E0 transitions:
where we have been,
where we are going,
where we would like to go

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E0: transition operator and matrix element --a model *independent* description

E0 transition strengths are a measure of the off-diagonal matrix elements of the mean-square charge radius operator.

Mixing of configurations with different mean-square charge radii produces E0 transition strength.

$$\rho^2(E0) = \frac{1}{\Omega\,\tau(E0)}$$

"Electronic factor"

$$\Omega = \Omega(Z,\Delta E) = \Omega_K + \Omega_{L_1} + \dots + \Omega_{e^+e^-}$$

Monopole strength parameter

$$\rho_{if}(E0) = \frac{\langle f|\sum_j e_j r_j^2|i\rangle}{eR^2} \equiv \frac{\langle f|m(E0)|i\rangle}{eR^2} \equiv \frac{M_{if}(E0)}{eR^2}$$

$$|i\rangle = \alpha|1\rangle + \beta|2\rangle\,, \quad |f\rangle = -\beta|1\rangle + \alpha|2\rangle$$

$$M_{if}(E0) = \alpha\beta\left\{\langle 2|m(E0)|2\rangle - \langle 1|m(E0)|1\rangle\right\} + (\alpha^2-\beta^2)\langle 1|m(E0)|2\rangle$$

$$\boxed{M_{if}(E0) \simeq \alpha\beta\,\Delta\langle r^2\rangle}$$

Ω values: http://bricc.anu.edu.au

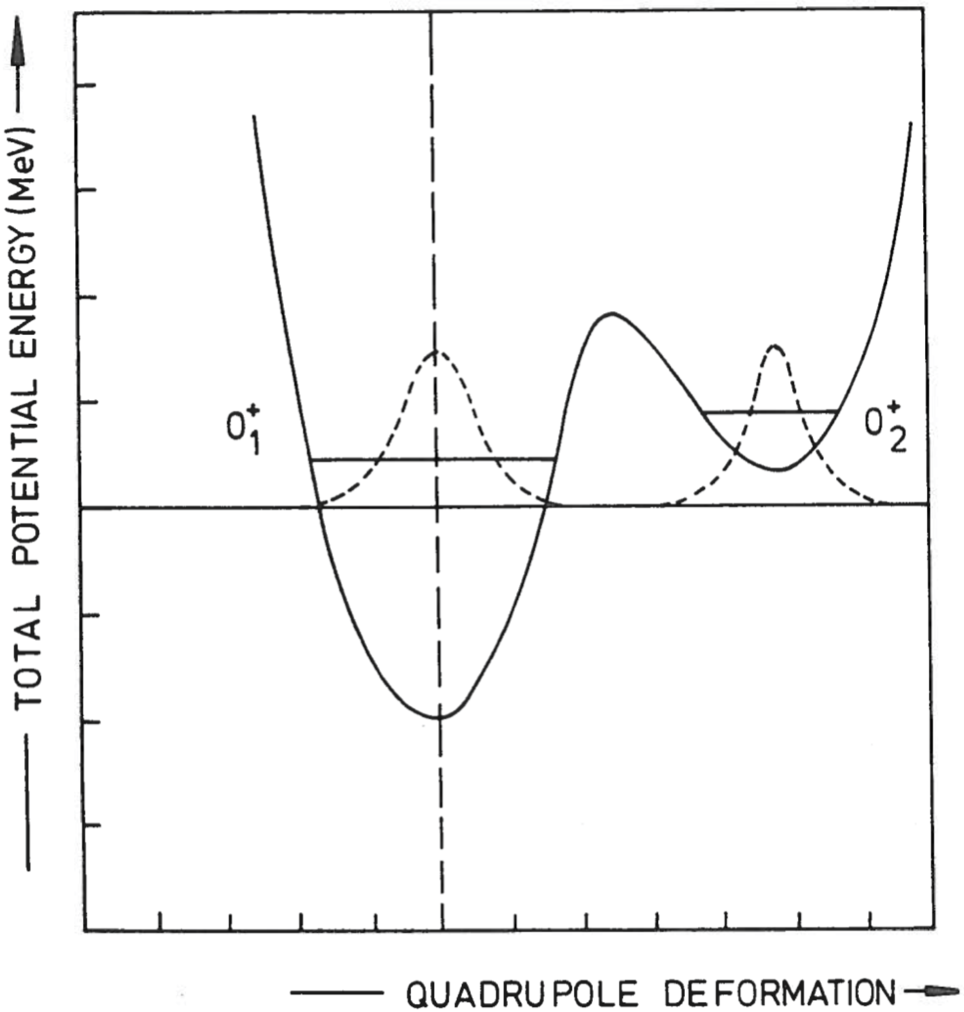
τ: partial lifetime for E0 decay branch

J. Kantele et al. Z. Phys. A289 157 1979
and see
JLW et al. Nucl. Phys. A651 323 1999

Comments on model-motivated research

- Research should be pursued to falsify models, not to promote them
- When a model fails, we have learned something—recall that failure of the Standard Model of particles and fields is being sought, avidly
- Models provide powerful schemes for organizing data

Wave functions must overlap for a transition to occur



solid line—schematic potential

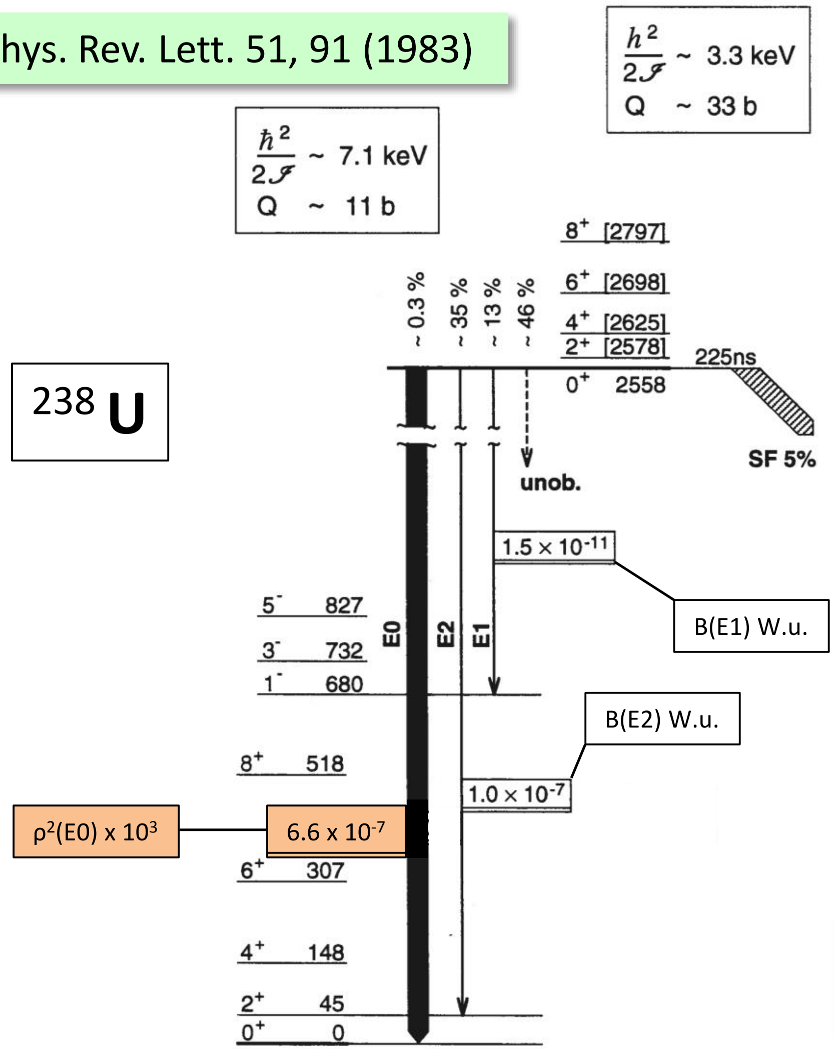
dashed line—schematic wave function

In the earlier literature there are some serious misconceptions on this point

Figure from JLW et al., Nucl. Phys. A651 323 1999

E0 transition between states with very different deformations and mean-square charge radii

J. Kantele et al., Phys. Rev. Lett. 51, 91 (1983)



The E0 strength from the ^{238}U "fission" isomer is the weakest known

T. Kibédi and R.H. Spear, ADNDT 89 77 2005

Figure from JLW et al., Nucl. Phys. A651 323 1999

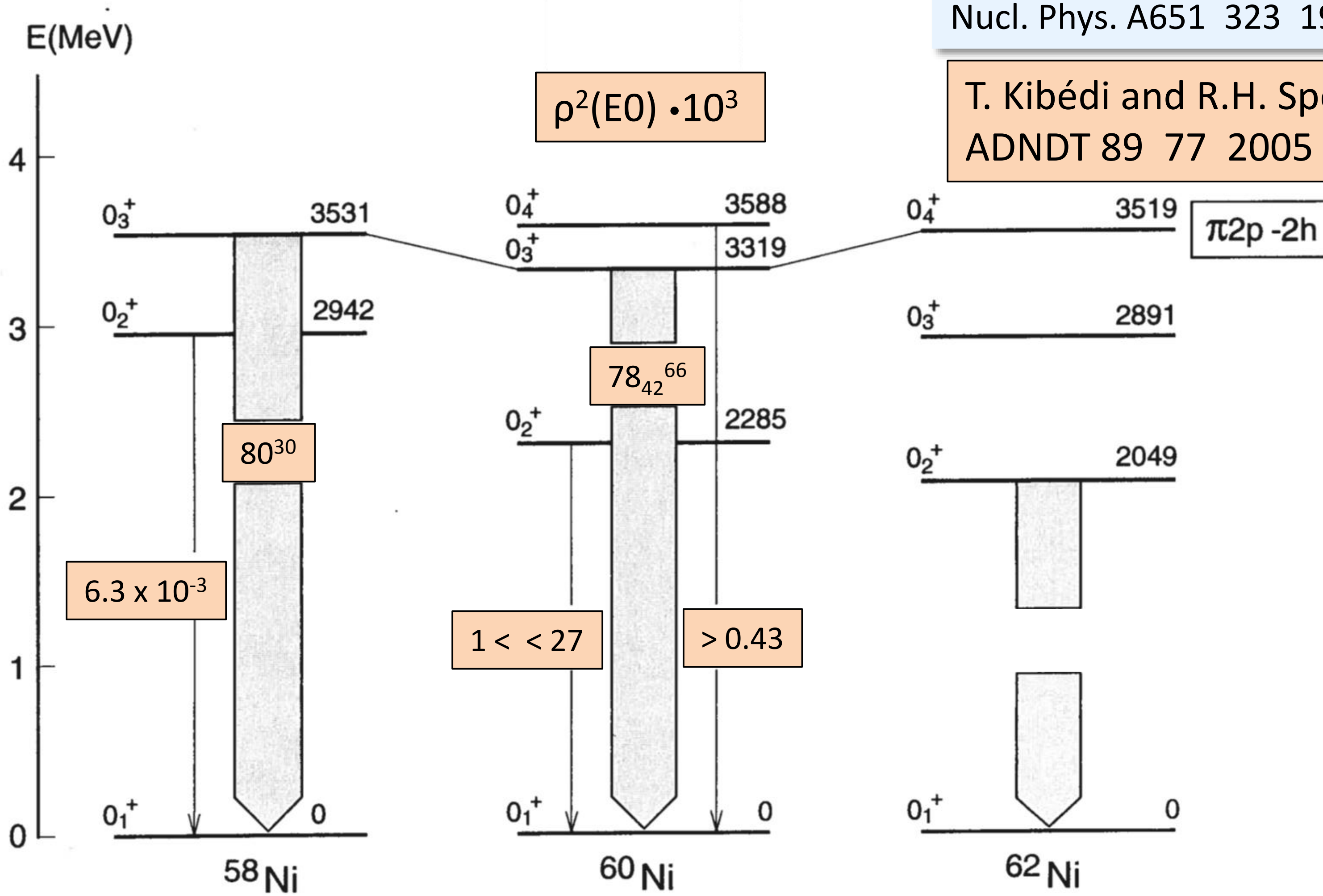
E0 transitions in the light Ni isotopes:

the $0_2 \rightarrow 0_1$ strength in ^{58}Ni is very small indicating near-pure neutron configurations are involved ($e_n = 0$)

π2p-2h from 2-, 4-proton transfer RX

Figure from JLW et al., Nucl. Phys. A651 323 1999

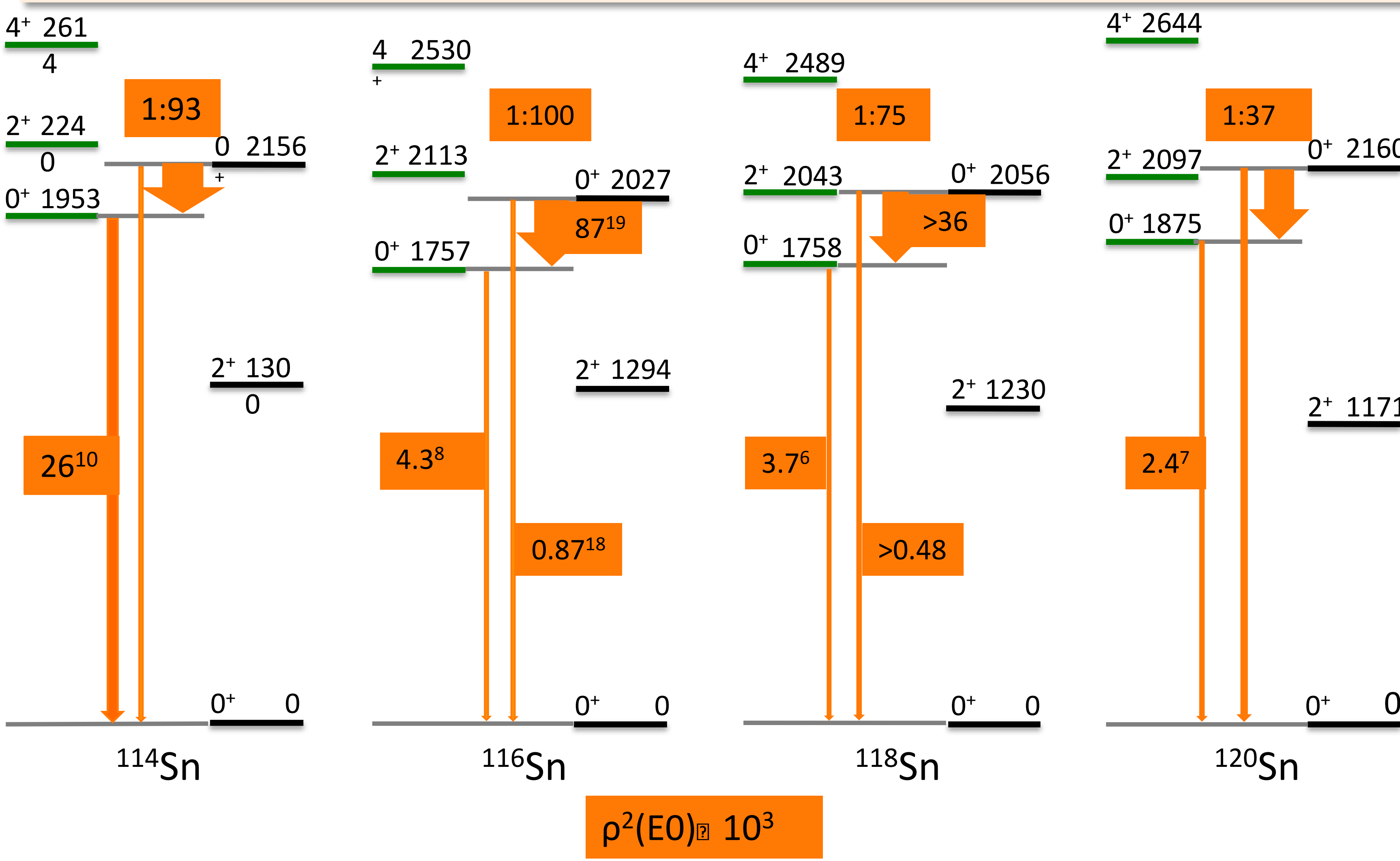
T. Kibédi and R.H. Spear, ADNDT 89 77 2005



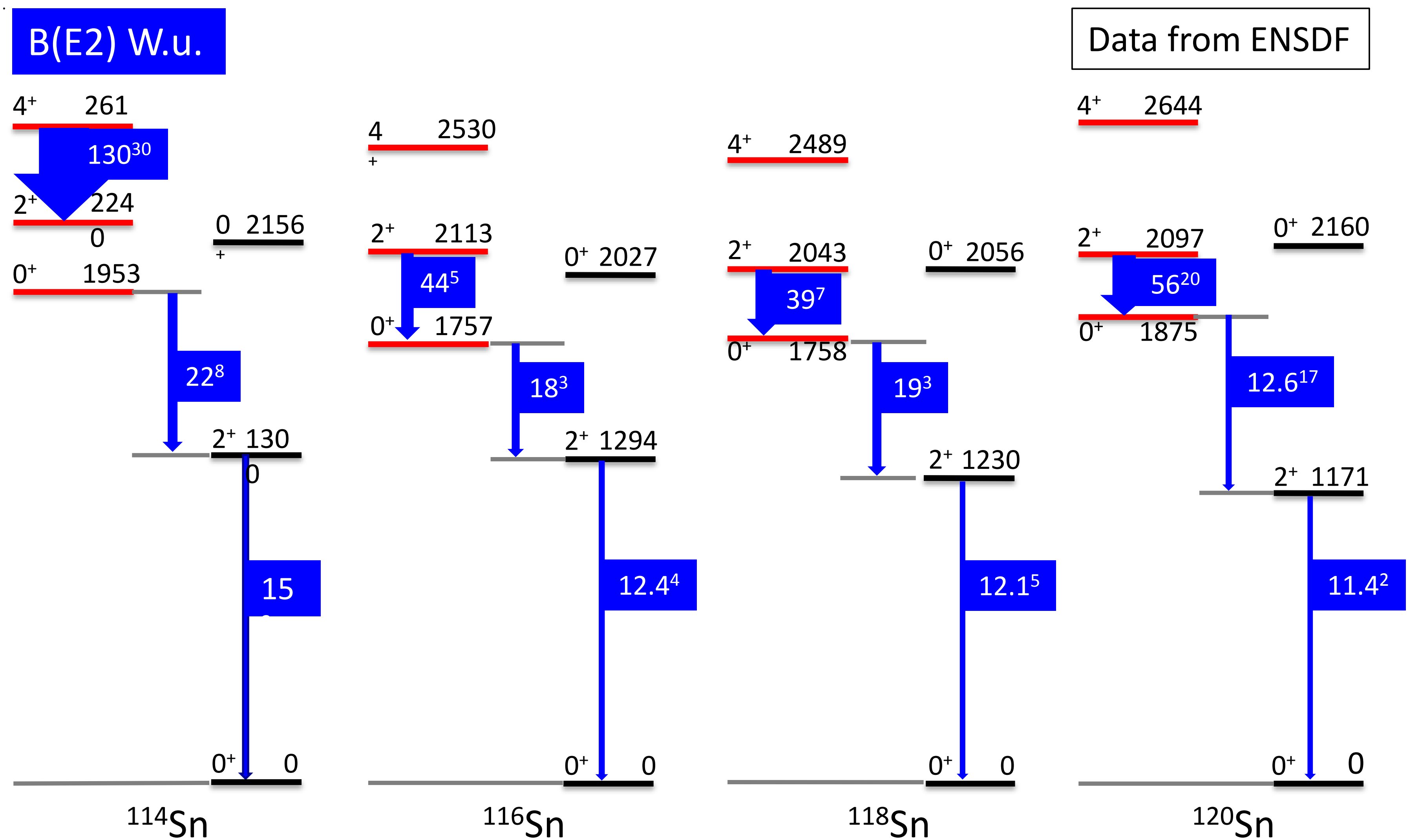
E0 transitions associated with shape coexistence in $^{114-120}Sn$

J. Kantele et al., ZP A289, 157 (1979) T. Kibedi and R.H. Spear, ADNDT 89, 277 (2005)

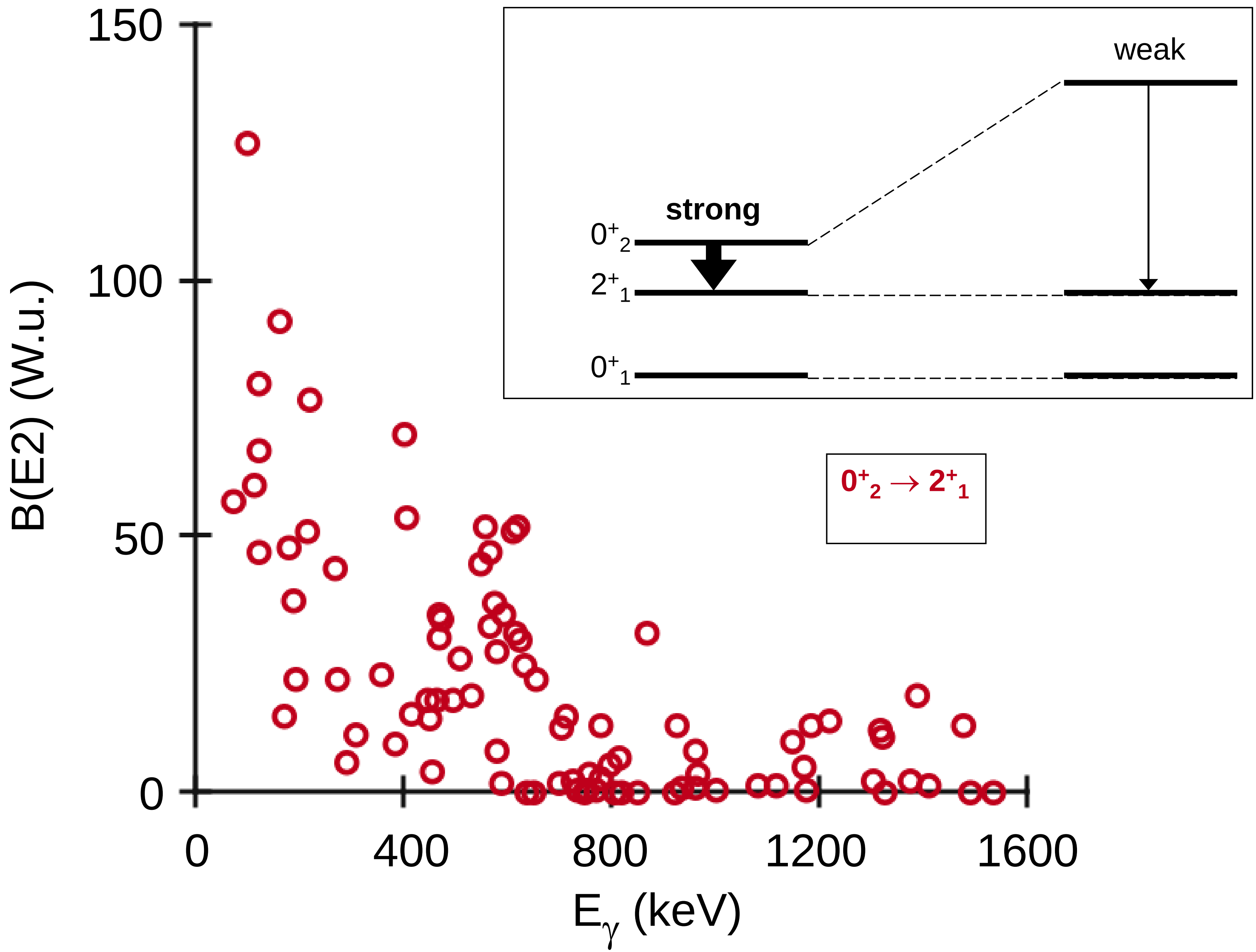
Mixing of close lying configurations with different mean-square charge radii produces E0 strength



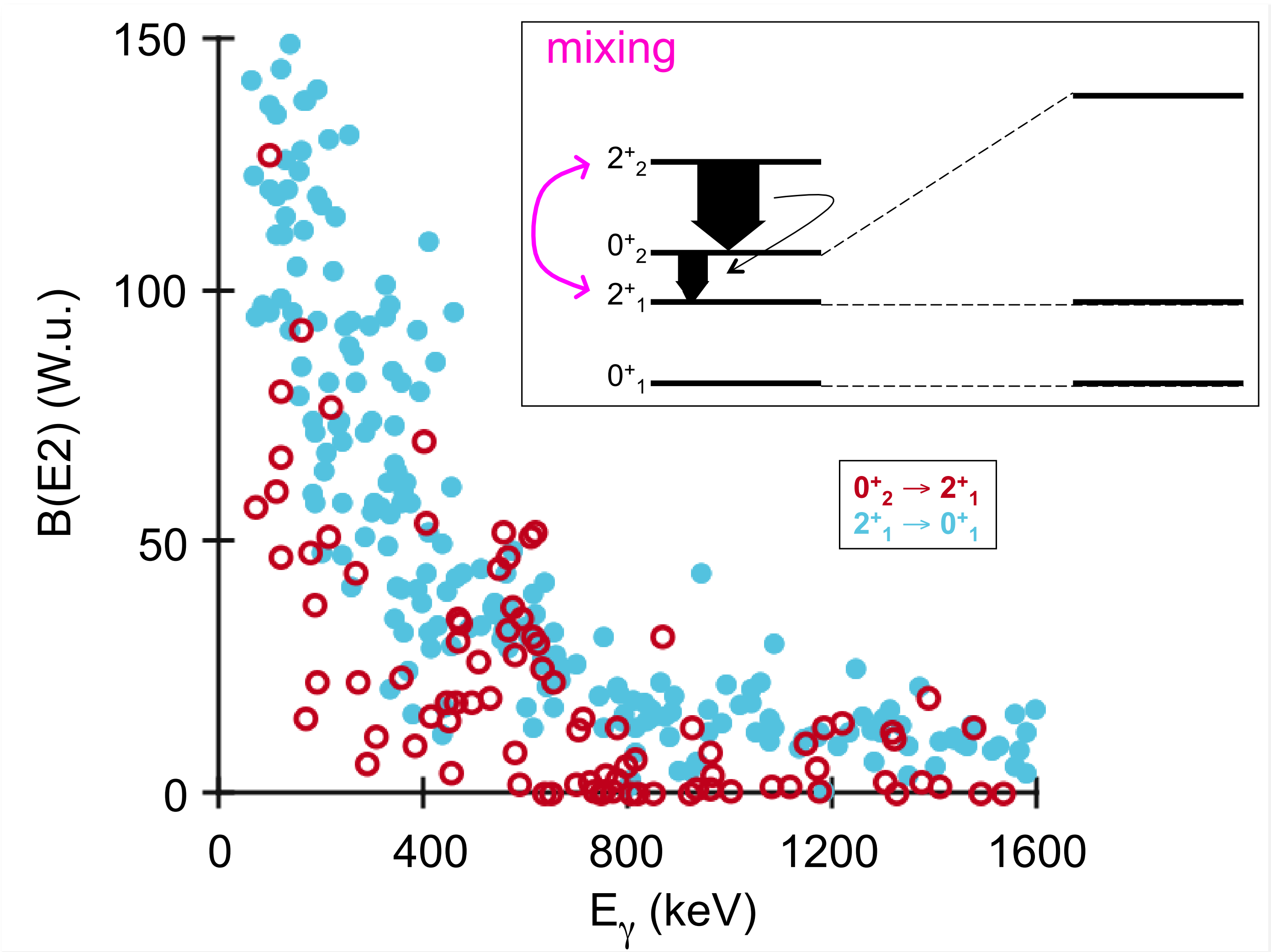
E2 transitions associated with shape coexistence in $^{114-120}Sn$



