)
c[i][j]	≡	(c[i] +j)
c[i]	≡	(c +i)

⟺

contiguous c[i][j][k]	over k=0:3
contiguous c[i][j]	over j=0:2
contiguous c[i]	over i=0:1

Pointer Array Approach

$(c[i][j] + k)_{1\cdot 4}$	skip k * 4 bytes from c[i][j]
$(c[i] + j)_{1\cdot 4}$	skip j * 4 bytes from c[i]
$(c + i)_{1\cdot 4}$	skip i * 4 bytes from c

k leaf elements	k * 4 bytes
j * 4 leaf elements	j * 4 * 4 bytes
i * 3 * 4 leaf elements	i * 3 * 4 * 4 bytes

Array Pointer Approach

$(c[i][j] + k)_{1\cdot 4}$	skip k * 4 bytes from c[i][j]
$(c[i] + j)_{4\cdot 4}$	skip j * 4*4 bytes from c[i]
$(c + i)_{3\cdot 4\cdot 4}$	skip i * 3*4*4 bytes from c

k leaf elements	k * 4 bytes
j * 4 leaf elements	j * 4 * 4 bytes
i * 3 * 4 leaf elements	i * 3 * 4 * 4 bytes

3-d Access **c**[**i**][**j**][**k**]

Accessing **c[i][j][k]** – Conditions

General requirements

```
c[i][j][k] = *(c[i][j]+k)
c[i][j]    = *(c[i]+j)
c[i]       = *(c+i)
```

```
&c[i][j][k] = c[i][j]+k
&c[i][j]    = c[i]+j
&c[i]       = c+i
```

```
&c[i][j][0] = c[i][j]
&c[i][0]    = c[i]
&c[0]       = c
```

Pointer array approach

```
int** c[2];
int*  b[2*3];
int   c[2*3*4];
```

```
c[i][j][k]   :: int
c[i][j]      :: int *
c[i]         :: int **
```

```
c[i]  ⇐  &b[i*3]
b[j]  ⇐  &a[j*4]
```

Hierarchical Pointer Arrays

Array pointer approach

```
int c[2][3][4];
```

```
c[i][j][k]   :: int
c[i][j]      :: int [4]
c[i]         :: int [3][4]
```

```
c      ⇐  &c[0][0][0]
c[i]   ⇐  &c[i][0][0]
c[i][j] ⇐  &c[i][j][0]
```

Virtual Array Pointers

Accessing **c[i][j][k]** – Pointer Array Approach (1)

c[i] ⇐ &b[i*3]
b[j] ⇐ &a[j*4]

[2][3][4]

c[i] ≡ *(c + i)
c[i][j] ≡ *(c[i] + j)
c[i][j][k] ≡ *(c[i][j] + k)

&c[i] ≡ (c + i)
&c[i][j] ≡ (c[i] + j)
&c[i][j][k] ≡ (c[i][j] + k)

b[j] ≡ (a+j*4)

*(b[j]+k) = *(a+j*4+k);

b[j][k] ≡ a[j*4+k]

c[i] ≡ (b+i*3)

*(c[i]+j) = *(b+i*3+j);

c[i][j] ≡ b[i*3+j]

*(c[i][j]+k) = *(b[i*3+j]+k);

c[i][j] ≡ (a+(i*3+j)*4)

*(c[i][j]+k) = *(a+(i*3+j)*4+k);

c[i][j][k] ≡ a[(i*3+j)*4+k]

```
int** c[2];
int*  b[2*3];
int   a[2*3*4];
```

Accessing **c[i][j][k]** – Pointer Array Approach (2)

c[i] ⟵ &b[i*3]
b[j] ⟵ &a[j*4]

[2][3][4]

(c + i)$_{1\cdot 4}$

skip **i** * sizeof(int **)
i * 4 bytes from **c**

(c[i] + j)$_{1\cdot 4}$

skip **j** * sizeof(int *)
j * 4 bytes from **c[i]**

(c[i][j] + k)$_{1\cdot 4}$

skip **k** * sizeof(int)
k * 4 bytes from **c[i][j]**

b[j] ≡ (a+j*4)

skip **j** elements of **b**
skip **j*4** elements of **a**

c[i] ≡ (b+i*3)

skip **i** elements of **c**
skip **i*3** elements of **b**

b[j][k] ≡ a[j*4+k]

skip **j** elements of **b** +
skip **k** elements of **a**

c[i][j] ≡ b[i*3+j]

skip **i** elements of **c** +
skip **j** elements of **b**

skip **i*3** elements of **b** +
skip **j** elements of **b**

c[i][j] ≡ (a+(i*3+j)*4)

skip **i*3*4** elements of **a** +
skip **j*4** elements of **a** +

```
int** c[2];
int*  b[2*3];
int   a[2*3*4];
```

c[i][j][j] ≡ a[(i*3+j)*4+k]

skip **i*3*4** elements of **a** +
skip **j*4** elements of **a** +
skip **k** elements of **a**

Accessing **c[i][j][k]** – Array Pointer Approach (1)

c	⇐	&**c**[0][0][0]
c[i]	⇐	&**c[i]**[0][0]
c[i][j]	⇐	&**c[i][j]**[0]

→

c[i]	≡ ***(c + i)**
c[i][j]	≡ ***(c[i] + j)**
c[i][j][k]	≡ ***(c[i][j] + k)**

&**c[i]**	≡ **(c + i)**
&**c[i][j]**	≡ **(c[i] + j)**
&**c[i][j][k]**	≡ **(c[i][j] + k)**

[2][3][4]

value(**c**)	= &**c**[0][0][0]
value(**c[i]**)	= &**c[i]**[0][0]
value(**c[i][j]**)	= &**c[i][j]**[0]
value(**c[i][j][k]**)	= &**c[i][j][k]**

→

sizeof(**c**)	= 2*3*4*sizeof(int)
sizeof(**c[i]**)	= 3*4*sizeof(int)
sizeof(**c[i][j]**)	= 4*sizeof(int)
sizeof(**c[i][j][k]**)	= sizeof(int)

→

value(**c[i]**)	= &**c**[0][0][0]	+ **i** * 3*4*sizeof(int)
value(**c[i][j]**)	= &**c[i]**[0][0]	+ **j** * 4*sizeof(int)
value(**c[i][j][k]**)	= &**c[i][j]**[0]	+ **k** * sizeof(int)

→

&**c[i]**	= value(**c**)	+ **i** * sizeof(***c**)
&**c[i][j]**	= value(**c[i]**)	+ **j** * sizeof(***c[i]**)
&**c[i][j][k]**	= value(**c[i][j]**)	+ **k** * sizeof(***c[i][j]**)

→

c[i]	≡ ***(c + i)**
c[i][j]	≡ ***(c[i] + j)**
c[i][j][k]	≡ ***(c[i][j] + k)**

Accessing **c[i][j][k]** – Array Pointer Approach (2)

c	⇐	&**c**[0][0][0]
c[i]	⇐	&**c[i]**[0][0]
c[i][j]	⇐	&**c[i][j]**[0]

[2][3][4]

$(\mathbf{c} + \mathbf{i})_{3 \cdot 4 \cdot 4}$

skip
i * sizeof(int [3][4])
i * 3*4*4 bytes
from **c**

$(\mathbf{c[i]} + \mathbf{j})_{4 \cdot 4}$

skip
j * sizeof(int [4])
j *4*4 bytes
from **c[i]**

$(\mathbf{c[i][j]} + \mathbf{k})_{1 \cdot 4}$

skip
k * sizeof(int)
k * 4 bytes
from **c[i][j]**

value(**c[i]**) = &**c[i]**[0][0]

skip **i** elements of **c[i]** — **c[i]** — skip **i*3*4** elements of **c[i][0][0]**

value(**c[i][j]**) = &**c[i][j]**[0]

skip **i** elements of **c[i]** + skip **j** elements of **c[i][j]** — **c[i]** — **c[i][j]** — skip **i*3*4** elements of **c[i][0][0]** + skip **j*4** elements of **c[i][j][0]**

