Matrix Transformation (2A)

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11/10/12

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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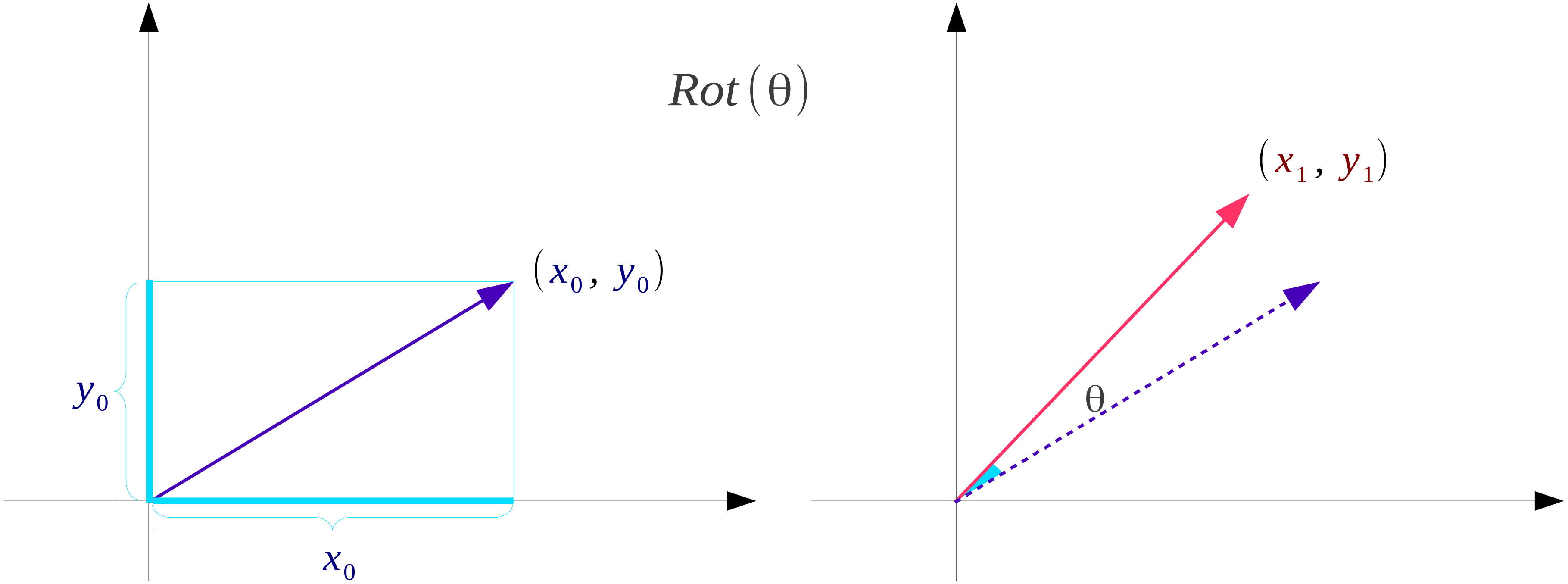
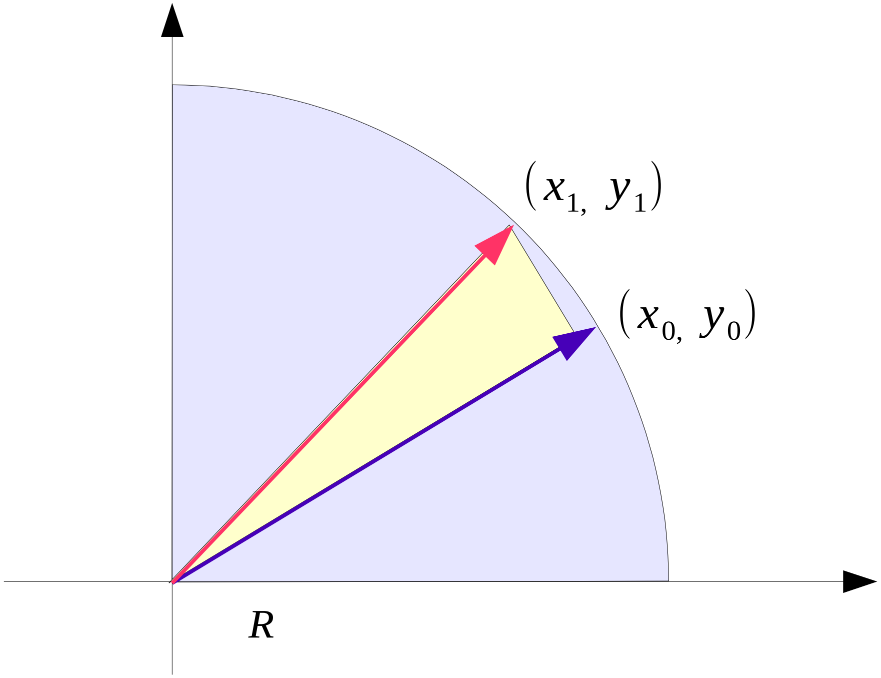
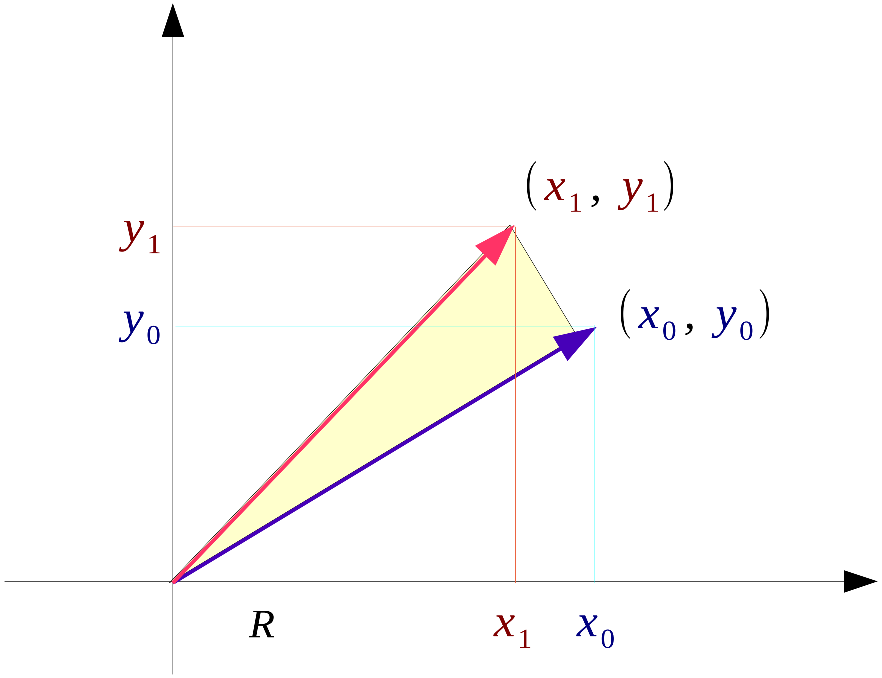
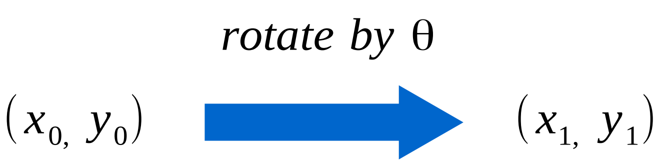
Vector Rotation (1)



Vector Rotation (2)



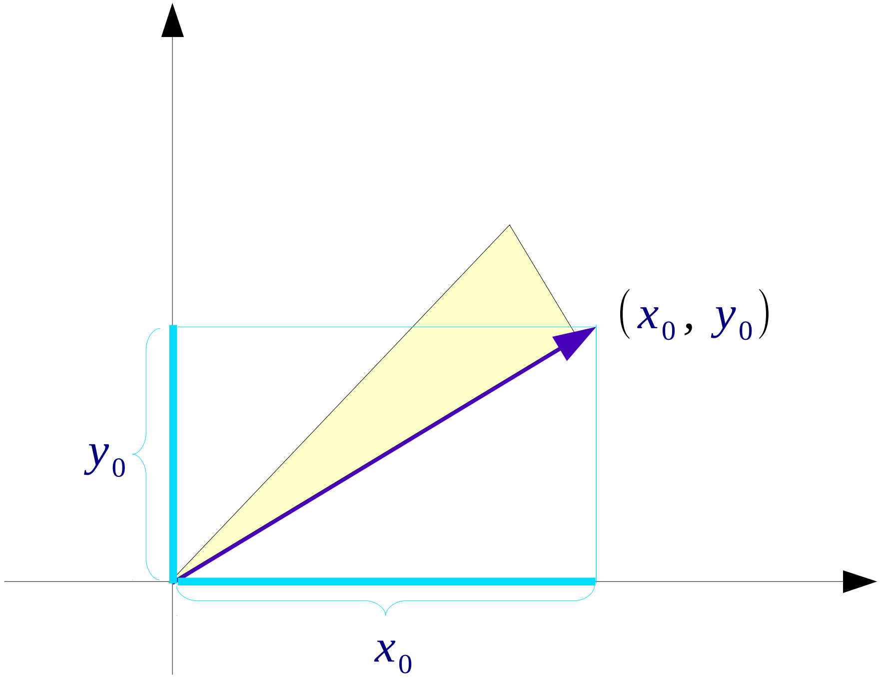


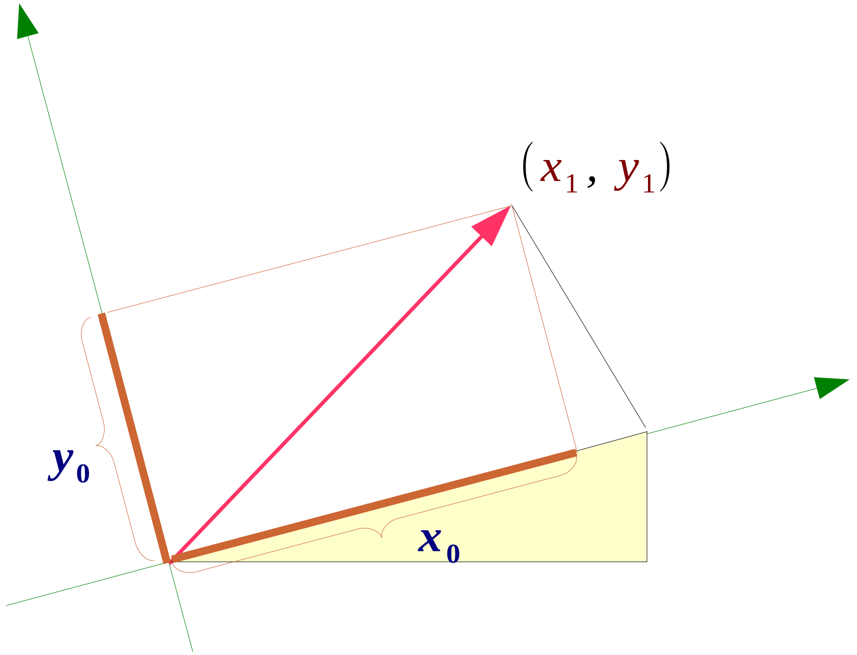


$$x_1 = x_0 \cos\theta - y_0 \sin\theta$$

$$y_1 = x_0 \sin\theta + y_0 \cos\theta$$

Vector Rotation (3)