Systematics of B(E2; $0^+_2 \rightarrow 2^+_1$) vs. E_γ ($0^+_2 - 2^+_1$)



$B(E2; 0_2^+ \rightarrow 2_1^+)$ vs. $E(0_2^+) - E(2_1^+)$: coexistence and mixing yields $B(E2; 0_2^+ \rightarrow 2_1^+) \sim \alpha^2 \beta^2 (\Delta Q)^2$

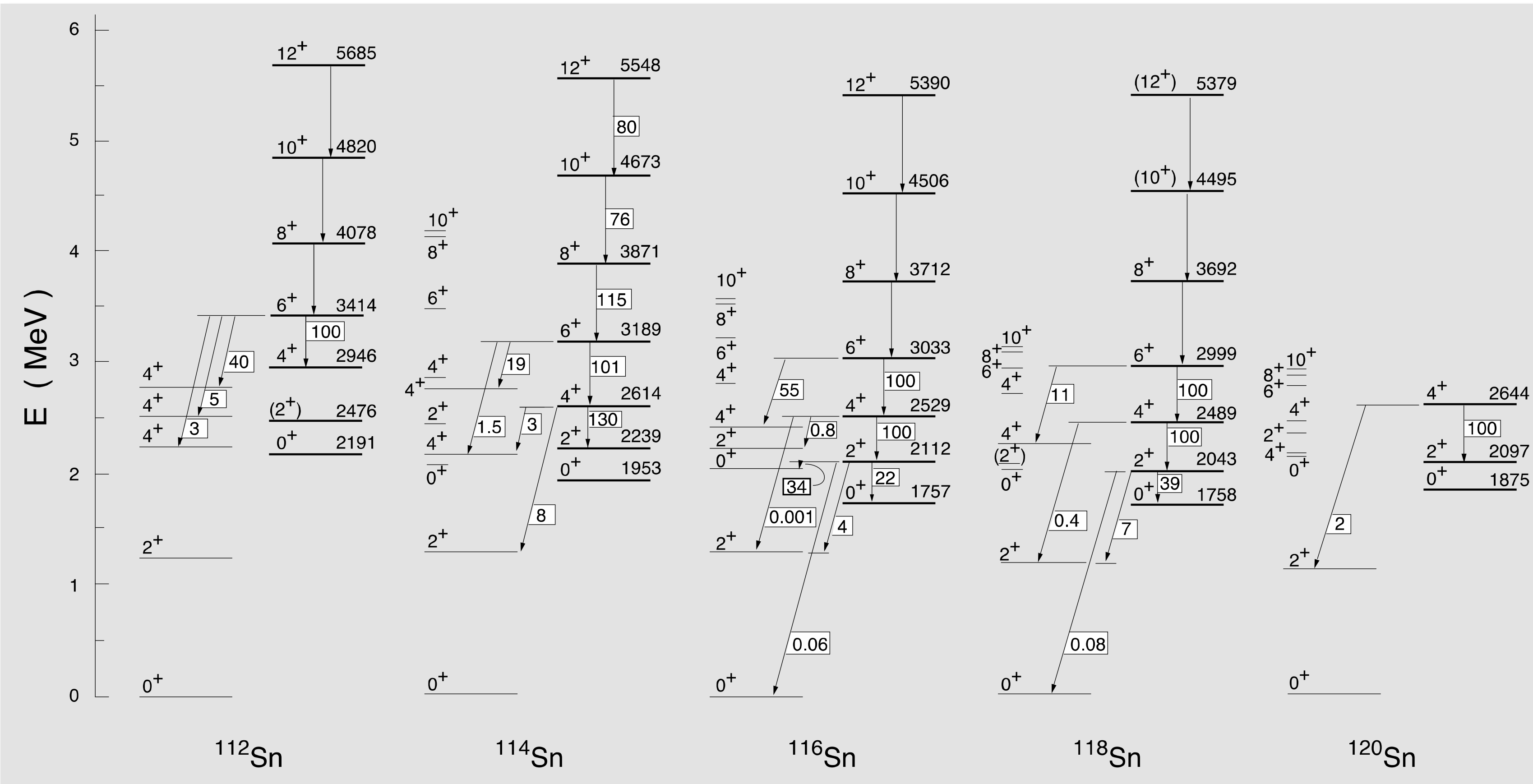


Recall:
$B_{02} = 5 \times B_{20}$

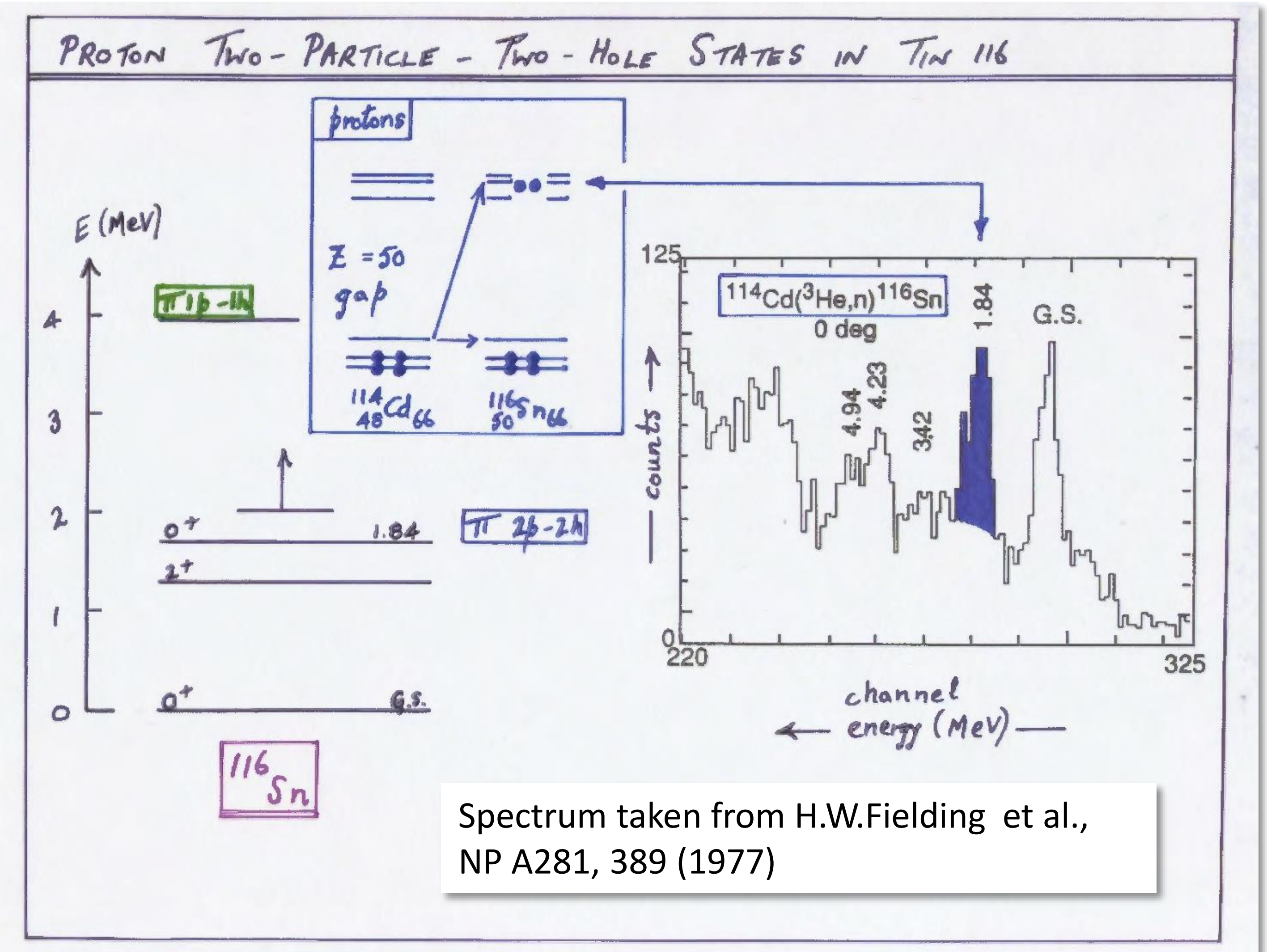
Deformed bands in $^{112-120}$Sn built on the first excited 0^+ states

Figure from Rowe & Wood

B(E2)'s in W.u. [100 = rel. value]

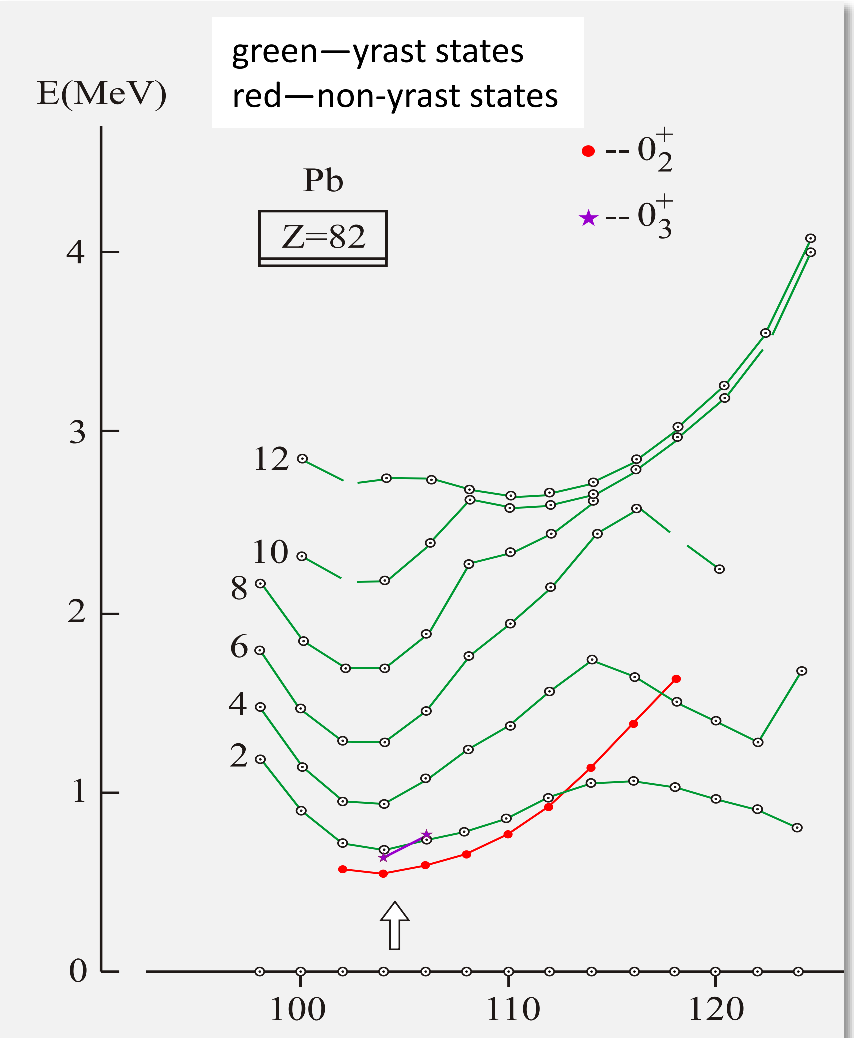


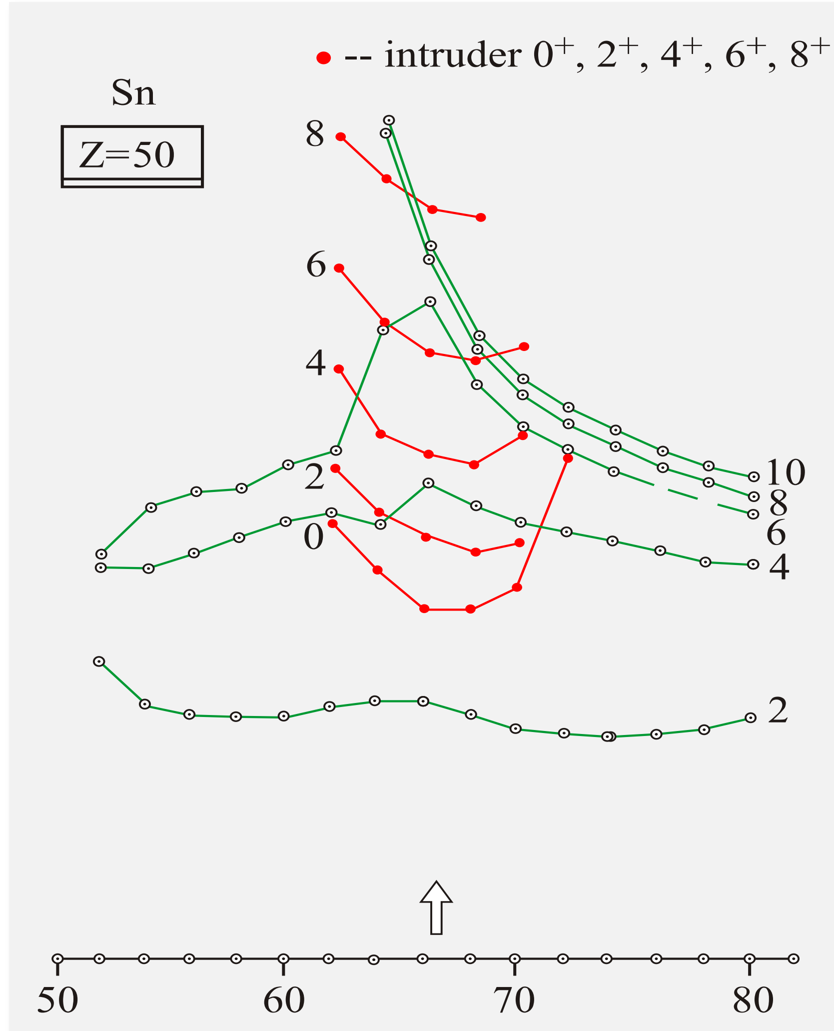
The nature of the shape coexisting state in ^{116}Sn revealed by (^{3}He,n) transfer reaction spectroscopy



Spectrum taken from H.W.Fielding et al., NP A281, 389 (1977)

Excited 0⁺ states at closed shells: intruder states in the Pb and Sn isotopes





mid shell

neutron number

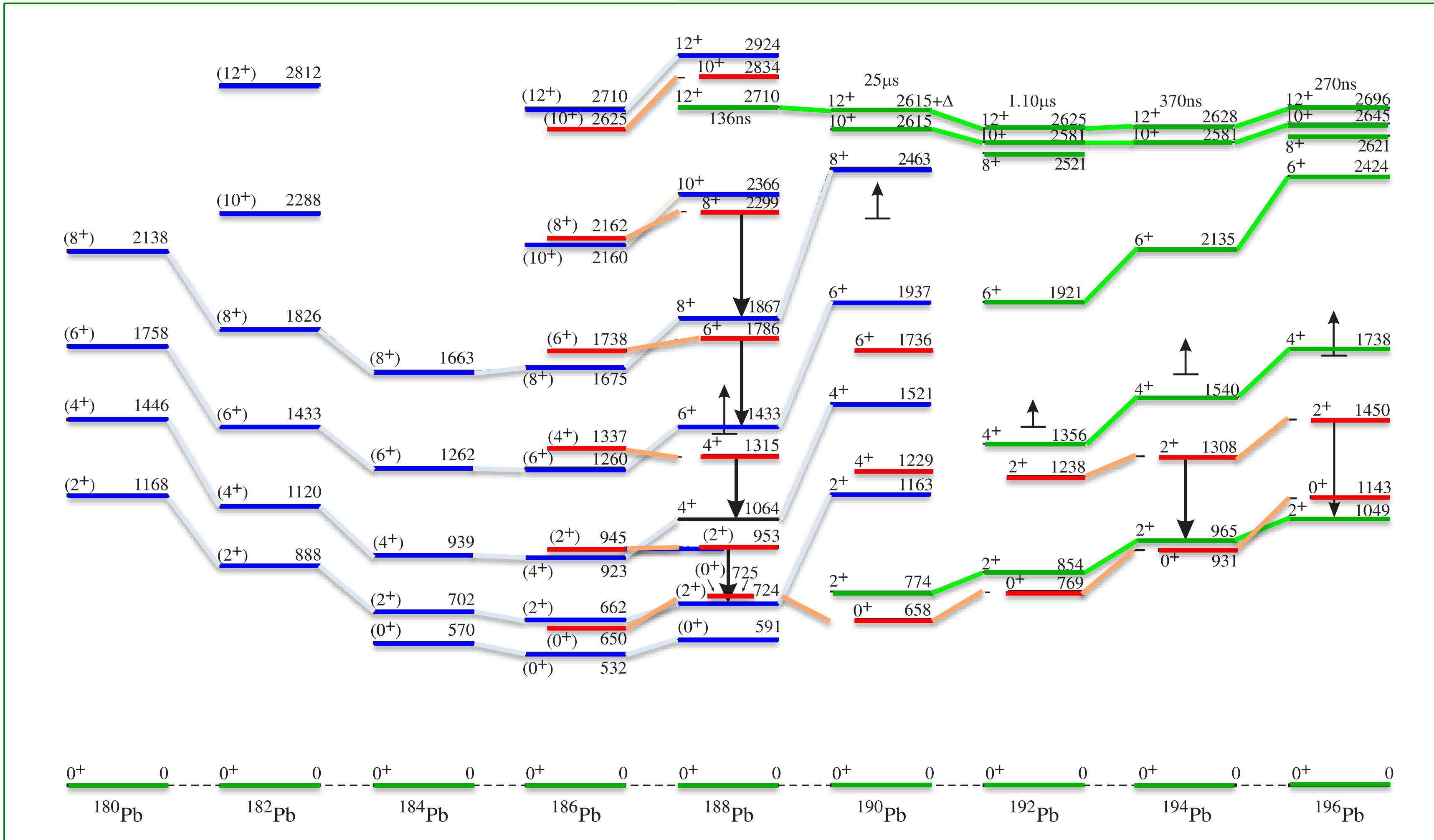
mid shell

Coexistence in even-Pb isotopes:

multiple parabolas and spherical (seniority) structure

Figure: Heyde & Wood

Heavy arrows indicate E0+M1+E2 transitions
^{188}Pb: G.D. Dracoulis et al., PR C67 R 051301 2003



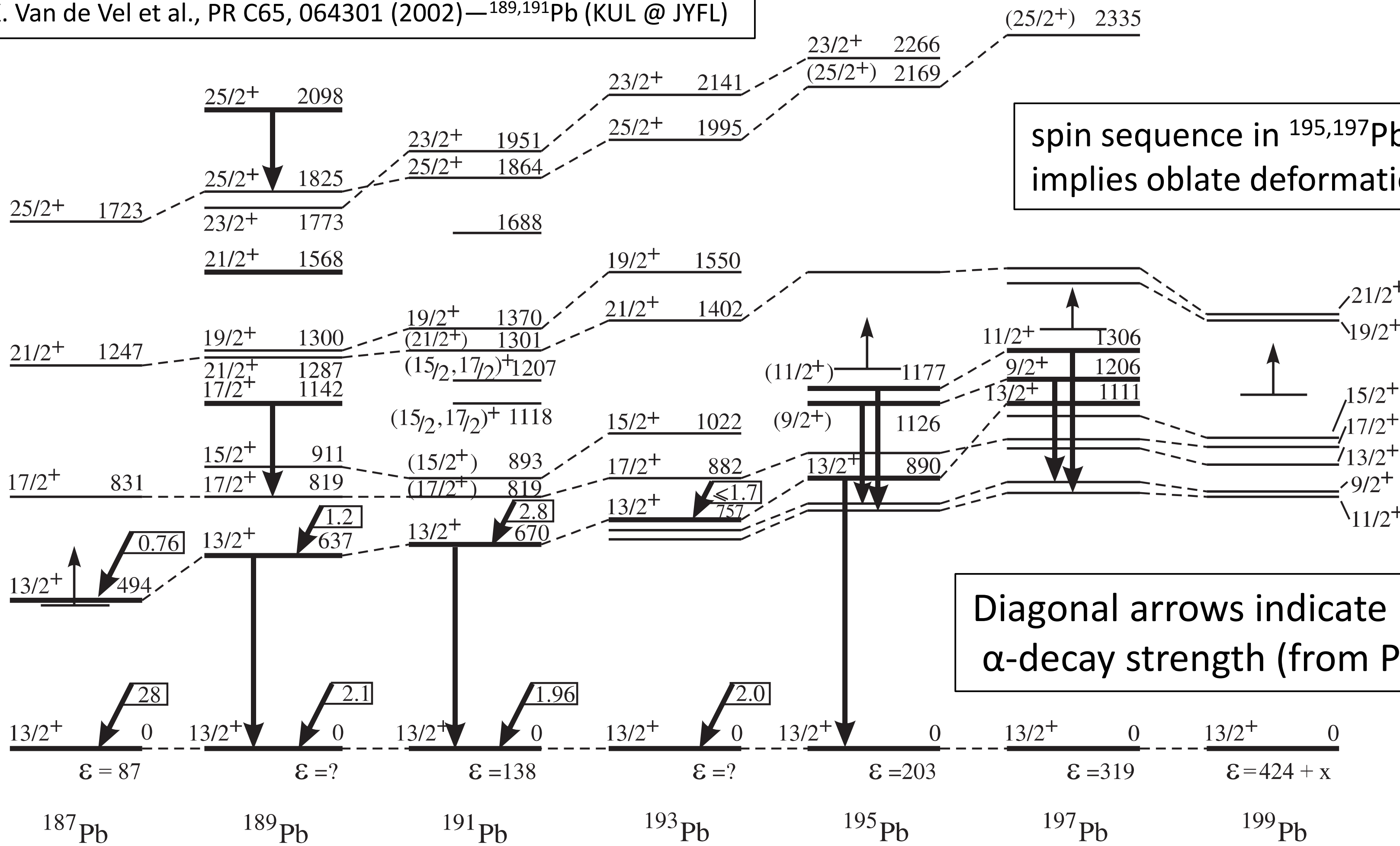
Coexistence in the odd-Pb isotopes:

Data for E0 transitions are from:
J.C. Griffin et al., NP A530 (1991) 401—^{195}Pb (UNISOR / LISOL);
J. Vanhorenbeeck et al., NP A531 (1991) 63—^{197}Pb (LISOL);
K. Van de Vel et al., PR C65, 064301 (2002)—^{189,191}Pb (KUL @ JYFL)

Heavy downward vertical arrows indicate E0+M1+E2 transitions

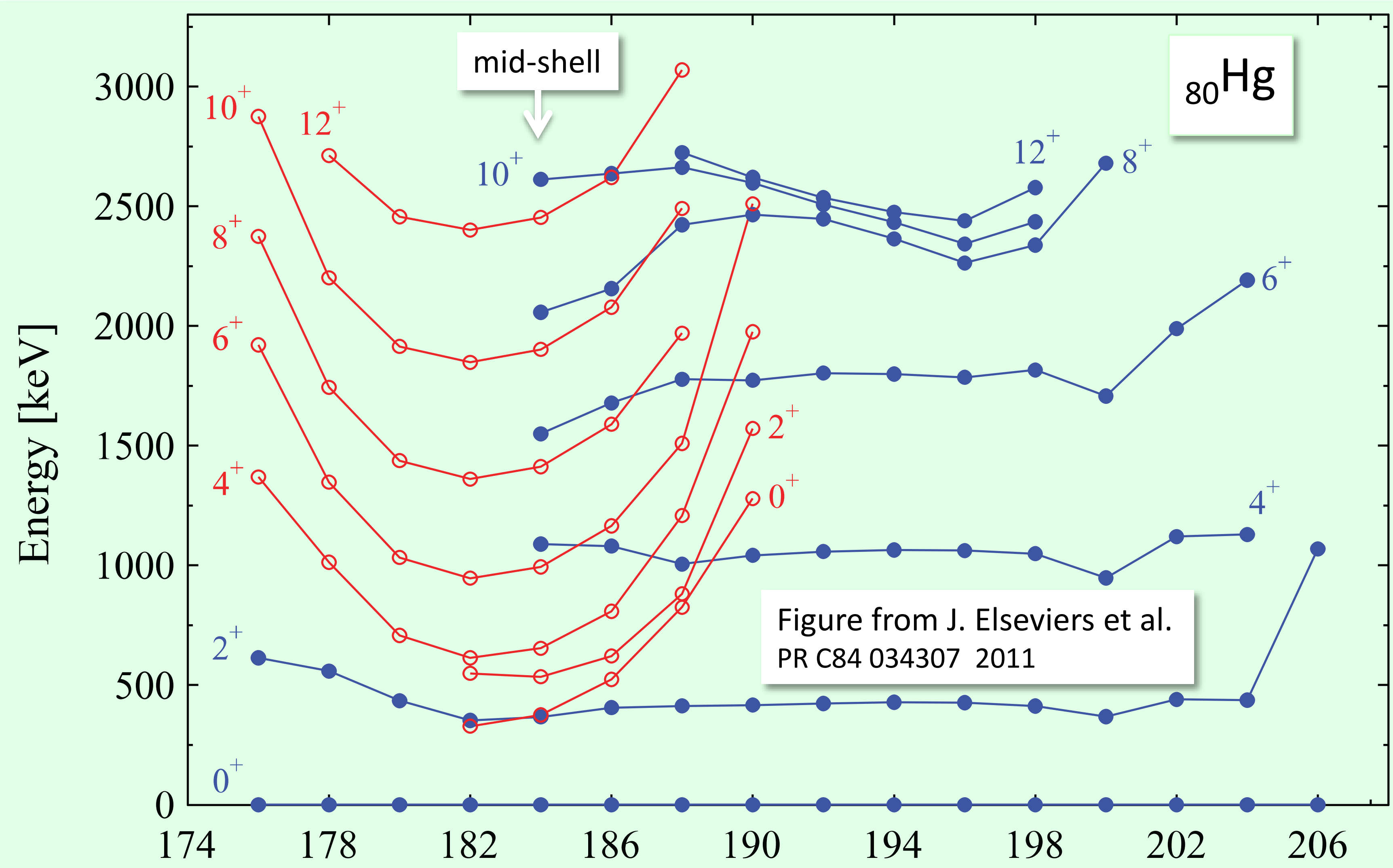
spin sequence in ^{195,197}Pb implies oblate deformation