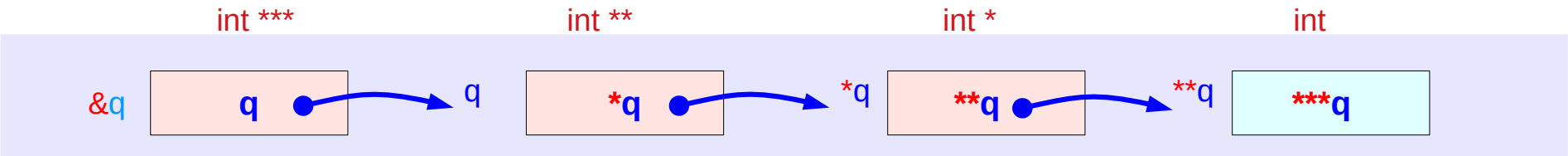
value(**c[i][j][k]**) = &**c[i][j][k]**

skip **i** elements of **c[i]** + skip **j** elements of **c[i][j]** + skip **k** elements of **c[i][j][k]** — **c[i]** — **c[i][j]** — **c[i][j][k]** — skip **i*3*4** elements of **c[i][0][0]** + skip **j*4** elements of **c[i][j][0]** + skip **k** elements of **c[i][j][k]**

Pointer Chain Types

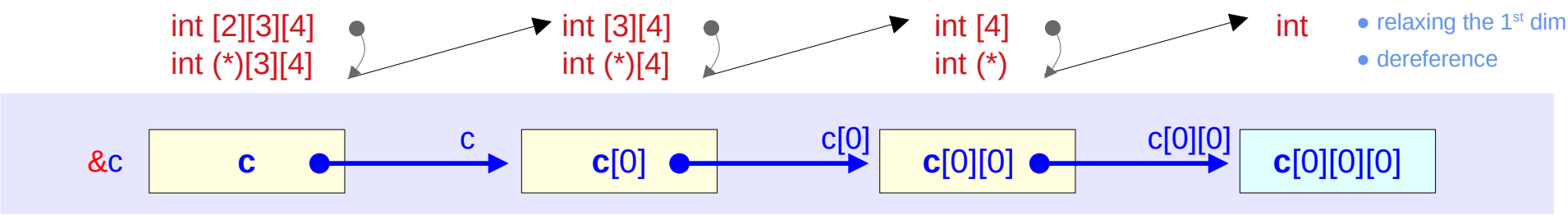
Pointer Chain Types

Pointer Chain Type I



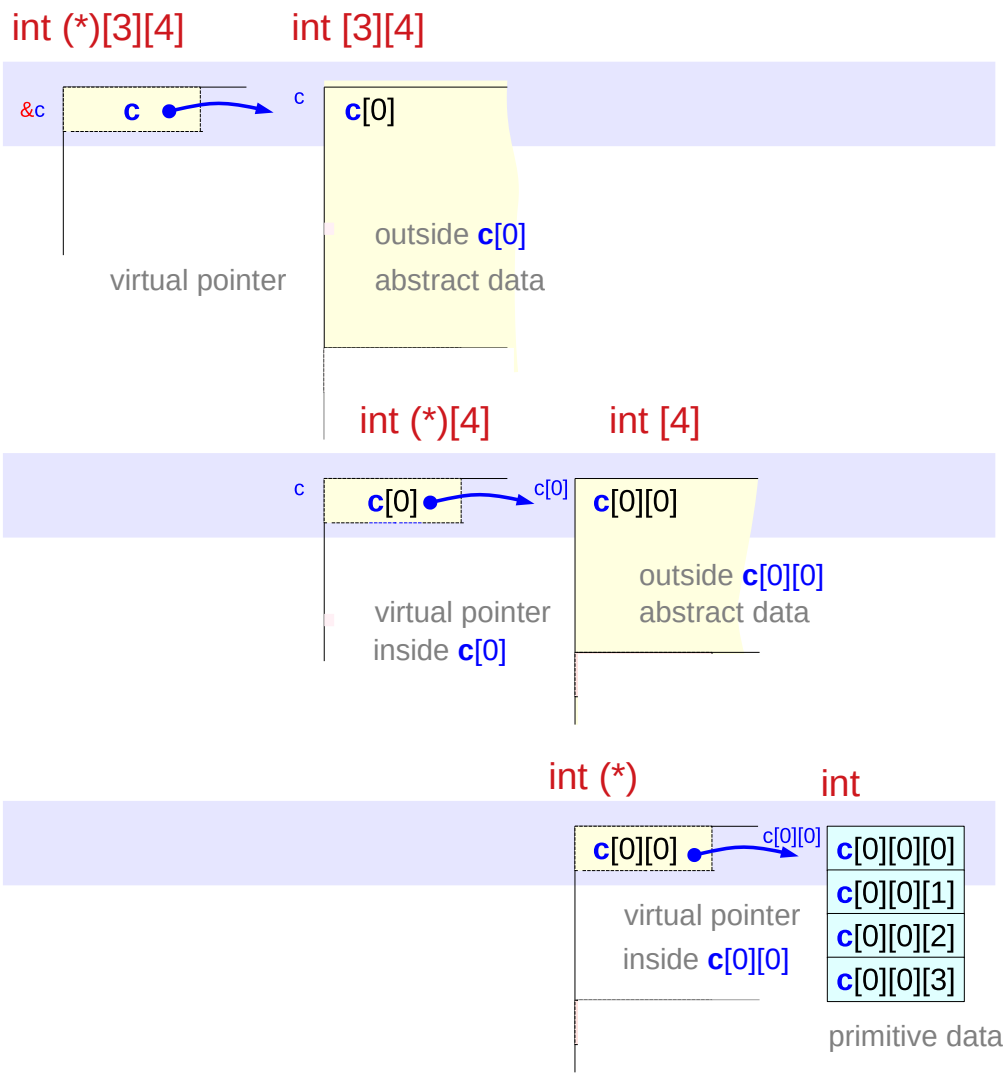
- **Pointer Array Approach** (index operations are handled by a user)

Pointer Chain Type II



- **Array Pointer Approach** (index operations are handled by a compiler)

Examples of two step dereferencing in type II



int [3][4]
c[0]
abstract data

relaxing the 1st dim

sizeof(**c**[0]) = 3 * sizeof(**c**[0][0])

within an array **c**[0] of int[3][4] type, **c**[0] can be relaxed to a pointer of int (*)[4] type

c[0][0] = ***(c**[0]+0)$_{4\cdot 4}$

Math Expression

int (*)[4]
c[0]
virtual pointer

int [4]
c[0][0]
abstract data

relaxing the 1st dim

sizeof(**c**[0][0]) = 4 * sizeof(**c**[0][0][0])

within an array **c**[0][0] of int [4] type, **c**[0][0] can be relaxed to a pointer of int (*) type

c[0][0][0] = ***(c**[0][0]+0)$_{1\cdot 4}$

Math Expression

int (*)
c[0][0]
virtual pointer

int
c[0][0][0]
abstract data

Pointer Chains in Type **II** (1)

