# Hypergeomtric Distribution 

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## Outline

(1) Hypergeometric Distribution

- Based on
- Examples
- Assumptions


## Based on

## "Probability with R: An Introduction with Computer Science Applications" Jane Horgan <br> https://en.wikipedia.org/wiki/Hypergeometric_distribution

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## Example (1)

The classical application of the hypergeometric distribution is sampling without replacement.
Think of an urn with two types of marbles, red ones and green ones.
Define drawing a green marble as a success and drawing a red marble as a failure (analogous to the binomial distribution).

- the variable N describes the number of all marbles in the urn (see contingency table below)
- K describes the number of green marbles
- then, N K corresponds to the number of red marbles.
- In this example, X is the random variable whose outcome is $k$, the number of green marbles actually drawn in the experiment


## Example (2)

- the variable N describes the number of all marbles in the urn (see contingency table below)
- $K$ describes the number of green marbles
- then, N K corresponds to the number of red marbles.
- In this example, X is the random variable whose outcome is k , the number of green marbles actually drawn in the experiment

|  | drawn | not drawn | total |
| :--- | :--- | :--- | :--- |
| green marbles | $k$ | $\mathrm{~K}-\mathrm{k}$ | K |
| red marbles | $\mathrm{n}-\mathrm{k}$ | $\mathrm{N}+\mathrm{k}-\mathrm{n}-\mathrm{K}$ | $\mathrm{N}-\mathrm{K}$ |
| total | n | $\mathrm{N}-\mathrm{n}$ | N |

## Example (3)

- assume (for example) that there are 5 green and 45 red marbles in the urn.
- Standing next to the urn, you close your eyes and draw 10 marbles without replacement.
- What is the probability that exactly 4 of the 10 are green?
- Note that although we are looking at success/failure, the data are not accurately modeled by the binomial distribution,
- because the probability of success on each trial is not the same, as the size of the remaining population changes as we remove each marble.

|  | drawn | not drawn | total |
| :--- | :--- | :--- | :--- |
| green marbles | $\mathrm{k}=4$ | $\mathrm{~K}-\mathrm{k}=1$ | $\mathrm{~K}=5$ |
| red marbles | $\mathrm{n}-\mathrm{k}=6$ | $\mathrm{~N}+\mathrm{k}-\mathrm{n}-\mathrm{K}=39$ | $\mathrm{~N}-\mathrm{K}=45$ |
| total | $\mathrm{n}=10$ | $\mathrm{~N}-\mathrm{n}=40$ | $\mathrm{~N}=50$ |

## Assumptions

- The result of each draw (the elements of the population being sampled) can be classified into one of two mutually exclusive categories (e.g. Pass/Fail or Employed/Unemployed).
- The probability of a success changes on each draw, as each draw decreases the population (sampling without replacement from a finite population).