Diagonal arrows indicate α-decay strength (from Po)

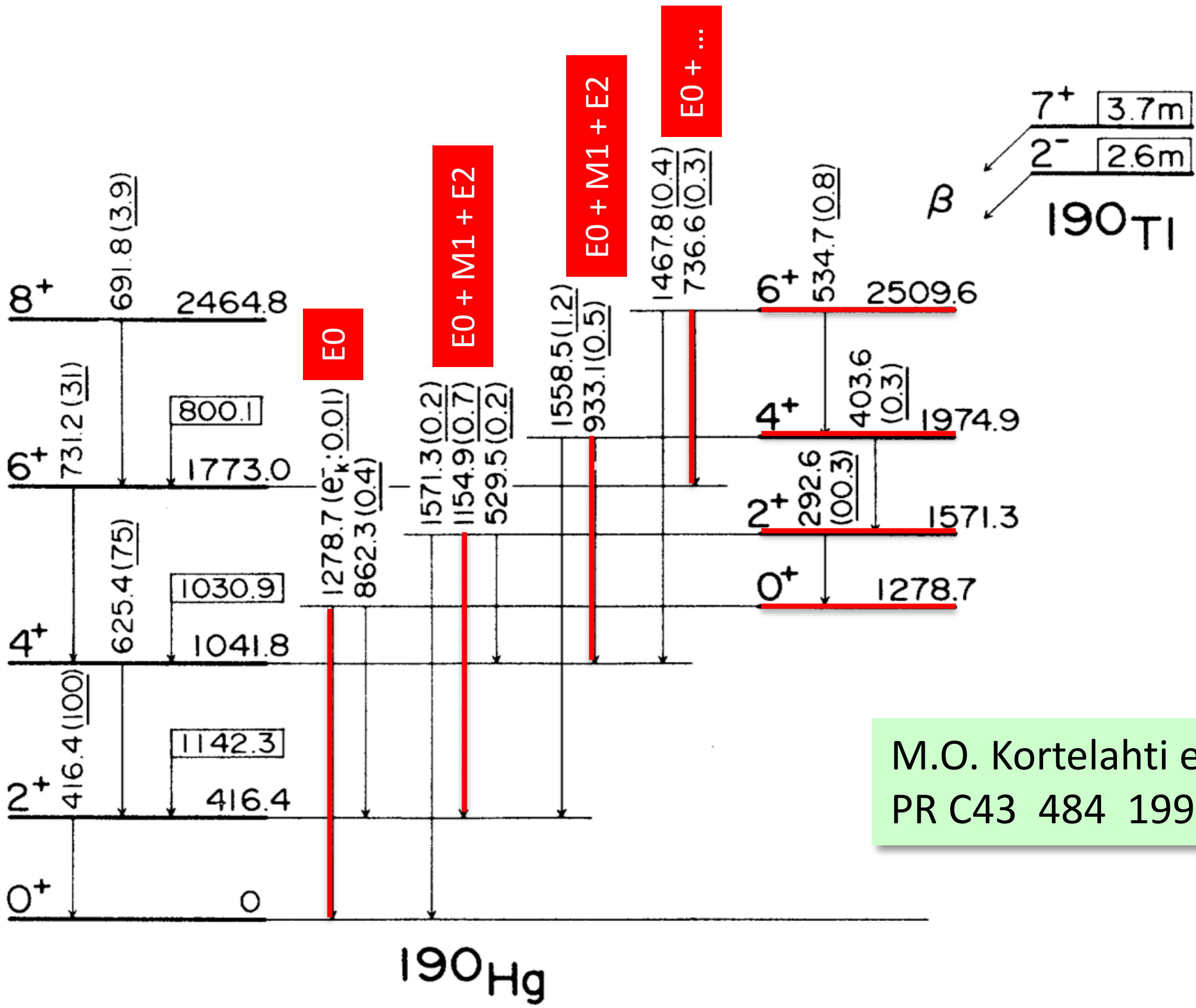


Shape coexistence in the even-Hg isotopes:

NOTE characteristic *parabolic energy* trend



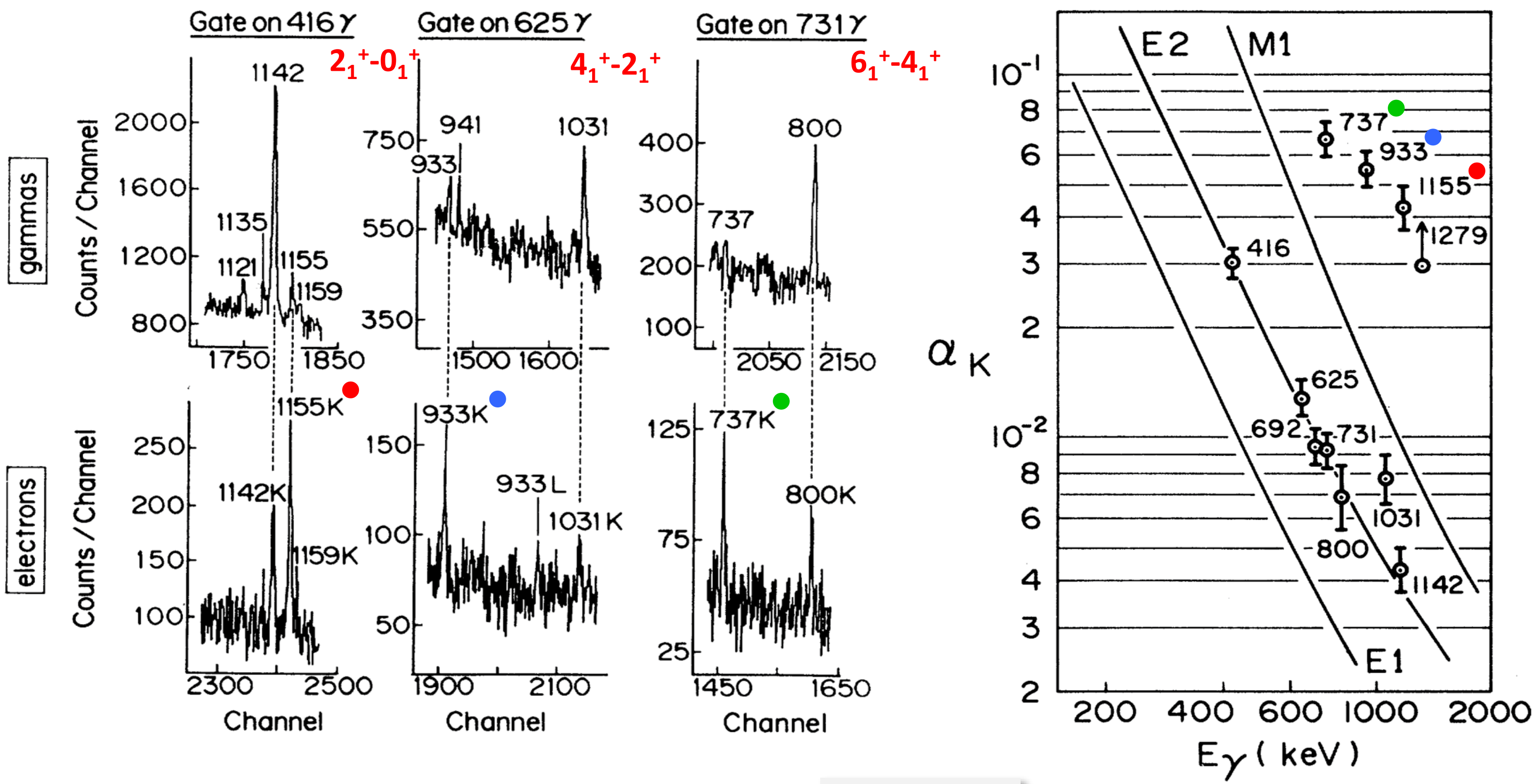
Conversion electron spectroscopy:
uniquely sensitive to E0 transitions, identifies shape coexistence



M.O. Kortelahti et al.,
PR C43 484 1991 UNISOR

E0 transitions: $\alpha_K > \alpha_K(M1)$

M.O. Kortelahti et al., PR C43 484 1991

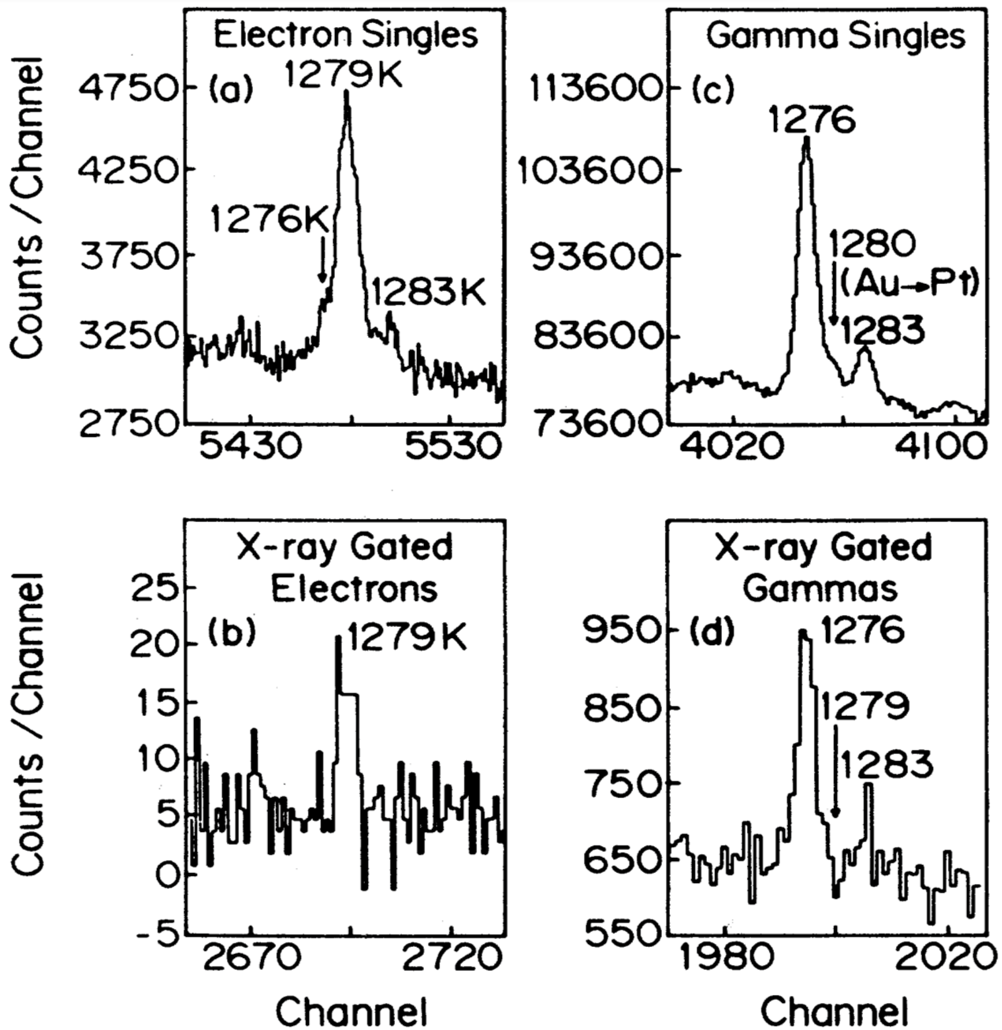


^{190}Hg

$0^+ \rightarrow 0^+$ decays are pure E0: no γ's (^{190}Hg)

M.O. Kortelahti et al. PR C43 484 1991

1279 keV pure E0 evidence



The ground states of $^{178-186}$Pt and $^{177-187}$Pt are intruder states

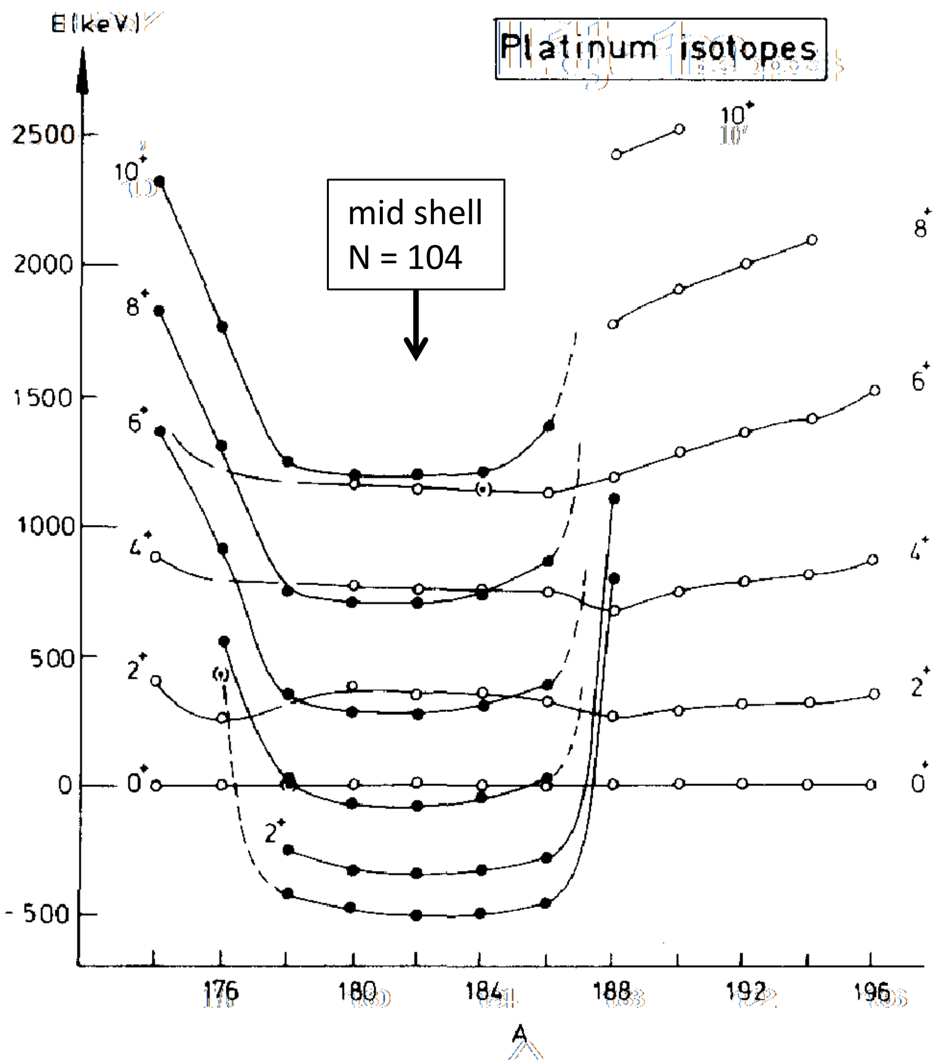
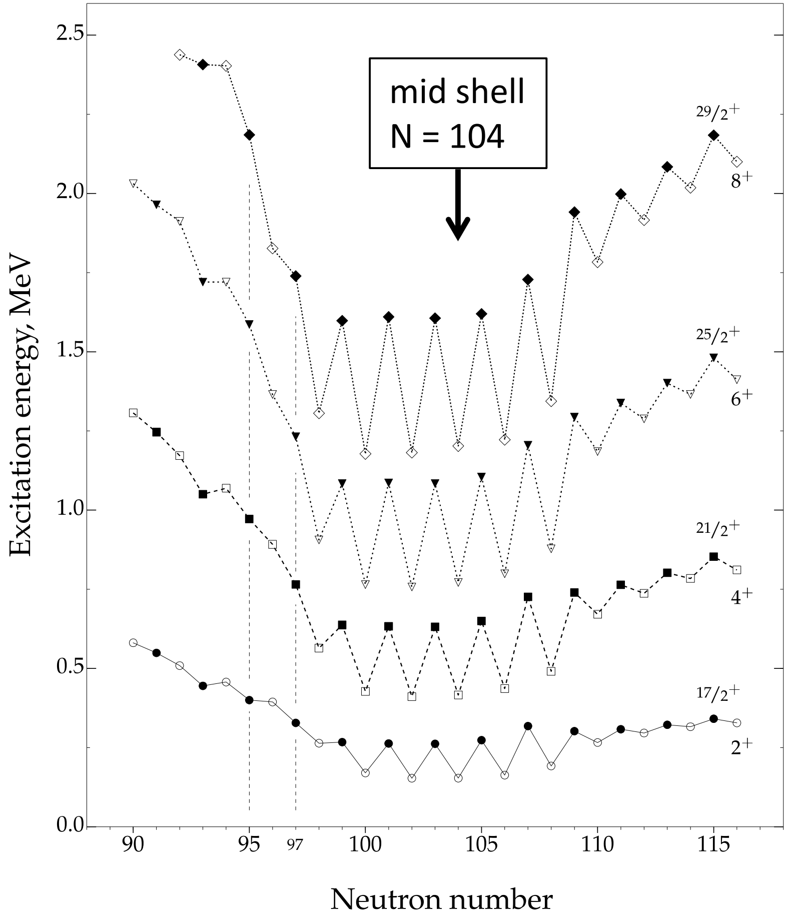


Figure from: JLW et al., Phys. Repts. 215, 101 (1992)

See P.M. Davidson et al., NP A657 219 1999 (ANU)

Coexistence in the odd-Pt isotopes



P. Peura, Ph.D. thesis, JYFL 2014

Coexistence in the even-Pt isotopes: mixing and E0 transition strength

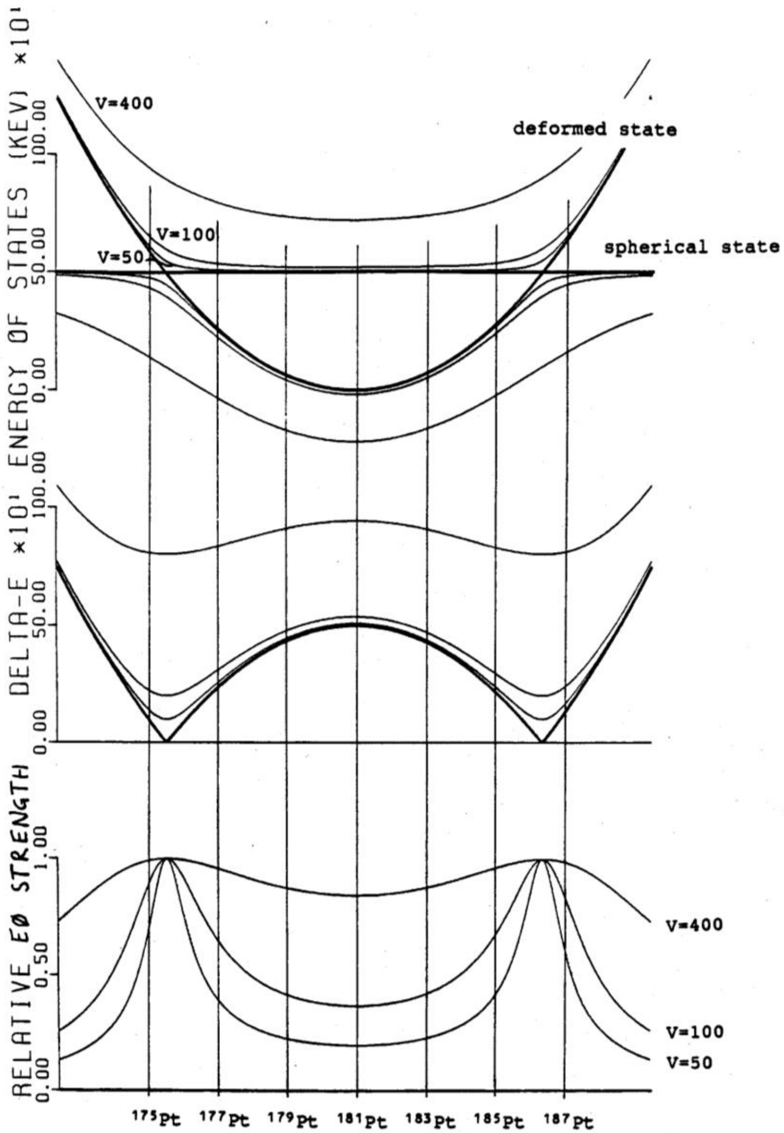
From: J. von Schwarzenberg, PhD thesis, Ga Tech 1991

V = 50, 100, 400 keV

E (sph.gs) = 0

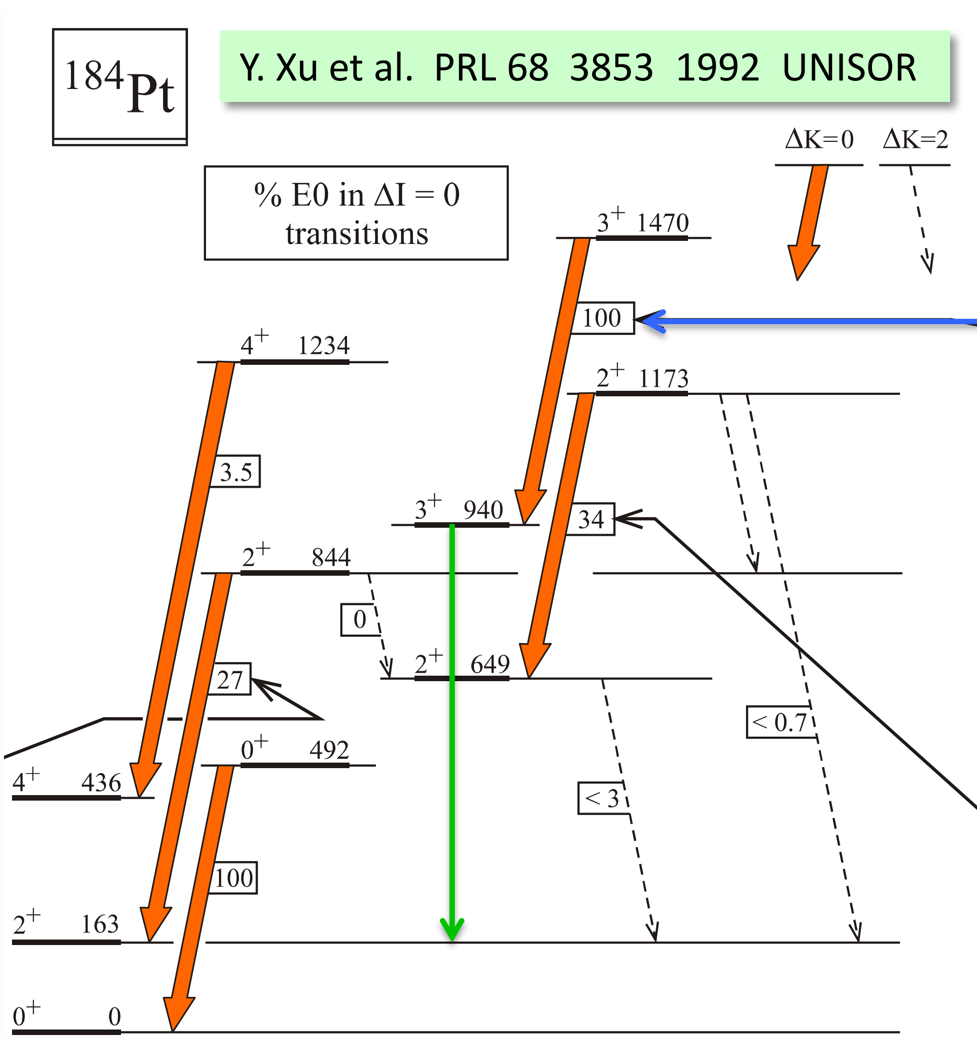
E (gs) = 0

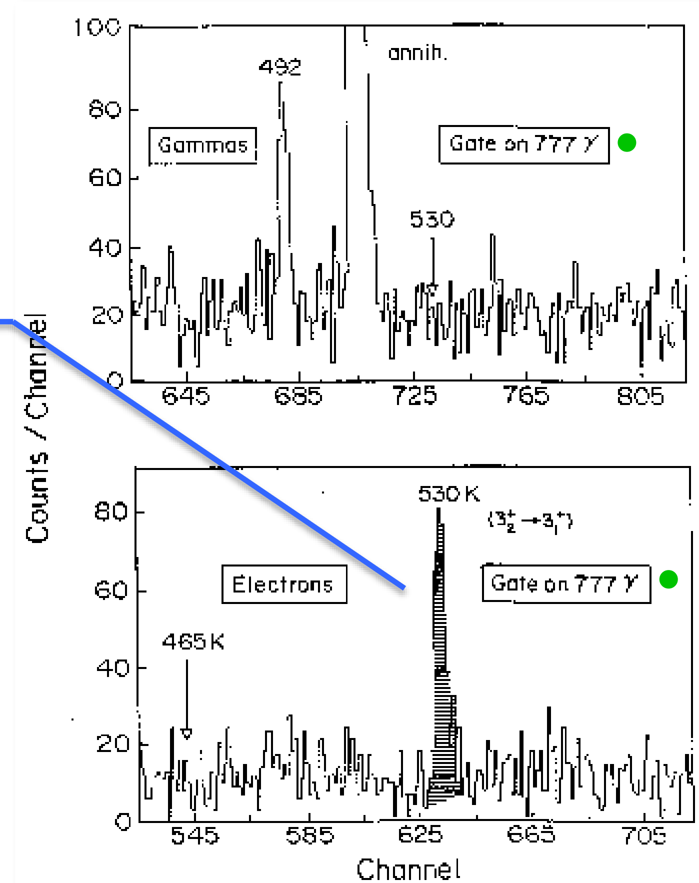
$\rho^2(E0)$



Coexistence in the even-Pt isotopes:

