Sampling Basics(1A)

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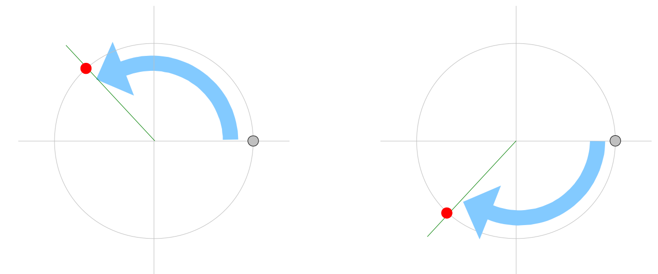
Measuring Rotation Rate

Angular Speed (Frequency)

$$\omega = \frac{2\pi}{T} \qquad \frac{\textit{angular displacement}}{\textit{time}}$$

$$\omega = 2\pi f \qquad 2\pi \times \textit{linear frequency}$$

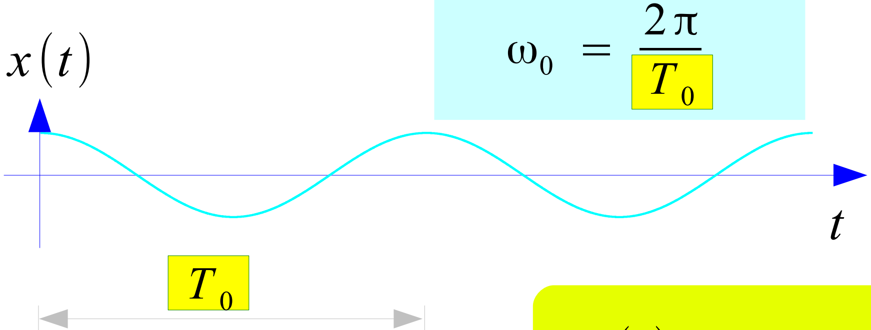
For 1 second | For 1 second



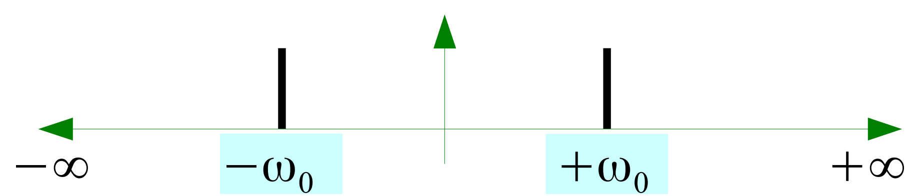
$+\omega_0 \ (rad/sec)$ | $-\omega_0 \ (rad/sec)$

Time Domain

$$\omega_0 = \frac{2\pi}{T_0}$$



Frequency Domain

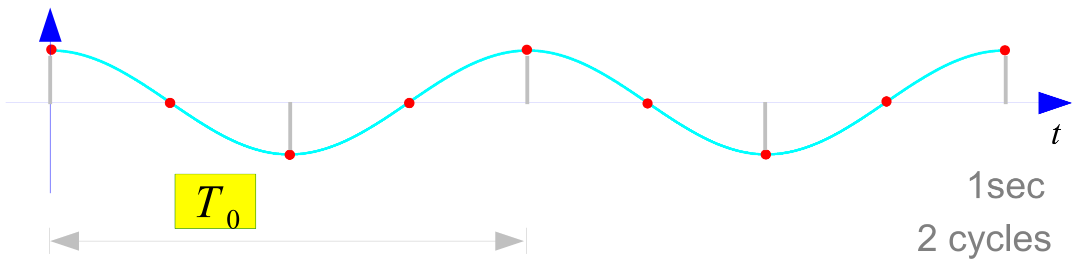


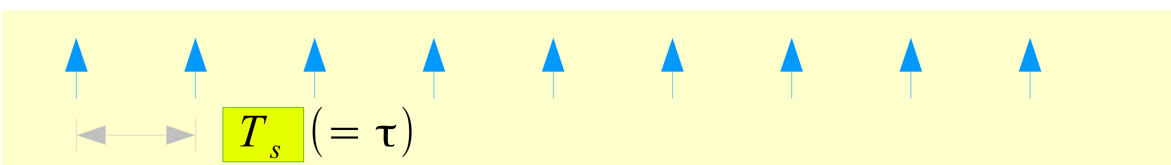
$$x(t) = A\cos(\omega_0 t) = \frac{A}{2}e^{j\omega_0 t} + \frac{A}{2}e^{-j\omega_0 t}$$