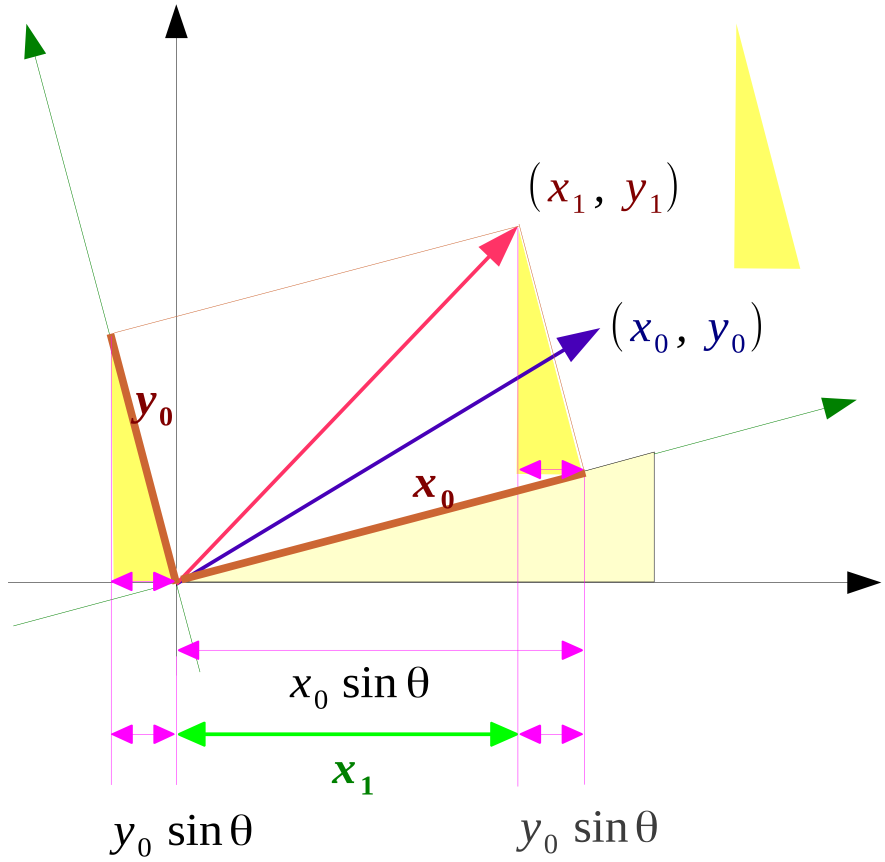
In the rotated coordinate

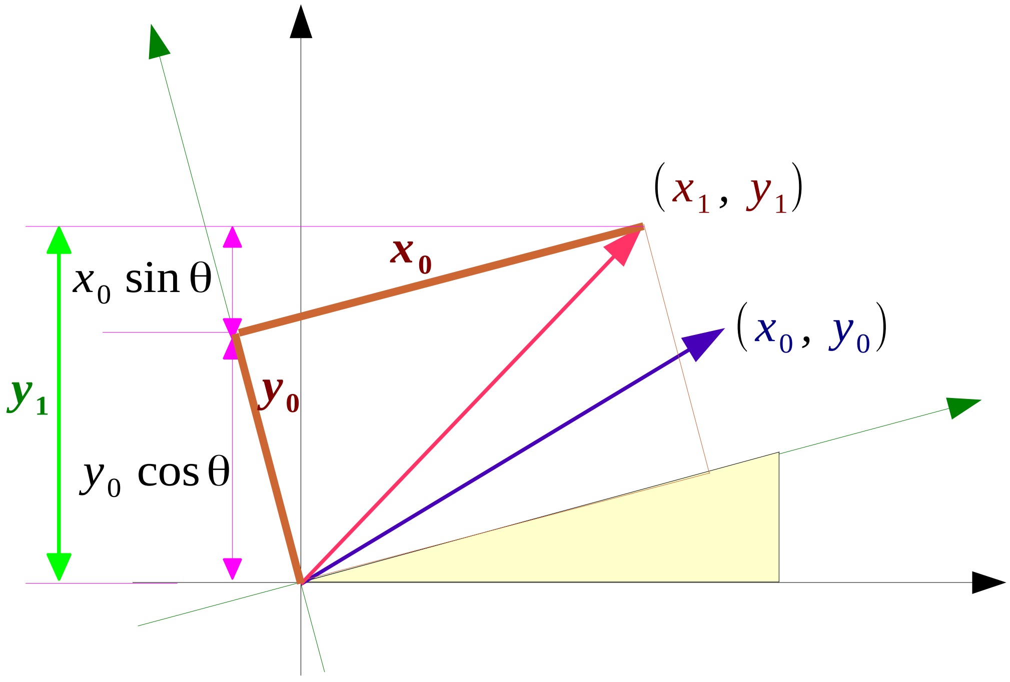
invariant length $\quad x_0, y_0$

Vector Rotation (4)

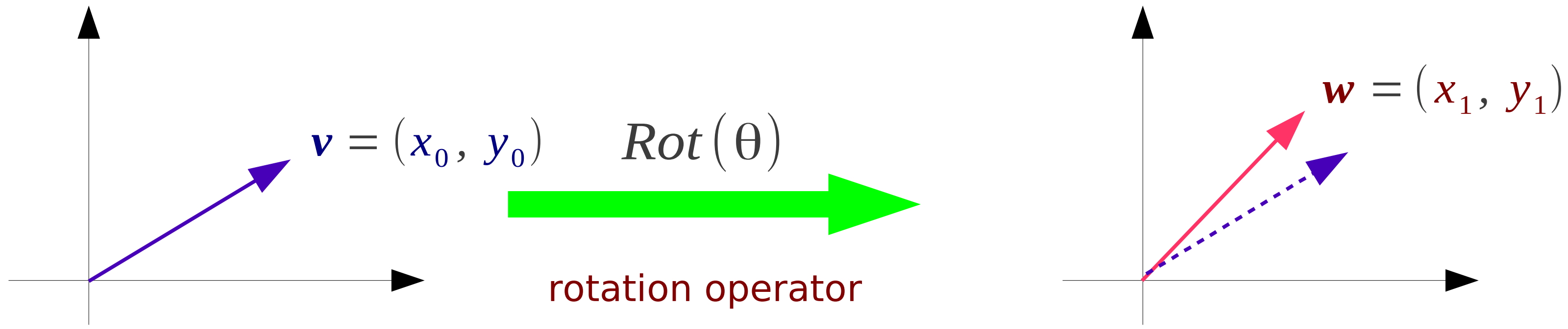
$$x_1 = x_0 \cos\theta - y_0 \sin\theta$$

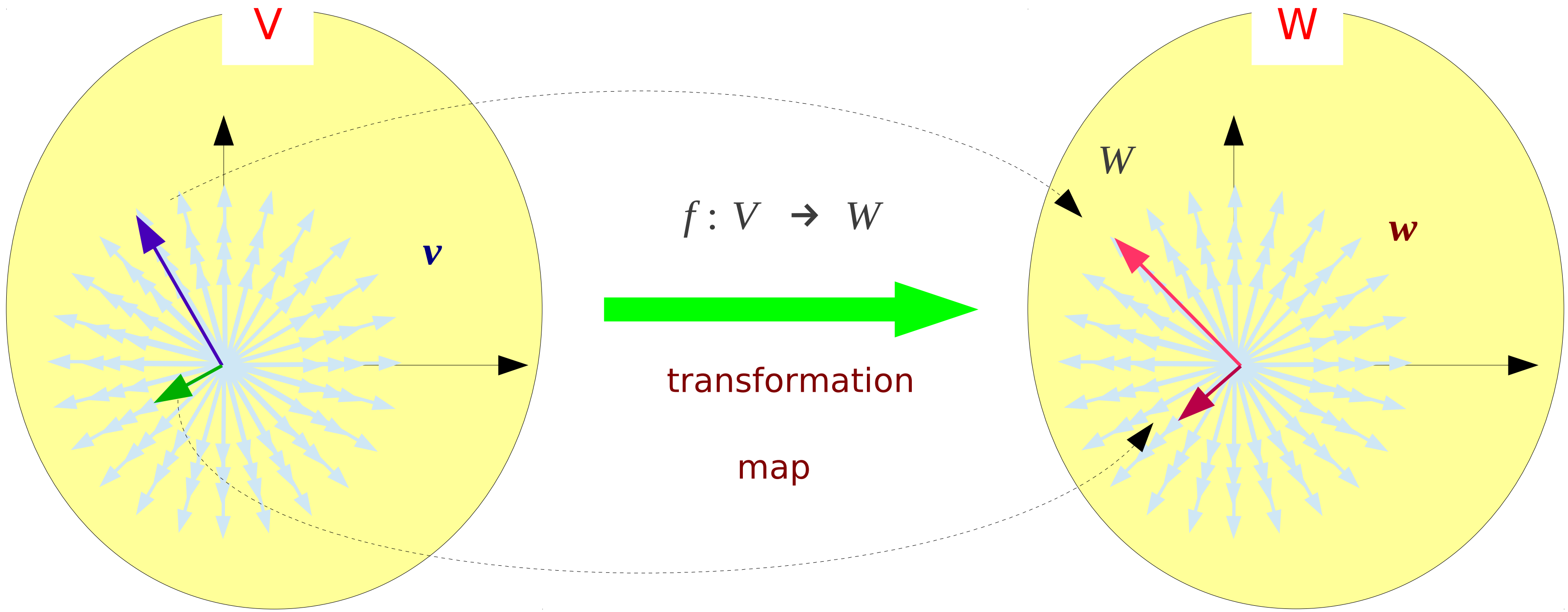
$$y_1 = x_0 \sin\theta + y_0 \cos\theta$$



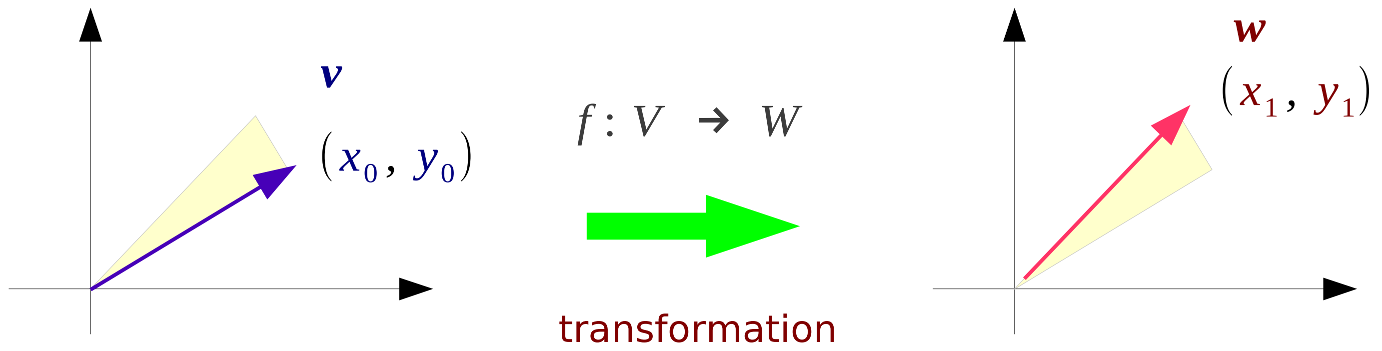


Transformation





Matrix Transformation



$$x_1 = x_0 \cos\theta - y_0 \sin\theta$$
$$y_1 = x_0 \sin\theta + y_0 \cos\theta$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_0 \\ y_1 \end{bmatrix}$$

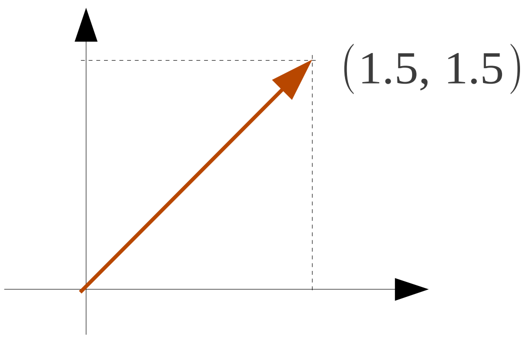
$$\boldsymbol{A} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

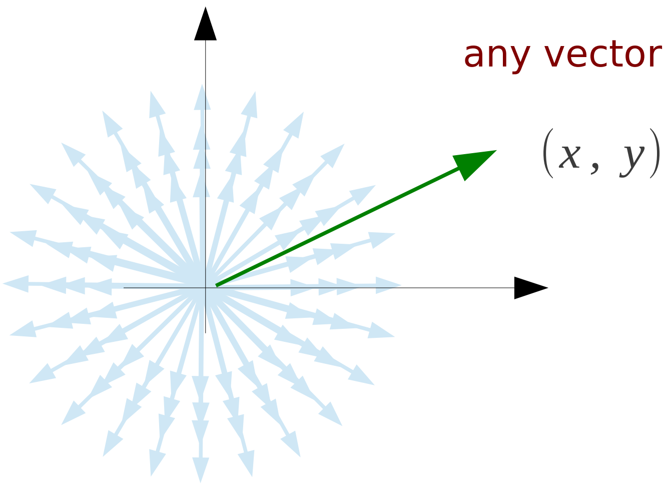
$$\boldsymbol{w} = \boldsymbol{A} \boldsymbol{x}$$
$$\boldsymbol{w} = T_{\boldsymbol{A}}(\boldsymbol{x})$$

$$\boldsymbol{x} \xrightarrow{T_{\boldsymbol{A}}} \boldsymbol{w}$$

Coordinates and Coordinates Systems

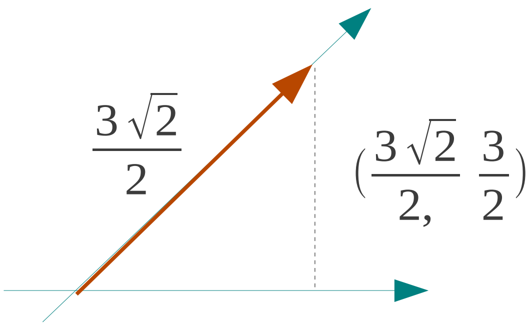
Rectangular Coordinate System

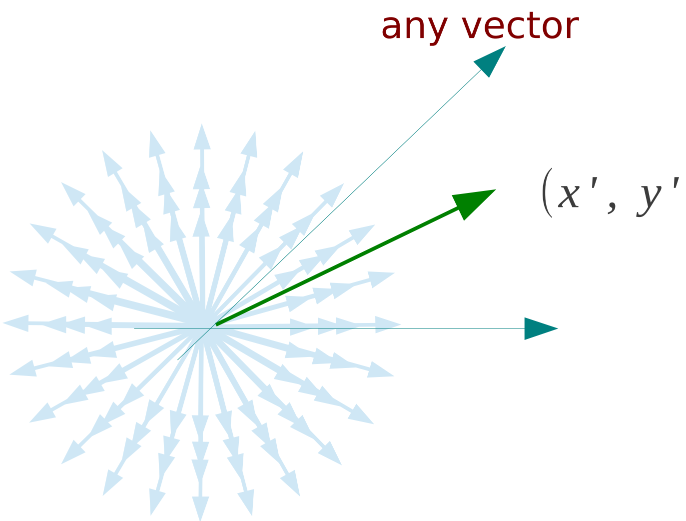




coordinates $(1.5\,,\ 1.5)$

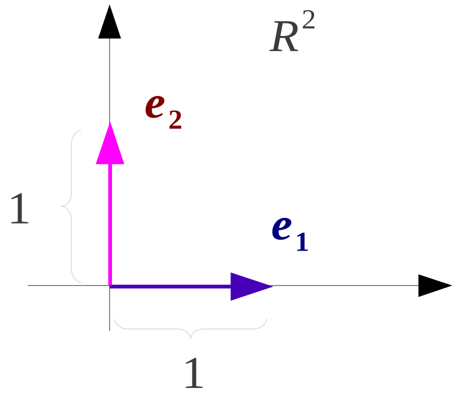
Non-Rectangular Coordinate System





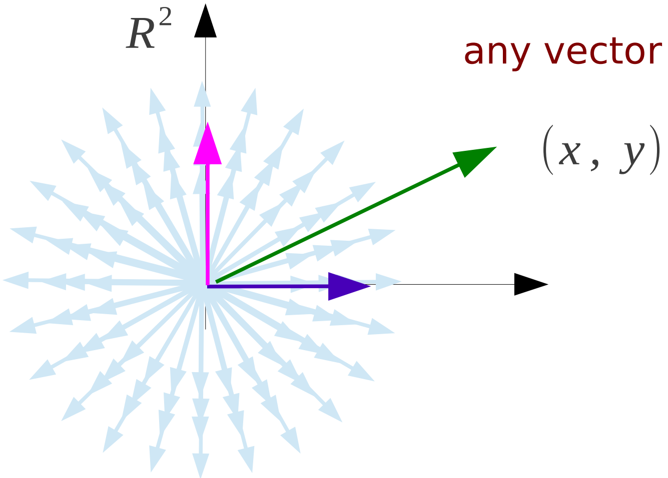
coordinates $(\frac{3\sqrt{2}}{2}, \frac{3}{2})$