Pointer Chain Type II

int [2][3][4]
int (*)[3][4]

int [3][4]
int (*)[4]

int [4]
int (*)

int

- relaxing the 1st dim
- dereference

&c | **c** —c→ **c**[0] —c[0]→ **c**[0][0] —c[0][0]→ **c**[0][0][0]

c points to **2** elements
sizeof(**c**) = 2 * sizeof(**c**[**i**])

c[**i**] **i**=0,1
***(c+i)**$_{3\cdot4\cdot4}$

c+**i** | **c**[**i**] —c[**i**]→ **c**[**i**][0] —c[**i**][0]→ **c**[**i**][0][0]

c[**i**] points to **3** elements
sizeof(**c**[**i**]) = 3 * sizeof(**c**[**i**][**j**])

c[**i**][**j**] **j**=0,1,2
***(c[i]+j)**$_{4\cdot4}$

c[**i**]+**j** | **c**[**i**][**j**] —c[**i**][**j**]→ **c**[**i**][**j**][0]

c[**i**][**j**] points to **4** elements
sizeof(**c**[**i**][**j**]) = 4 * sizeof(**c**[**i**][**j**][**k**])

c[**i**][**j**][**k**] **k**=0,1,2,3
***(c[i][j]+k)**$_{1\cdot4}$

Pointer Chains in Type II (2)

- relaxing the 1st dim
- dereference

i = 0,1

int [2][3][4] → int [3][4]

int (*)[3][4]

c points to **2** elements

sizeof(**c**) = 2 * sizeof(**c[i]**)

c[i] **i**=0,1

***(c+i)**$_{3\cdot4\cdot4}$

j = 0,1,2

int [3][4] → int [4]

int (*)[4]

c[i] points to **3** elements

sizeof(**c[i]**) = 3 * sizeof(**c[i][j]**)

c[i][j] **j**=0,1,2

***(c[i]+j)**$_{4\cdot4}$

k = 0,1,2,3

int [4] → int

int (*)

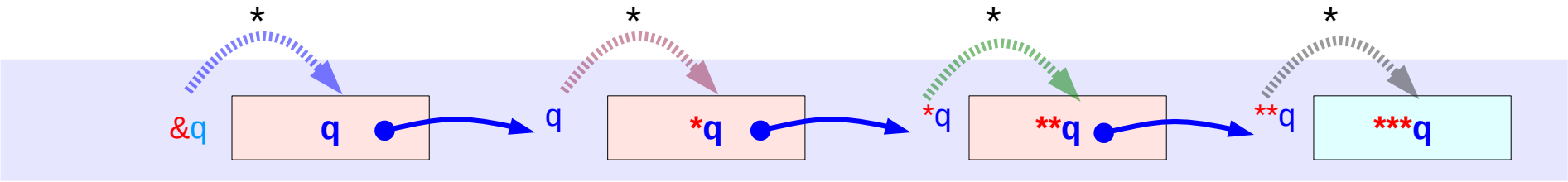
c[i][j] points to **4** elements

sizeof(**c[i][j]**) = 4 * sizeof(**c[i][j][k]**)

c[i][j][k] **k**=0,1,2,3

***(c[i][j]+k)**$_{1\cdot4}$

Pointer Chain Type I – * and & operators



*(&q) ≡ q

C expression *(&q) equals to the variable q

*(q) ≡ *q

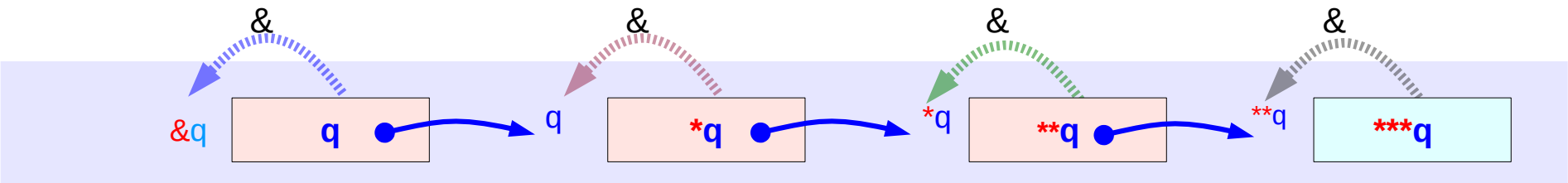
C expression *(q) equals to the variable q

*(*q) ≡ **q

C expression *(*q) equals to the variable **q

*(**q) ≡ ***q

C expression *(**q) equals to the variable ***q



&q

C expression &q equals to value(&q) which is the address value of a variable q

&(*q) ≡ value(q)

C expression &(*q) equals to value(q) which is the address value of a variable *q

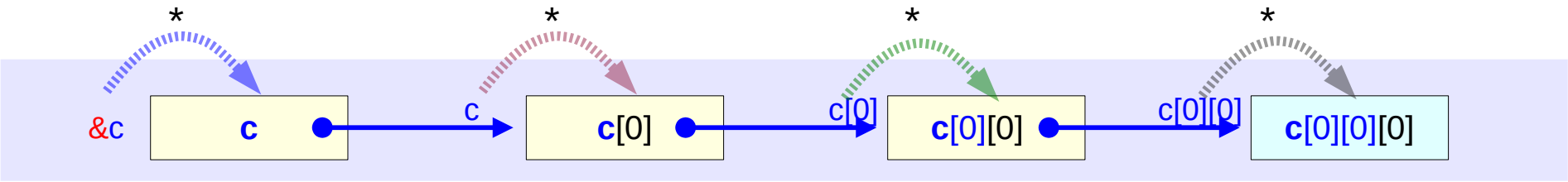
&(**q) ≡ value(*q)

C expression &(**q) equals to value(*q) which is the address value of a variable **q

&(***q) ≡ value(**q)

C expression &(***q) equals to value(**q) which is the address value of a variable ***q

Pointer Chain Type II – * and & operators



*(&c) ≡ c

*(c) ≡ c[0]

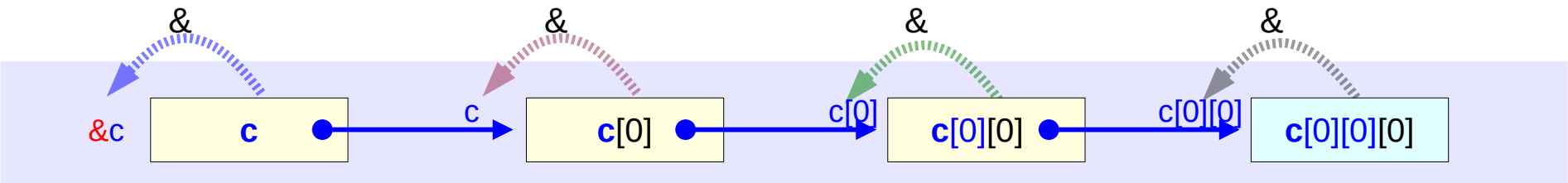
(int (*)[3][4]) c can be viewed as a pointer to (int [3][4]) c[0]

*(c[0]) ≡ c[0][0]

(int (*)[4]) c[0] can be viewed as a pointer to (int [4]) c[0][0]

*(c[0][0]) ≡ c[0][0][0]

(int (*)) c[0][0] can be viewed as a pointer to (int) c[0][0][0]



&c

&(c[0]) ≡ value(c)

(int (*)[3][4]) c has the address value of (int [3][4]) c[0]

&(c[0][0]) ≡ value(c[0])

