Propositional Logic– Arguments (5A)

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Based on

Contemporary Artificial Intelligence,
R.E. Neapolitan & X. Jiang

Logic and Its Applications,
Burkey & Foxley

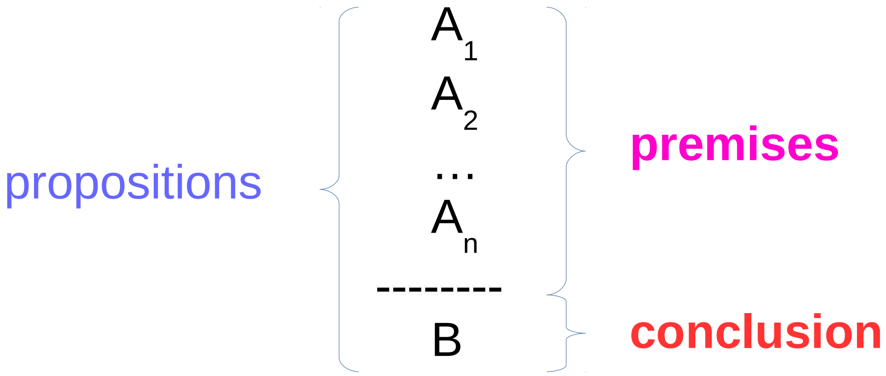
Arguments

An **argument** consists of a set of propositions :

The **premises** propositions

The **conclusion** proposition

List of **premises** followed by the **conclusion**



propositions

A_1

A_2

…

A_n

\-\-\-\-\-\-\-\-

B

premises

conclusion

Entailment Definition

If **the truth of a statement** **P** **guarantees** that
another **statement** **Q** must be **true**,

then we say that **P** **entails** **Q**, or that **Q** is **entailed** by **P**.

the key term, "must be true"
"Must be" is stating that
something is **necessary**,
something for which **no other option** or **possibility** exists.
"Must be" encodes the concept of **logical necessity**.

http://www.kslinker.com/VALID-AND-INVALID-ARGUMENTS.html

Entailment Examples

P: “Mary knows all the capitals of the United States”
Q: “Mary knows the capital of Kentucky”

P: “Every one in the race ran the mile in under 5 minutes”
Q: “John, a runner in the race, ran the mile in less than 5 minutes”

P: “The questionnaire had a total of 20 questions, and Mary answered only 13”
Q: “The questionnaire answered by Mary had 7 unanswered questions”

P: “The winning ticket starts with 3 7 9”
Q: “Mary's ticket starts with 3 7 9 so Mary's ticket is the winning ticket”

conceptually familiarity
true **conditional statements**, or true **implications** are examples of **entailment**.

any conditional statement, (“if . . .. then”) which is true, is an example of **entailment**.
In Mathematics, the more familiar term is “**implication**”.

http://www.kslinker.com/VALID-AND-INVALID-ARGUMENTS.html

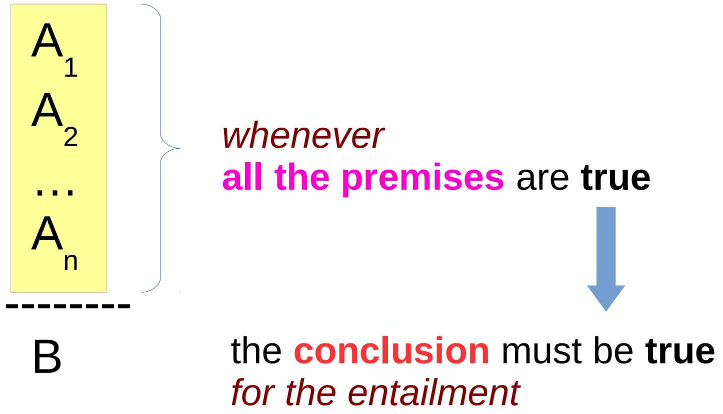
Entail

The **premises** is said to **entail** the **conclusion**

If in **every model** in which **all the premises** are **true**,

the **conclusion** is also **true**

List of **premises** followed by the **conclusion**



A_1

A_2

…

A_n

\-\-\-\-\-\-\-\-\-

B

whenever
all the premises are **true**

the **conclusion** must be **true**
for the entailment

A Model

A **model** or a **possible world**:

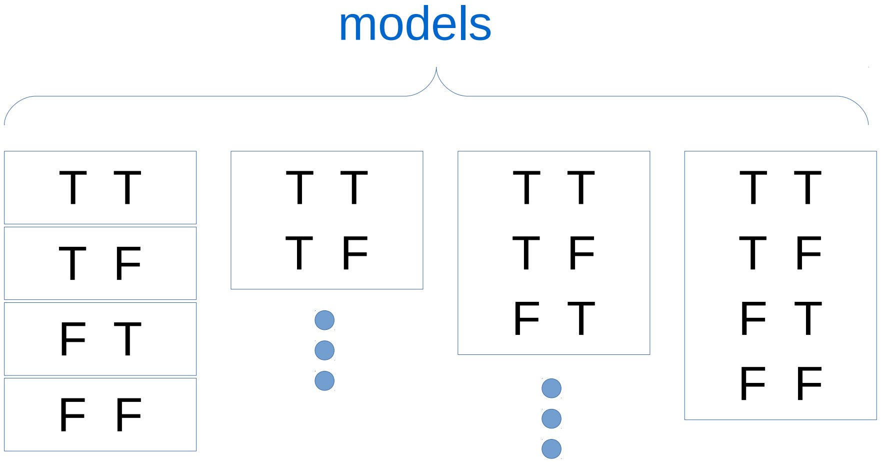
Every atomic proposition is assigned a value **T** or **F**

The set of all these assignments constitutes
A **model** or a **possible world**

All possible worlds (assignments) are permissiable

A	B	$A \wedge B$	$A \wedge B \Rightarrow A$
T	T	**T**	T
T	F	F	T
F	T	F	T
F	F	F	T

Every atomic proposition : A, B



Interpretation

An **interpretation** of a formal system is
the assignment of meanings to the symbols,
and **truth values** to the **sentences** of a formal system.

The study of interpretations is called formal semantics

Giving an interpretation is synonymous with
constructing a model.

An interpretation is expressed in a metalanguage,
which may itself be a formal language,
and as such itself is a syntactic entity.

https://en.wikipedia.org/wiki/Syntax_(logic)#Syntactic_consequence_within_a_formal_system

Entailment Notation

Suppose we have *an argument*
whose **premises** are $A_1, A_2, \ldots, A_n$
whose **conclusion** is B

Then

$A_1, A_2, \ldots, A_n \vDash B$ if and only if
$A_1 \land A_2 \land \ldots \land A_n \Rrightarrow B$ (**logical implication**)

logical implication: if $A_1 \land A_2 \land \ldots \land A_n \Rrightarrow B$ is tautology (*always true*)

The **premises** is said to **entail** the **conclusion**
If in every model in which
all the premises are **true**,
the **conclusion** is also **true**

Entailment and Logical Implication

$A_1, A_2, \ldots, A_n \vDash B$

$\Leftrightarrow$ $A_1 \wedge A_2 \wedge \ldots \wedge A_n \Rightarrow B$

$\Leftrightarrow$ $A_1 \wedge A_2 \wedge \ldots \wedge A_n \Rightarrow B$ is a tautology

(**logical implication**)

If all the premises are **true**,
then the **conclusion** must be **true**

$T \wedge T \wedge \ldots \wedge T \Rightarrow T$

$T \wedge T \wedge \ldots \wedge T \Rightarrow \xcancel{F}$

$F \wedge X \wedge \ldots \wedge X \Rightarrow T$

Sound Argument and Fallacy

A **sound argument**

$A_1, A_2, \ldots, A_n \models B$

If the **premises** **entails** the **conclusion**

A **fallacy**

$A_1, A_2, \ldots, A_n \not\models B$

If the **premises** does **not** **entail** the **conclusion**

Modal Accounts

Modal accounts of logical consequence are variations on the following basic idea:

$\Gamma \vdash A$ is true if and only if

it is *necessary* that if all of the elements of Γ are **true**, then A is **true**.

Alternatively :

$\Gamma \vdash A$ is true if and only if

it is *impossible* for all of the elements of Γ to be **true** and A **false**.