coexistence of K = 0 and K = 2 bands in ^{184}Pt

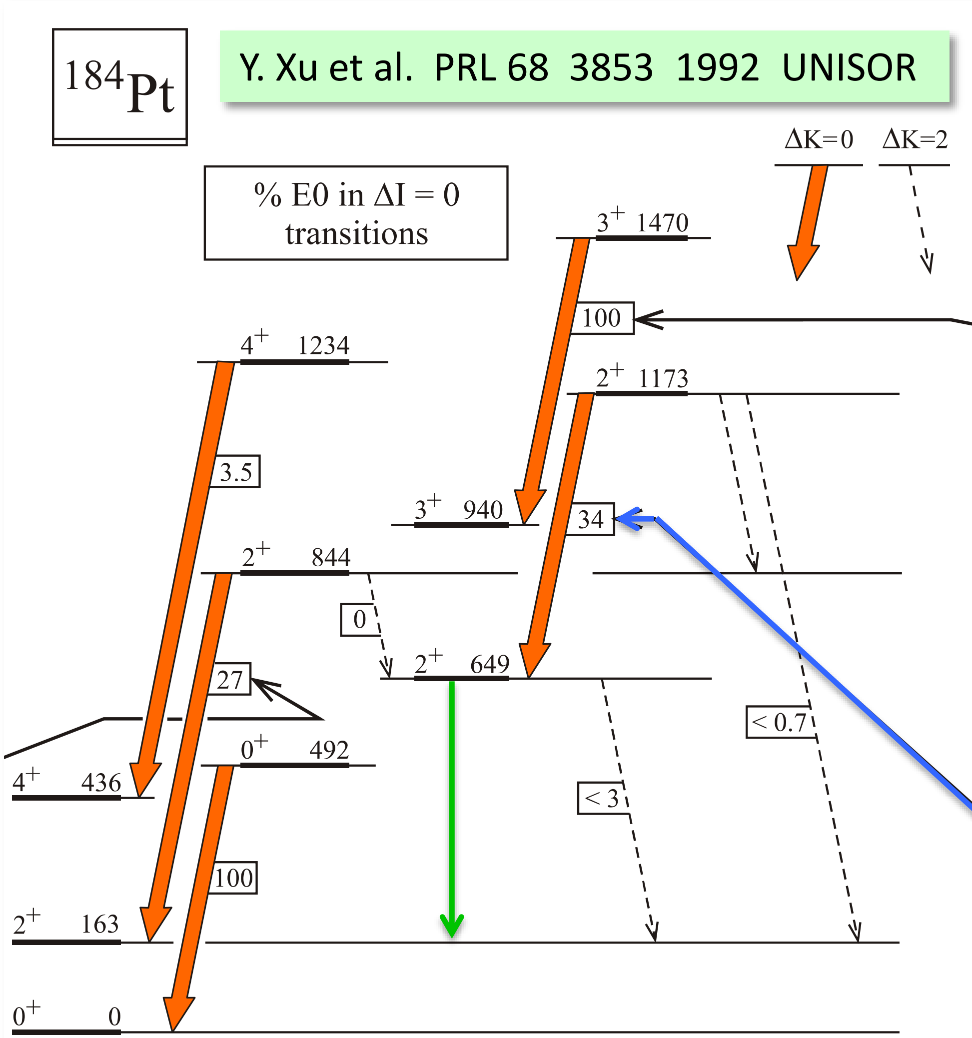


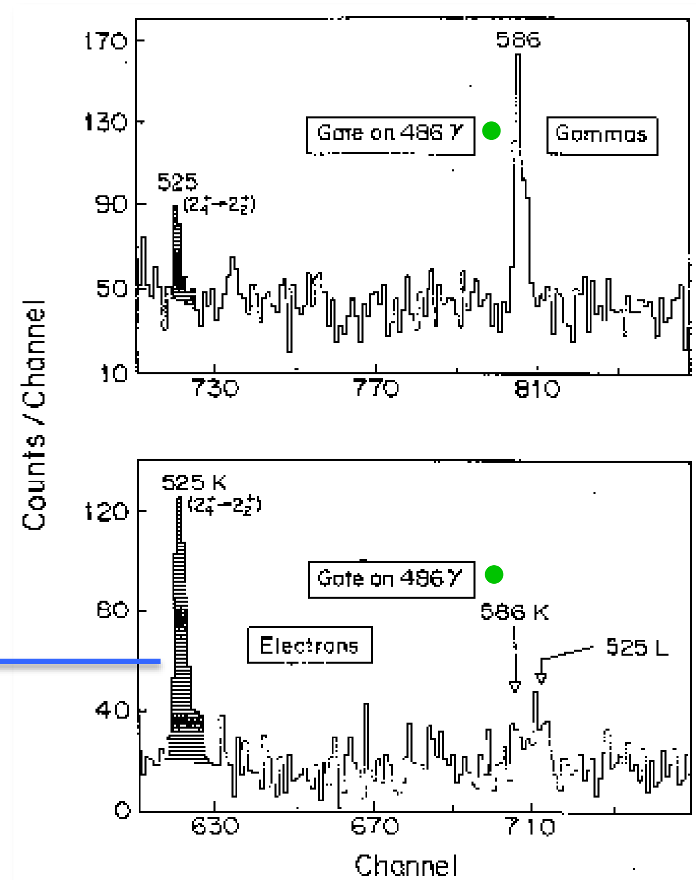


E2 / M1 from low-temperature nuclear orientation on-line

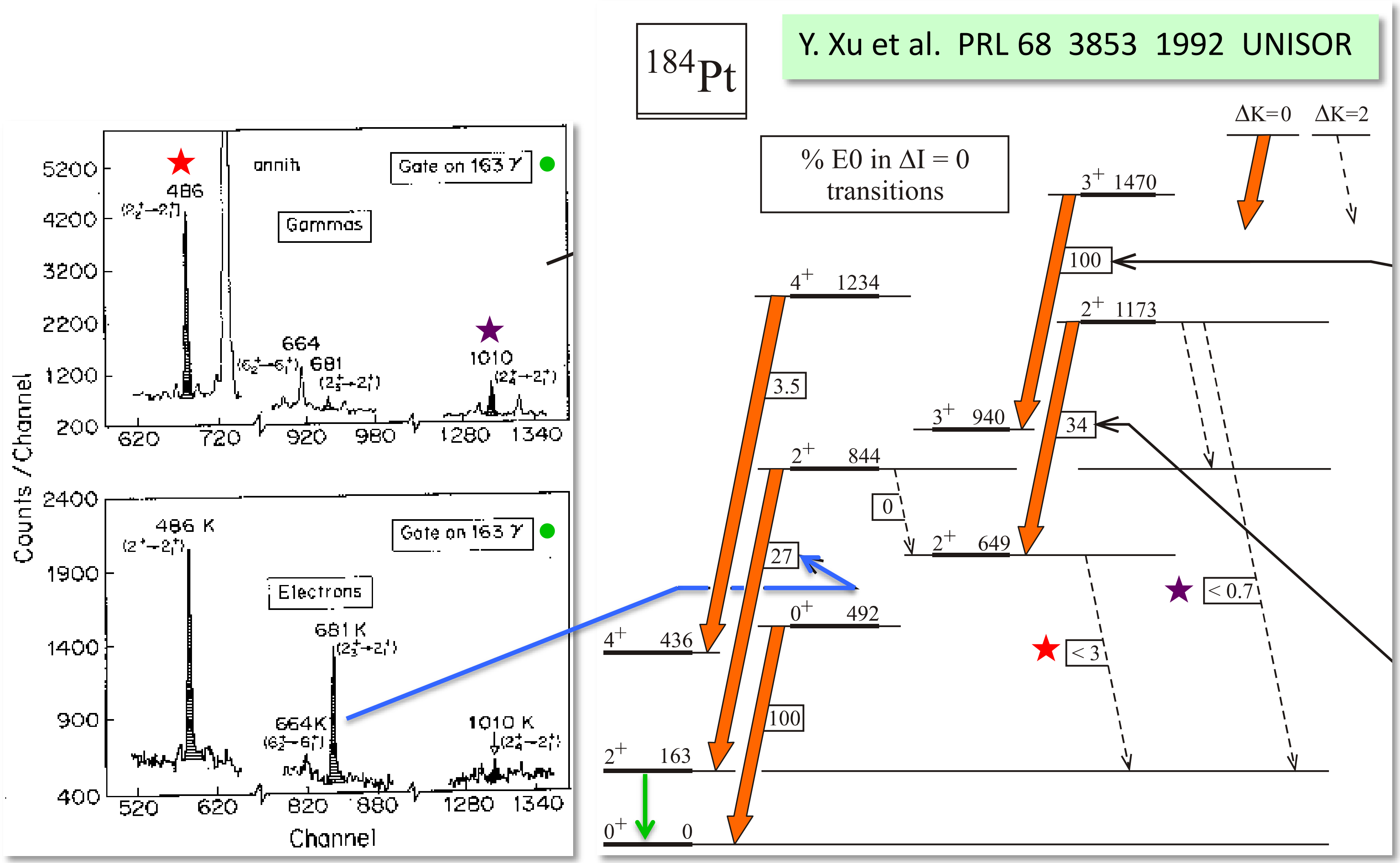
Coexistence in the even-Pt isotopes:
K = 0 and K = 2 bands

Y. Xu et al. PRL 68 3853 1992 UNISOR

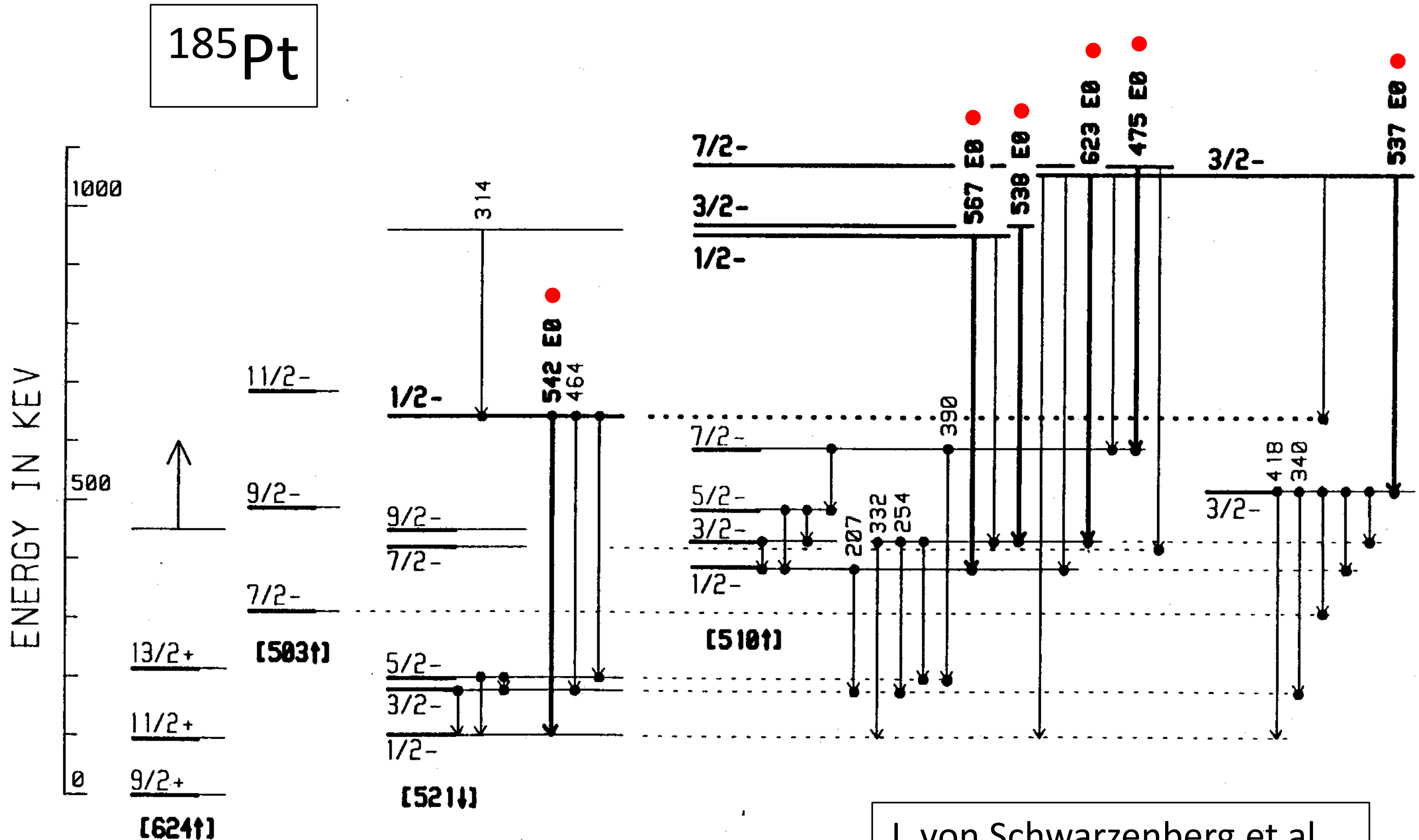




Coexistence in the even-Pt isotopes:
K = 0 and K = 2 bands

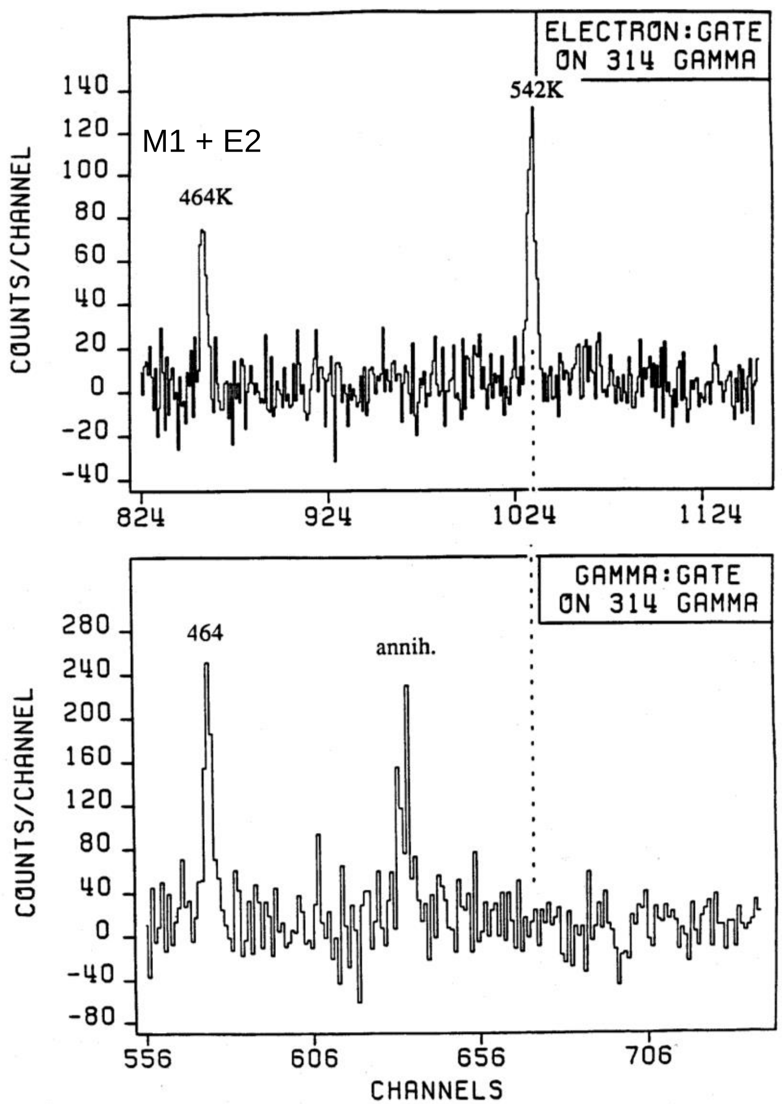


Pure E0's in an odd-mass nucleus?



J. von Schwarzenberg et al.,
PR C45 R896 1992 UNISOR

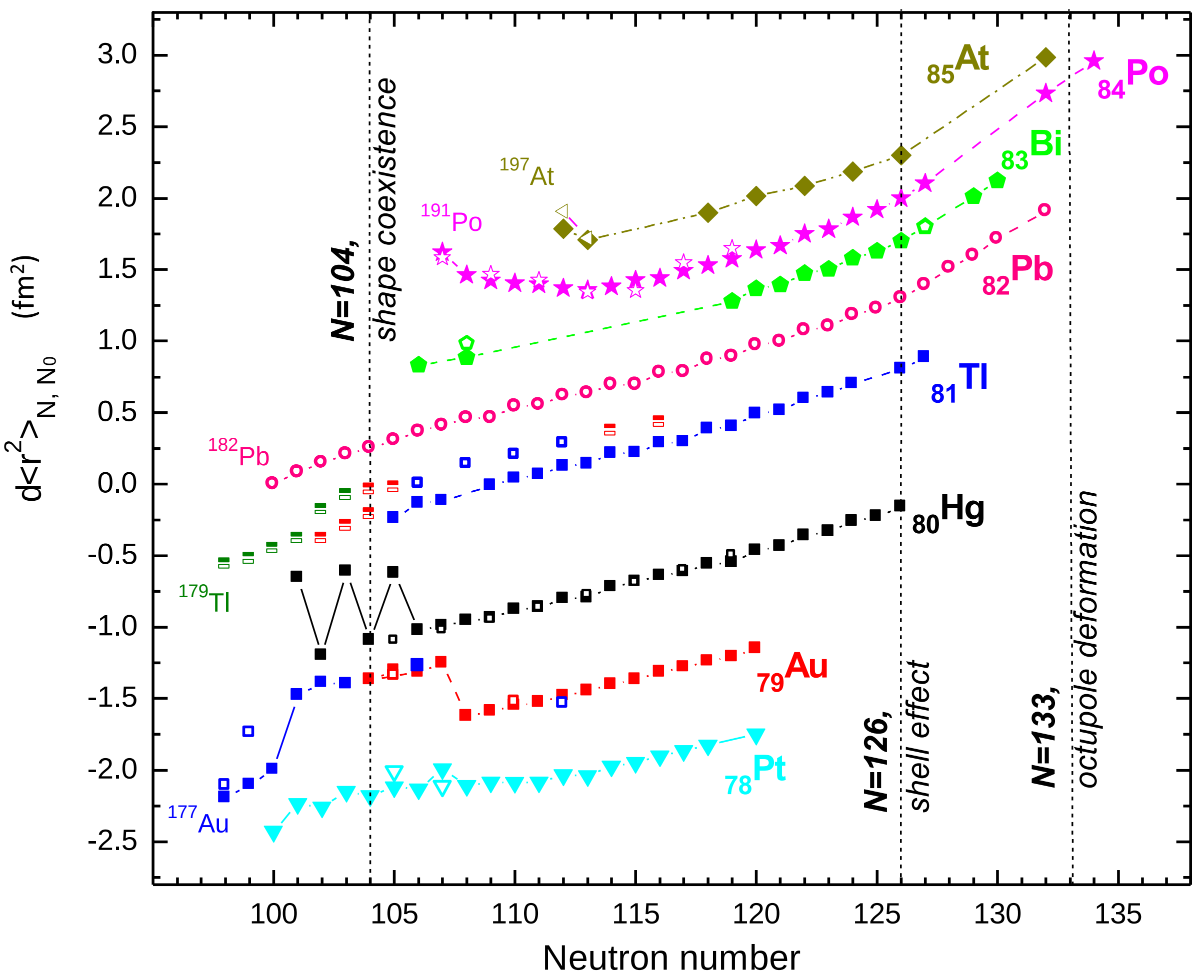
Pure E0's in an odd-mass nucleus



$^{185}Au \rightarrow {}^{185}Pt$

J. von Schwarzenberg et al., PR C45 R896 1992 UNISOR

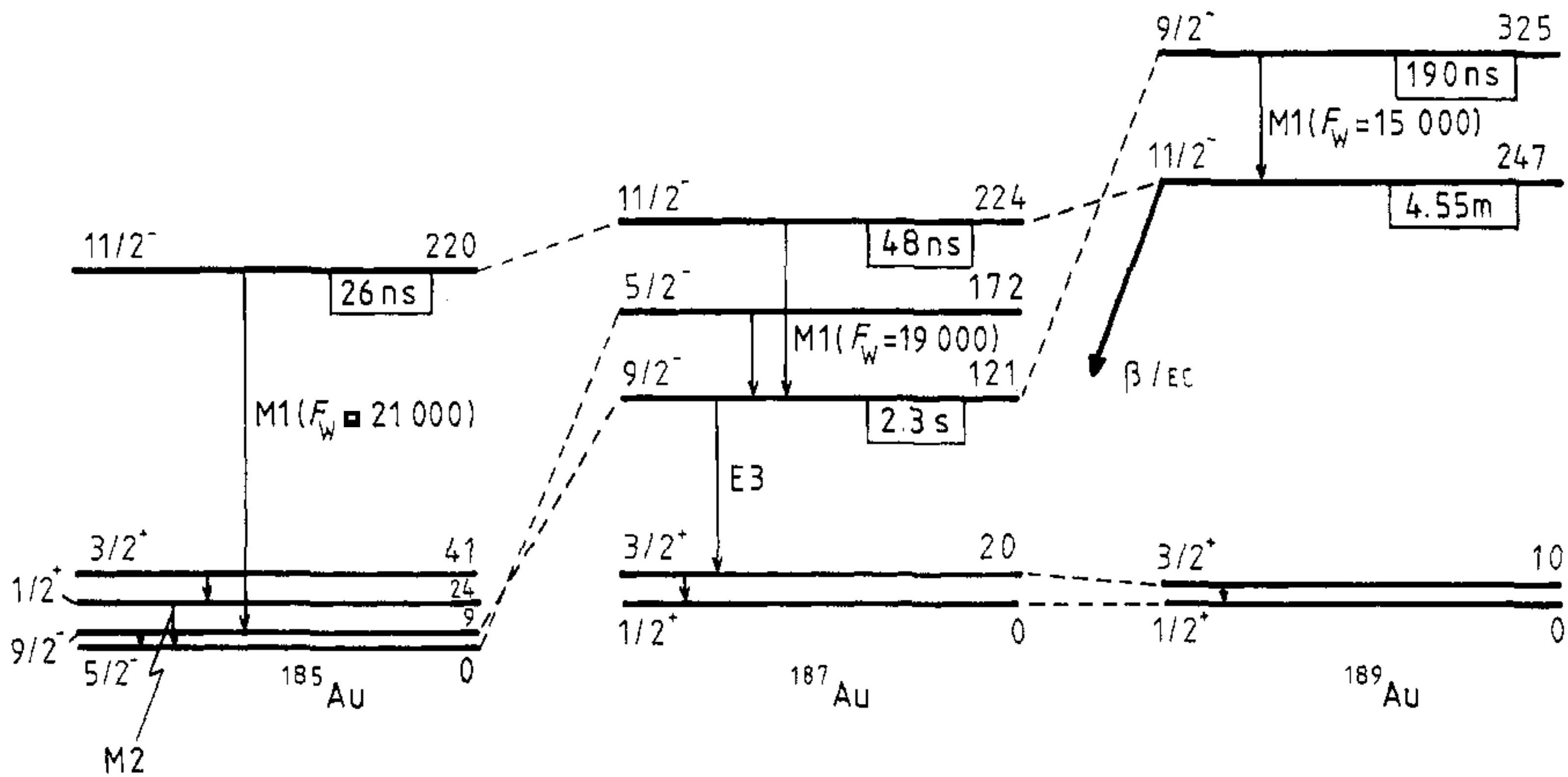
Isotope shifts: Pt, Au, Hg, Tl, Pb, Bi, Po, At



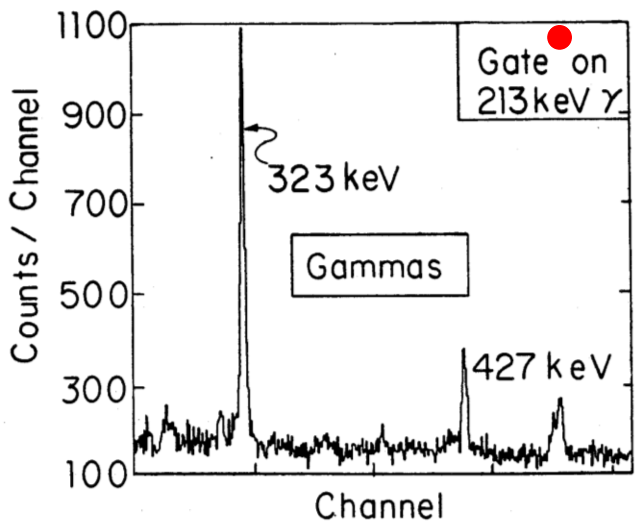
From: Barzakh INPC 2013

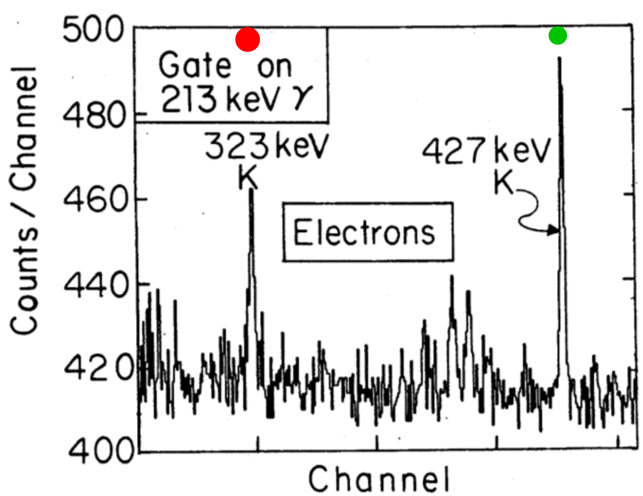
Odd-mass Au systematics showing the $h_{9/2}$ intruder state