Standard Basis

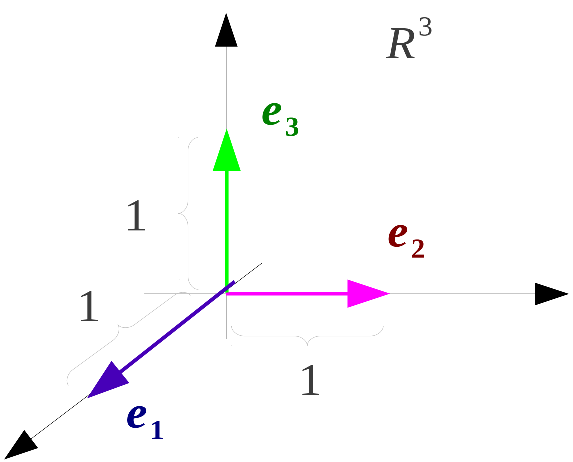


$e_1 = (1,\ 0)$

$e_2 = (0,\ 1)$



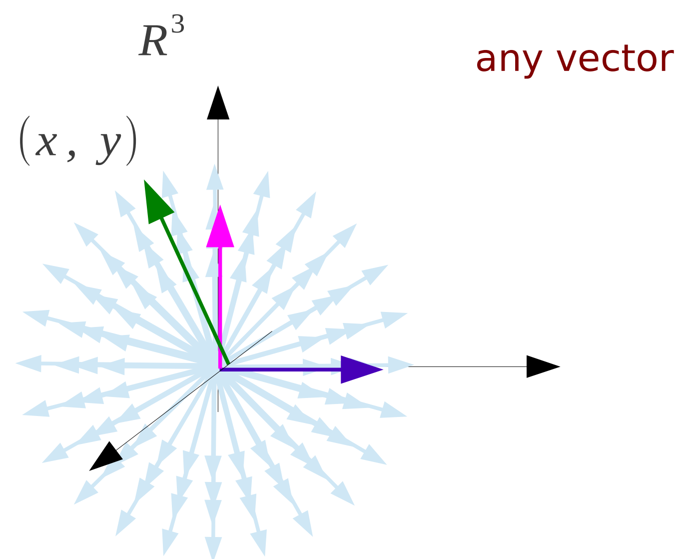
standard basis $\{e_1,\ e_2\}$ **spans** R^2



$e_1 = (1,\ 0,\ 0)$

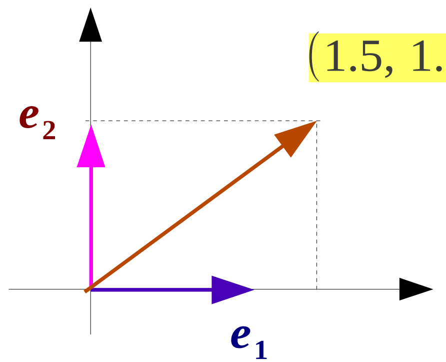
$e_2 = (0,\ 1,\ 0)$

$e_3 = (0,\ 0,\ 1)$



standard basis $\{e_1,\ e_2,\ e_3\}$ **spans** R^3

Basis and Coordinates



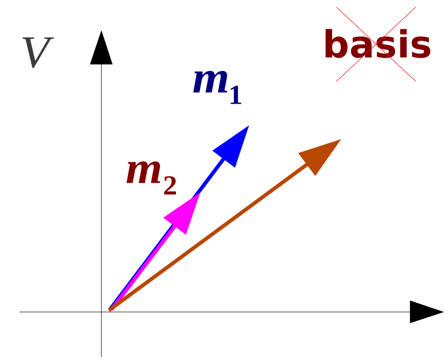
$$(1.5,\ 1.0) = 1.5\,\boldsymbol{e}_1 + 1.0\,\boldsymbol{e}_2$$

$$= 1.5\begin{bmatrix}1\\0\end{bmatrix} + 1.0\begin{bmatrix}0\\1\end{bmatrix}$$

$$= \begin{bmatrix}1.5 & 1.0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

basis $\{\boldsymbol{e}_1,\ \boldsymbol{e}_2\}$

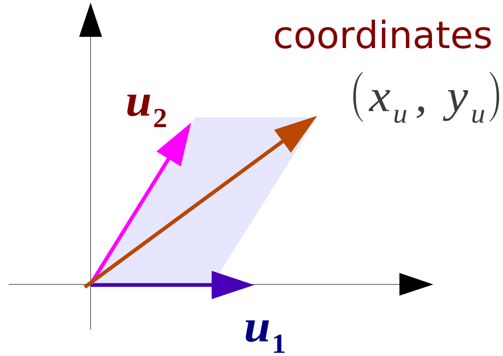
coordinates $(1.5,\ 1.0)$



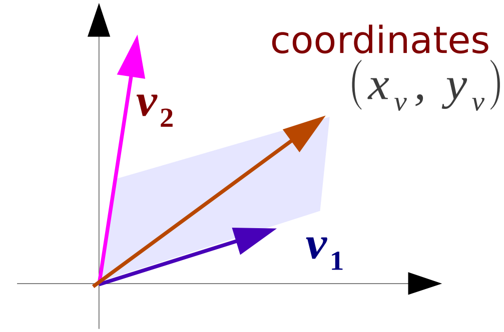
collinear vectors → linearly dependent vectors

many bases but the same number of basis vectors

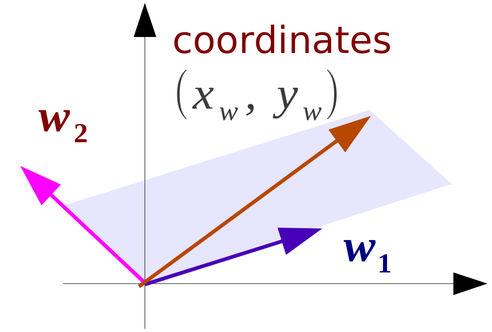
basis $\{\boldsymbol{u}_1,\ \boldsymbol{u}_2\}$



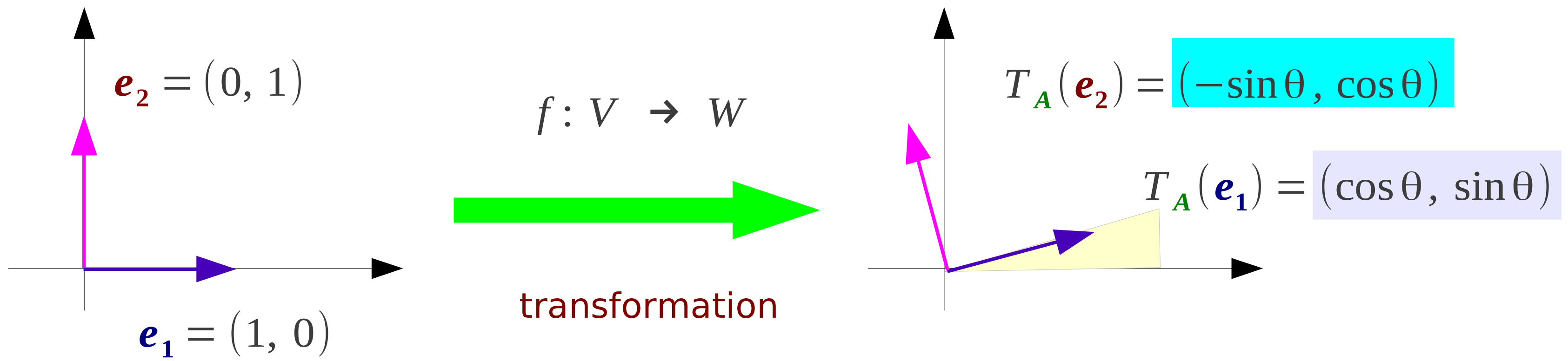
basis $\{\boldsymbol{v}_1,\ \boldsymbol{v}_2\}$



basis $\{\boldsymbol{w}_1,\ \boldsymbol{w}_2\}$



Standard Basis & Standard Matrix



standard basis $\{e_1, e_2\}$

standard matrix $A = [\, T_A(e_1) \mid T_A(e_2) \,]$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_0 \\ y_1 \end{bmatrix}$$

$$w = A \quad x$$

$$w = T_A(x)$$

$$x \xrightarrow{T_A} w$$

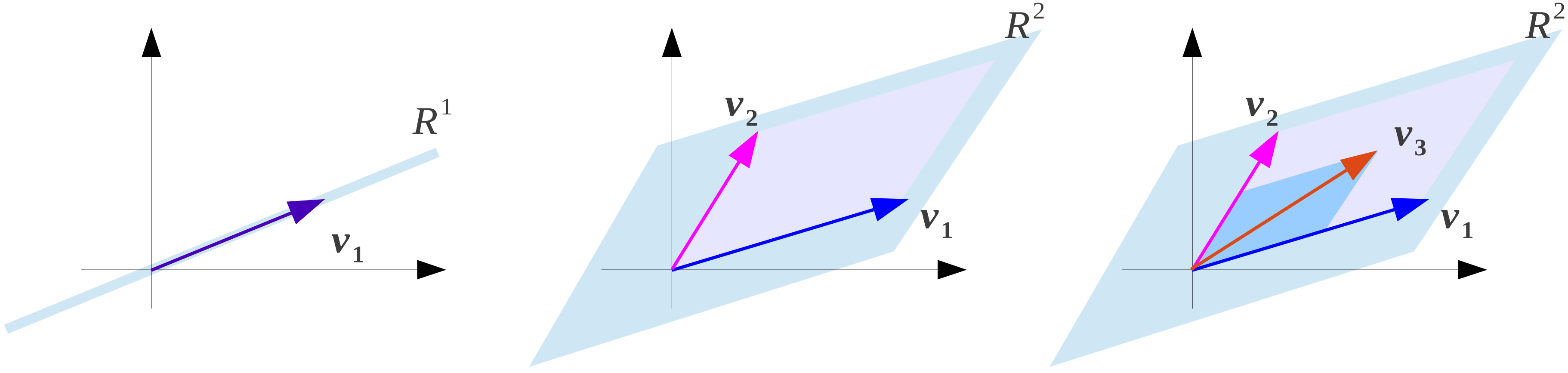
$$A = \begin{pmatrix} T_A(e_1) & T_A(e_2) & \cdots & T_A(e_n) \end{pmatrix}$$

Dimension

In vector space R^2

any one vector	line	R^1	linearly independent
any two non-collinear vectors	plane	R^2	linearly independent
any three or more vectors			linearly dependent

$$\mathbf{v}_3 = k_1 \mathbf{v}_1 + k_2 \mathbf{v}_2$$

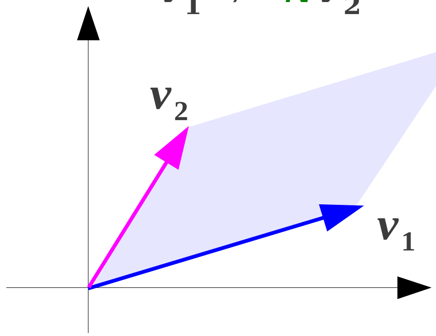


Basis

$S = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_n\}$ non-empty finite set of vectors in V