Such accounts are called "modal" because they appeal to the modal notions of logical necessity and logical possibility. 'It is necessary that' is often expressed as a universal quantifier over possible worlds, so that the accounts above translate as:

$\Gamma \vdash A$ is true if and only if

there is *no possible world* at which all of the elements of Γ are **true** and A is **false** (untrue).

https://en.wikipedia.org/wiki/Syntax_(logic)#Syntactic_consequence_within_a_formal_system

Valid Argument Criteria

- If all the premises are true, then the conclusion must be **true**.
- the truth of the conclusion is guaranteed

  if all the premises are **true**
- It is impossible to have a false conclusion

  if all the premises are **true**
- The premises of a valid argument **entail** the conclusion.

http://www.kslinker.com/VALID-AND-INVALID-ARGUMENTS.html

Valid Argument Examples

If John makes this field goal, then the U of A will win. John makes the field goal . Therefore the U of A wins **Modus Ponens**	If **P** then **Q** **P** **Therefore Q**
If the patient has malaria, then a blood test will indicate that his blood harbors at least one of these parasites: P. falciparum, P. vivax , P. ovale and P. malaria Blood test indicate that the patient harbors **none** of these parasites Therefore the patient does **not** have malaria. **Modus Tollens**	If **P** then **Q** Not **Q** **Therefore** Not **P**
Either The Patriots or the Philadelphia Eagles will **win** the Superbowl The Patriots **lost** Therefore The Eagles **won** **Disjunctive Syllogism (Process of Elimination)**	Either **P** or **Q** Not **P** **Therefore Q**
If John gets a raise, then he will buy a house. If John buys a house, he will run for a position on the neighborhood council. Therefore, if John gets a raise, he will run for a position on the neighborhood council **Hypothetical Syllogism**	If **P** then **Q** If **Q** then **R** **Therefore** If **P** then **R**

http://www.kslinker.com/VALID-AND-INVALID-ARGUMENTS.html

Sound Arguments

An argument is **sound** if it is **valid** and all the **premises** are actually **true**.

for an argument to be sound, two conditions must be meet:
1) the argument must be **valid**, and
2) the argument must actually have **all true premises**.

What can be said about the conclusion to a sound argument?

Since the argument is **sound**, then it is both **valid** and actually has **all true premises**, so the **conclusion must be true**, by definition of validity.

an example of a sound argument:

If a number is greater than 7 it is greater than 3.
8 is greater than 7.
Therefore 8 is greater than 3.

http://www.iep.utm.edu/val-snd/

Validity and Soundness (1)

An argument form is **valid** if and only if

whenever the premises are **all true**, then conclusion is **true**.

An argument is valid if its argument form is valid.

If	premises : **true**	→	**then**	conclusion : **true**
	false			**true**
	false			**false**
If	**true**	→	**then never**	**false**

An argument is **sound** if and only if

it is valid and all its premises are true.

Always premises : **true** → **therefore** conclusion : **true**

http://math.stackexchange.com/questions/281208/what-is-the-difference-between-a-sound-argument-and-a-valid-argument

Validity and Soundness (2)

A deductive argument is said to be **valid** if and only if

it takes a form that makes it *impossible*
for the premises to be **true** and the conclusion nevertheless to be **false**.

If true → then never false

Otherwise, a deductive argument is said to be **invalid**.
for the premises to be **true** and the conclusion is **false**.

A deductive argument is **sound** if and only if

it is both **valid**, and all of its premises are **actually true**.

Otherwise, a deductive argument is **unsound**.

Always premises : **true** → **therefore** conclusion : **true**

http://www.iep.utm.edu/val-snd/

Validity and Soundness (3)

A	B	$A \Rightarrow B$	$A \land (A \Rightarrow B)$	$A \land (A \Rightarrow B) \Rightarrow B$
T	T	T	T	**T**
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

valid

If premises : **true then never** conclusion : **false**

A	B	$A \Rightarrow B$	$A \land (A \Rightarrow B)$	$A \land (A \Rightarrow B) \Rightarrow B$
T	T	**T**	**T**	➡ **T**
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

sound

Always premises : **true therefore** conclusion : **true**

http://www.iep.utm.edu/val-snd/

Validity and Soundness (4)

the author of a deductive argument always intends
that the premises provide the sort of justification
for the conclusion whereby if the premises are **true**,
the conclusion is guaranteed to be **true** as well.

if the author's process of reasoning is a good one,
if the premises actually do provide this sort of justification
for the conclusion, then the argument is valid.

an argument is valid
if the **truth** of the premises logically guarantees
the **truth** of the conclusion.