Figure from: M.O. Kortelahti et al., JP G14 1361 (1988)



E0 transitions between "single"and "double" intruder states in ^{185}Au

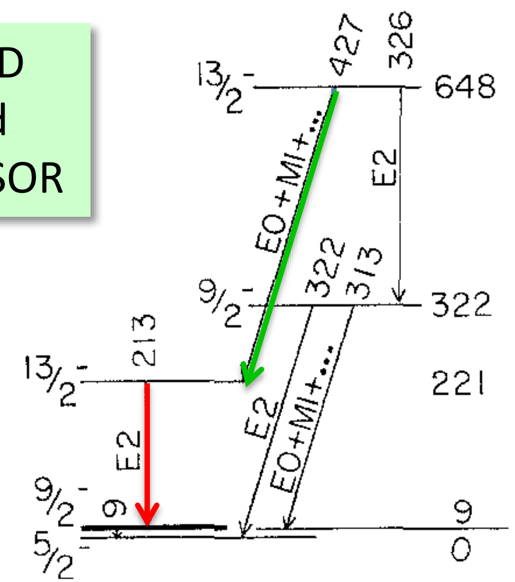




C.D. Papanicolopulos PhD thesis Ga Tech 1987 and ZP A330 371 1988 UNISOR

Z = 79 427 keV

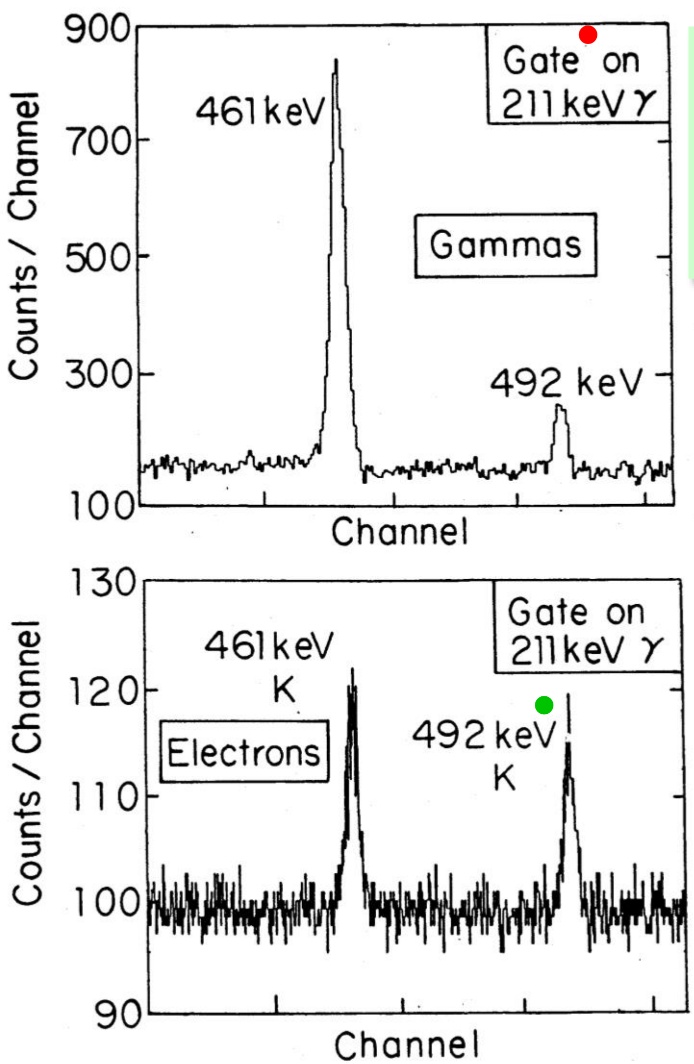
mult	α_K
E1	0.010
E2	0.027
M1	0.11
expt.	0.33



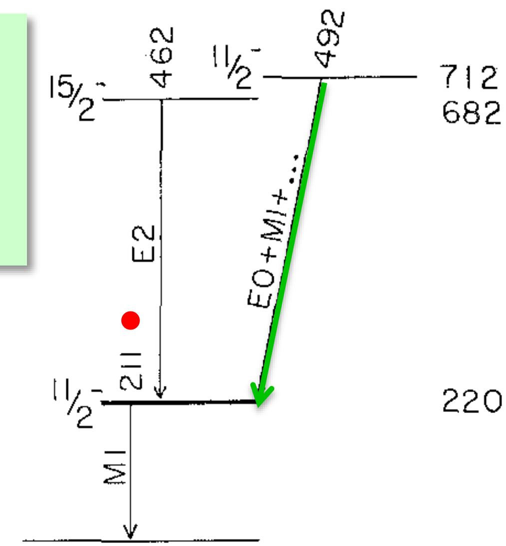
$9/2^-$ state @ 9 keV: "double" intruder state: $\pi h_{9/2}$ (1p) × ^{184}Pt [π(2p-6h)] = π(3p-6h)

$9/2^-$ state @ 322 keV: "single" intruder state: $\pi h_{9/2}$ (1p) × ^{184}Pt [π(4h)] = π(1p-4h)

E0 transitions between “spherical” states and “core” intruder states in ^{185}Au



C.D. Papanicolopulos
PhD thesis Ga Tech 1987
ZP A330 371 1988
UNISOR



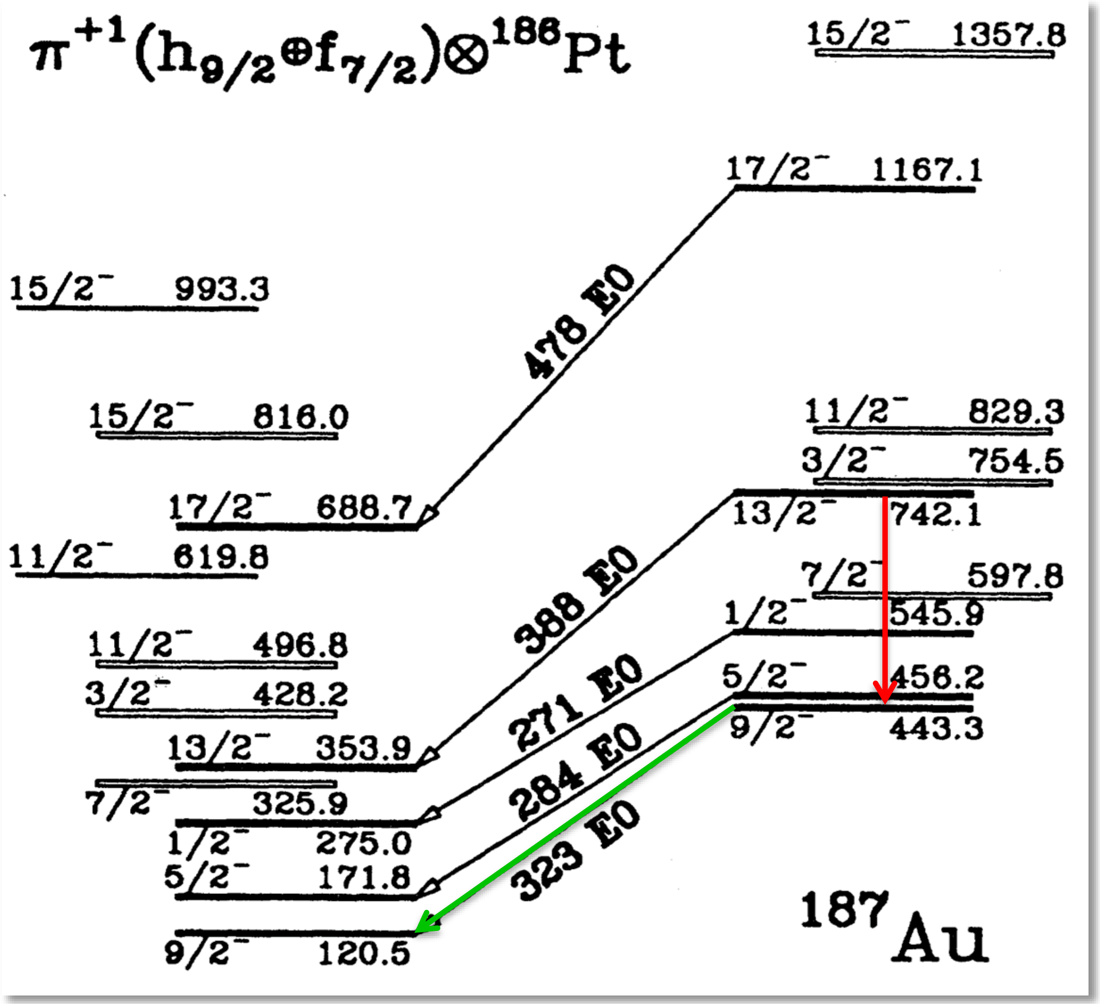
Z = 79 492 keV

mult	α_K
E1	0.007
E2	0.020
M1	0.073
expt.	0.21

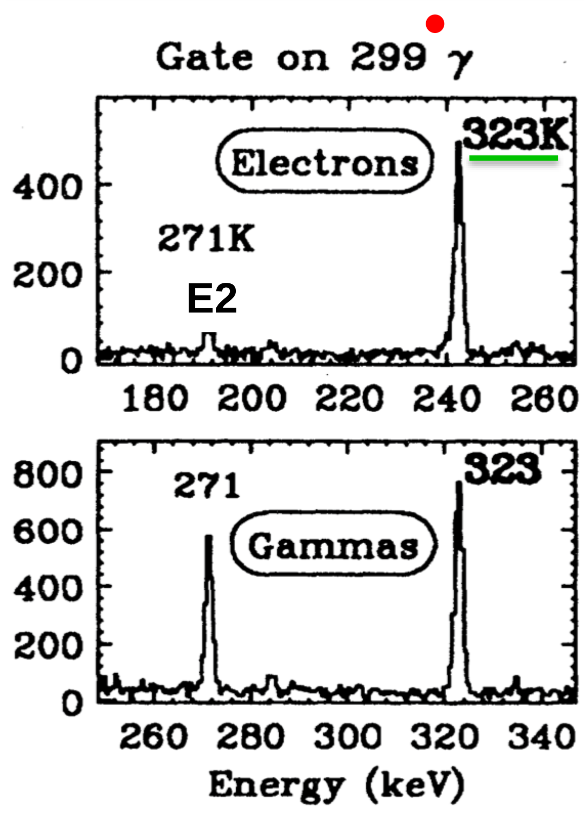
$11/2^-$ state @ 220 keV: “spherical” state:
$\pi h_{11/2}$ (1h) × ^{186}Hg [π(2h)] = π(3h)

$11/2^-$ state @ 712 keV: “core” intruder state:
$\pi h_{11/2}$ (1h) × ^{186}Hg [π(2p-4h)] = π(2p-5h)

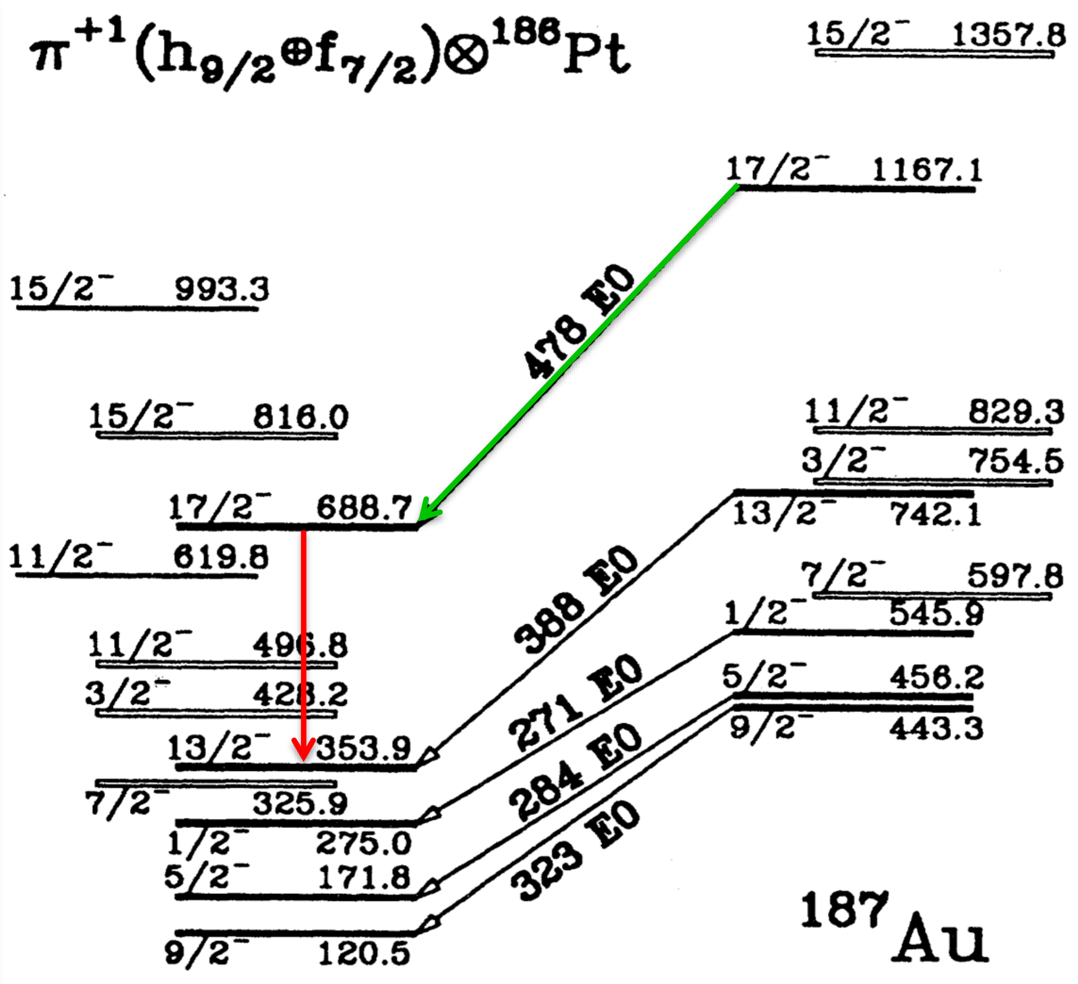
E0 transitions between “single”and “double” intruder states in ^{187}Au



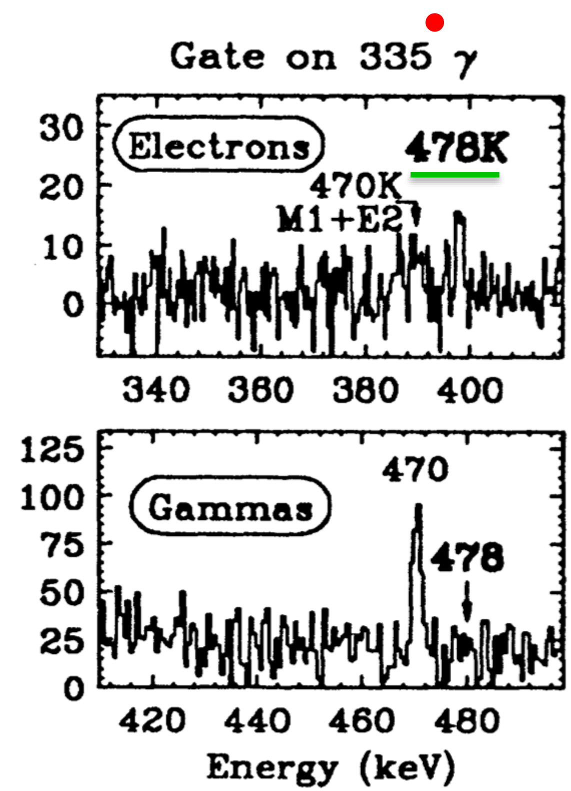
D. Rupnik et al.
PR C51 R2867 1995 UNISOR



E0 transitions between “single”and “double” intruder states in ^{187}Au



D. Rupnik et al.
PR C51 R2867 1995 UNISOR



WHEN STUDYING THE QUANTUM MECHANICAL MANY-BODY PROBEM, ALWAYS BE MINDFUL OF:

- "We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances.

  Therefore, to the same natural effects we must, so far as possible, assign the same causes."

  --Isaac Newton

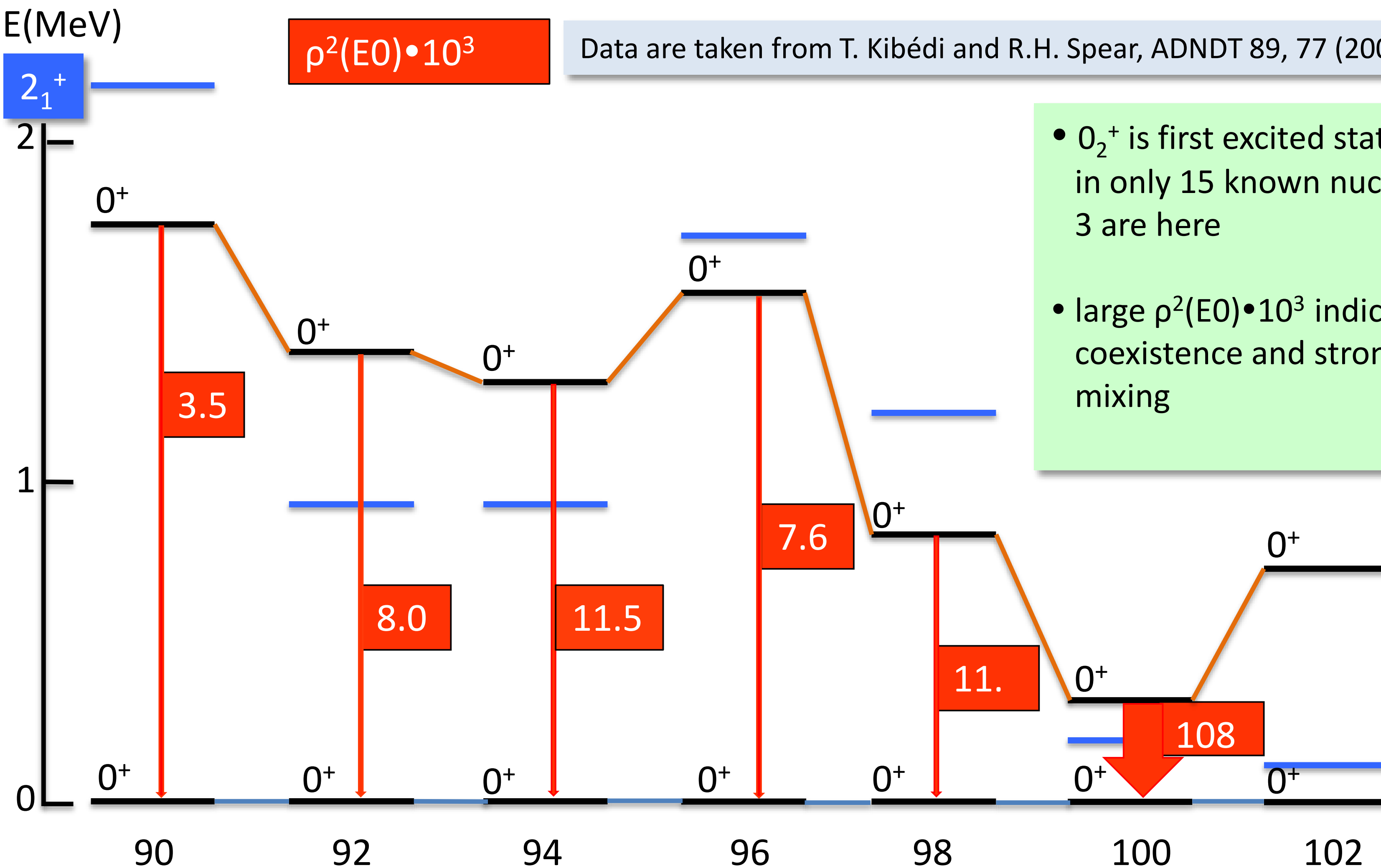
  (From William of Ockham [near Guildford, UK], ca. 1320)

- "Everything should be made as simple as possible, but not simpler."

  -- Albert Einstein

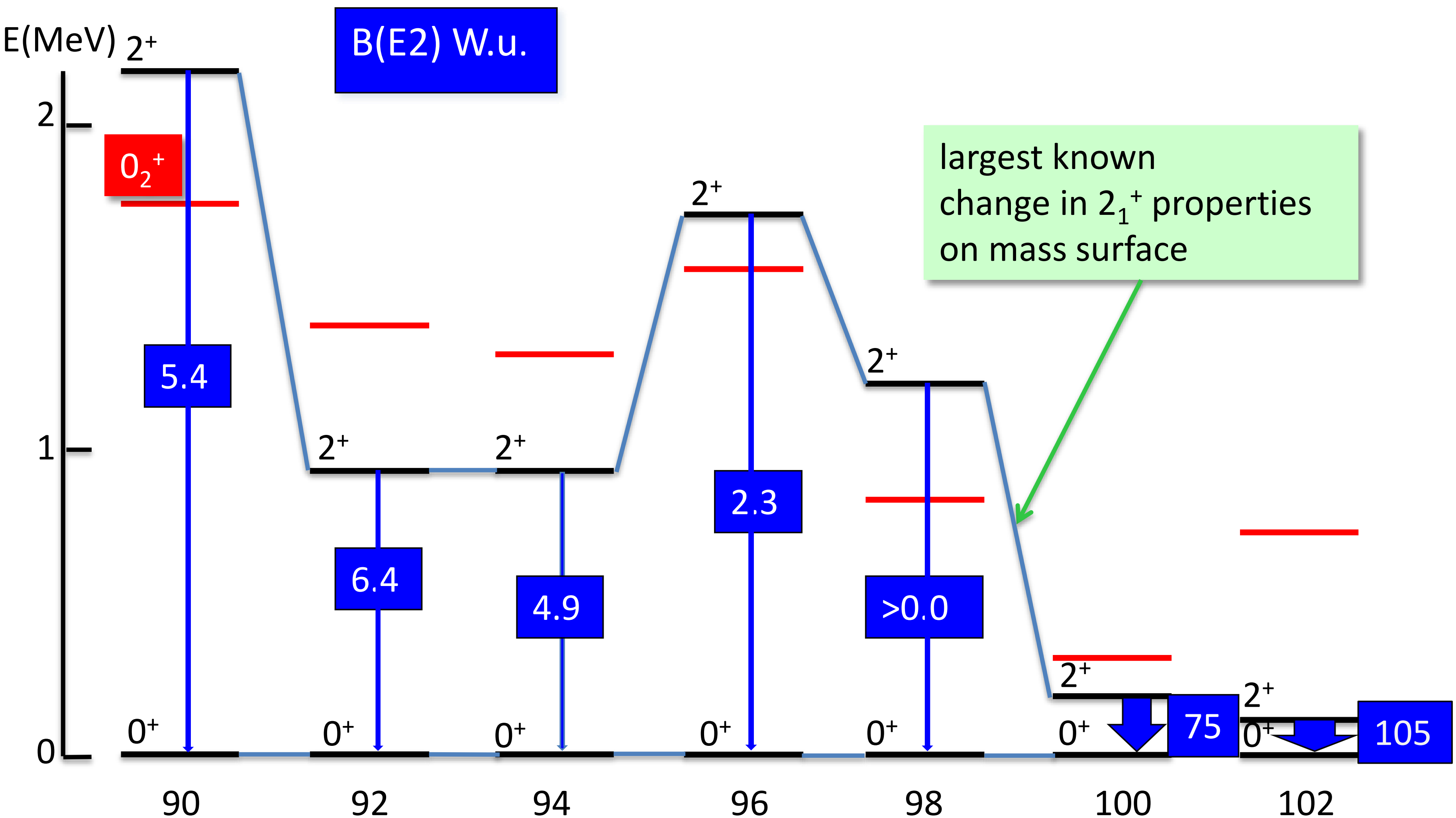
Systematics of 0_2^+ states in Zr isotopes, 50 ≤ N ≤ 62: electric monopole transition strengths

Data are taken from T. Kibédi and R.H. Spear, ADNDT 89, 77 (2005)