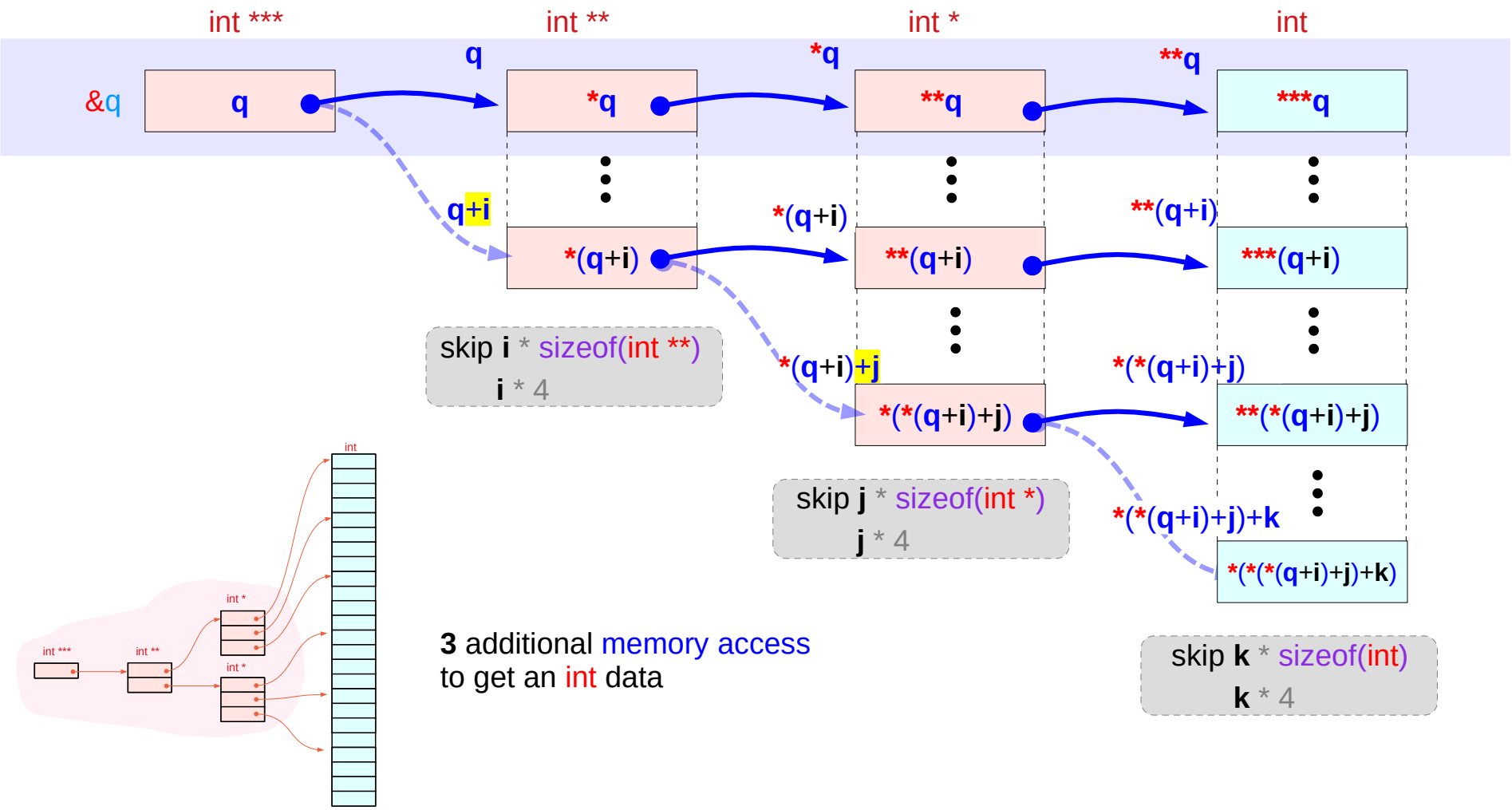
(int (*)[4]) c[0] has the address value of (int [4]) c[0][0]

&(c[0][0][0]) ≡ value(c[0][0])

(int (*)) c[0][0] has the address value of (int) c[0][0][0]

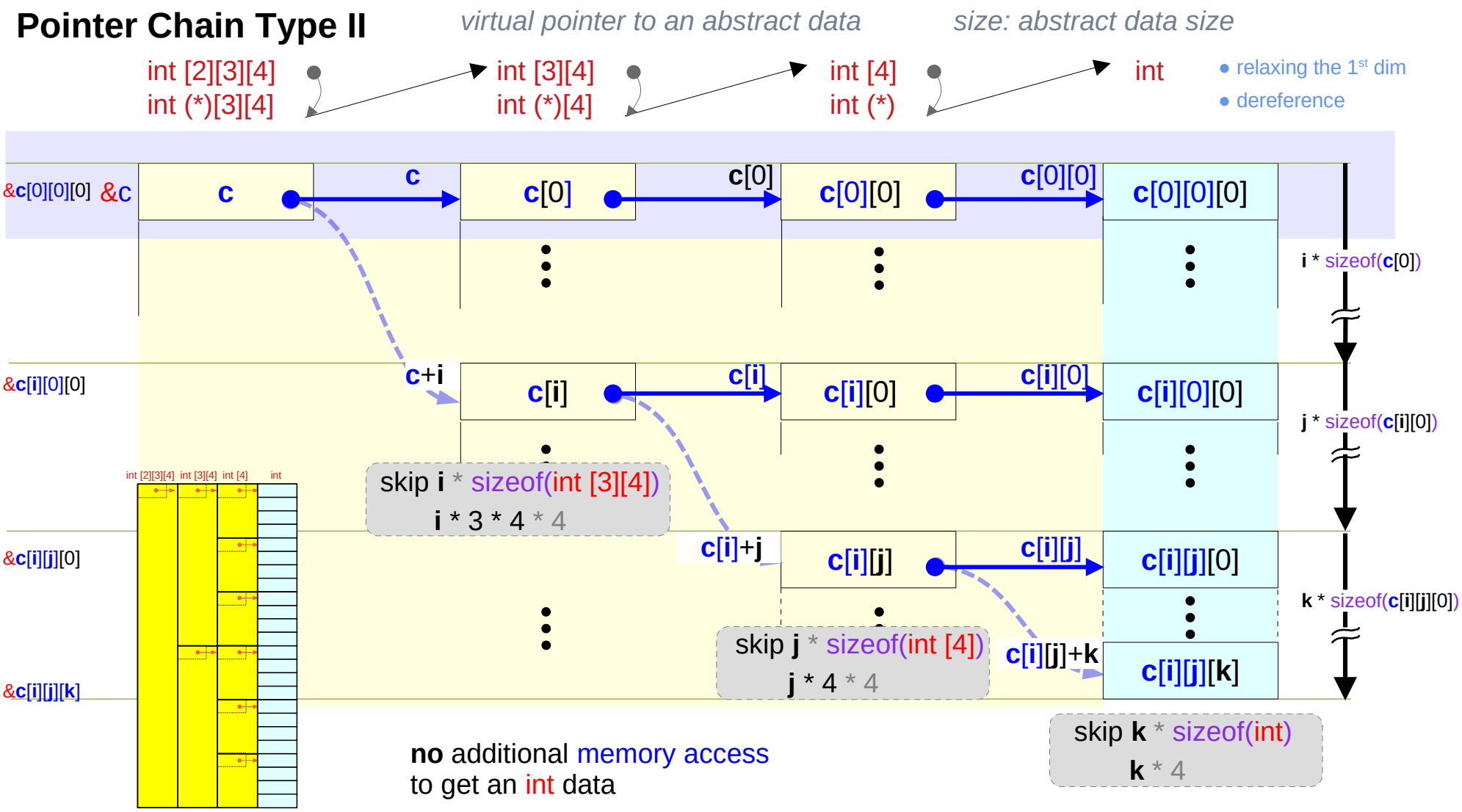
Pointer Chain Type I – skipping elements