Sampling

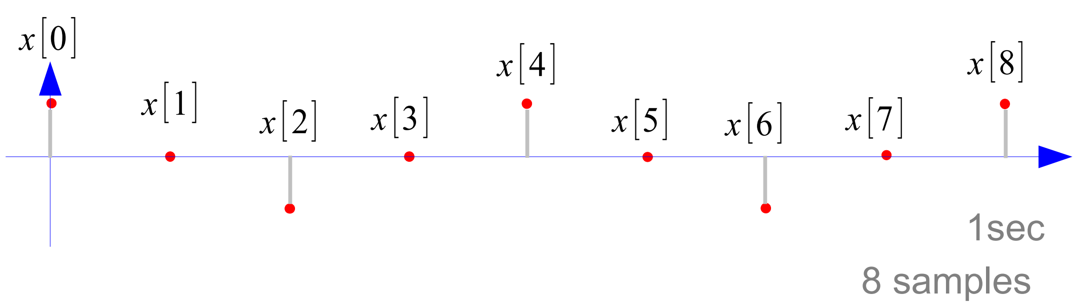
continuous-time signals

$$x(t) = A\cos(\omega_0 t)$$





discrete-time sequence



Sampling Time

$$T_s \ (= \tau)$$

Sequence Time Length

$$T = N \cdot T_s$$

Sampling Frequency

$$f_s = \frac{1}{T_s}$$ ***(samples/sec)***

Signal's Frequency

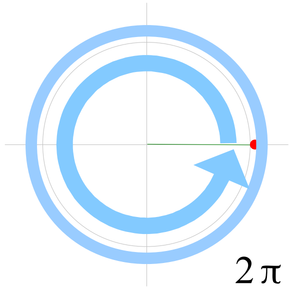
$$f_0 = \frac{1}{T_0}$$ ***(cycles/sec)***