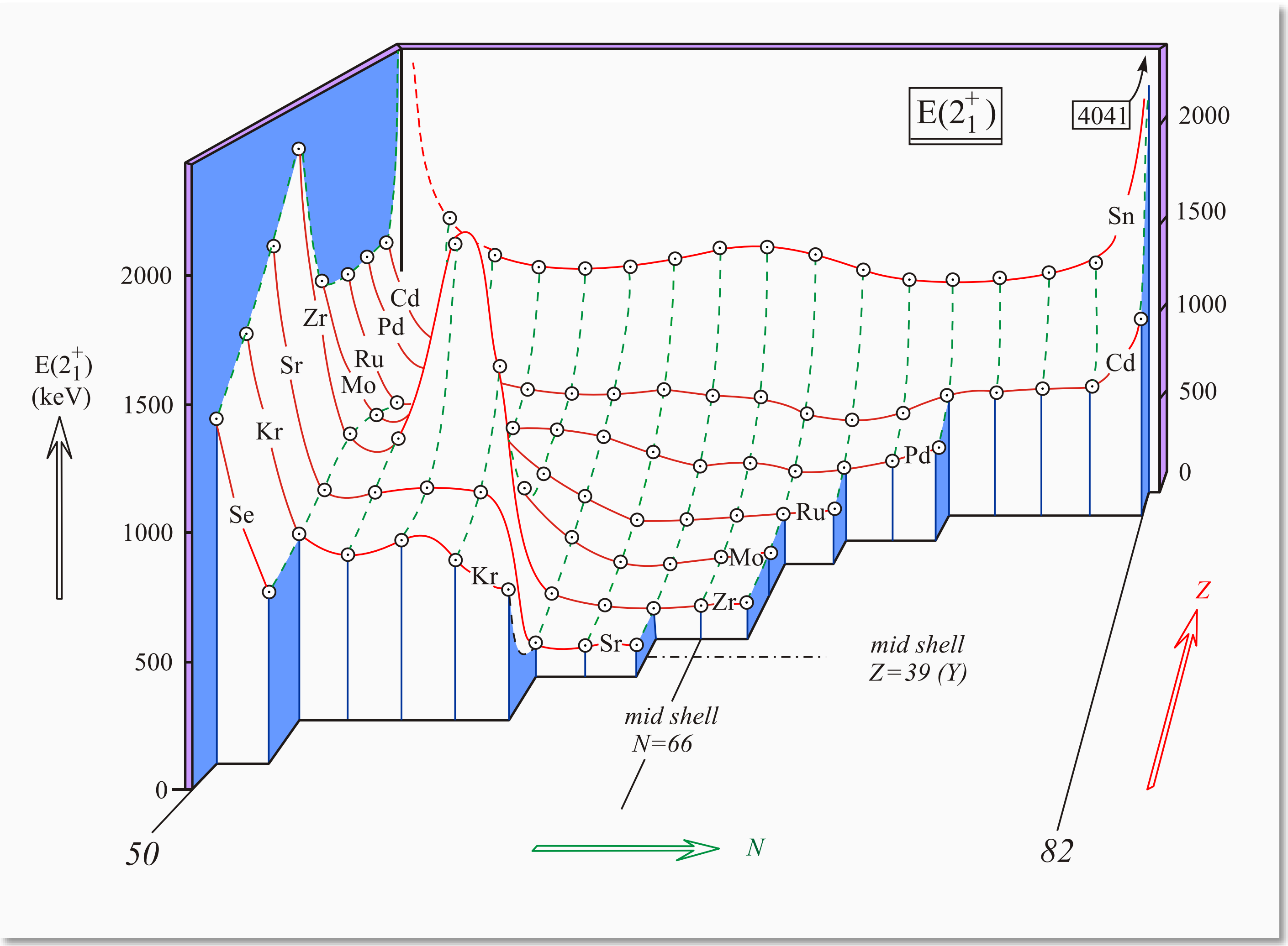


- 0_2^+ is first excited state in only 15 known nuclei: 3 are here
- large $\rho^2(E0)\bullet 10^3$ indicates coexistence and strong mixing

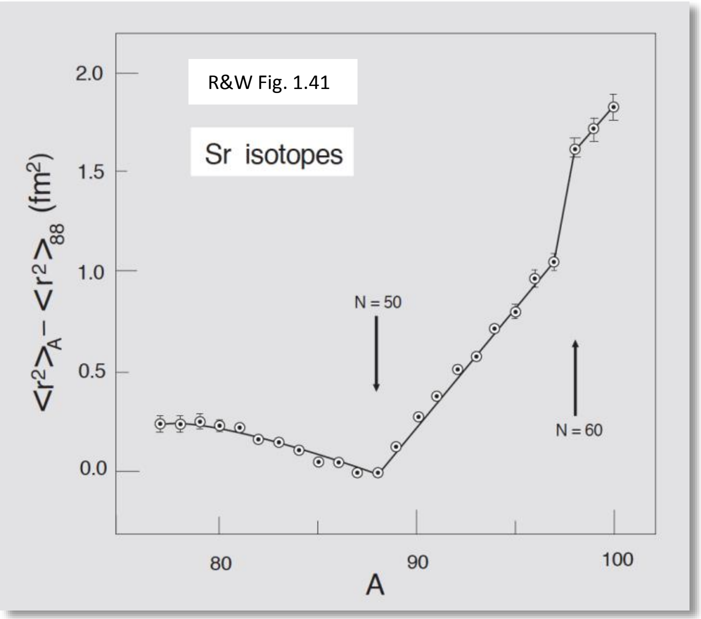
Systematic of 2_1^+ states in Zr isotopes, $50 \leq N \leq 62$: electric quadrupole transition strengths



Systematic of $E(2_1^+)$ for $N \geq 50$, $Z \leq 50$

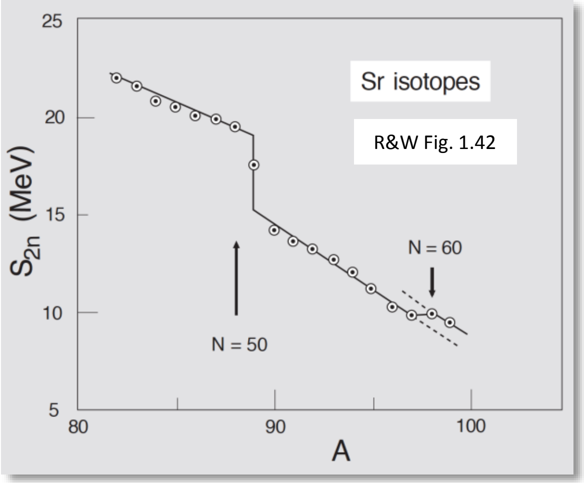


Ground-state properties are a direct signature of shell and deformation structures



Differences in mean-square charge radii (isotope shifts) determined by:

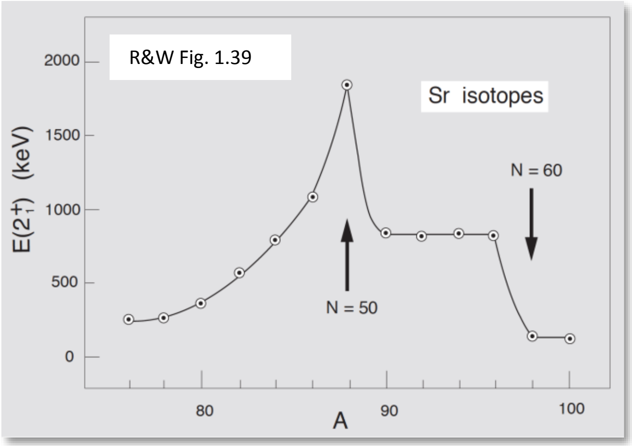
optical hyperfine spectroscopy using lasers



Two-neutron separation energies deduced from nuclear masses determined by:

direct mass measurements

2_1^+ state properties are a strong signature of shell and deformed structures

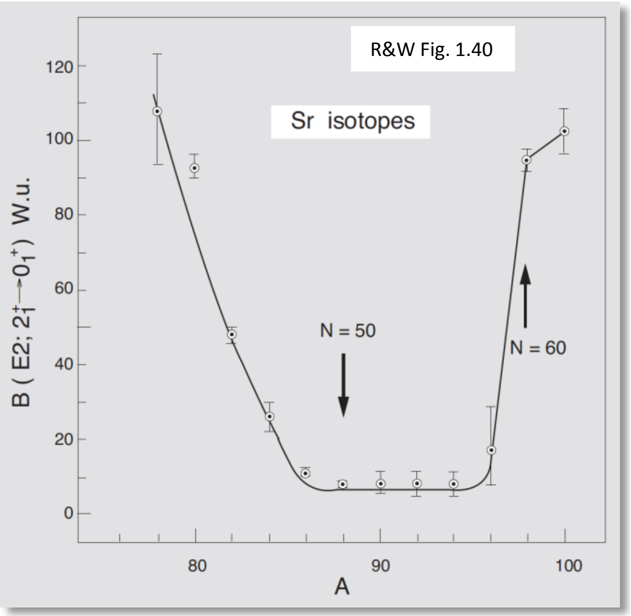


Energies of 2_1^+ states determined by:

gamma-ray spectroscopy following β decay

problem—β-decaying parent is further from stability and yield will be (much) lower than nucleus of interest

gamma-ray spectroscopy following Coulomb excitation



Reduced E2 transition rates, B(E2) from 2_1^+ states determined by:

lifetime measurements using fast β-γ timing following β decay

problem--see above

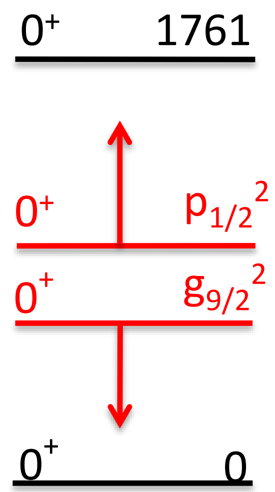
gamma-ray yields following Coulomb excitation

Excited 0^+ states at closed shells--mixing and repulsion of pair configurations in ^{90}Zr

N=50: $g_{9/2}$ seniority structure

j = ½ orbitals can only contribute to v = 0 states, at low energy

^{90}Zr $E(2_1^+)$ is high: suggests a closed subshell, BUT is due to *depression* of the ground-state energy



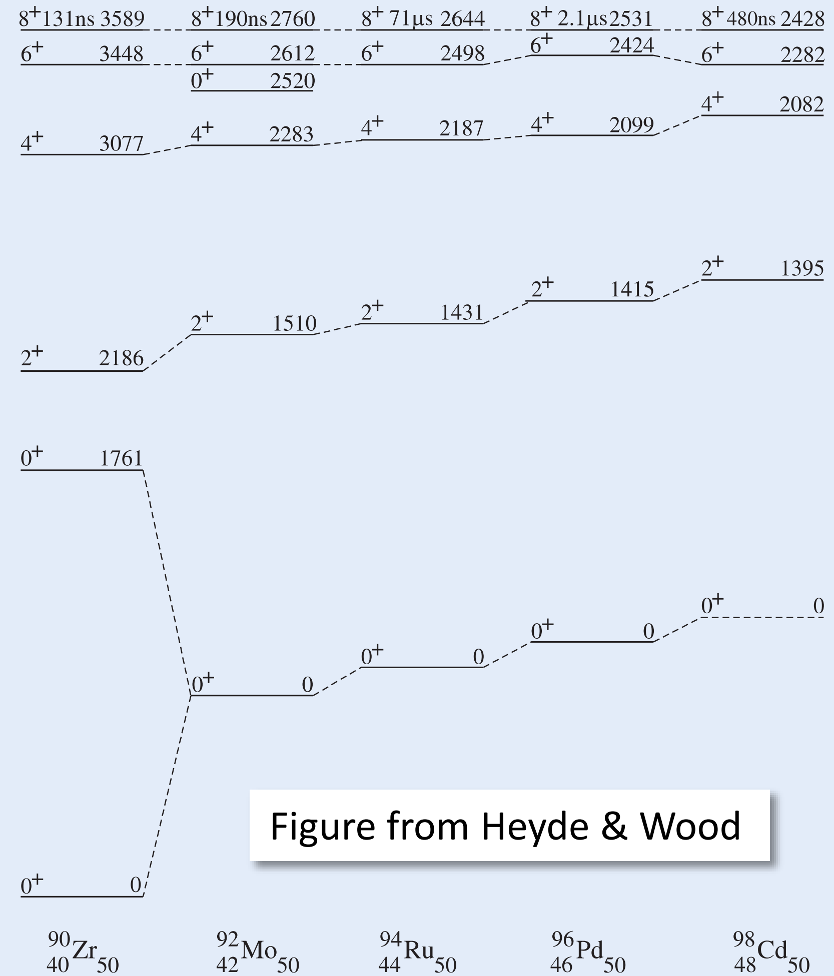
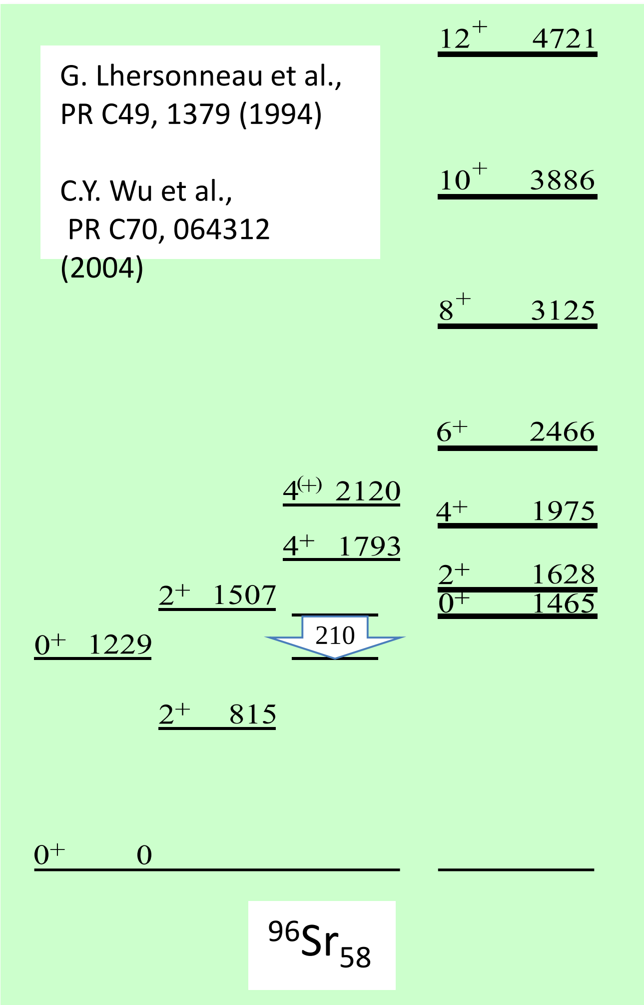


Figure from Heyde & Wood

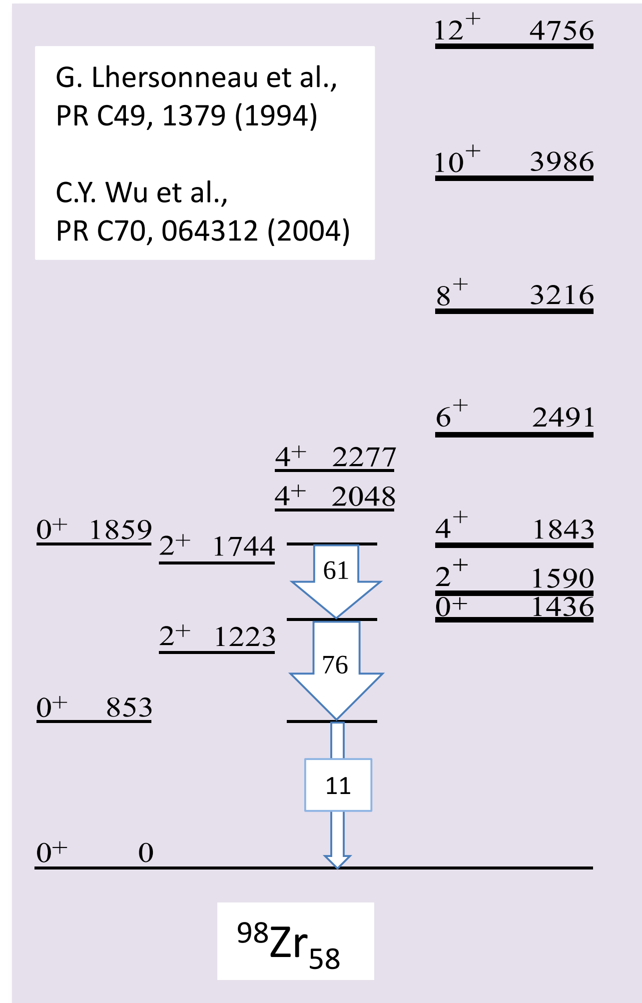
Shape coexistence at and near closed subshells: the nuclei ^{96}Sr and ^{98}Zr

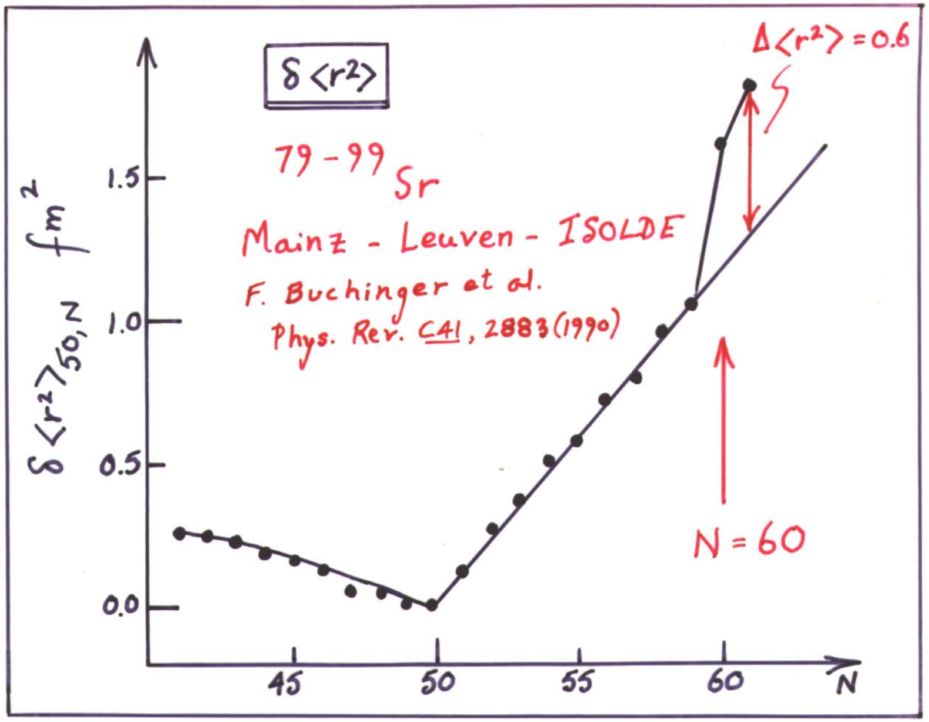
Figure from K. Heyde and J.L. Wood, Rev. Mod. Phys. 83, 1467 (2011)



E0 transitions: $\rho^2(E0)\bullet10^3$ values are shown

$\rho^2(E0)\bullet10^3 = 210^{31}$ in ^{96}Sr is largest known for A > 56





expt. ^{96}Sr $\rho^2 = 0.210^{31}$ (H. Mach et al. PR C41, 350 (1990))

theory:

$$\Delta\langle r^2\rangle = 0.6$$

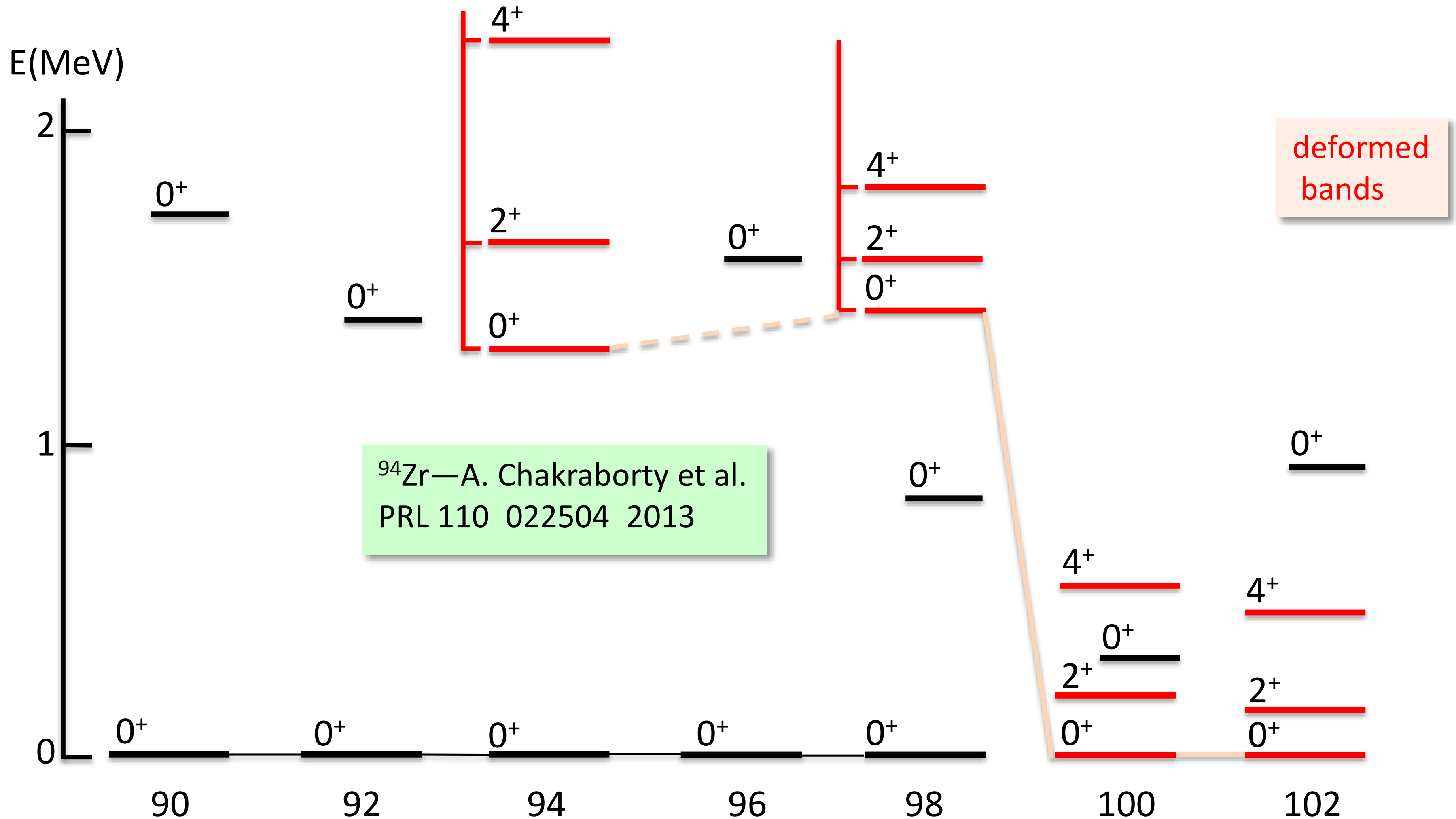
$$\rho^2 = \frac{\alpha^2\beta^2 Z^2 (\Delta\langle r^2\rangle)^2}{R^4}$$

for $\alpha^2 = \beta^2 = 0.5$ (maximal mixing)

$$\rho^2 = \frac{0.25 \times 38^2 \times 0.36}{(1.2 \times 96^{1/3})^4}$$

$$\therefore \rho^2 = 0.143$$

Deformation in Zr isotopes, 50 ≤ N ≤ 62

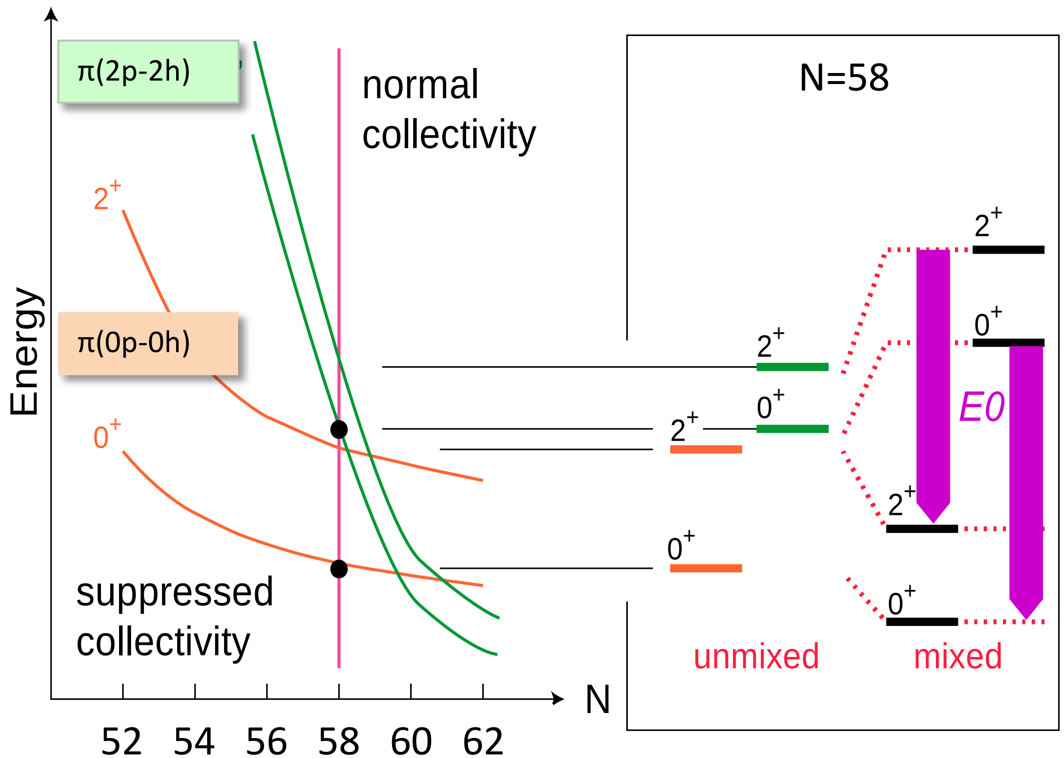


A deformed structure can intrude to become a ground state:

appears to produce a “collective phase change”

Nuclei are manifestations of coexisting structures
that may invert by addition of a few nucleons, and may mix.