it is **impossible** for the premises to be **true**
and the conclusion to nevertheless be **false**:

http://www.iep.utm.edu/val-snd/

Entailment Examples

$A, B \models A$

$A, B \Rrightarrow A$

$A \wedge B \Rrightarrow A$

$A, (A \Rightarrow B) \models B$

$A, (A \Rightarrow B) \Rrightarrow B$

$A \wedge (A \Rightarrow B) \Rrightarrow B$

Entailment Examples and Truth Tables

A	B	$A \land B$	$A \land B \Rightarrow A$
T	T	**T**	T
T	F	F	T
F	T	F	T
F	F	F	T

The **premises** is said to **entail** the **conclusion**
If in every model in which
all the premises are **true**,
the **conclusion** is also **true**

any of the premises are **false**,
still **premises** ⇒ **conclusion** is **true**
(F⇒T and F⇒F always T)

$A \land B \Rightarrow A$

Tautology

A	B	$A \Rightarrow B$	$A \land (A \Rightarrow B)$	$A \land (A \Rightarrow B) \Rightarrow B$
T	T	T	**T**	**T**
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

$A \land (A \Rightarrow B) \Rightarrow B$

Deduction System

Propositional logic

Given propositions (statements) : T or F
Deductive inference of T or F of *other propositions*

Deductive Inference

A process by which the truth of the conclusion
is shown to *necessarily follow*
from the truth of the premises

A	B	$A \Rightarrow B$	$A \land (A \Rightarrow B)$	$A \land (A \Rightarrow B) \Rightarrow B$
T	T	**T**	**T**	➡ **T**
T	F	**F**	**F**	T
F	T	**T**	**F**	T
F	F	**T**	**F**	T

$A \land (A \Rightarrow B) \Rightarrow B$

Deductive Inference

Entailment
(logical implication)

Deduction System

Deduction System : a set of inference rules

Inference rules are used to reason deductively

Sound Deduction System :

if it derives only sound arguments

Each of the inference rules is sound

A **sound** **argument:**

If the **premises** **entails** the **conclusion**

A **fallacy:**

If the **premises** does **not** **entail** the **conclusion**

Complete Deduction System :

It can drive every sound argument

Must contain deduction theorem rule

Inference Rules

Combination Rule	$A, B \models A \wedge B$
Simplification Rule	$A \wedge B \models A$
Addition Rule	$A \models A \vee B$
Modus Pones	$A, A \Rightarrow B \models B$
Modus Tolens	$\neg B, A \Rightarrow B \models \neg A$
Hypothetical Syllogism	$A \Rightarrow B,\ B \Rightarrow C \models A \Rightarrow C$
Disjunctive Syllogism	$A \vee B, \neg A \models B$
Rule of Cases	$A \Rightarrow B, \neg A \Rightarrow B \models B$
Equivalence Elimination	$A \Leftrightarrow B \models A \Rightarrow B$
Equivalence Introduction	$A \Rightarrow B,\ B \Rightarrow A \models A \Leftrightarrow B$
Inconsistency Rule	$A, \neg A \models B$
AND Commutivity Rule	$A \wedge B \models B \wedge A$
OR Commutivity Rule	$A \vee B \models B \vee A$
Deduction Theorem	If $A_1, A_2, \ldots, A_n, B \models C$ then $A_1, A_2, \ldots, A_n, \models B \Rightarrow C$

Deduction Theorem

$A_1,\ A_2,\ \ldots,\ A_n,\ B \vDash C$

➡ $A_1,\ A_2,\ \ldots,\ A_n,\ \vDash B \Rightarrow C$

$A_1 \wedge A_2 \wedge \ldots \wedge A_n \wedge B \vDash C$

➡ $A_1 \wedge A_2 \wedge \ldots \wedge A_n \vDash B \Rightarrow C$

The **premises** is said to **entail** the **conclusion**
If in every model in which
all the premises are **true**,
the **conclusion** is also **true**

$A_1, A_2, \ldots, A_n \vDash B$ if and only if
$A_1 \wedge A_2 \wedge \ldots \wedge A_n \Rightarrow B$
($A_1 \wedge A_2 \wedge \ldots \wedge A_n \Rightarrow B$ is a tautology)

$A,B \vDash C \ \rightleftarrows\ A \vDash B \Rightarrow C$

If A is T, then B⇒C is always T (for the tautology)
Even if A,B is T, then B⇒C is always T
And if A,B is T, then B is T
By modus ponens in the RHS, A,B is T, then C is true

$B \vDash C \ \rightleftarrows\ \vDash B \Rightarrow C$

Deduction Theorem

$\Delta, A \models B$ iff $\Delta \models A \Rightarrow B$

where Δ : a set of formulas,

if the formula B is deducible
from a set Δ of assumptions,
together with the assumption A,
then the formula $A \Rightarrow B$ is deducible from Δ alone.