Pointer Chain Type I *multiple indirection* *size: pointer size*

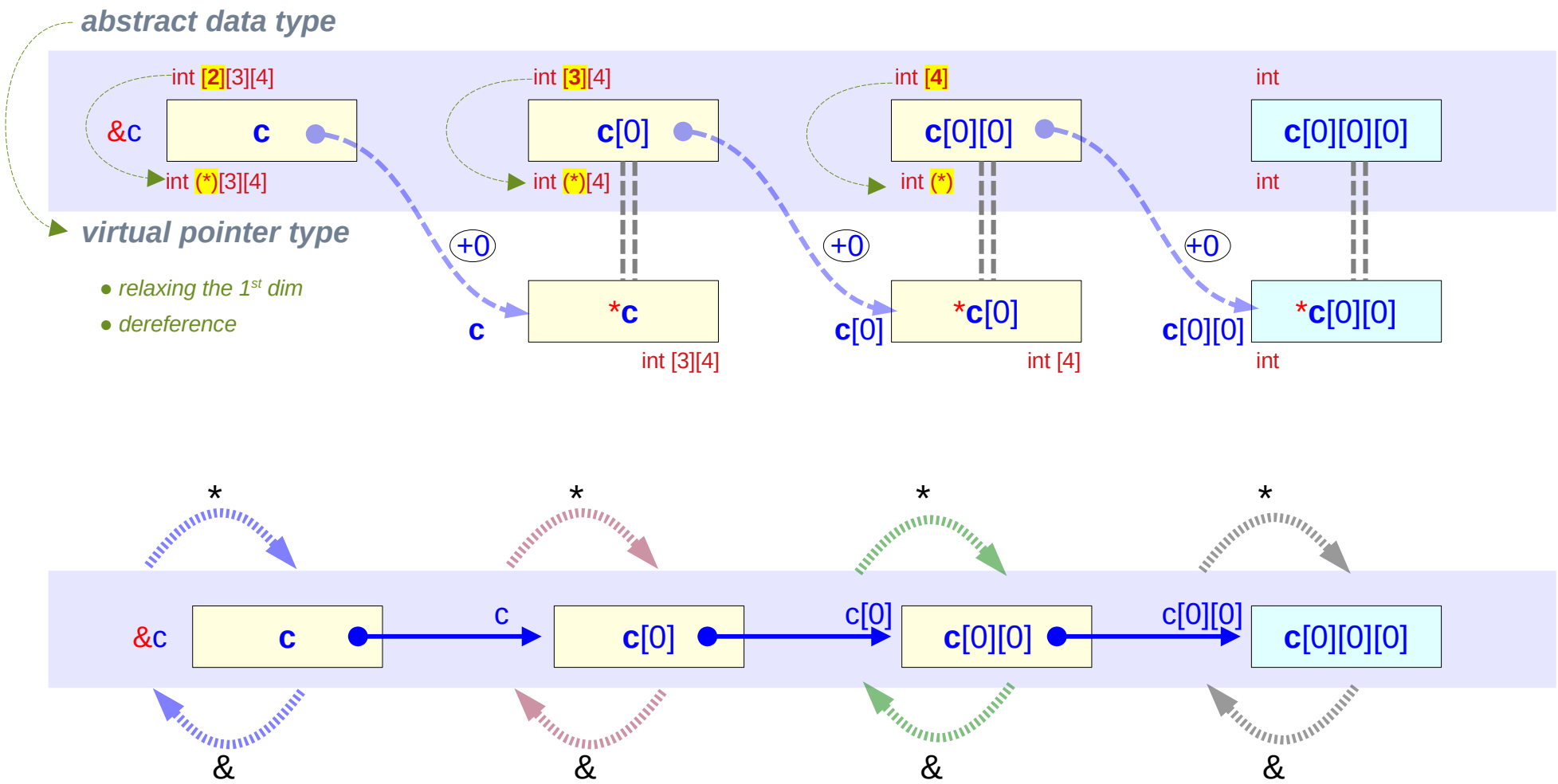


3 additional memory access
to get an int data

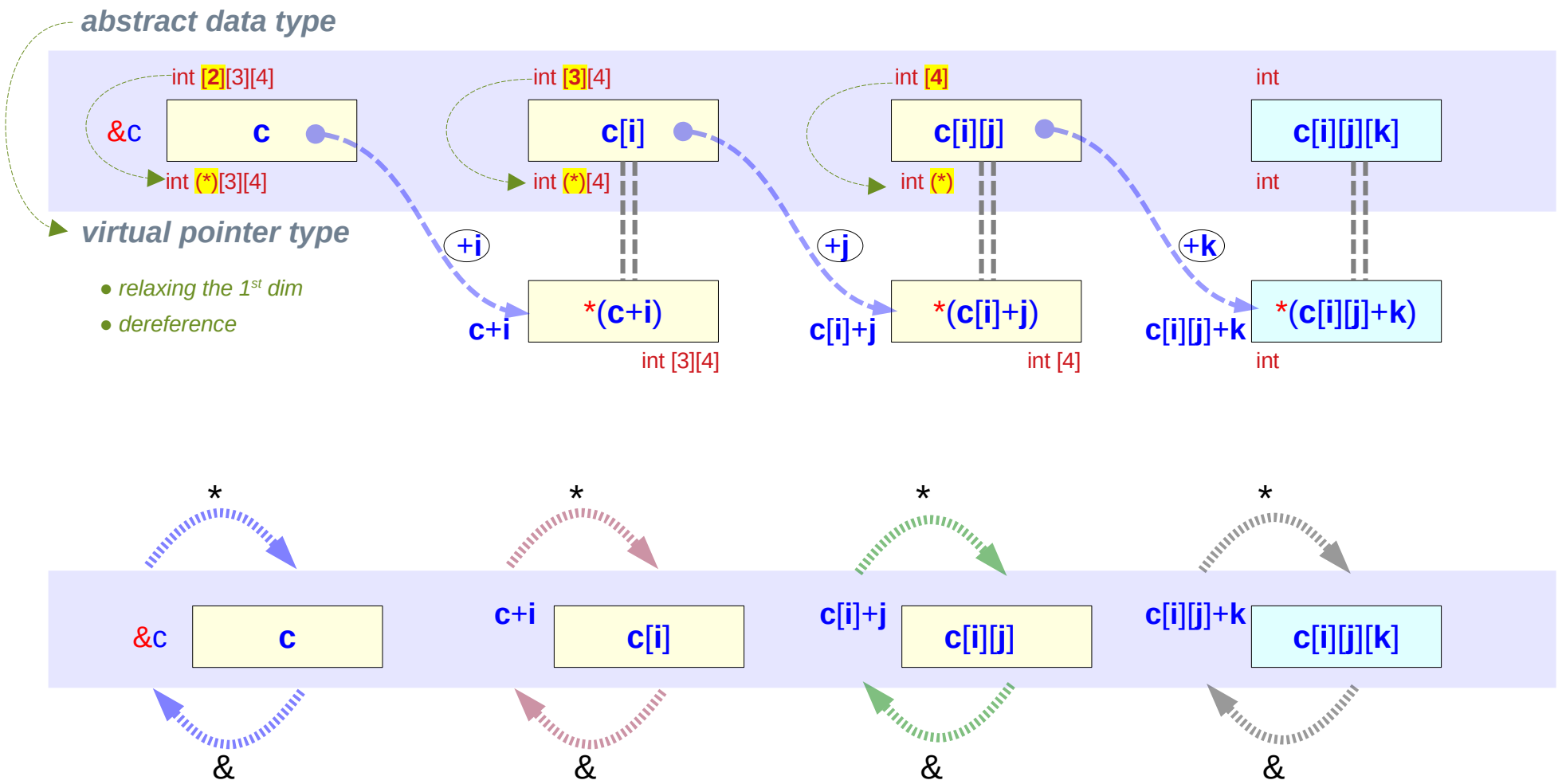
Pointer Chain Type **II** – skipping elements