Proton pair excitations with respect to the Z = 40 subshell



Ground state properties, S_{2n} and $\delta\langle r^2\rangle$, in the regions of N = 60, 90 are very similar

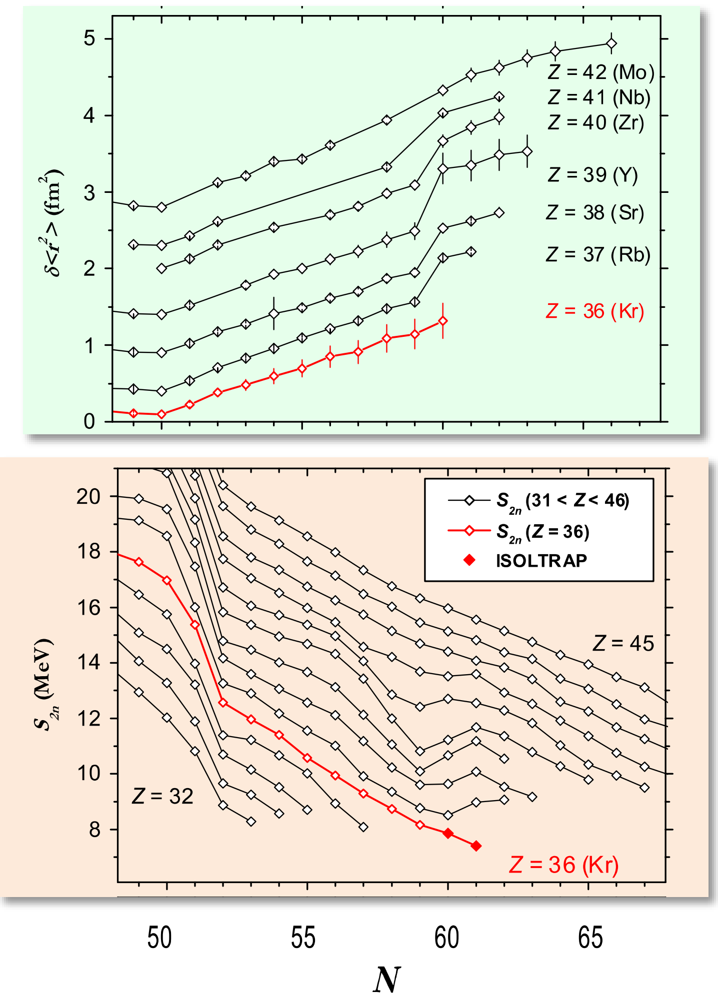


Figure from S. Naimi et al. Phys. Rev. Lett. 105 032502 (2010)

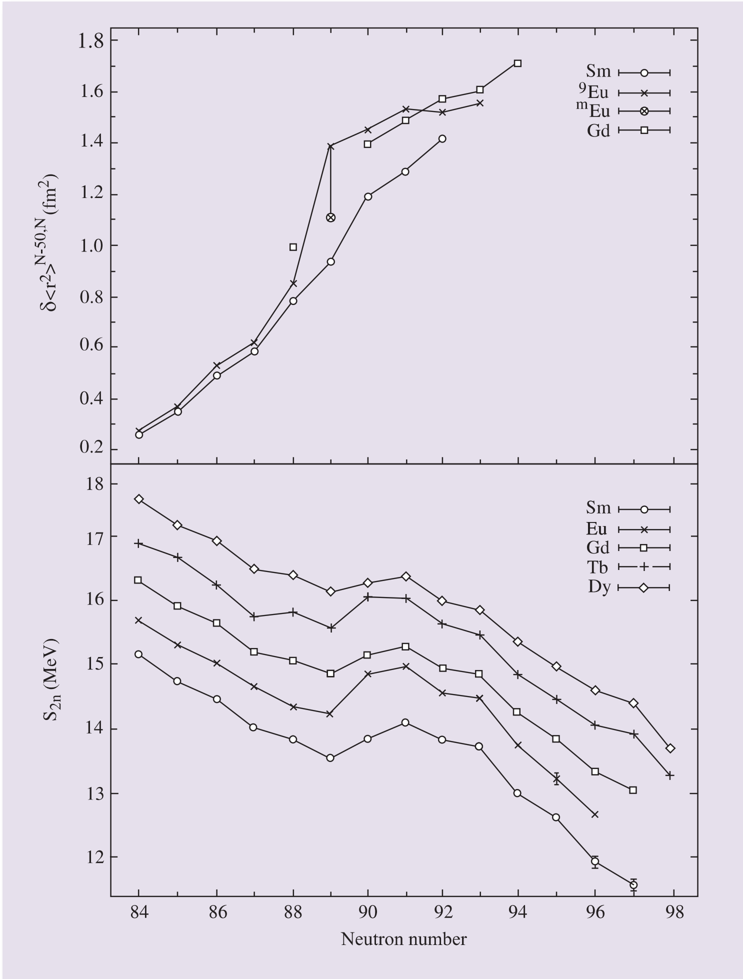
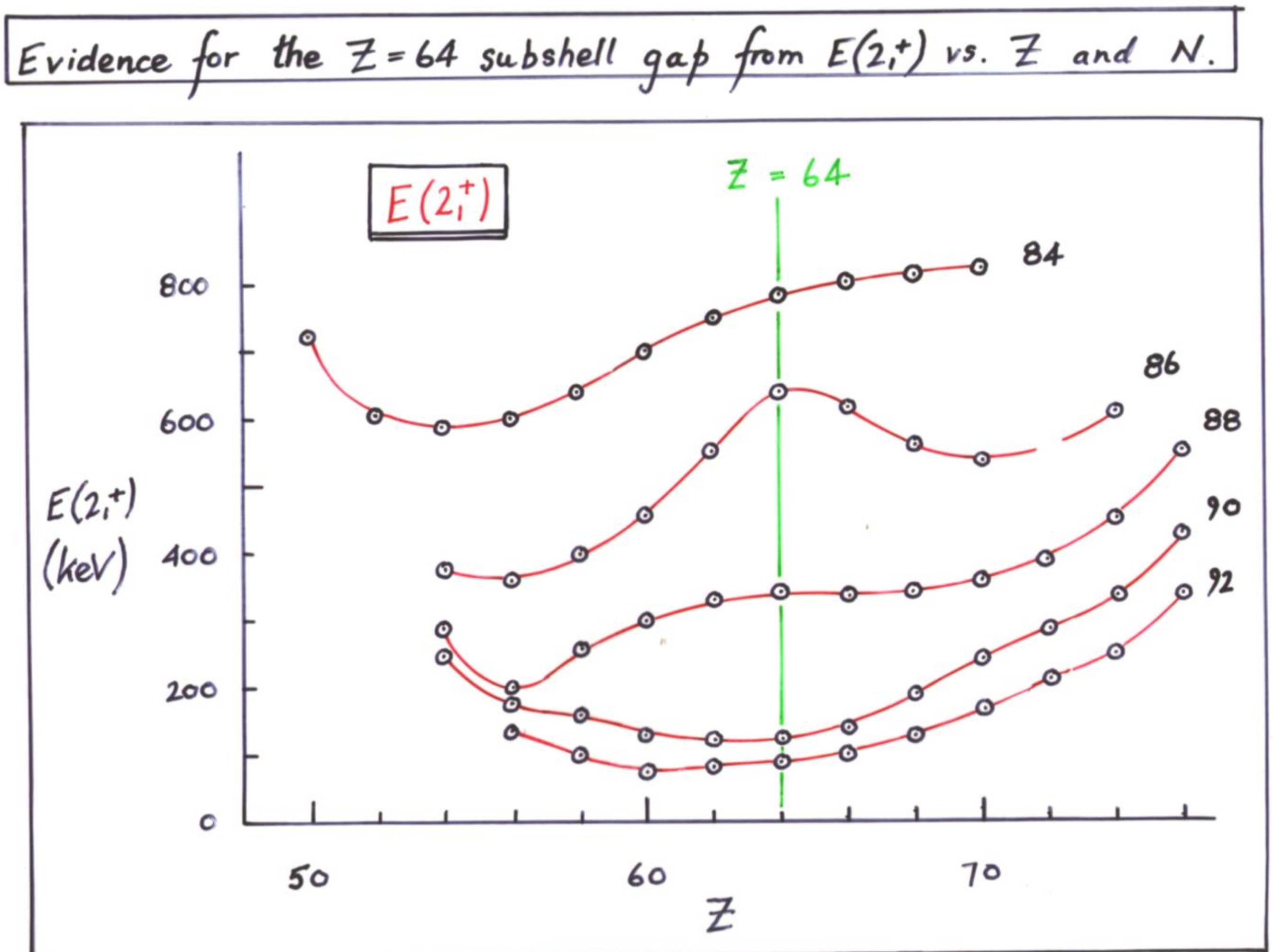
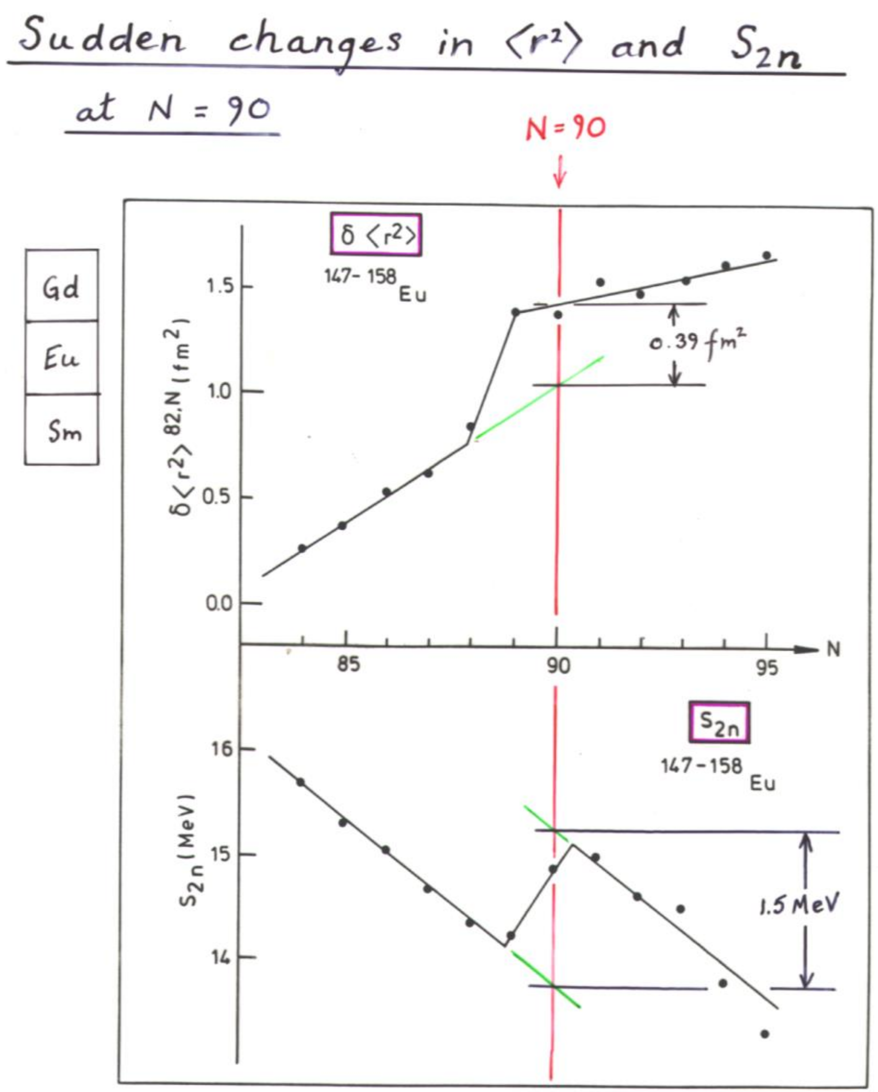


Figure from Heyde & Wood

$E(2_1^+)$ systematics for N ~90 and Z ~64

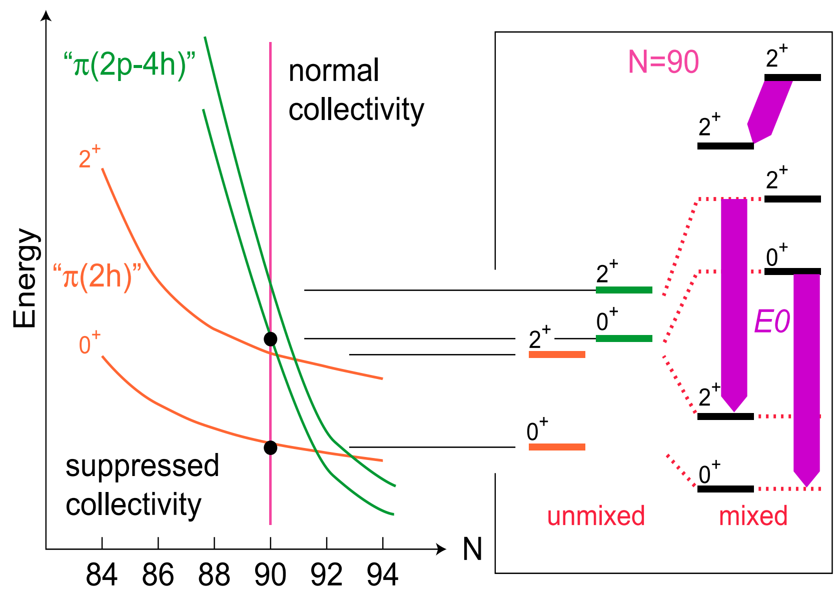


Systematics of <r^2> and S_{2n} for the Eu isotopes



^{152}Sm and the neighboring N = 90 isotones are a manifestation of shape coexistence

Proton particle-hole excitations across the Z = 64 gap may be the source of the coexisting shapes.



Less-deformed 2h and more-deformed 2p-4h structures coexist at low energy at N=90.

Strong mixing obscures the energy differences that are indicative of different shapes.

Strong *E0* transitions are a key signature of the mixing of coexisting structures.

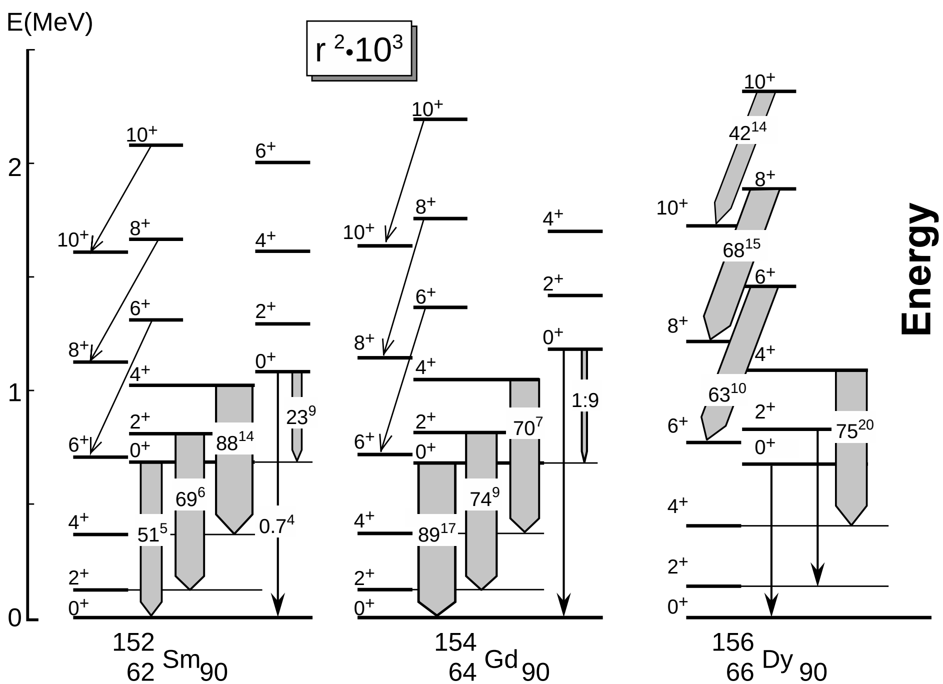
As observed, the K=2 bands will also mix strongly, resulting in *E0* transitions.

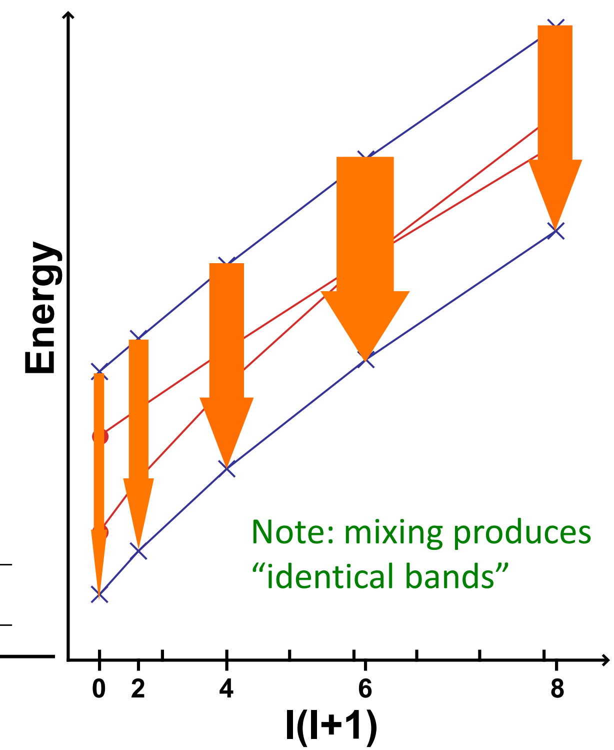
Shape coexistence in the N = 90 isotones:

revealed by E0 transition strengths

Strong mixing of coexisting shapes produces strong electric monopole (E0) transitions and identical bands.

Data from Heyde and Wood (2011)



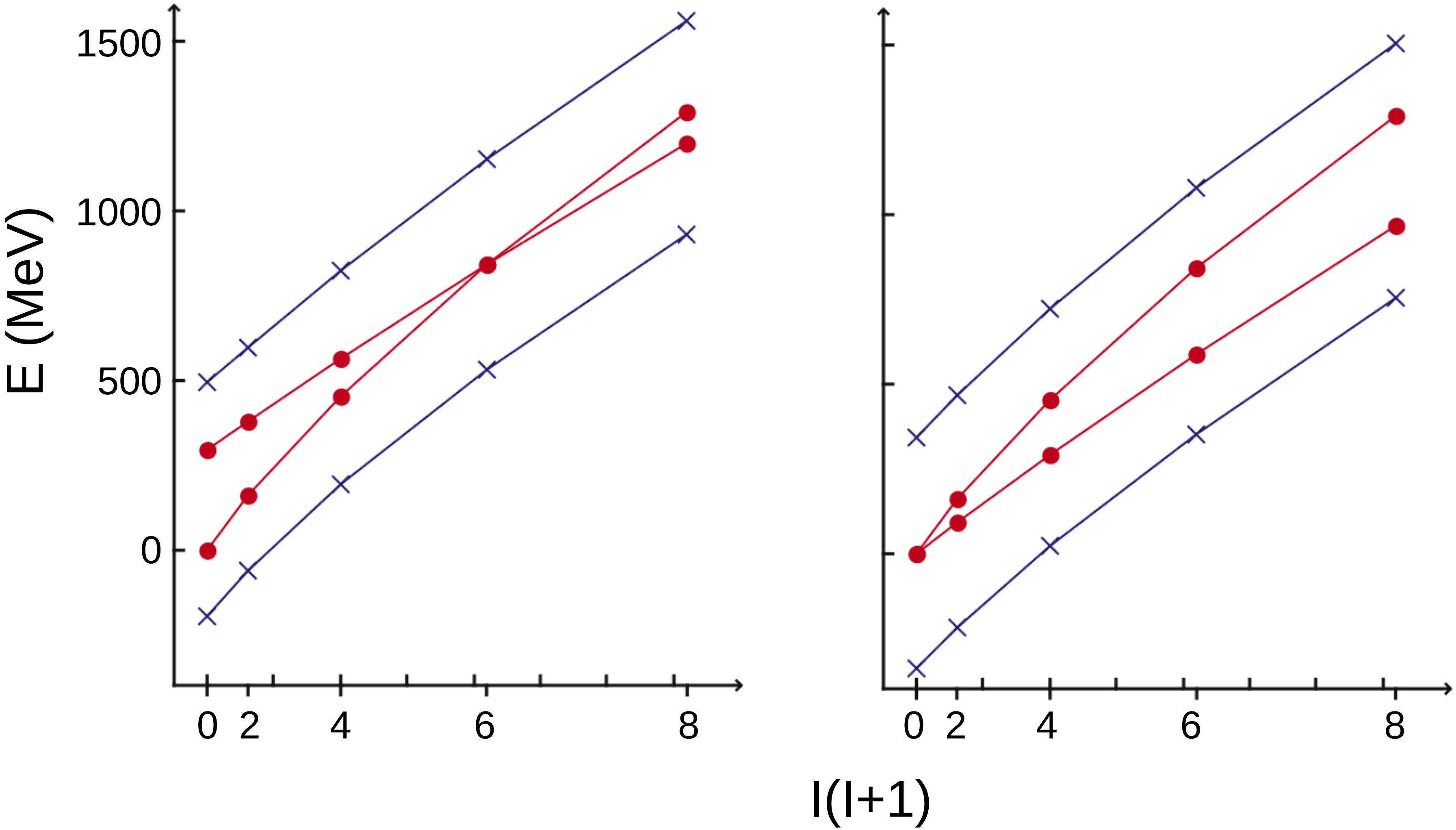


$$\rho^2 \cdot 10^3 = \alpha^2\beta^2 \left(\Delta\langle r^2\rangle\right)^2 \cdot 10^3 \frac{Z^2}{R_0^4}$$

$\Rightarrow$ *E0* strength is a function of mixing.

$$R_0 = 1.2A^{1/3} \text{ fm}$$

Strong mixing produces (near) identical bands



Mixing of coexisting structures in ^{152}Sm

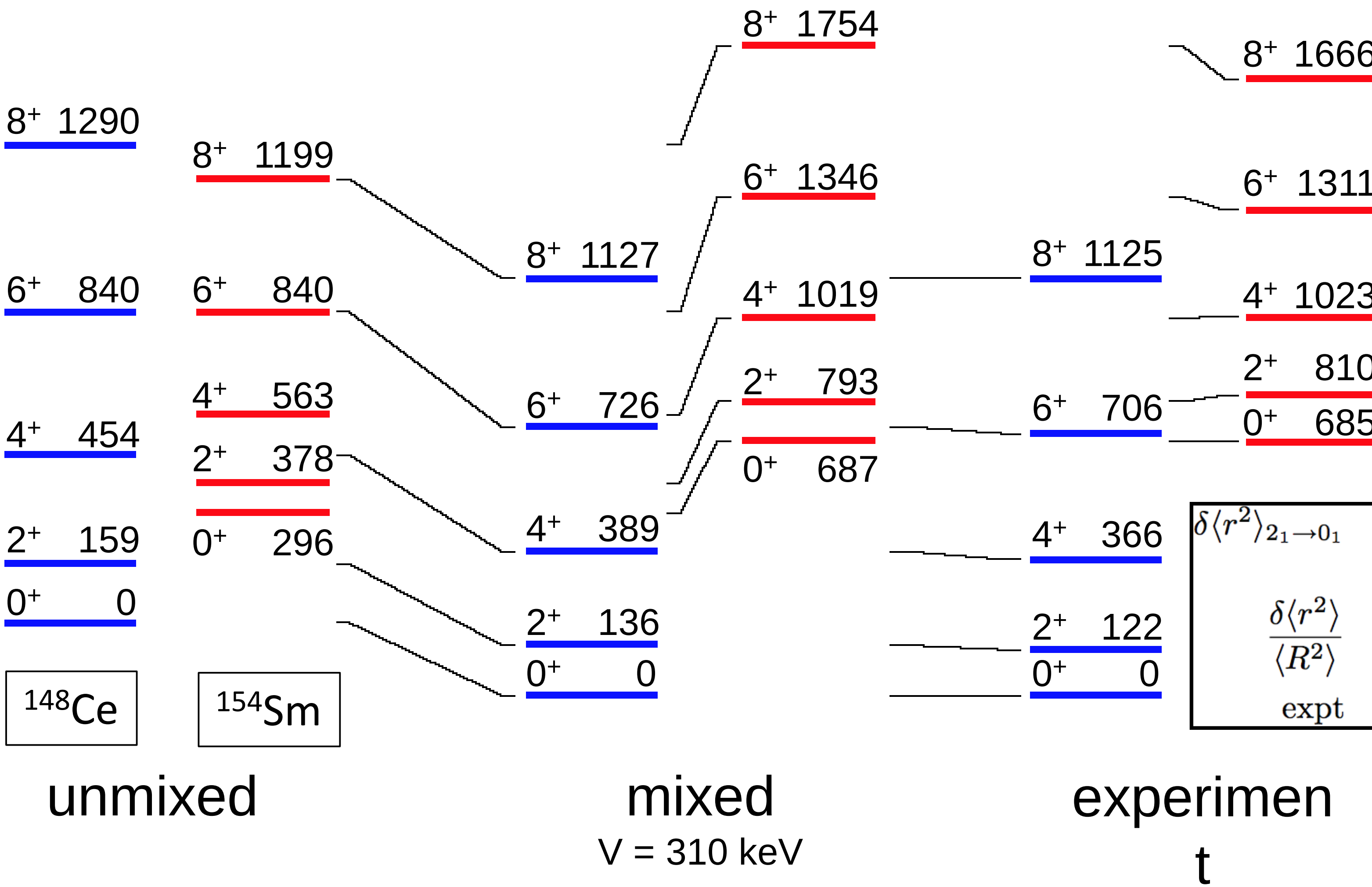
J	0	2	4	6	8
α_J	0.8458	0.8167	0.7656	0.7071	0.6536
β_J	0.5334	0.5770	0.6433	0.7071	0.7569

