

Multiple Random Variables

Young W Lim

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles, Jr. and B. Shi

Outline

1 Conditional Distribution and Density

Conditional Distribution and Density

for a single random variable X

Definition

Let A denote the event $\{X \leq x\}$ in $P(A | B) = \frac{P(A \cap B)}{P(B)}$
the conditional distribution function of X is defined as

$$F_X(x | B) = P\{X \leq x | B\} = \frac{P\{X \leq x \cap B\}}{P(B)}$$

$$f_X(x | B) = \frac{dF_X(x | B)}{dx}$$

the density function of the random variable X

the derivative of the distribution function $F_X(x | B)$

Point Conditioning

for 2 random variables X and Y

Definition

the distribution function of a random variable X conditioned by the fact that a second random variable Y has some specific value y

$$F_X(x|B) = \lim_{\Delta y \rightarrow 0} F_X(x|y - \Delta y < Y \leq y + \Delta y)$$

$$F_X(x|B) = \lim_{\Delta y \rightarrow 0} \frac{\int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^x f_{X,Y}(\xi_1, \xi_2) d\xi_1 d\xi_2}{\int_{y-\Delta y}^{y+\Delta y} f_Y(\xi) d\xi}$$

where event B is defined as $\{y - \Delta y < Y \leq y + \Delta y\}$

Point Conditioning (1)

for 2 **discrete** random variables X and Y

Definition

assume X and Y are both discrete random variables and have values $x_i, i = 1, 2, \dots, N$ and $y_j, j = 1, 2, \dots, M$. with the corresponding probabilities $P(x_i)$ and $P(y_j)$, respectively $P(x_i, y_j)$ denotes the probability of joint occurrence of x_i and y_j

$$f_X(x|y = y_k) = \sum_{i=1}^N \frac{P(x_i, y_k)}{P(y_k)} \delta(x - x_i)$$

Point Conditioning (2)

for 2 **discrete** random variables X and Y

$$f_Y(y) = \sum_{j=1}^M P(y_j) \delta(y - y_j)$$

$$f_{X,Y}(x, y) = \sum_{i=1}^N \sum_{j=1}^M P(x_i, y_j) \delta(x - x_i) \delta(y - y_j)$$

$$B = \{y - \Delta y < Y \leq y + \Delta y\}$$

$$F_X(x|B) = \lim_{\Delta y \rightarrow 0} \frac{\int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^x f_{X,Y}(\xi_1, \xi_2) d\xi_1 d\xi_2}{\int_{y-\Delta y}^{y+\Delta y} f_Y(\xi) d\xi}$$

$$F_X(x|B) = \frac{\int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^x \sum_{i=1}^N \sum_{j=1}^M P(x_i, y_j) \delta(x - x_i) \delta(y - y_j) dx dy}{\int_{y-\Delta y}^{y+\Delta y} \sum_{j=1}^M P(y_j) \delta(y - y_j) dy}$$

Point Conditioning (3)

for 2 **discrete** random variables X and Y

$$F_X(x|B) = \frac{\sum_{i=1}^N \sum_{j=1}^M \int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^x P(x_i, y_j) \delta(x - x_i) \delta(y - y_j) dx dy}{\sum_{j=1}^M \int_{y-\Delta y}^{y+\Delta y} P(y_j) \delta(y - y_j) dy}$$

$$F_X(x|y = y_k) = \frac{\sum_{i=1}^N \int_{y-\Delta y}^{y+\Delta y} \int_{-\infty}^x P(x_i, y_k) \delta(x - x_i) \delta(y - y_k) dx dy}{\int_{y-\Delta y}^{y+\Delta y} P(y_j) \delta(y - y_k) dy}$$

$$F_X(x|y = y_k) = \frac{\sum_{i=1}^N \int_{-\infty}^x P(x_i, y_k) \delta(x - x_i) dx}{P(y_k)}$$

Point Conditioning (4)

for 2 **discrete** random variables X and Y

$$F_X(x|y = y_k) = \sum_{i=1}^N \frac{P(x_i, y_k)}{P(y_k)} u(x - x_i)$$

$$f_X(x|y = y_k) = \sum_{i=1}^N \frac{P(x_i, y_k)}{P(y_k)} \delta(x - x_i)$$

Marginal Density Functions

for **continuous** N random variable X_1, X_2, \dots, X_n

Definition

$$f_{X_1, X_2, \dots, X_k}(x_1, x_2, \dots, x_k) =$$

$$\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} f_{X_1, X_2, \dots, X_N}(x_1, x_2, \dots, x_N) dx_{k+1} dx_{k+2} \cdots dx_N$$