Angular Frequencies in Sampling

continuous-time signals

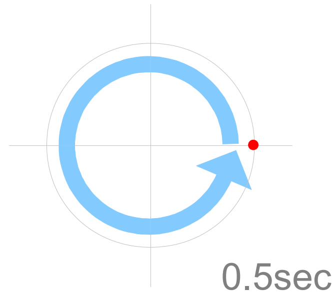
For 1 second

$\omega_0 = 2\pi f_0 \ (rad/sec)$

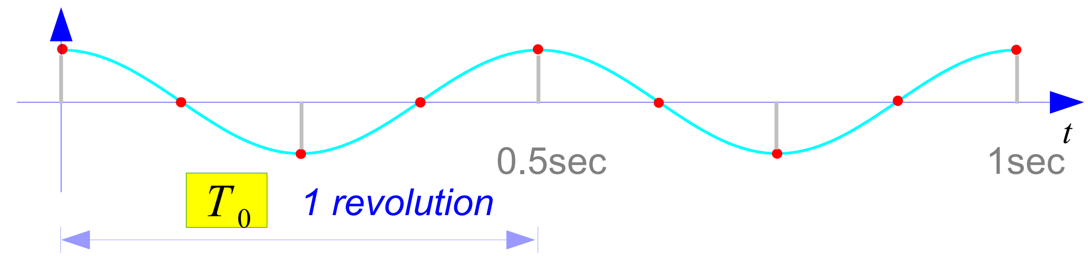


For 1 revolution

$2\pi \ (rad) \ / \ T_0 \ (sec)$



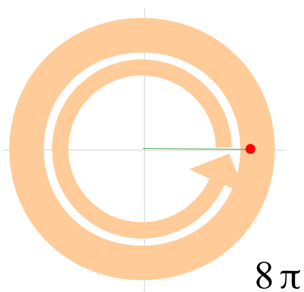
$x(t) = A\cos(\omega_0 t)$



sampling sequence

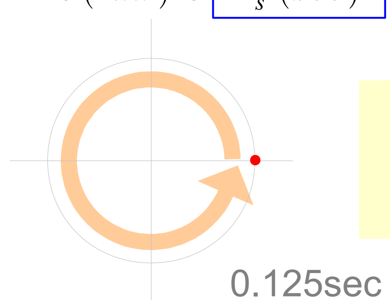
For 1 second

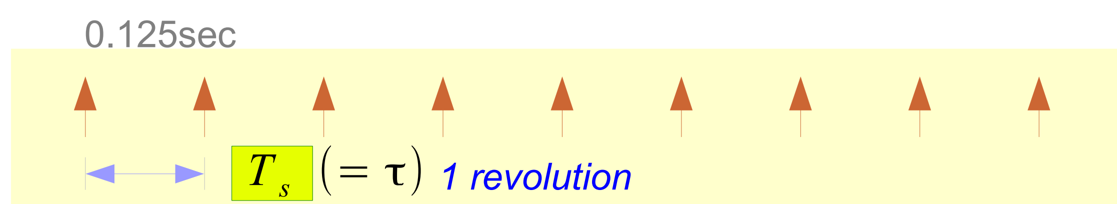
$\omega_s = 2\pi f_s \ (rad/sec)$



For 1 revolution

$2\pi \ (rad) \ / \ T_s \ (sec)$





Sampling of Sinusoid Functions

$$x(t) = A\cos(\omega t + \phi)$$

$$t \rightarrow nT_s$$

$$x[n] = x(nT_s)$$

$$= A\cos(\omega \cdot nT_s + \phi)$$

$$= A\cos(\omega \cdot T_s n + \phi)$$

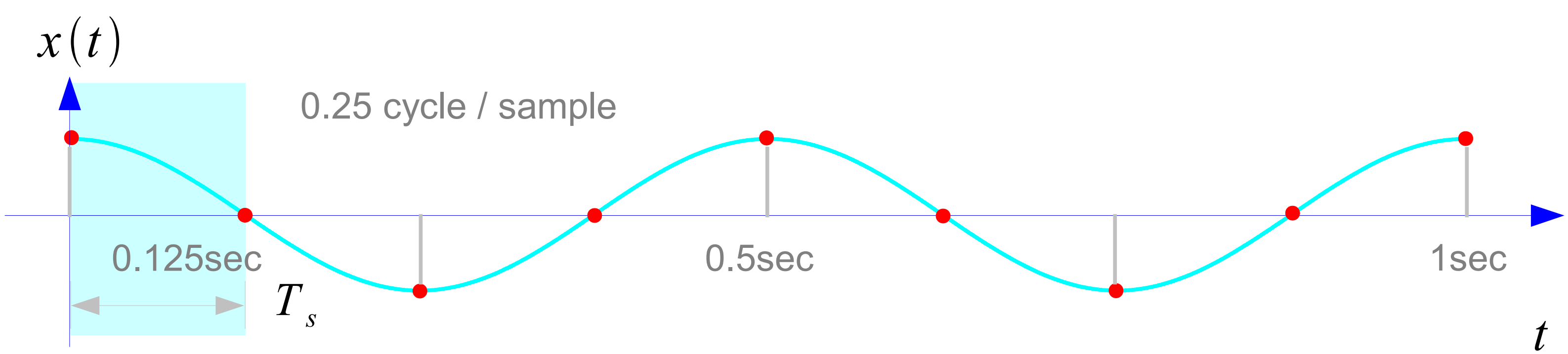
$$= A\cos(\hat{\omega} \cdot n + \phi)$$

$$\hat{\omega} = \omega \cdot T_s = \frac{\omega}{1/T_s}$$

$$\hat{\omega} = \frac{\omega}{f_s} = 2\pi\frac{f}{f_s}$$

Normalized to f_s

Normalized Radian Frequency

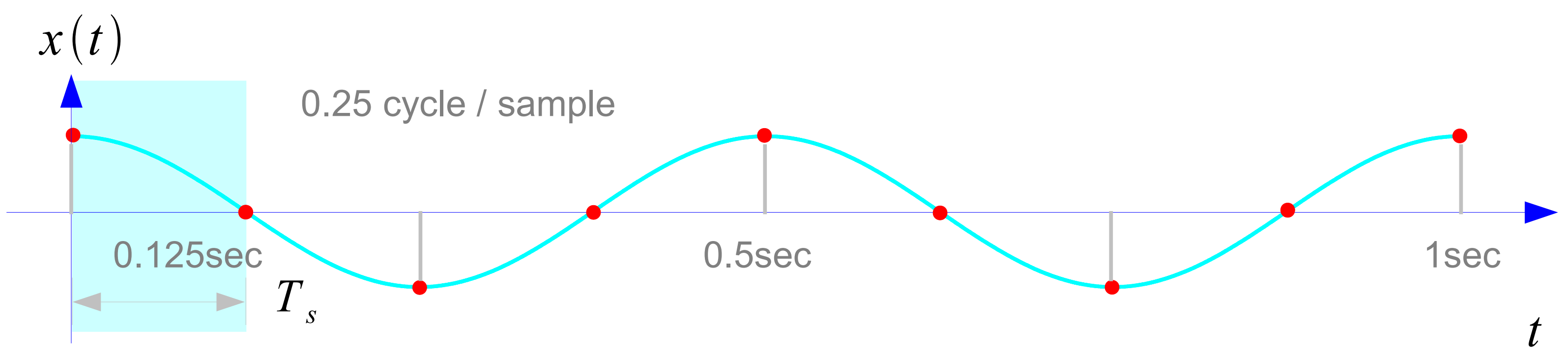


Normalized Radian Frequency (1)

continuous-time signals	Sampling	discrete-time sequence
$x(t)$	$t \rightarrow nT_s$	$x[n] = x(nT_s)$

