Cross Power Density Spectrum and Cross-Correlation Function

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

Definition

$$S_{XY}(\omega) = \int_{-\infty}^{+\infty} \left\{ \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} R_{XY}(t, t+\tau)dt \right\} e^{-j\omega\tau} d\tau$$

Definition

$$X_T(\omega) = \int_{-T}^{+T} X(t) e^{-j\omega t} dt$$

$$Y_T(\omega) = \int_{-T}^{+T} Y(t_1) e^{-j\omega t_1} dt_1$$

$$X_T^*(\omega) Y_T(\omega) = \int_{-T}^{+T} X(t) e^{+j\omega t} dt \int_{-T}^{+T} Y(t_1) e^{-j\omega t_1} dt_1$$

Definition

$$S_{XY}(\omega) = \lim_{T\to\infty} \frac{1}{2T} E[X_T^*(\omega) Y_T(\omega)]$$

$$= \lim_{T\to\infty} \frac{1}{2T} E\left[\int_{-T}^{+T} X(t) e^{+j\omega t} dt \int_{-T}^{+T} Y(t_1) e^{-j\omega t_1} dt_1\right]$$

$$= \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{+T} \int_{-T}^{+T} R_{XY}(t, t_1) e^{-j\omega(t_1 - t)} dt dt_1$$