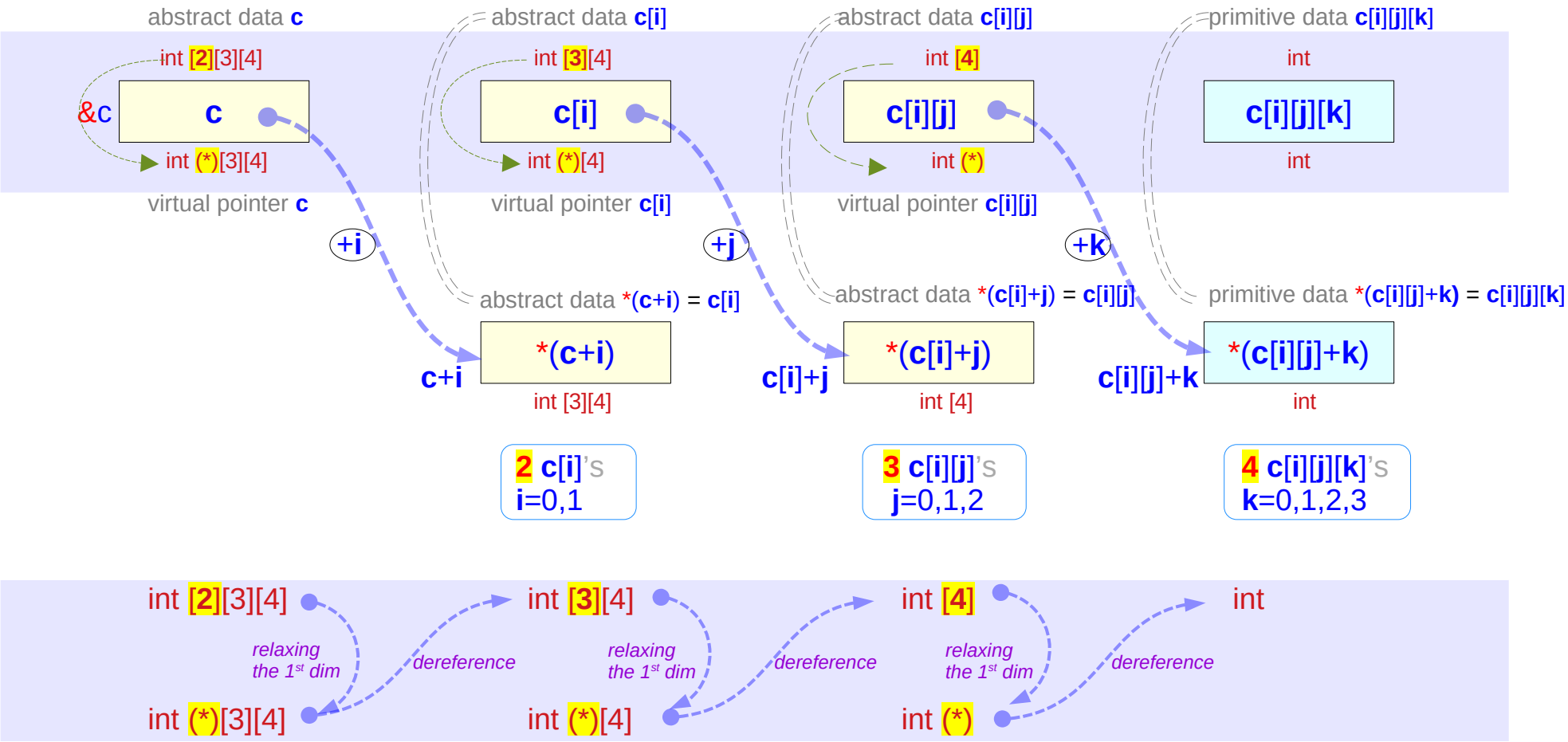
Two step deferencing in type **II** (1) – without skipping



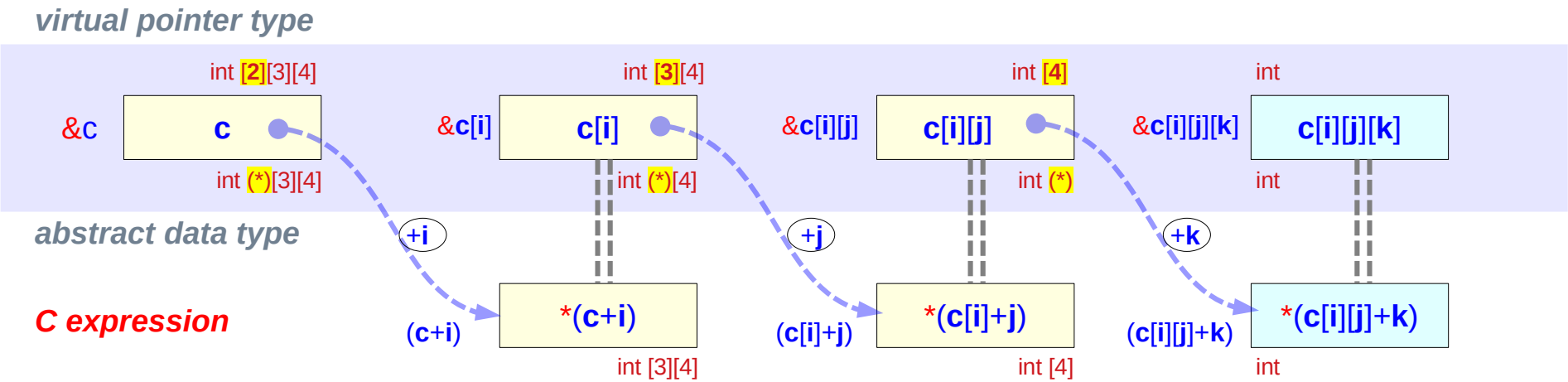
Two step deferencing in type **II** (2) – with skipping



Two step deferencing in type **II** (3)