S is a basis ⟺

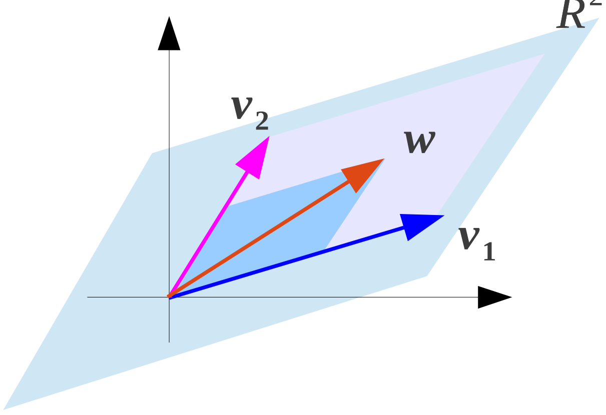
- S linearly independent
- S spans V



span(S) $= span\{\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_n\}$ ⟺

all possible linear combination of the vectors in in S

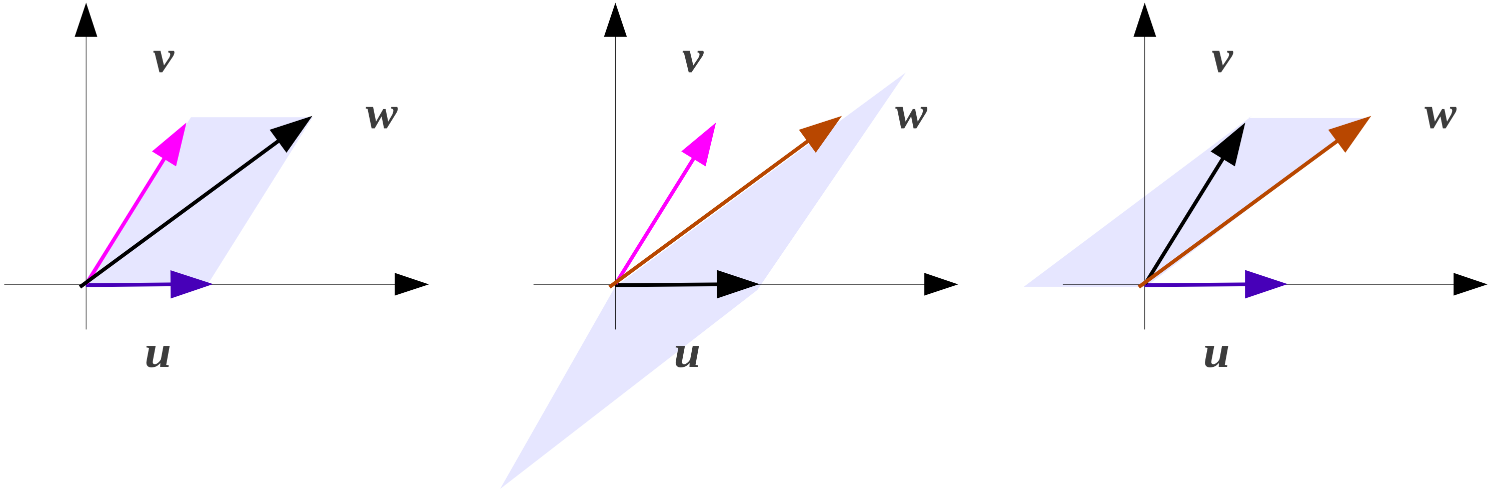
$\{\ \boldsymbol{w} = k_1\boldsymbol{v}_1 + k_2\boldsymbol{v}_2 + \cdots + k_n\boldsymbol{v}_n\ \}$



Linear Dependent (1)

$\{\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}\}$ linearly dependent

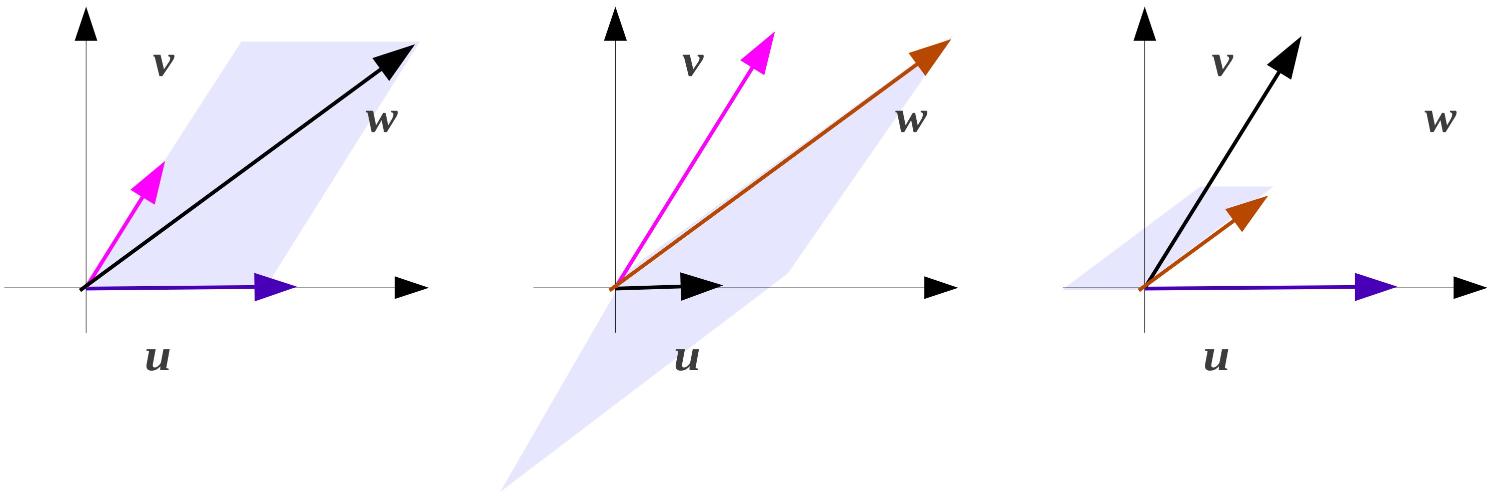
$\boldsymbol{w} = \boldsymbol{u} + \boldsymbol{v}$ $\boldsymbol{u} = \boldsymbol{w} - \boldsymbol{v}$ $\boldsymbol{v} = \boldsymbol{w} - \boldsymbol{u}$



$\boldsymbol{u} + \boldsymbol{v} - \boldsymbol{w} = \boldsymbol{0}$ $\boldsymbol{u} + \boldsymbol{v} - \boldsymbol{w} = \boldsymbol{0}$ $\boldsymbol{u} + \boldsymbol{v} - \boldsymbol{w} = \boldsymbol{0}$

Linear Dependent (2)

$\{\boldsymbol{u}\ ,\ \boldsymbol{v}\ ,\ \boldsymbol{w}\}$ linearly dependent



$k_1\boldsymbol{u} + k_2\boldsymbol{v} + k_3\boldsymbol{w} = \boldsymbol{0}$

$m_1\boldsymbol{u} + m_2\boldsymbol{v} + m_3\boldsymbol{w} = \boldsymbol{0}$

$n_1\boldsymbol{u} + n_2\boldsymbol{v} + n_3\boldsymbol{w} = \boldsymbol{0}$

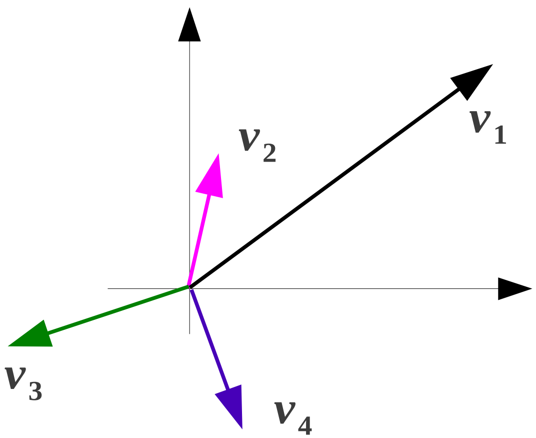
$(k_1 = 0)\wedge(k_2 = 0)\wedge(k_3 = 0)$
$(k_1 \neq 0)\vee(k_2 \neq 0)\vee(k_3 \neq 0)$

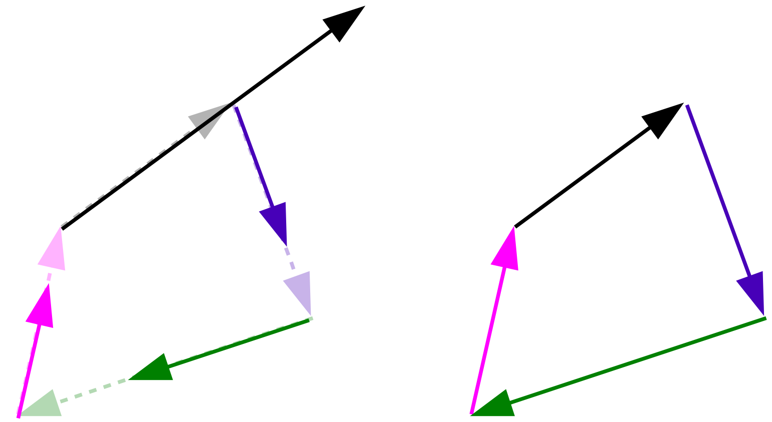
$(m_1 = 0)\wedge(m_2 = 0)\wedge(m_3 = 0)$
$(m_1 \neq 0)\vee(m_2 \neq 0)\vee(m_3 \neq 0)$

$(n_1 = 0)\wedge(n_2 = 0)\wedge(n_3 = 0)$
$(n_1 \neq 0)\vee(n_2 \neq 0)\vee(n_3 \neq 0)$

Linear Dependent (3)

$\{\boldsymbol{v}_1 , \boldsymbol{v}_2 , \boldsymbol{v}_3 , \boldsymbol{v}_4\}$ linearly dependent



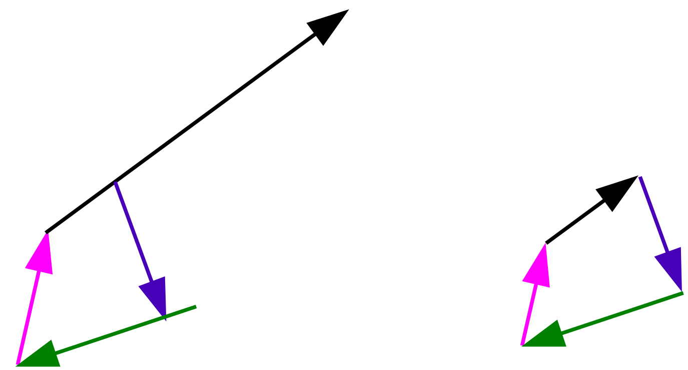


$$k_1\boldsymbol{v}_1 + k_2\boldsymbol{v}_2 + k_3\boldsymbol{v}_3 + k_4\boldsymbol{v}_4 = \boldsymbol{0}$$

$$(k_1 \neq 0)\vee(k_2 \neq 0)\vee(k_3 \neq 0)\vee(k_4 \neq 0)$$

$$0\,\boldsymbol{v}_1 + m_2\boldsymbol{v}_2 + m_3\boldsymbol{v}_3 + m_4\boldsymbol{v}_4 = \boldsymbol{0}$$

$$(m_1 = 0)\vee(m_2 \neq 0)\vee(m_3 \neq 0)\vee(m_4 \neq 0)$$

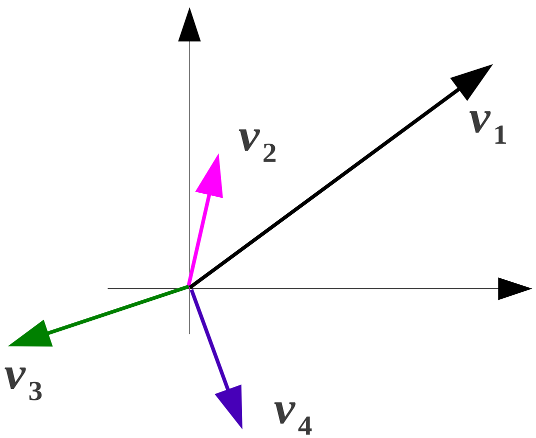


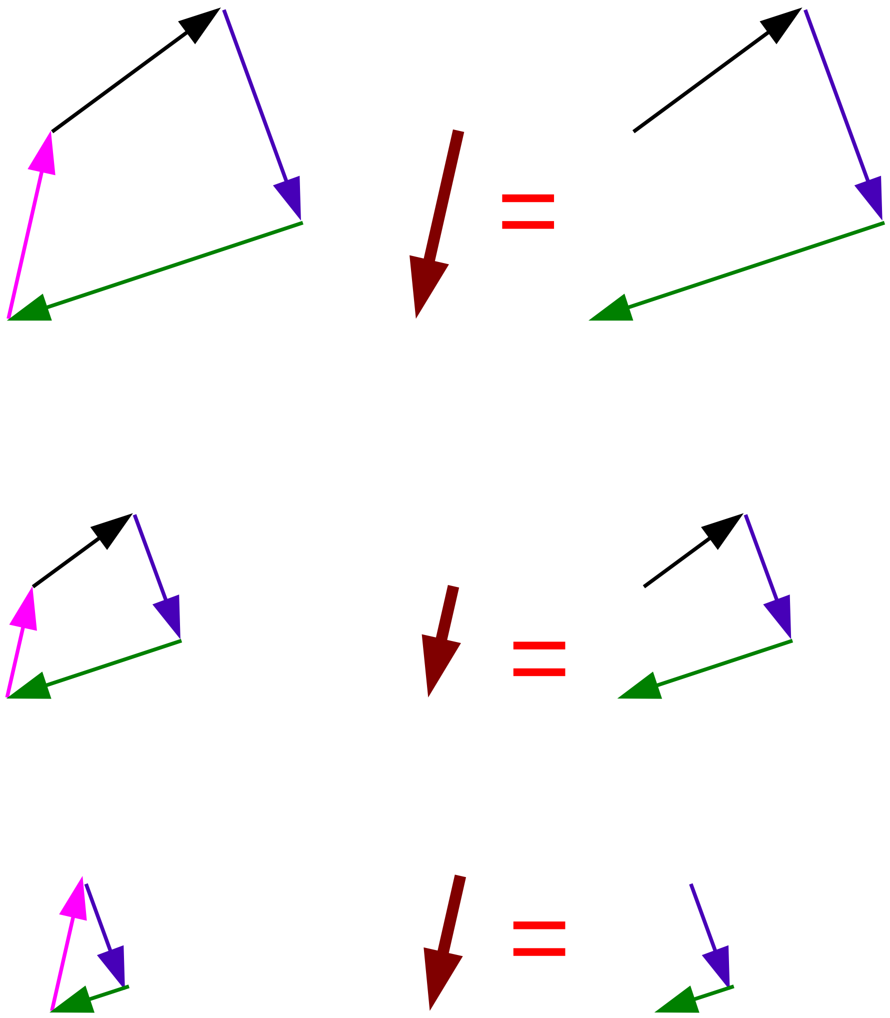
$$m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2 + m_3\boldsymbol{v}_3 + m_4\boldsymbol{v}_4 = \boldsymbol{0}$$

$$(m_1 \neq 0)\vee(m_2 \neq 0)\vee(m_3 \neq 0)\vee(m_4 \neq 0)$$

Linear Dependent (4)

