

# Cross Power Density Spectrum and Cross-Correlation Function

Young W Lim

November 12, 2019

Copyright (c) 2018 Young W. Lim. Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

This work is licensed under a Creative Commons "Attribution-NonCommercial-ShareAlike 3.0 Unported" license.



Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles, Jr. and B. Shi



# Cross Power Spectrum and Cross Correlation

$N$  Gaussian random variables

## Definition

$$S_{XY}(\omega) = \int_{-\infty}^{+\infty} \left\{ \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} R_{XY}(t, t + \tau) dt \right\} e^{-j\omega\tau} d\tau$$

# Two truncated processes

$N$  Gaussian random variables

## Definition

$$X_T(\omega) = \int_{-T}^{+T} X(t) e^{-j\omega t} dt$$

$$Y_T(\omega) = \int_{-T}^{+T} Y(t_1) e^{-j\omega t_1} dt_1$$

$$X_T^*(\omega) Y_T(\omega) = \int_{-T}^{+T} X(t) e^{+j\omega t} dt \int_{-T}^{+T} Y(t_1) e^{-j\omega t_1} dt_1$$

# Cross Power Density Spectrum

$N$  Gaussian random variables

## Definition

$$\begin{aligned} S_{XY}(\omega) &= \lim_{T \rightarrow \infty} \frac{1}{2T} E[X_T^*(\omega) Y_T(\omega)] \\ &= \lim_{T \rightarrow \infty} \frac{1}{2T} E \left[ \int_{-T}^{+T} X(t) e^{+j\omega t} dt \int_{-T}^{+T} Y(t_1) e^{-j\omega t_1} dt_1 \right] \\ &= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} \int_{-T}^{+T} R_{XY}(t, t_1) e^{-j\omega(t_1-t)} dt dt_1 \end{aligned}$$