Angular Frequency	Sampling	Normalized Radian Frequency
$\omega \quad (rad/sec)$	$\times T_s$	$\hat{\omega} = \omega \cdot T_s \quad (rad/sample)$

X Sampling Time



$\hat{\omega} \quad (rad)$

discrete-time sequence

Normalized Radian Frequency (2)

Normalized Frequency

$$\frac{f_0}{f_s} \quad \frac{(cycle / sec)}{(sample / sec)} \Rightarrow \frac{f_0}{f_s} \quad (cycle / sample)$$

Normalized Radian Frequency

$$2\pi \frac{(rad)}{(cycle)} \cdot \frac{f_0}{f_s} \frac{(cycle)}{(sample)} \Rightarrow \frac{\omega_0}{f_s} \quad (rad / sample)$$

Signal's relative angle position after each of T_s second

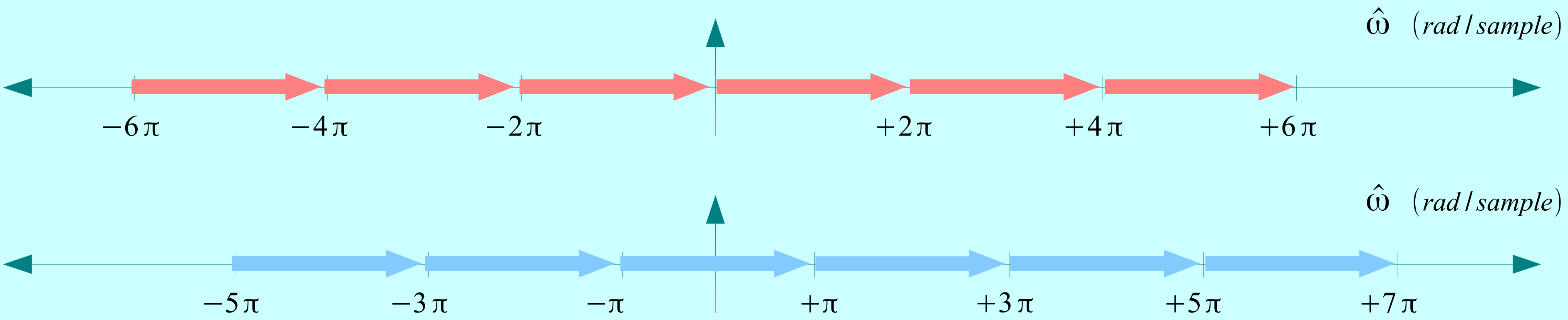
$$\hat{\omega} = \omega T_s$$

Normalized Radian Frequency
can be viewed as
"the angular displacement of a signal during the period of its sample time T_s"

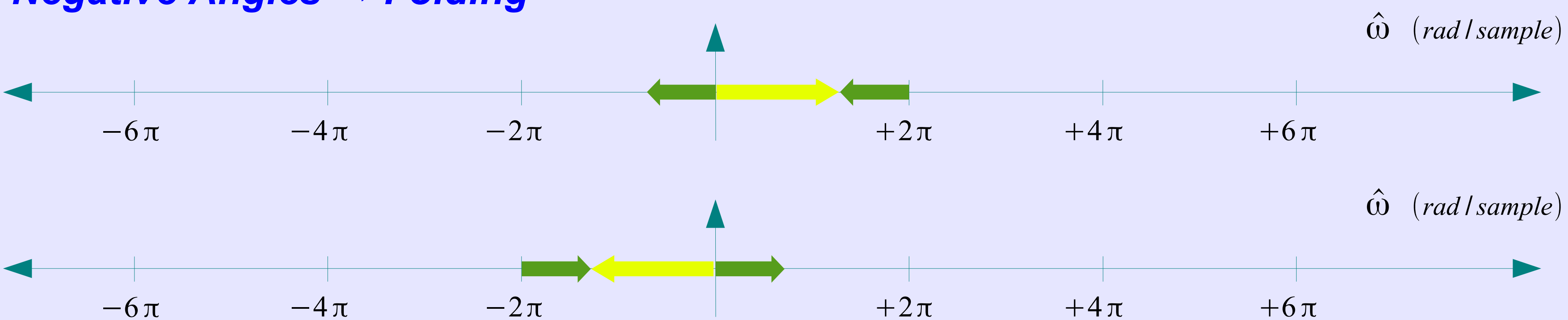
- Negative Angles
  → folding
- Co-terminal Angles
  → periodic

Periodic and Folding

Co-terminal Angles → Periodic



Negative Angles → Folding



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A “graphical interpretation” of the DFT and FFT, y Steve Mann