$\{\mathbf{v}_1 , \mathbf{v}_2 , \mathbf{v}_3 , \mathbf{v}_4\}$ linearly dependent





Linear Independent (1)

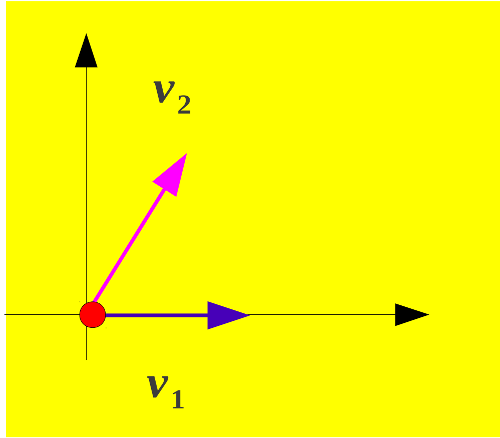
$S = \{\boldsymbol{v_1}, \boldsymbol{v_2}, \cdots, \boldsymbol{v_n}\}$ non-empty set of vectors in V

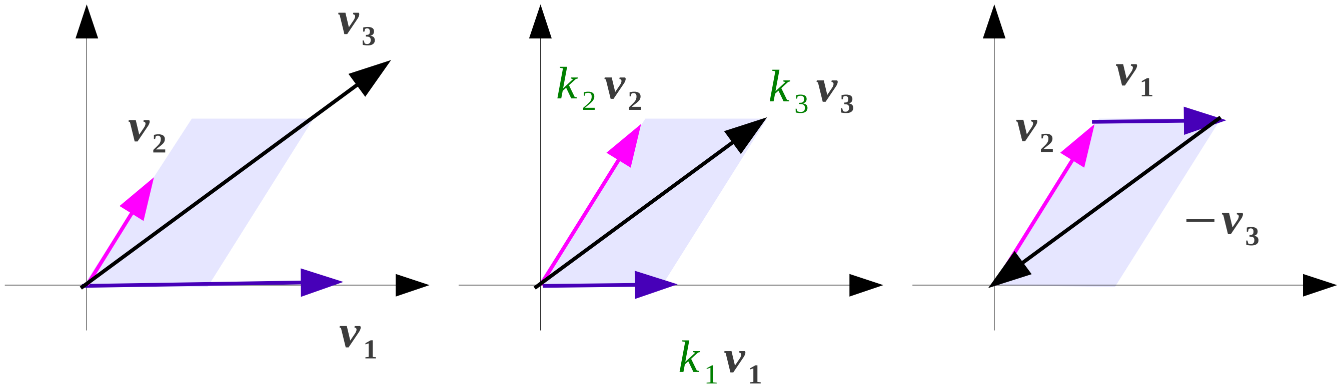
$k_1\boldsymbol{v_1} + k_2\boldsymbol{v_2} + \cdots + k_n\boldsymbol{v_n} = \boldsymbol{0}$

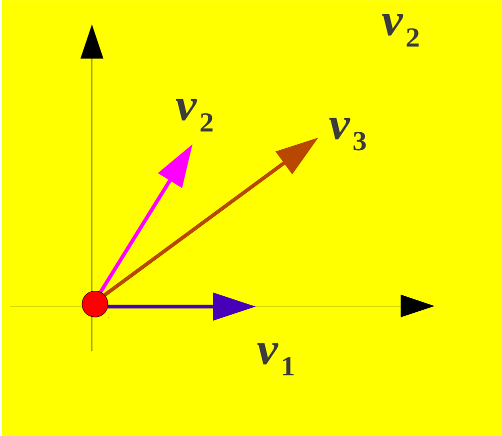
the solution of the above equation

trivial solution: $k_1 = k_2 = \cdots = k_n = 0$

- if other solution exists — S linearly dependent
- if no other solution exists — S linearly independent







Linear Independent (2)

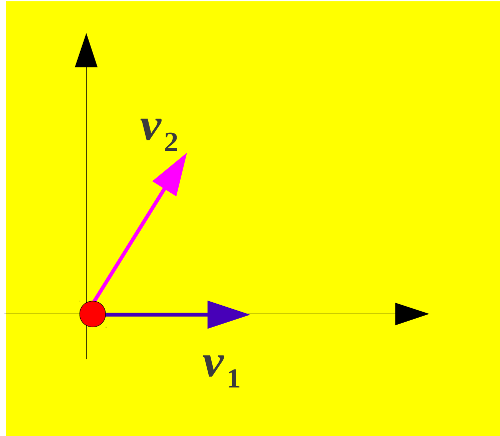
$S = \{\mathbf{v_1}, \mathbf{v_2}, \cdots, \mathbf{v_n}\}$ non-empty set of vectors in V

$k_1\mathbf{v_1} + k_2\mathbf{v_2} + \cdots + k_n\mathbf{v_n} = \mathbf{0}$

the solution of the above equation

$$k_1 = k_2 = \cdots = k_n = 0$$

- if other solution exists ⟺ S linearly dependent
- if no other solution exists ⟺ S linearly independent



at least one vector in S is a linear combination of the other vectors in S

$$\mathbf{v_i} = k_1\mathbf{v_1} + k_2\mathbf{v_2} + \cdots + k_{i-1}\mathbf{v_{i-1}} + k_{i+1}\mathbf{v_{i+1}} + \cdots + k_n\mathbf{v_n}$$

⟺ S linearly dependent

no vector in S is a linear combination of the other vectors in S

$$\mathbf{v_i} \neq k_1\mathbf{v_1} + k_2\mathbf{v_2} + \cdots + k_{i-1}\mathbf{v_{i-1}} + k_{i+1}\mathbf{v_{i+1}} + \cdots + k_n\mathbf{v_n}$$

⟺ S linearly independent

Linear Independent (3)

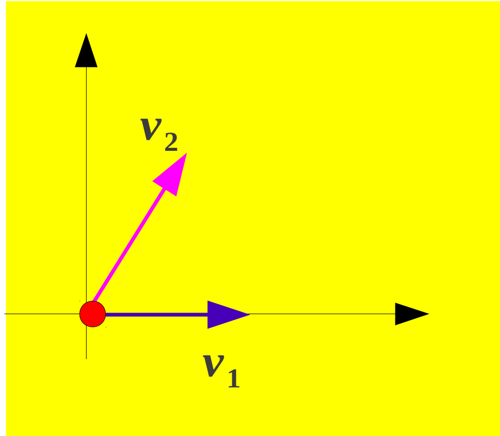
$S = \{\mathbf{v_1}, \mathbf{v_2}, \cdots, \mathbf{v_n}\}$ non-empty set of vectors in V

$k_1\mathbf{v_1} + k_2\mathbf{v_2} + \cdots + k_n\mathbf{v_n} = \mathbf{0}$

the solution of the above equation

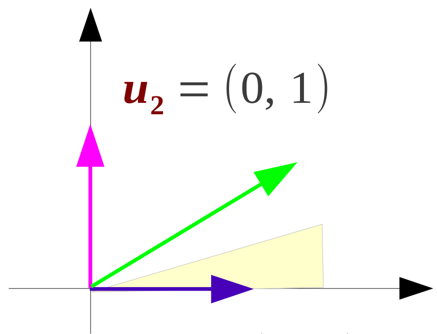
$$k_1 = k_2 = \cdots = k_n = 0$$

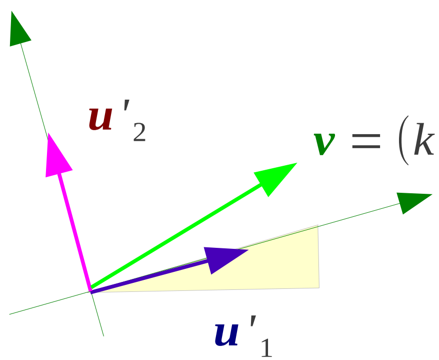
- if other solution exists ⟺ S linearly dependent
- if no other solution exists ⟺ S linearly independent



$S = \{\mathbf{0}\}$	linearly dependent
$S = \{\mathbf{v_1}\}$	linearly independent
$S = \{\mathbf{v_1}, \mathbf{v_2}\}$ $\mathbf{v_1} \neq k\mathbf{v_2}$	linearly independent

Change of Basis





Old Basis $B = \{u_1, u_2\}$

New Basis $B' = \{u'_1, u'_2\}$

$[u'_1]_B = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$ coordinate of **u'$_1$** with respect to B

$[u'_2]_B = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$ coordinate of **u'$_2$** with respect to B

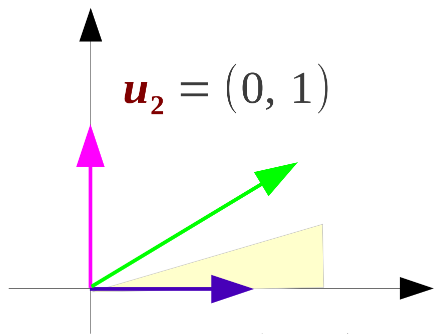
$$u'_1 = \cos\theta\, u_1 + \sin\theta\, u_2$$
$$u'_2 = -\sin\theta\, u_1 + \cos\theta\, u_2$$

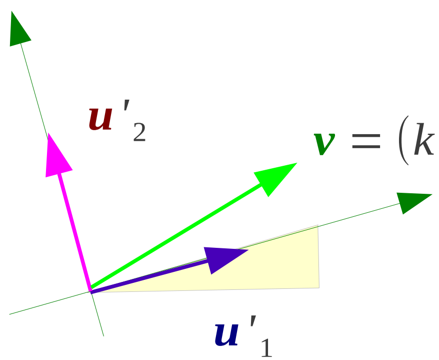
$$v = k_1 u'_1 + k_2 u'_2$$
$$= k_1(\cos\theta\, u_1 + \sin\theta\, u_2) + k_2(-\sin\theta\, u_1 + \cos\theta\, u_2)$$
$$= (k_1\cos\theta - k_2\sin\theta)u_1 + (k_1\sin\theta + k_2\cos\theta)u_2$$

$[v]_{B'} = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$ coordinate of **v** with respect to B'

$[v]_B = \begin{bmatrix} k_1\cos\theta - k_2\sin\theta \\ k_1\sin\theta + k_2\cos\theta \end{bmatrix}$ coordinate of **v** with respect to B

Change of Basis





Old Basis $B = \{u_1, u_2\}$

New Basis $B' = \{u'_1, u'_2\}$

$$[v]_{B'} = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$$

coordinate of **v** with respect to B'

$$[v]_B = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$$

coordinate of **v** with respect to B

$$[v]_B = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} [v]_{B'}$$

$$[u'_1]_B = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

coordinate of **u'$_1$** with respect to B

$$[v]_B = P_{B' \to B} [v]_{B'}$$

$$[u'_2]_B = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

coordinate of **u'$_2$** with respect to B

$$P_{B' \to B} = \begin{bmatrix} [u'_1]_B & [u'_2]_B \end{bmatrix}$$

Transition Matrix

$$P_{B' \to B} = \begin{bmatrix} [u'_1]_B & [u'_2]_B & \cdots & [u'_n]_B \end{bmatrix}$$

$$[v]_B = P_{B' \to B} [v]_{B'}$$

$[u'_1]_B$ coordinate of u'_1 with respect to B

$[u'_2]_B$ coordinate of u'_2 with respect to B

$[v]_{B'}$ coordinate of v with respect to B'