Skipping elements



Math expression

$(c+i)_{3\cdot4\cdot4}$

skip $i\cdot3\cdot4$
integer
elements
from **c**

$(c[i]+j)_{4\cdot4}$

skip $j\cdot4$
integer
elements
from **c[i]**

$(c[i][j]+k)_{1\cdot4}$

skip $k\cdot1$
integer
elements
from **c[i][j]**

sizeof(*c) = 3*4*4

value($(c+i)_{3\cdot4}$)
= value(**c**) + **i** *3*4*4

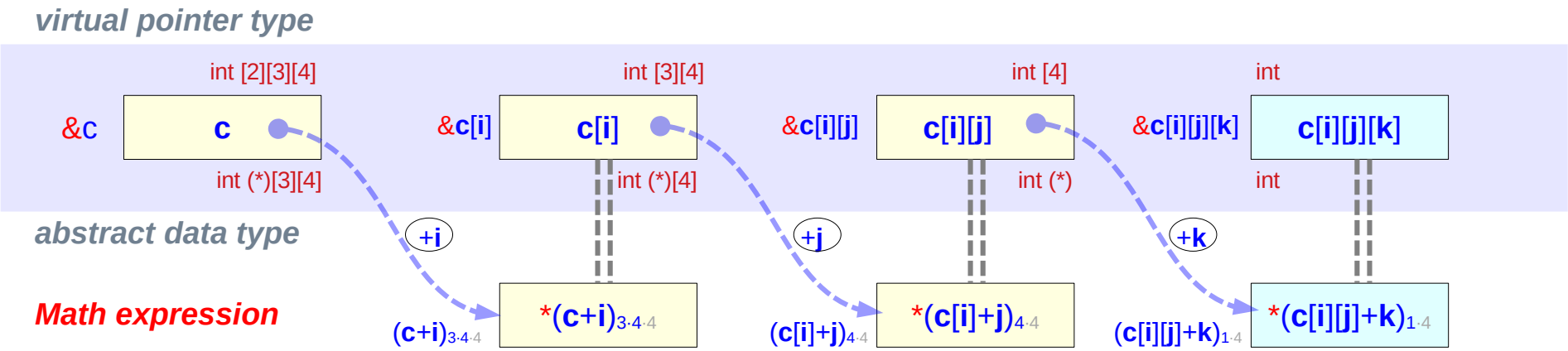
sizeof(*c[i]) = 4*4

value($(c[i]+j)_4$)
= value(**c[i]**) + **j** *4*4

sizeof(*c[i][j]) = 1*4

value($(c[i][j]+k)_1$)
= value(**c[i][j]**) + **k** *4

Address replication



equivalence relations – c expressions

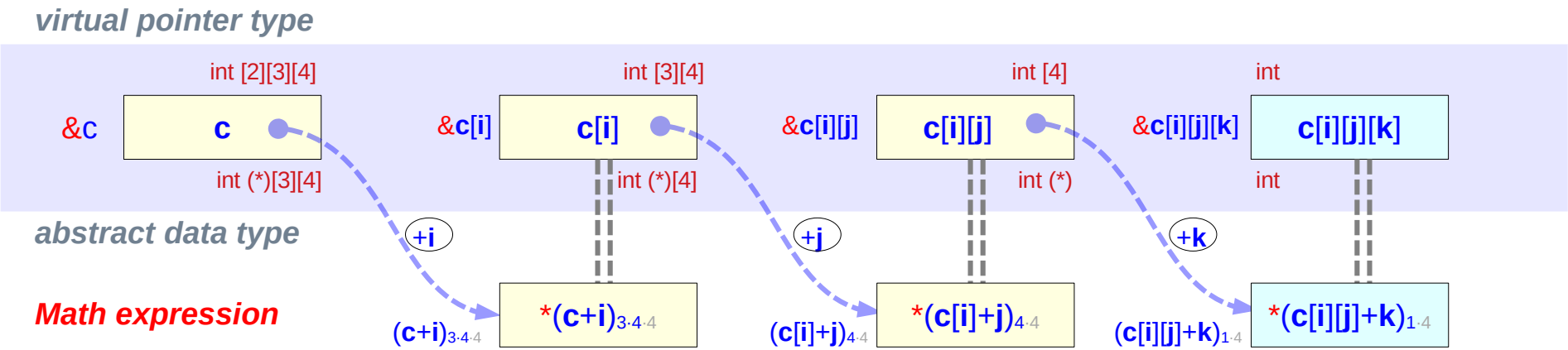
c[i][j][k]	= *(c[i][j] + k)
c[i][j]	= *(c[i] + j)
c[i]	= *(c + i)

&c[i][j][k]	= (c[i][j] + k)
&c[i][j]	= (c[i] + j)
&c[i]	= (c + i)

address replication – math expressions

value(c[i][j]) = value($(c[i] + j)_{4\cdot4}$)
value(c[i]) = value($(c + i)_{3\cdot4\cdot4}$)

Applying address replication



element size
= sizeof(int)
= 4 bytes

address replication

value((c+i)$_{3\cdot4\cdot4}$)
= value(c)
+ i *3*4*4

address replication

value((c[i]+j)$_{4\cdot4}$)
= value(c[i])
+ j *4*4

value((c[i][j]+k)$_{1\cdot4}$)
= value(c[i][j])
+ k *4

const pointers

const type, const pointer type (1)

```
const int  *  p;
int * const q  ;
const int  *  const r  ;
```

⟺

```
int  *  p;
int  *  q  ;
int  *   r  ;
```

constant

must not be changed
must not be updated
must not be written
must not be assigned

const type, const pointer type (2)

```
const int * p ;
```
constant integer

```
int * const q ;
```
constant pointer

```
const int * const r ;
```
constant integer

```
const int * const r ;
```
constant pointer

*p : constant integer value

q : constant (int *) pointer

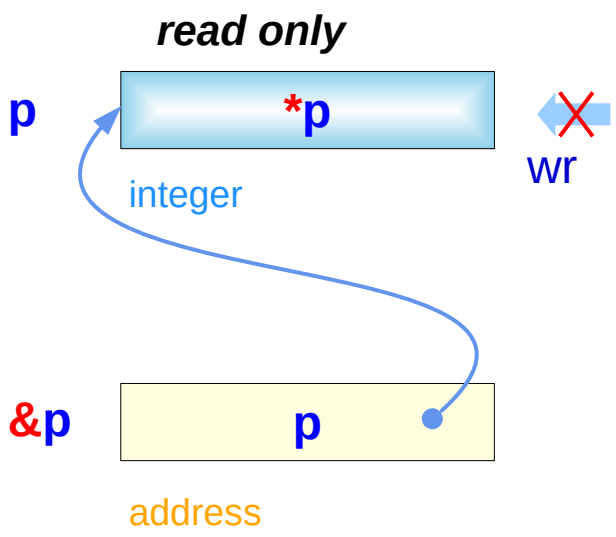
*r : constant integer value

r : constant (int *) pointer

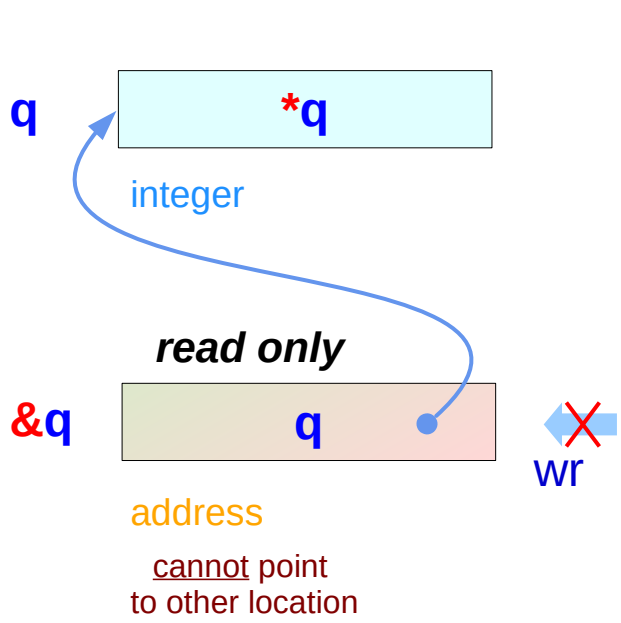
```
const [   ]
```
group with the following

const type, const pointer type (3)

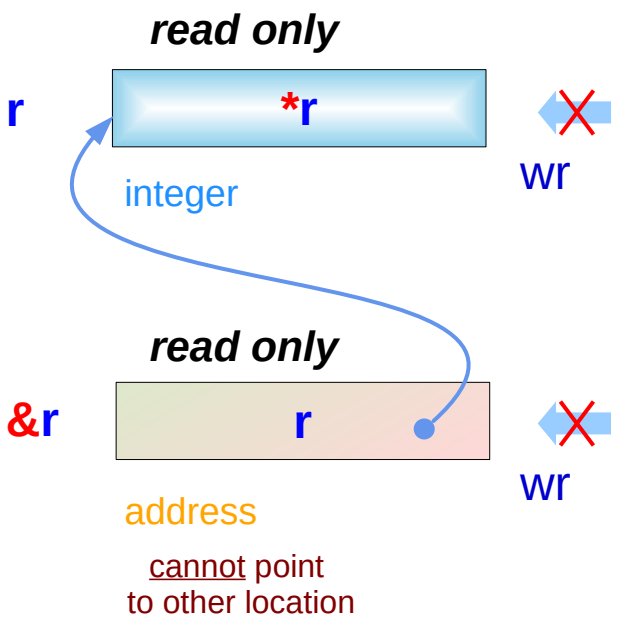
const int *p;



int * const q ;



const int * const r ;



const examples (1)

```
const int * pc;
      int * p, i;
const int   ic;
```

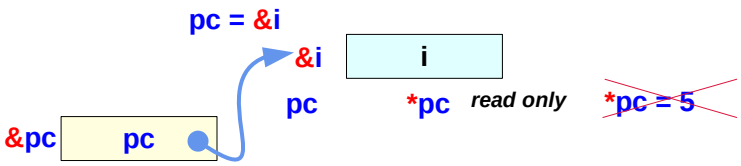
```
 pc = &i;      // (const int *) ← (int *)
*pc  = 5;      // (const int) error
```