$$\begin{tikzcd}
\Delta, A \models B \arrow[d] \\
\Delta \models A \Rightarrow B
\end{tikzcd}$$

Conversely, if we can deduce $A \Rightarrow B$ from Δ,
and if in addition we assume A,
then B can be deduced.

$$\begin{tikzcd}
\Delta \models A \Rightarrow B \arrow[d] \\
\Delta, A \models B
\end{tikzcd}$$

http://planetmath.org/deductiontheorem

Deduction Theorem

The deduction theorem conforms with
our intuitive understanding
of how mathematical proofs work:

if we want to prove the statement “A implies B”,
then by assuming A, if we can prove B,
we have established “A implies B”.

http://planetmath.org/deductiontheorem

Deduction Theorem

The converse statement of the deduction theorem
turns out to be a trivial consequence of **modus ponens**: $A, A \Rightarrow B \vDash B$

if $\Delta \vDash A \Rightarrow B$,
then certainly $\Delta, A \vDash A \Rightarrow B$
Since $\Delta, A \vDash A$, we get,
via **modus ponens**, $\Delta, A \vDash B$ as a result.

$$\frac{\begin{array}{l}\Delta, A \vDash A \\ \Delta, A \vDash A \Rightarrow B\end{array}}{\Delta, A \vDash B}$$

http://planetmath.org/deductiontheorem

Deduction Theorem

Deduction theorem is needed to derive **arguments** that has **no premises**

An argument without premises is simply a **tautology**

$\models A \lor \neg A$

no premises appear before the ⊨ symbol

an argument without premises

Tautology if it is sound

Argument Example

1. $\neg Q$
2. $P \Rightarrow Q$
3. $\neg P \Rightarrow R \lor S$
4. $R \Rightarrow T$
5. U
6. $U \Rightarrow \neg T$

7.	$\neg P$	$\neg Q, P \Rightarrow Q \models \neg P$	(1, 2)
8.	$R \lor S$	$\neg P \Rightarrow R \lor S, \neg P \models R \lor S$	(3, 7)
9.	$\neg T$	$U, U \Rightarrow \neg T \models \neg T$	(5, 6)
10.	$\neg R$	$\neg T, R \Rightarrow T \models \neg R$	(9, 4)
11.	S	$R \lor S, \neg R \models S$	(8, 10)

Argument without premises

A

$A \lor \neg A$

$A \Rightarrow A \lor \neg A$

$\neg A$

$\neg A \lor A$

$A \lor \neg A$

$\neg A \Rightarrow A \lor \neg A$

$A \lor \neg A$

assume A

$X \models X \lor Y$

$A \models A \lor \neg A$ ➡ $\models A \Rightarrow A \lor \neg A$

discharge A

assume ¬A

$X \models X \lor Y$

$X \lor Y \models Y \lor X$

$\neg A \models A \lor \neg A$ ➡ $\models \neg A \Rightarrow A \lor \neg A$

discharge A

$X \Rightarrow Y, \neg X \Rightarrow Y \models Y$

$A \Rightarrow A \lor \neg A, \neg A \Rightarrow A \lor \neg A \models A \lor \neg A$

$\models A \lor \neg A$

Argument without premises

A	A	assume A
$A \lor \neg A$	$A \models A \lor \neg A$	
$A \Rightarrow A \lor \neg A$	$\models A \Rightarrow A \lor \neg A$	
	discharge A	
$\neg A$	$\neg A$	assume $\neg A$
$\neg A \lor A$	$\neg A \models \neg A \lor A$	
$A \lor \neg A$	$\neg A \models A \lor \neg A$	
$\neg A \Rightarrow A \lor \neg A$	$\models \neg A \Rightarrow A \lor \neg A$	
	discharge A	
$A \lor \neg A$	$\models A \lor \neg A$	

To prove a sound argument

Prove using truth tables

Whether an argument is sound or fallacy

1. time complexity (2^n)
2. not the way which humans do

Prove using inference rules

To reason deductively

Logical Equivalences

¬, ∧ ,
∨

¬, ∧ ,
∨

∧∨⌐¬⇒
⇔≡⇒ ⊨

∧∨⌐¬⇒
⇔≡⇒⊨

⇒
⇔
≡

⇒
⇔
≡

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