$[v]_B$ coordinate of v with respect to B

$$P_{B \to B'} = \begin{bmatrix} [u_1]_{B'} & [u_2]_{B'} & \cdots & [u_n]_{B'} \end{bmatrix}$$

$$[v]_{B'} = P_{B \to B'} [v]_B$$

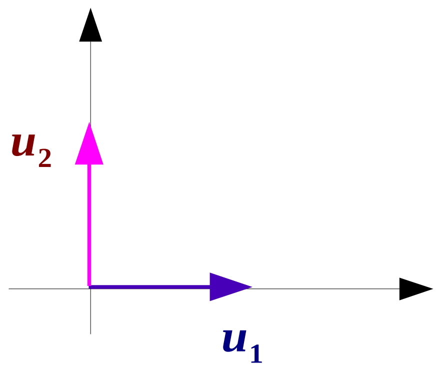
$[u_1]_{B'}$ coordinate of u_1 with respect to B'

$[u_2]_{B'}$ coordinate of u_2 with respect to B'

$[v]_B$ coordinate of v with respect to B

$[v]_{B'}$ coordinate of v with respect to B'

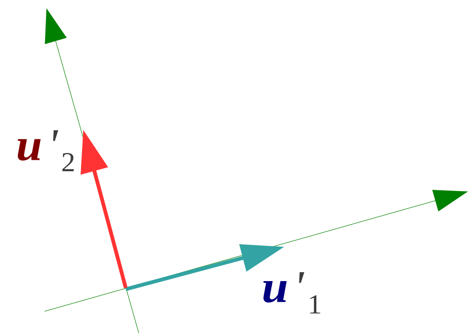
Change of Basis Example (1)



$$Rot(30°)$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$Rot(-30°)$$



$$P_{B \to B'} = \begin{bmatrix} [\boldsymbol{u}_1]_{B'} & [\boldsymbol{u}_2]_{B'} & \cdots & [\boldsymbol{u}_n]_{B'} \end{bmatrix}$$

$$P_{B' \to B} = \begin{bmatrix} [\boldsymbol{u}'_1]_{B} & [\boldsymbol{u}'_2]_{B} & \cdots & [\boldsymbol{u}'_n]_{B} \end{bmatrix}$$

$[\boldsymbol{u}_1]_{B'}$ coordinate of $\mathbf{u}_1$ with respect to B' $\begin{bmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{bmatrix}$

$[\boldsymbol{u}_2]_{B'}$ coordinate of $\mathbf{u}_2$ with respect to B' $\begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$

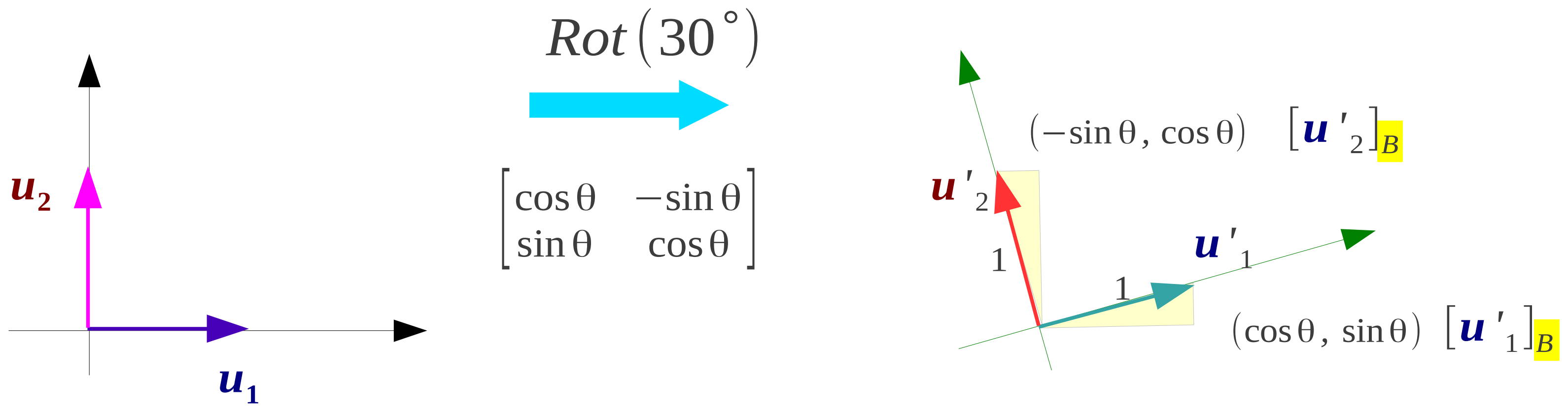
$$P_{B \to B'} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

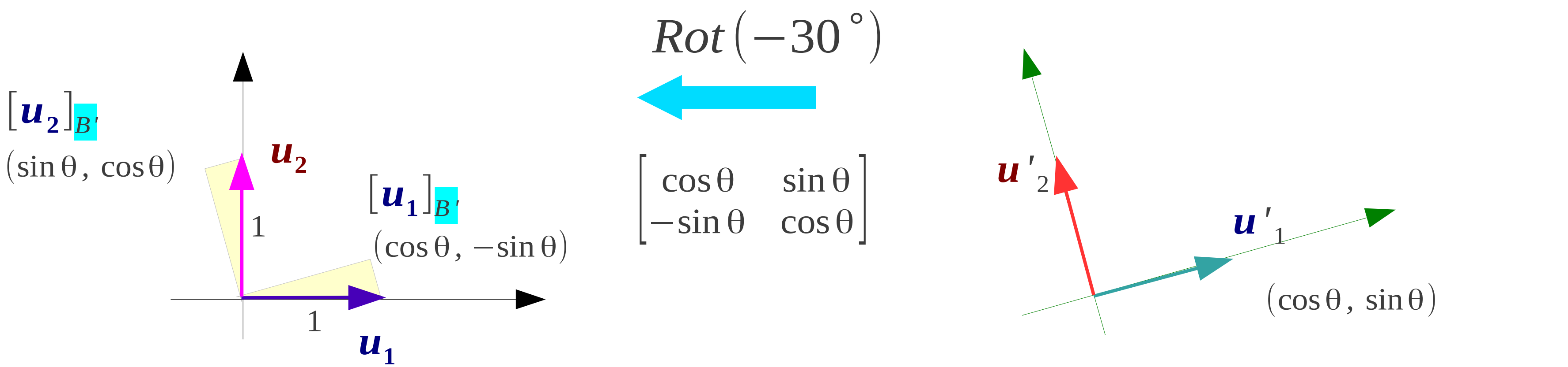
$[\boldsymbol{u}'_1]_{B}$ coordinate of $\mathbf{u'}_1$ with respect to B $\begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}$

$[\boldsymbol{u}'_2]_{B}$ coordinate of $\mathbf{u'}_2$ with respect to B $\begin{bmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$

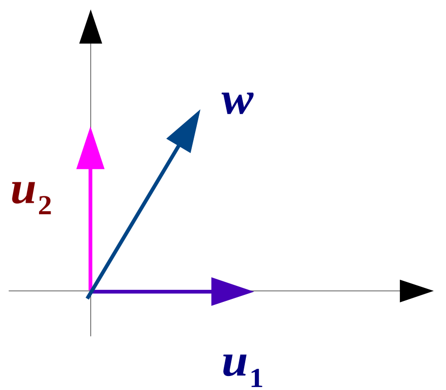
$$P_{B' \to B} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

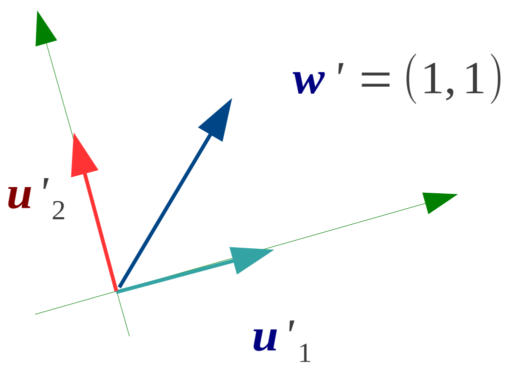
Change of Basis Example (2)





Change of Basis Example (3)





$[\mathbf{v}]_B = P_{B' \to B} [\mathbf{v}]_{B'}$

$P_{B' \to B} = \begin{bmatrix} [\mathbf{u}'_1]_B & [\mathbf{u}'_2]_B & \cdots & [\mathbf{u}'_n]_B \end{bmatrix}$

$[\mathbf{v}]_{B'}$ coordinate of **v** with respect to B'

$[\mathbf{v}]_B$ coordinate of **v** with respect to B

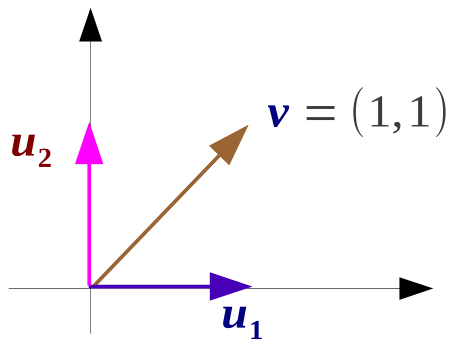
$[\mathbf{u}'_1]_B$ coordinate of **u'**$_1$ with respect to B $\begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix}$

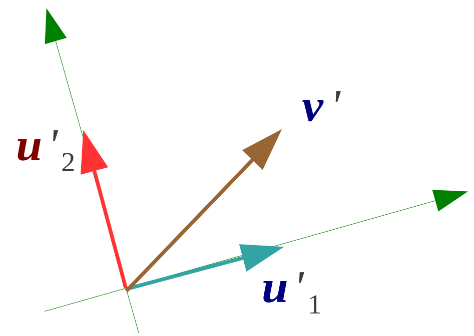
$[\mathbf{u}'_2]_B$ coordinate of **u'**$_2$ with respect to B $\begin{bmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$

$$\mathbf{w} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{-1+\sqrt{3}}{2} \\ \frac{1+\sqrt{3}}{2} \end{bmatrix}$$

$$P_{B' \to B} = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

Change of Basis Example (4)





$$P_{B \to B'} = \left[\; [\boldsymbol{u}_1]_{B'} \quad [\boldsymbol{u}_2]_{B'} \quad \cdots \quad [\boldsymbol{u}_n]_{B'} \right]$$

$[\boldsymbol{u}_1]_{B'}$ coordinate of $\mathbf{u}_1$ with respect to B' $\begin{bmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{bmatrix}$

$[\boldsymbol{u}_2]_{B'}$ coordinate of $\mathbf{u}_2$ with respect to B' $\begin{bmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{bmatrix}$

$$P_{B \to B'} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

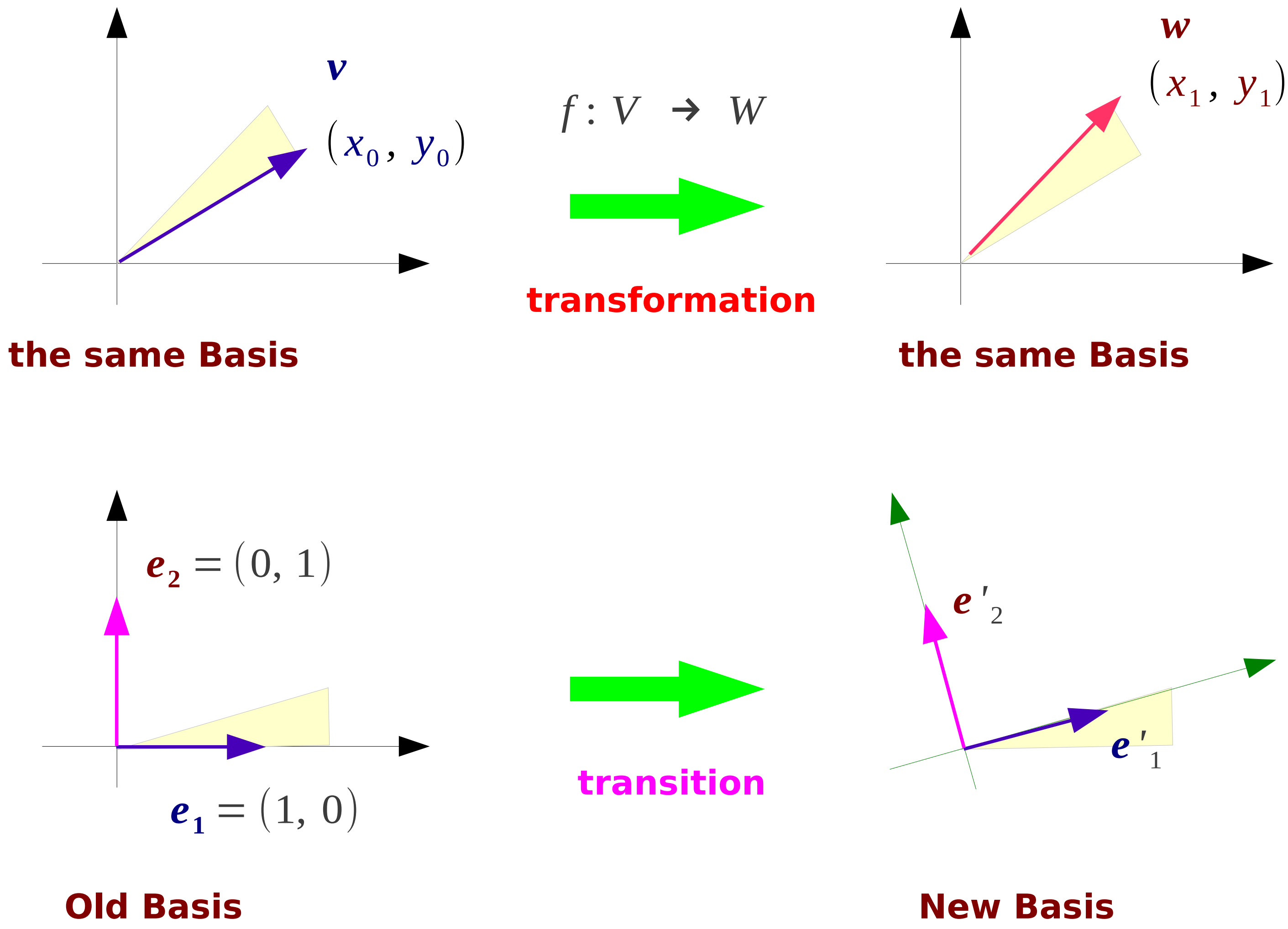
$$[\boldsymbol{v}]_{B'} = P_{B \to B'} [\boldsymbol{v}]_B$$