Writing to the writable memory location (**i**)
is forbidden via **pc** … (no harm, OK)

```
 p   = &ic;    // (int *) ← (const int *) warning
*p   = 5;      // (int)
```

Writing to the read only memory location (**ic**)
is not forbidden via **p** … (hazardous, not OK)

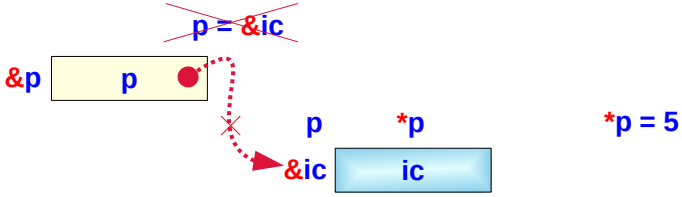
C A Reference Manual, Harbison & Steele Jr.



pc can point to **i**
***pc** must be **const**

the same memory location
that can be written via **i**
cannot be written via ***pc**

***pc** should not write
the writable memory location



Assume **p** points to **const ic**

the same memory location
that cannot be written via **ic,**
can be written via ***p**

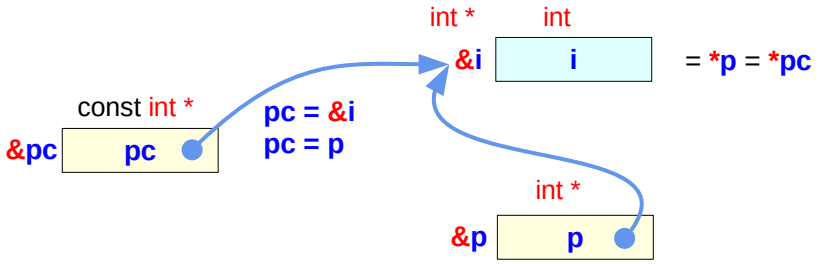
thus ***p** can write
the **const** memory location

therefore, **p** should not point to **const ic**

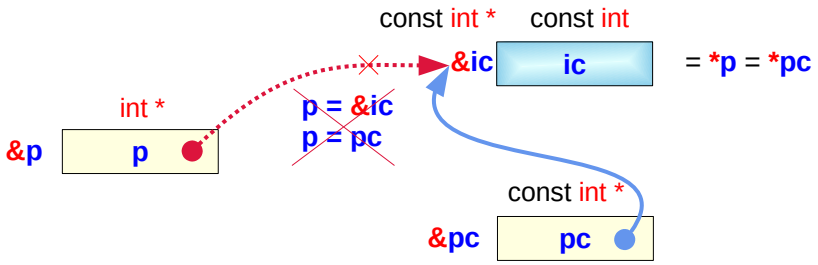
const examples (2)

```
const int * pc;
        int * p, i = 5;
const int   ic = 7;
```

```
p   = &i;
pc  = &ic
```




```
// more constrained type ← general type (O)
pc  = &i;     // (const int * ← int *)
pc  =  p;     // (const int * ← int *)
```

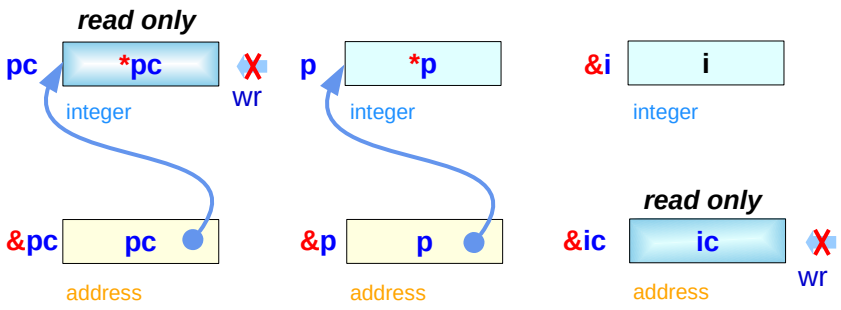



```
// general type ← more constrained type (X)
p   = &ic;    // (int * ← const int *)  warning
p   =  pc;    // (int * ← const int *)  warning
```

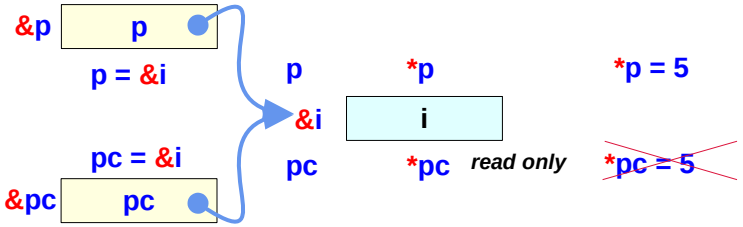
C A Reference Manual, Harbison & Steele Jr.

const examples (3)

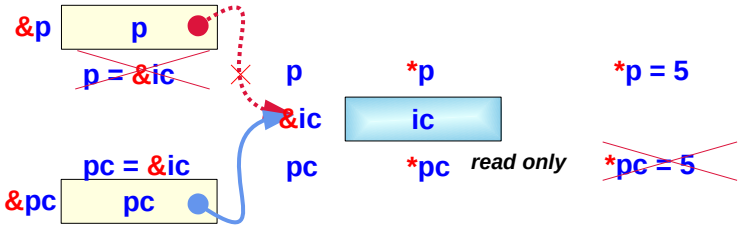
```
const int * pc;
      int * p, i;
const int   ic;
```




```
 p   = &i;     // (int *) ← (int *)
*p   = 5;      // (int)
 pc  = &i;     // (const int *) ← (int *)
*pc  = 5;      // (const int) error
```



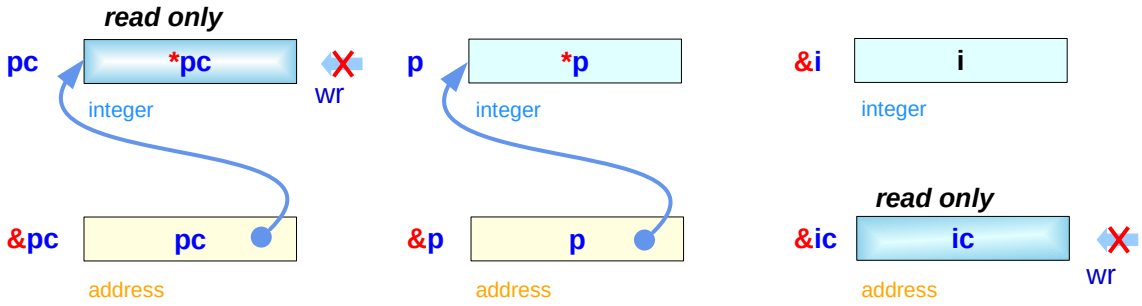

```
 p   = &ic;    // (int *) ← (const int *) warning
*p   = 5;      // (int)
 pc  = &ic;    // (const int *) ← (const int *)
*pc  = 5;      // (const int) error
```



C A Reference Manual, Harbison & Steele Jr.

const examples (4)

```
const int * pc;
        int * p, i;
const int   ic;
```




```
 pc  =   p = &i;
 pc  = &ic
*p   =   5;
*pc  =   5;              // invalid      *pc :: cons int

pc   = &i;               //              (const int * ← int *)
pc   =   p;              //              (const int * ← int *)
p    = &ic;              // invalid      (int * ← const int *)
p    =   pc;             // invalid      (int * ← const int *)
p    = (int *) &ic;      // type cast
p    = (int *)   pc;     // type cast
```

C A Reference Manual, Harbison & Steele Jr.

References

[1] Essential C, Nick Parlante
[2] Efficient C Programming, Mark A. Weiss
[3] C A Reference Manual, Samuel P. Harbison & Guy L. Steele Jr.
[4] C Language Express, I. K. Chun