$[\boldsymbol{v}]_B$ coordinate of $\mathbf{v}$ with respect to B

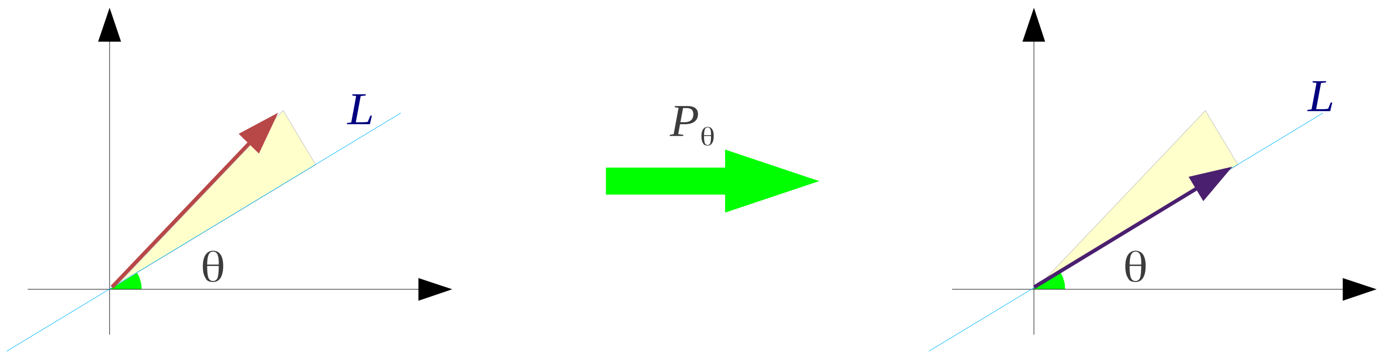
$[\boldsymbol{v}]_{B'}$ coordinate of $\mathbf{v}$ with respect to B'

$$\boldsymbol{v}' = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1+\sqrt{3}}{2} \\ \frac{-1+\sqrt{3}}{2} \end{bmatrix}$$

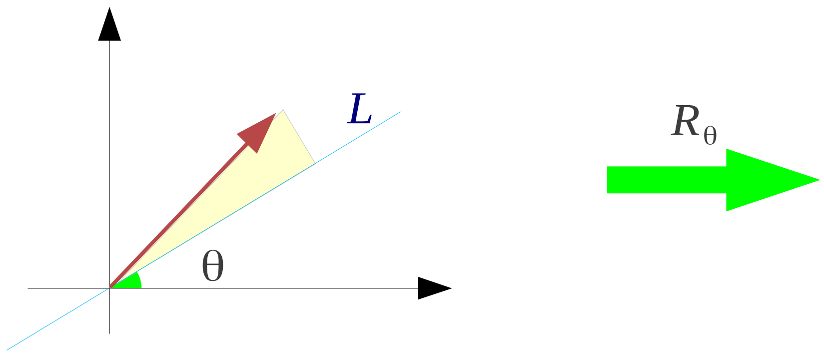
Transformation & Transition Matrix

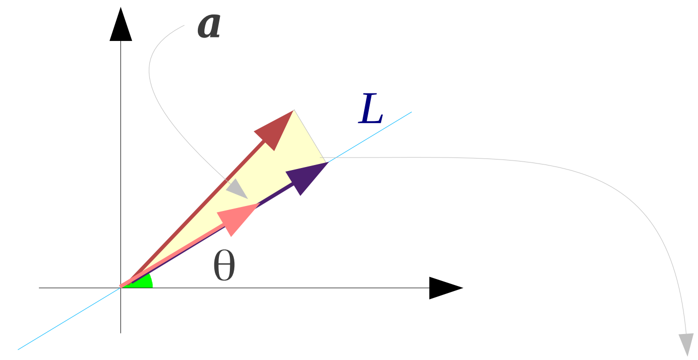


Projection onto the Lines Through Zero (1)



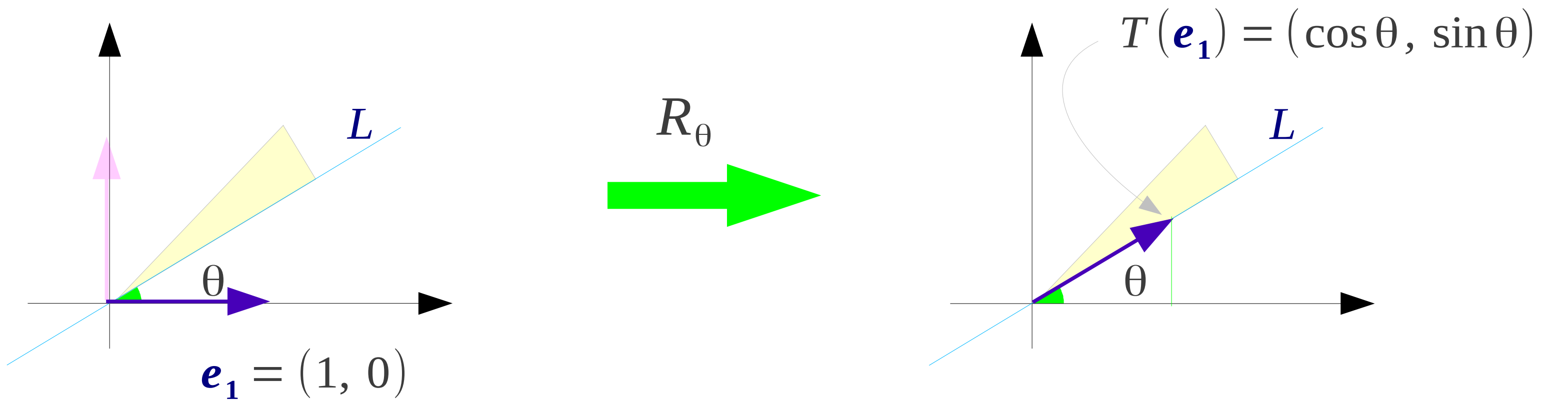
line is represented by a vector **a**



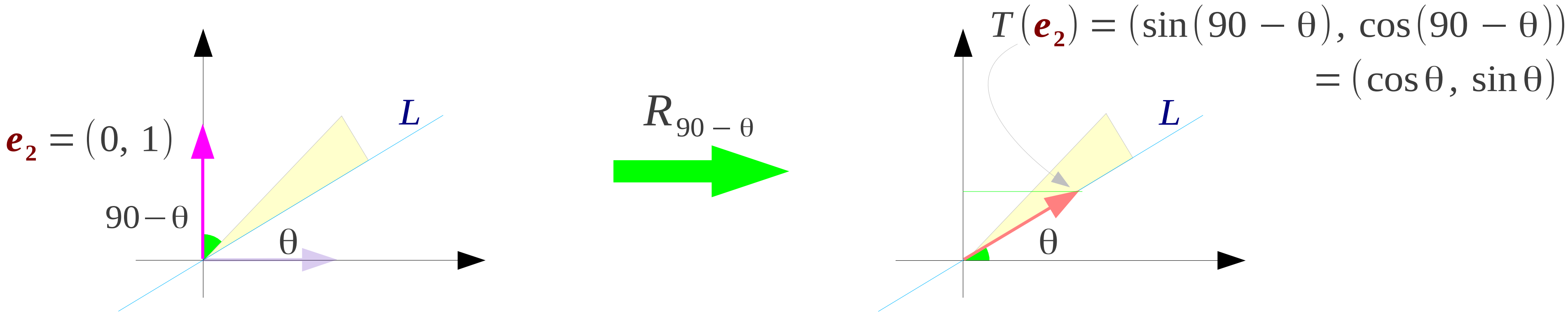


$$proj_{\boldsymbol{a}}\boldsymbol{u} = \frac{\boldsymbol{u}\cdot\boldsymbol{a}}{\|\boldsymbol{a}\|^2}\,\boldsymbol{a}$$

Finding the vector **a**



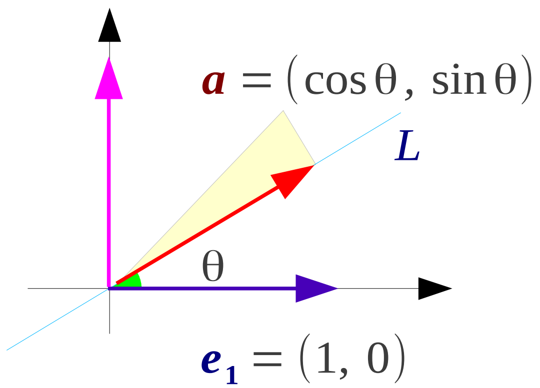
vector component of $\mathbf{e}_1$ along $\mathbf{a}$

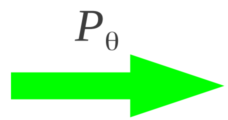


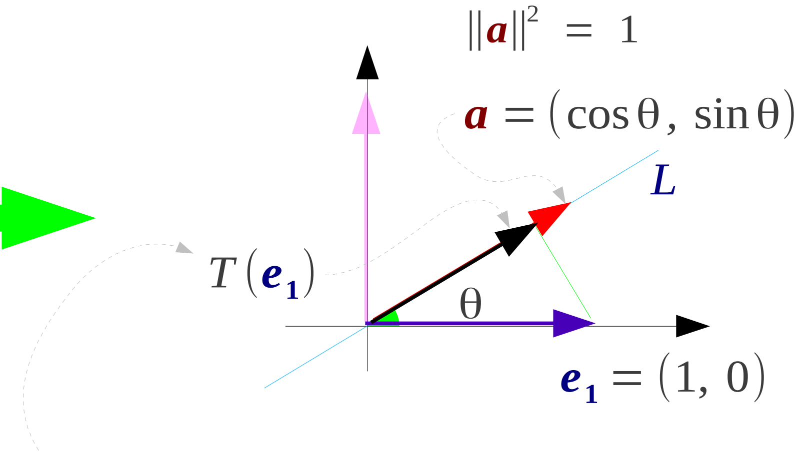
vector component of $\mathbf{e}_2$ along $\mathbf{a}$

Projection onto the Lines Through Zero (3)

Finding the projection of the unit vectors







vector component of **e$_1$** along **a**

$$proj_{\boldsymbol{a}}\boldsymbol{u} = \frac{\boldsymbol{u}\cdot\boldsymbol{a}}{\|\boldsymbol{a}\|^2}\ \boldsymbol{a}$$

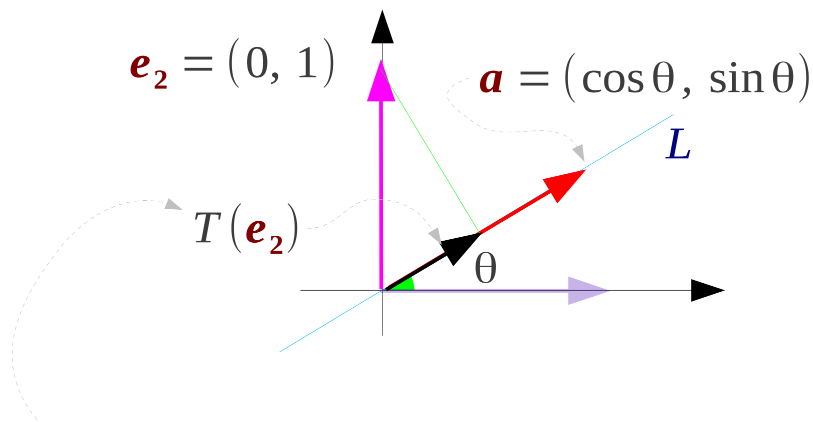
$$\|\boldsymbol{a}\|^2 = \cos^2\theta + \sin^2\theta = 1$$

$$\boldsymbol{e}_1\cdot\boldsymbol{a} = (1,\ 0)\cdot(\cos\theta,\ \sin\theta) = \cos\theta$$

$$T(\boldsymbol{e}_1) = \frac{\boldsymbol{e}_1\cdot\boldsymbol{a}}{\boldsymbol{a}}\ \boldsymbol{a} = (\cos^2\theta,\ \cos\theta\sin\theta)$$

$$\boldsymbol{e}_1\cdot\boldsymbol{a} = (0,\ 1)\cdot(\cos\theta,\ \sin\theta) = \sin\theta$$

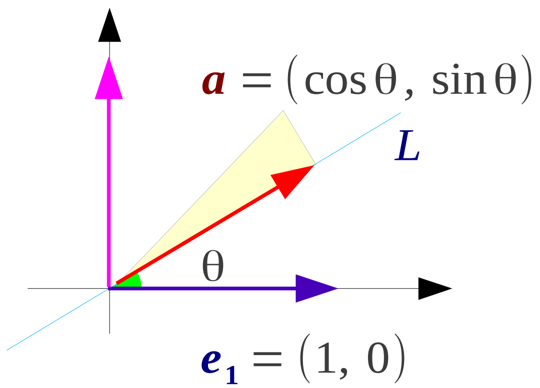
$$T(\boldsymbol{e}_2) = \frac{\boldsymbol{e}_2\cdot\boldsymbol{a}}{\boldsymbol{a}}\ \boldsymbol{a} = (\cos\theta\sin\theta,\ \sin^2\theta)$$

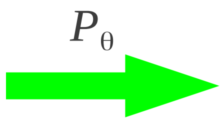


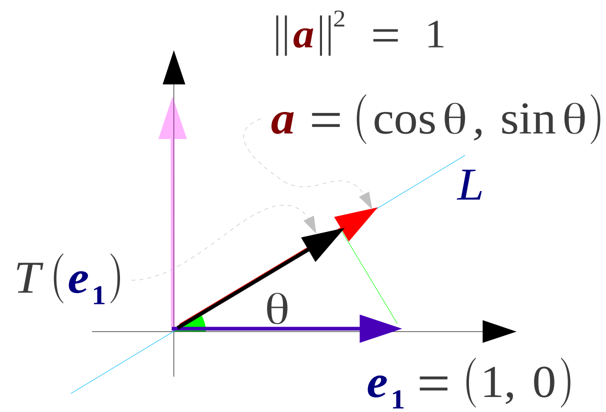
vector component of **e$_2$** along **a**

Projection onto the Lines Through Zero (3)

Finding the projection of the unit vectors







vector component of **e$_1$** along **a**

$$proj_{\boldsymbol{a}}\boldsymbol{u} = \frac{\boldsymbol{u}\cdot\boldsymbol{a}}{\|\boldsymbol{a}\|^2}\,\boldsymbol{a}$$

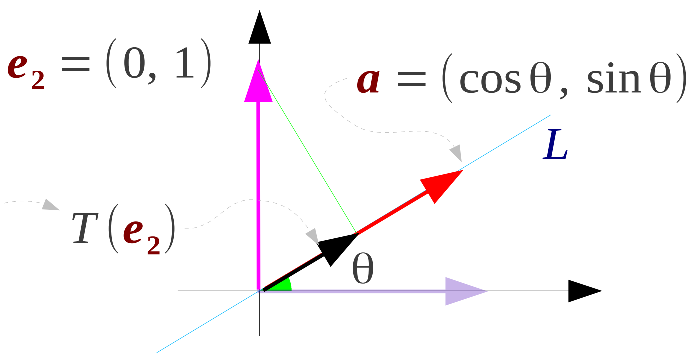
$$\|\boldsymbol{a}\|^2 = \cos^2\theta + \sin^2\theta = 1$$

$$\boldsymbol{e}_1\cdot\boldsymbol{a} = (1,\ 0)\cdot(\cos\theta,\ \sin\theta) = \cos\theta$$

$$T(\boldsymbol{e}_1) = \frac{\boldsymbol{e}_1\cdot\boldsymbol{a}}{\boldsymbol{a}}\,\boldsymbol{a} = (\cos^2\theta,\ \cos\theta\sin\theta)$$

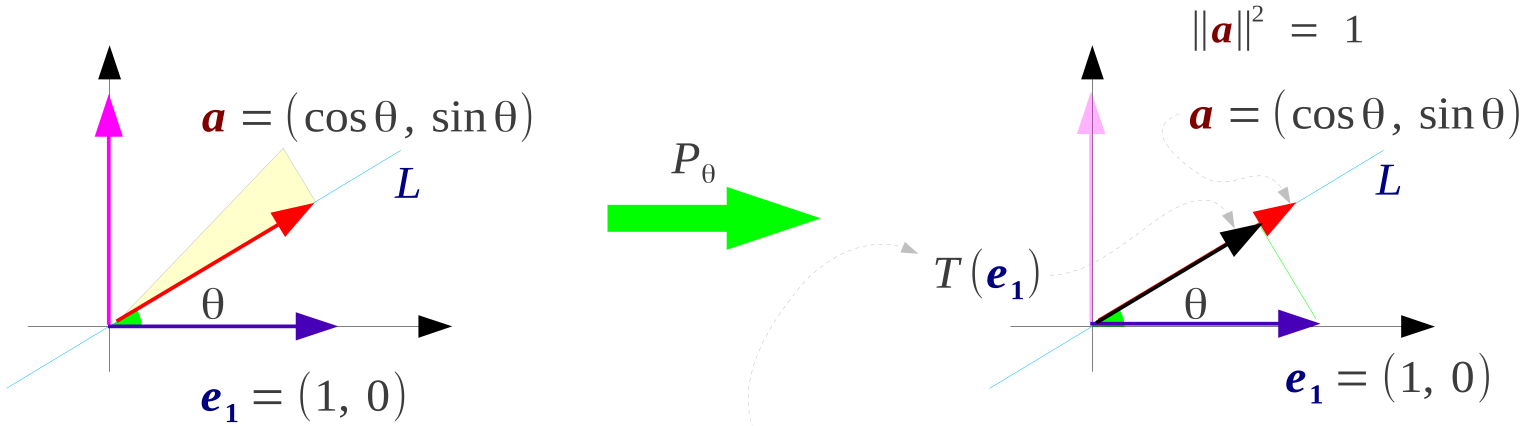
$$\boldsymbol{e}_1\cdot\boldsymbol{a} = (0,\ 1)\cdot(\cos\theta,\ \sin\theta) = \sin\theta$$

$$T(\boldsymbol{e}_2) = \frac{\boldsymbol{e}_2\cdot\boldsymbol{a}}{\boldsymbol{a}}\,\boldsymbol{a} = (\cos\theta\sin\theta,\ \sin^2\theta)$$



vector component of **e$_2$** along **a**

The Standard Matrix

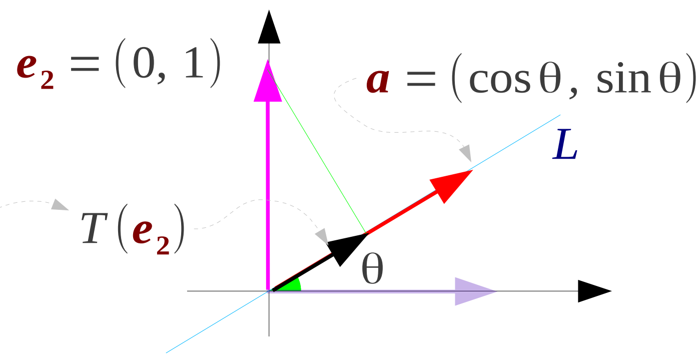


vector component of $\mathbf{e}_1$ along **a**

$$[T] = [T(\boldsymbol{e}_1) \mid T(\boldsymbol{e}_2)]$$

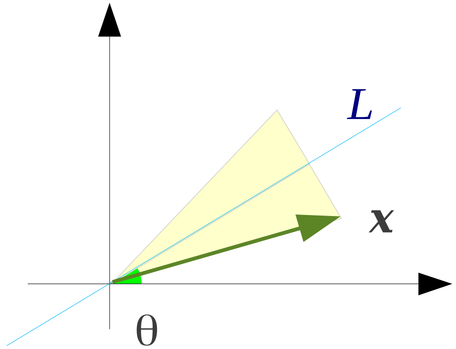
$$= \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

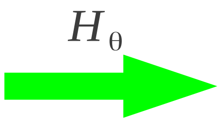
$$= \begin{bmatrix} \cos^2\theta & \frac{1}{2}\sin 2\theta \\ \frac{1}{2}\sin 2\theta & \sin^2\theta \end{bmatrix} = \boldsymbol{P}_\theta$$

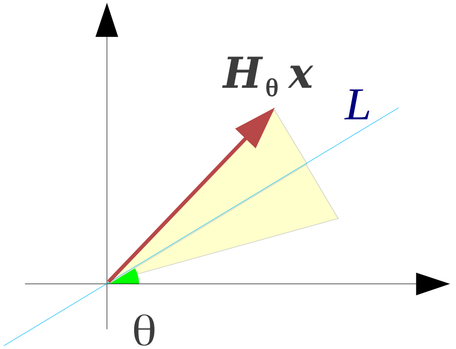


vector component of $\mathbf{e}_2$ along **a**

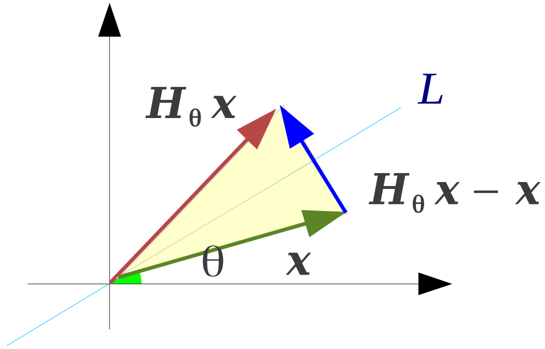
Reflections About the Lines Through Zero (1)

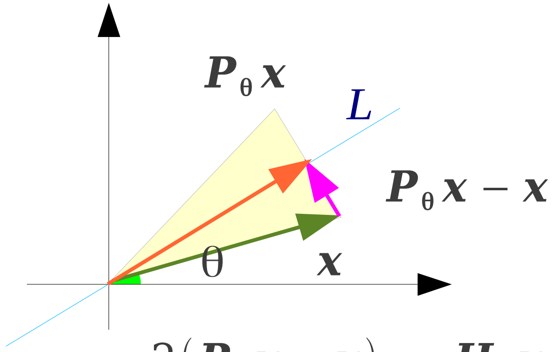






line is represented by a vector **a**



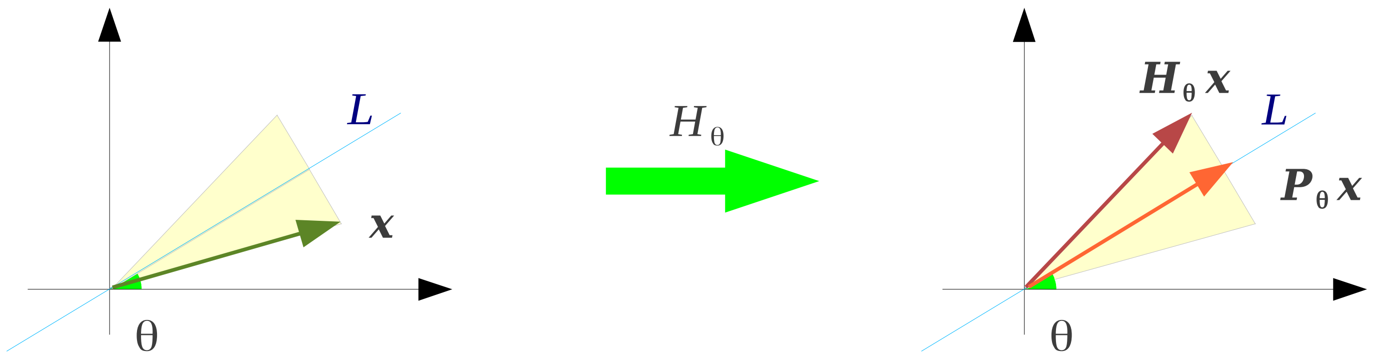


$$2(\boldsymbol{P}_\theta \boldsymbol{x} - \boldsymbol{x}) = \boldsymbol{H}_\theta \boldsymbol{x} - \boldsymbol{x}$$

$$2\boldsymbol{P}_\theta \boldsymbol{x} - \boldsymbol{x} = \boldsymbol{H}_\theta \boldsymbol{x}$$

$$(2\boldsymbol{P}_\theta \boldsymbol{x} - \boldsymbol{I})\boldsymbol{x} = \boldsymbol{H}_\theta \boldsymbol{x}$$

Reflections About the Lines Through Zero (2)



$$(2\boldsymbol{P}_\theta \boldsymbol{x} - \boldsymbol{I})\boldsymbol{x} = \boldsymbol{H}_\theta \boldsymbol{x}$$

$$\boldsymbol{P}_\theta = \begin{bmatrix} \cos^2\theta & \frac{1}{2}\sin 2\theta \\ \frac{1}{2}\sin 2\theta & \sin^2\theta \end{bmatrix}$$

$$2\boldsymbol{P}_\theta - \boldsymbol{I} = \begin{bmatrix} 2\cos^2\theta - 1 & \sin 2\theta \\ \sin 2\theta & 2\sin^2\theta - 1 \end{bmatrix}$$

$$\boldsymbol{H}_0 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$$

References

[1] http://en.wikipedia.org/
[2] Anton, et al., Elementary Linear Algebra, 10th ed, Wiley, 2011
[3] Anton, et al., Contemporary Linear Algebra,