$\Delta\langle r^2\rangle = 0.39\ \mathrm{fm}^2$ (from Eu) $R = 1.20\ A^{1/3}\ \mathrm{fm}$

$$\frac{Z^2}{R^4}\left[\Delta\langle r^2\rangle\right]^2 \cdot 10^3$$

$$\rho_J^2(E0) = 348\ \alpha_J^2\ \beta_J^2$$

calc	expt
85.2	
87.0	
84.4	88^{14}
77.3	69^{6}
70.8	51^{5}

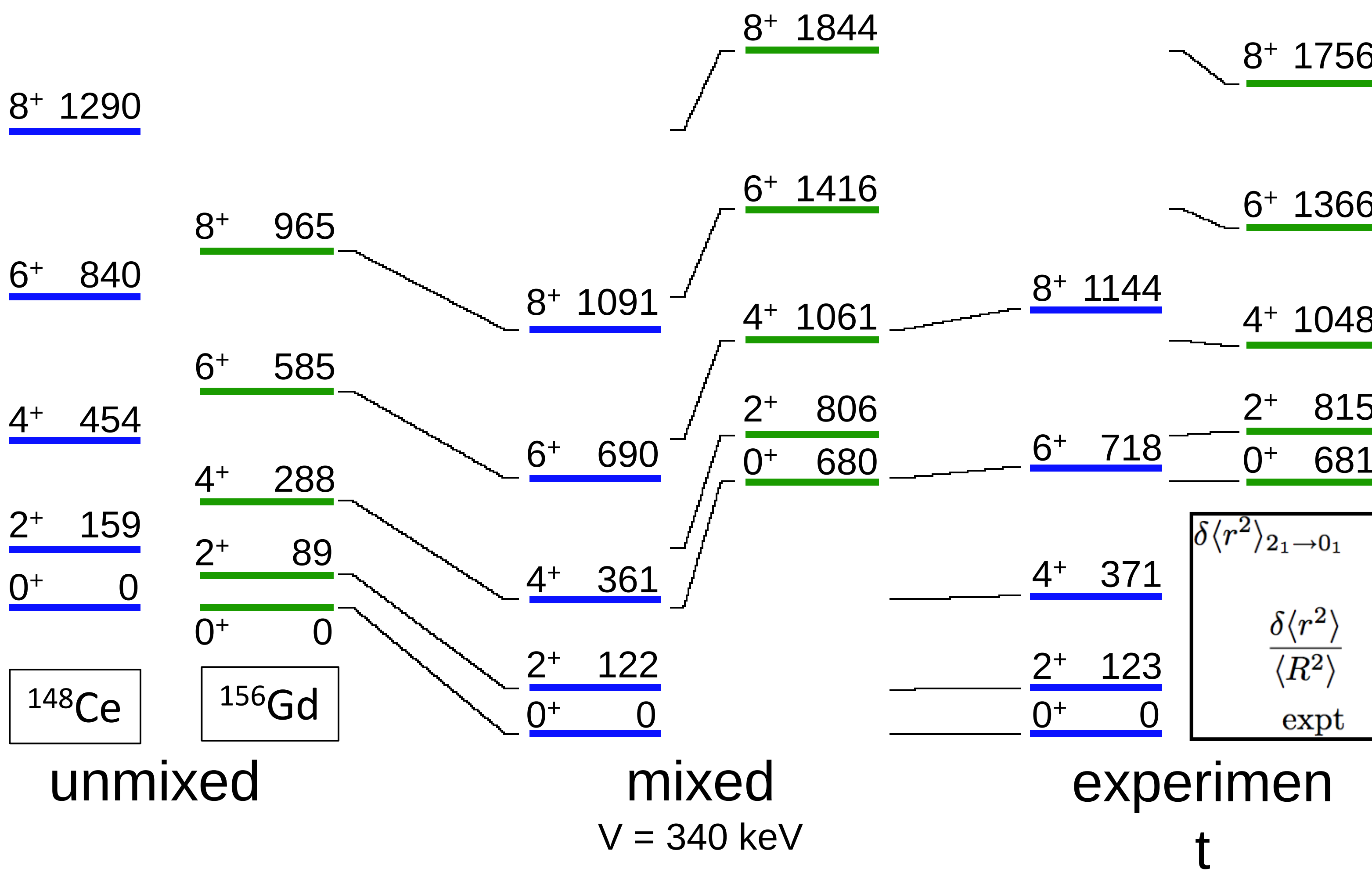


$$\delta\langle r^2\rangle_{2_1\to 0_1} = \Delta\langle r^2\rangle(\beta_2^2 - \beta_0^2)$$
$$= 0.0189\ \mathrm{fm}^2$$
$$\frac{\delta\langle r^2\rangle}{\langle R^2\rangle} = 4.6(-4)$$
expt $5.2^{6}(-4)$

Mixing of coexisting structures in ^{154}Gd

J	0	2	4	6	8
α_J	0.7071	0.7422	0.7865	0.8226	0.8463
β_J	0.7071	0.6702	0.6176	0.5686	0.5327

$\Delta\langle r^2\rangle = 0.39\ \mathrm{fm}^2$ (from Eu) $\qquad R = 1.20\ A^{1/3}\ \mathrm{fm}$



$$\frac{Z^2}{R^4}\left[\Delta\langle r^2\rangle\right]^2 \cdot 10^3$$

$$\rho_J^2(E0) = 364\ \alpha_J^2\ \beta_J^2$$

calc	expt
74.0	
79.6	
85.9	70^{7}
90.1	74^{9}
91.0	89^{17}

$$\delta\langle r^2\rangle_{2_1\to 0_1} = \Delta\langle r^2\rangle(\beta_2^2 - \beta_0^2) = 0.0198\ \mathrm{fm}^2$$

$$\frac{\delta\langle r^2\rangle}{\langle R^2\rangle} = 4.8(-4)$$

expt $\quad 5.7^{8}(-4)$

^{152}Sm: B(E2) values

Grodzins' rule for quadrupole strength

$$[E(2_1^+)\ \text{keV}\ ][B(E2;0_1^+\rightarrow 2_1^+)e^2\cdot b^2]\frac{A}{Z^2}\approx 16.0$$

$$B(E2)=\frac{M^2}{2J_i+1}\times 206.4\ \text{W.u.}$$

Rotor matrix elements

$$M_{J,J-2} = \sqrt{\frac{3J(J-1)}{2(2J-1)}}\,M_{20}$$

$$M_{J,J} = -\sqrt{\frac{J(J+1)(2J+1)}{(2J-1)(2J+3)}}\,M_{20}$$

$$M_{20} = \sqrt{B(E2;0_1^+\rightarrow 2_1^+)}$$

$$M_{20}^a = 1.595\ e\cdot b\ (^{148}\text{Ce: } E(2_1^+))$$

$$M_{20}^b = 2.221\ e\cdot b\ (^{154}\text{Sm: } E(2_1^+))$$

e.g.,

$$M_{2_10_1} = \alpha_0\alpha_2 M_{20}^a+\beta_0\beta_2 M_{20}^b = 1.785\quad e\cdot b$$

$$M_{4_12_1} = (\alpha_2\alpha_4 M_{20}^a+\beta_2\beta_4 M_{20}^b)(1.604) = 2.922$$

$$M_{2_20_2} = \beta_0\beta_2 M_{20}^a+\alpha_0\alpha_2 M_{20}^b = 2.025$$

$$M_{2_20_1} = -\alpha_0\beta_2 M_{20}^a+\alpha_2\beta_0 M_{20}^b = 0.1891$$

$$M_{2_22_1} = \alpha_2\beta_2(M_{20}^b-M_{20}^a)(-1.195) = -0.3525$$

$$M_{0_22_1} = -\alpha_2\beta_0 M_{20}^a+\alpha_0\beta_2 M_{20}^b = 0.3891$$

	calc.	expt.	calc.*
$2_1\rightarrow 0_1$	132	143[2]	143
$4_1\rightarrow 2_1$	196	209[3]	212
$6_1\rightarrow 4_1$	228	245[5]	246
$8_1\rightarrow 6_1$	252	285[14]	272
$2_2\rightarrow 0_2$	170	169[15]	
$4_2\rightarrow 2_2$	233	265[44]	
$0_2\rightarrow 2_1$	31.4	33.1[21]	
$2_2\rightarrow 4_1$	22.8	17.8[14]	
$4_2\rightarrow 6_1$	21.5	16.7[28]	
$2_2\rightarrow 2_1$	5.16	5.80[47]	
$4_2\rightarrow 4_1$	5.12	4.9[8]	
$2_2\rightarrow 0_1$	1.48	0.94[8]	
$4_2\rightarrow 2_1$	0.47	0.75[13]	

*Grodzins +8%

Mottelson, Tokyo Conf., 1967: comment on breakdown of ΔK = 0 Alaga rules at N = 90

I have discussed in some detail the phenomena associated with the coupling between $K=2$ and $K=0$ bands in order to illustrate the wealth of quantitative relationships which can be brought to bear in analyzing the rotational effects. I shall now consider, much more briefly, the data concerning the coupling of the ground state and excited $K=0$ bands (beta vibrations) in even-even nuclei. The first step in the analysis, as in the $\Delta K=2$ case, is to consider the general form of the matrix elements as obtained from the expansion in powers of I; for the E2 transitions between the bands we get (including up to linear terms in I)

$$B(\mathrm{E2};\,K=0_2 I_2 \to K=0_1 I_1) = \langle I_2 0;\, 20 | I_1 0\rangle^2 |M_1 + M_2[I_2(I_2+1) - I_1(_1+1)]|^2\,. \qquad (17)$$

The intensity rule (17) has been much less tested than the relation (6) for $\Delta K=2$ transitions, but a similar accuracy is expected. During the past year, intensities of transitions from excited $K=0$ bands in ^{152}Sm, ^{154}Gd and ^{156}Gd have been measured and found to be in disagreement with the predictions of (17) (Ewan and Graham: Moscow Conference 1966; Liu, Nielsen, Salling and Skilbreid: Moscow Conference 1966; Ewan and Anderson: Contribution No. 4.146, this conference; Johnson, Riedinger and Hamilton: Contribution No. 4.144; similar data on ^{178}Hf has been obtained by Loft Nielsen: private communication). Since a failure of (17) would imply a breakdown in the fundamental rotational relationships (*i.e.* this is not a result that depends on any detailed model for the intrinsic structure), I think that everyone is reluctant to believe that the fault lies there. Indeed it has been noticed that all the deviations could be explained if in the transition $0_2 I=2 \to 0_1 I=2$ there is a significant contribution from M1 radiation. Such radiation is forbidden in the I-independent approximation, but the familiar rotational contribution to the nuclear magnetic moments is already a term linear in I and if the g-factor depends somewhat on the deformation we obtain a transition operator

$$\mathscr{M}(\mathrm{M1},\mu) = \sqrt{\frac{3}{4\pi}}\left\{g_R(\beta_0) + (\beta-\beta_0)\frac{\partial g_R(\beta_0)}{\partial\beta} + \cdots\right\} I_\mu \left(\frac{e\hbar}{2Mc}\right)$$

and a transition matrix element for decay of a β-vibrational state

$$B(\mathrm{M1};\, n_\beta=1, I_2 \to n_\beta=0, I_1) = \frac{3}{4\pi}\left(\frac{e\hbar}{2Mc}\right)^2 \frac{\hbar\omega_\beta}{2C_\beta}\left(\frac{\partial g_R}{\partial\beta}\right)^2 \delta(I_1, I_2) I_1(I_1+1)\,, \qquad (17a)$$

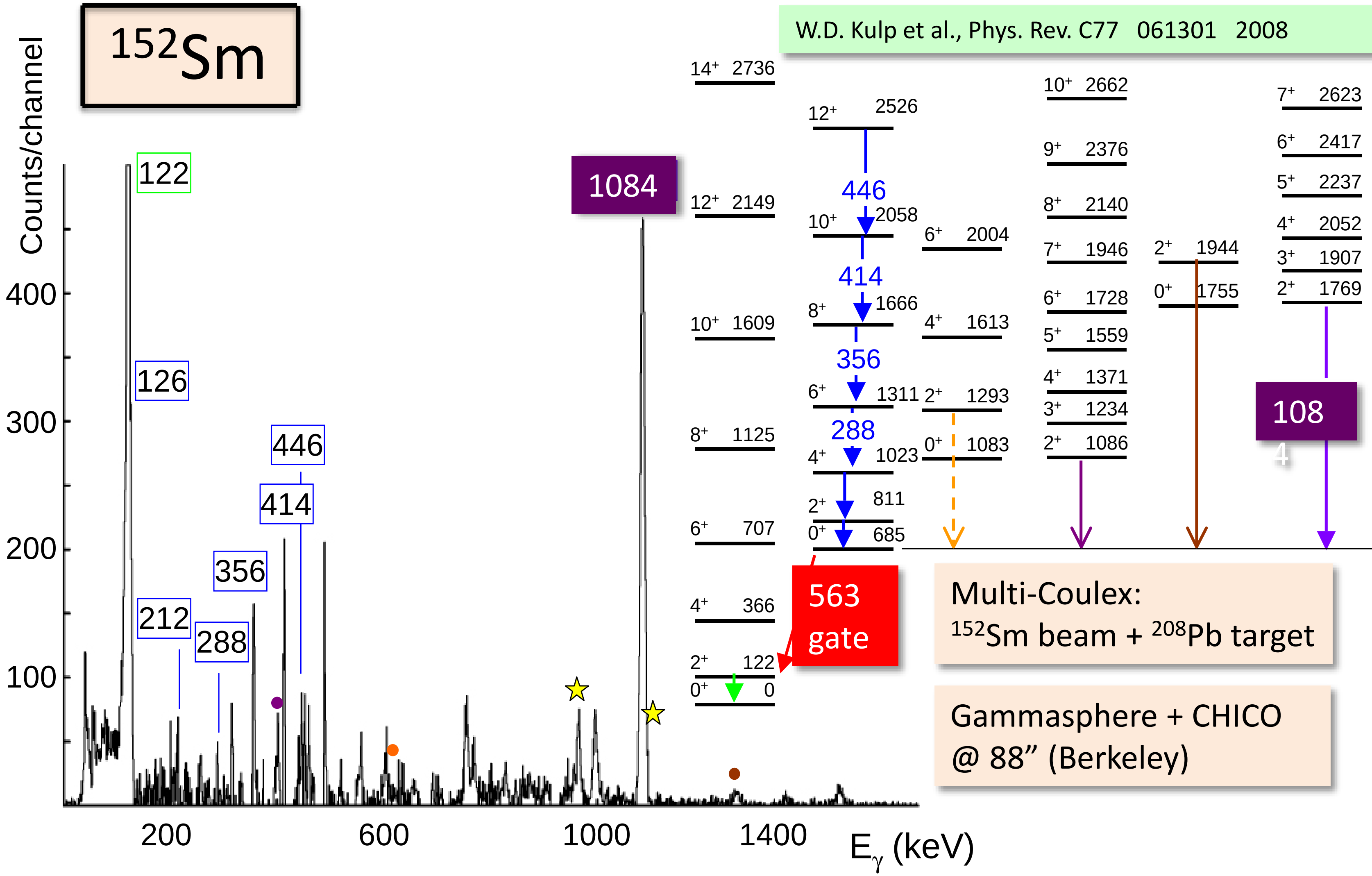
where $(\hbar\omega_\beta/2C_\beta)$ is the amplitude of the β-vibrational motion as measured in the E2 transition matrix elements connecting the two bands. Values of $\partial g_R/\partial\beta$ of order unity are sufficient to explain the postulated M1 intensities. The situation looks promising, but the crucial measurement is obviously a direct determination of the M1 contribution to the $\Delta I=0$ transitions between these bands. Tentative evidence against the expected M1 admixtures has been submitted to this conference by Hamilton, Ramayya, Whitlock and Meulenberg: Contribution No. 4.145. I cannot judge the finality of this measurement, but I must emphasize that if the M1 intensity is not found, we face a major crisis in the application of the rotational relationships to these nuclei.

Multi-Coulex of ^{152}Sm 0_2^+(685 keV):

strongest response is to head of K=2$^+$ band at 1769 keV

(in-band response attenuated by 99.7% decay out @ 811 level)

W.D. Kulp et al., Phys. Rev. C77 061301 2008

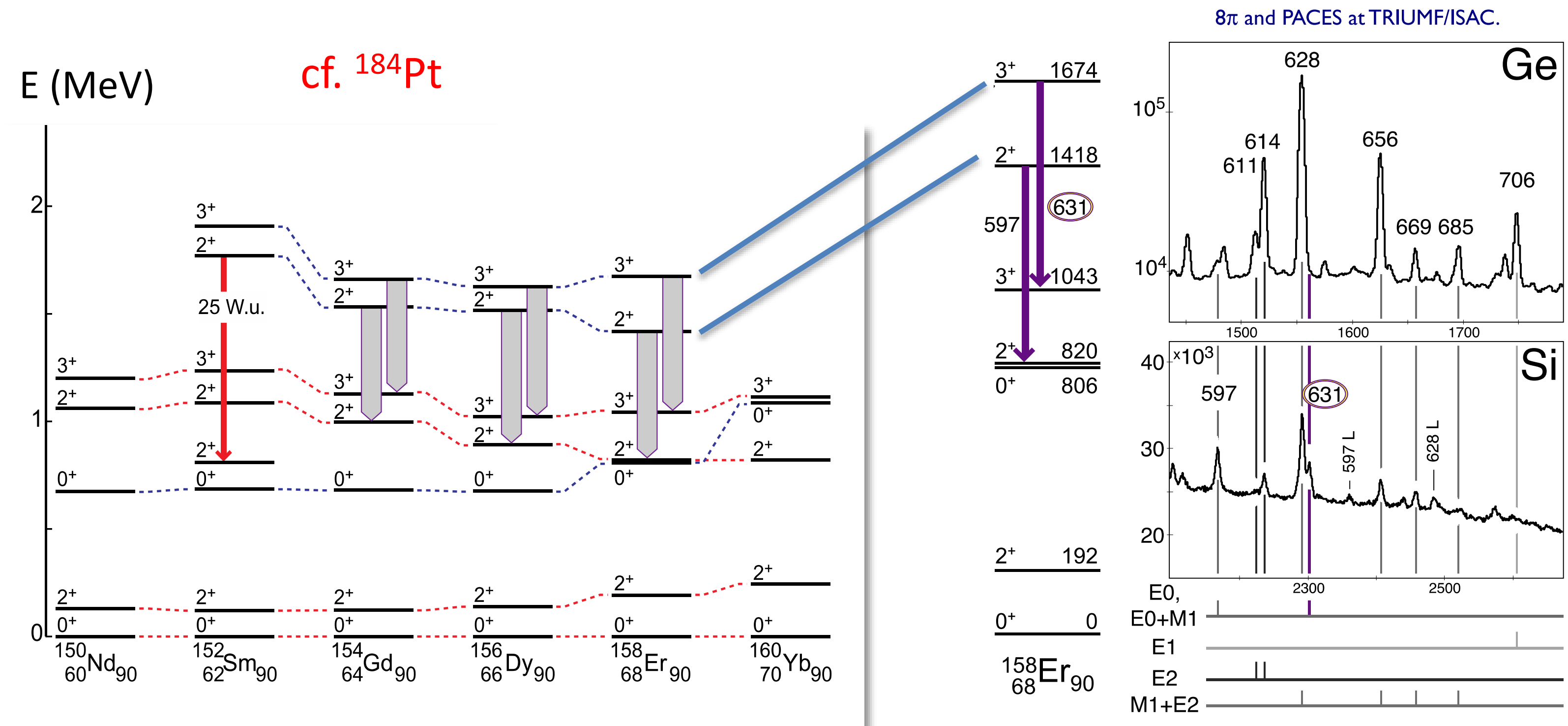


Multi-Coulex:
^{152}Sm beam + ^{208}Pb target

Gammasphere + CHICO
@ 88” (Berkeley)

Shape coexistence in the N = 90 isotones: coexisting K = 2 bands revealed by E0 transitions

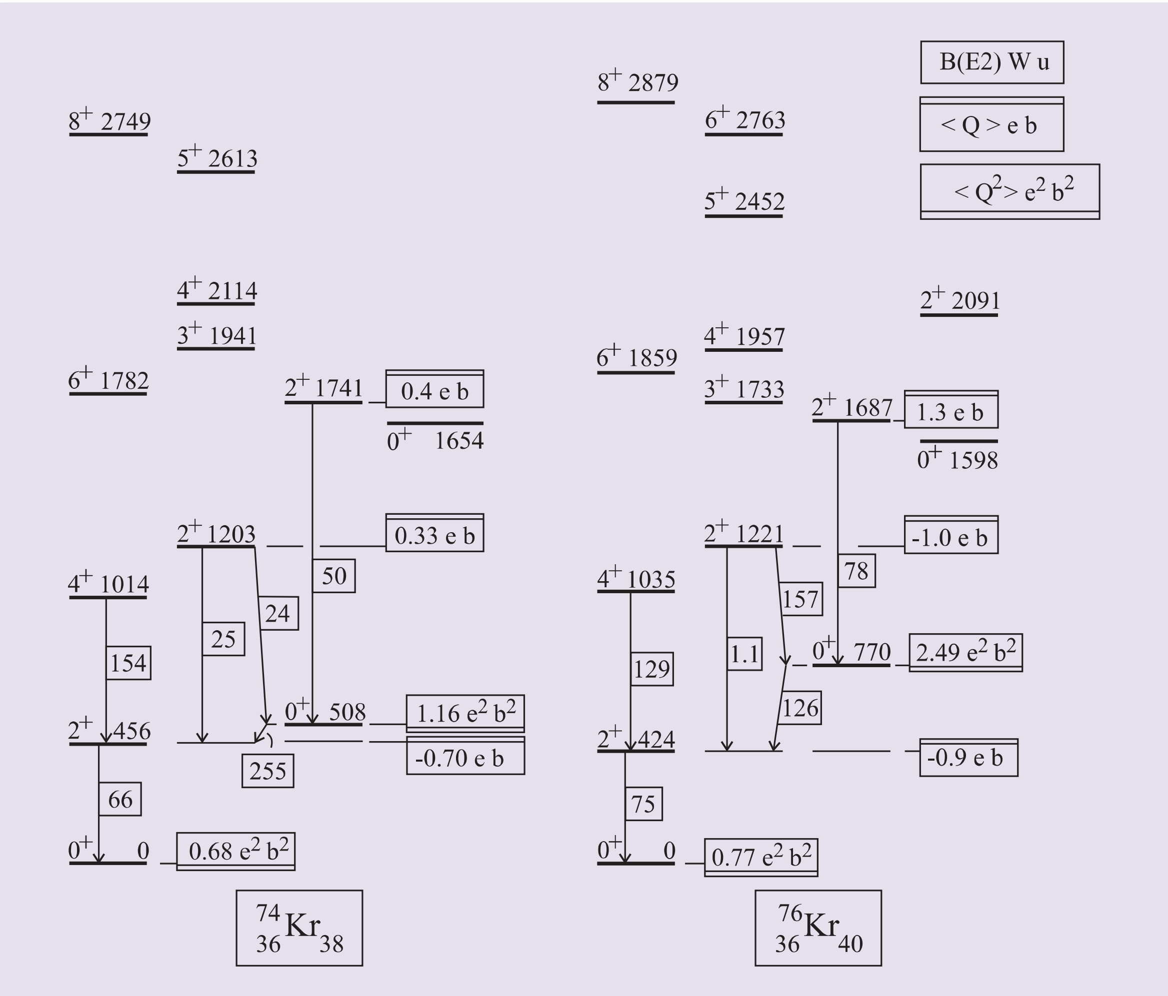
3^+, K = 2 → 3^+, K = 2: 631 keV transition in ^{158}Er has no observable γ-ray strength, only ce's are observed --accidental cancellation of E2; M1 is very weak.
[$3K^2 - I(I+1) = 0$]



^{152}Sm: W.D. Kulp et al., Phys. Rev. C77 061301 2008

Kulp, Wood, Garrett, Zganjar and others

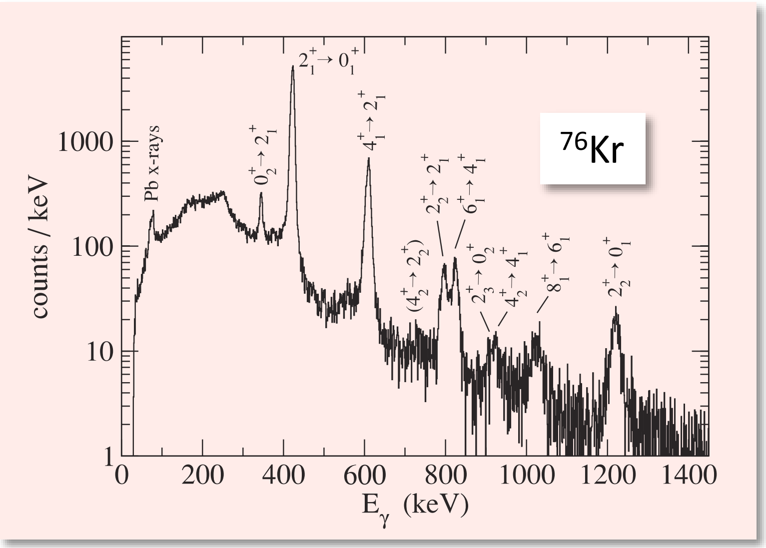
Neutron-deficient Kr isotopes: puzzling collectivity

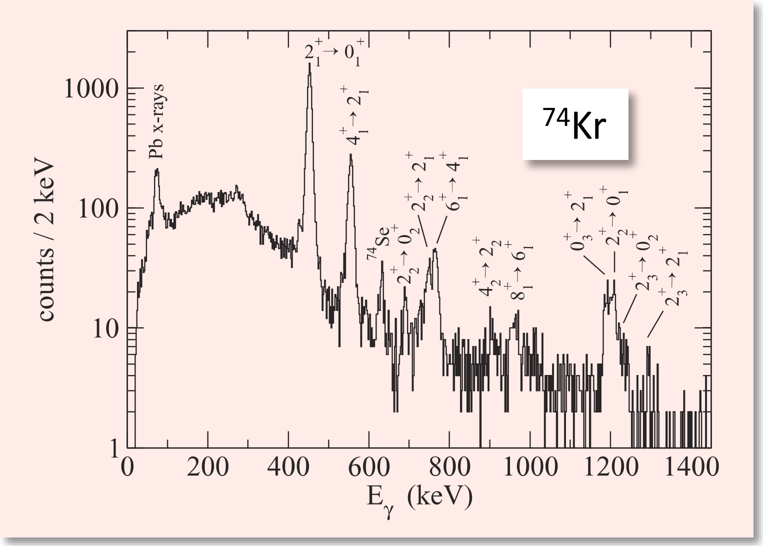


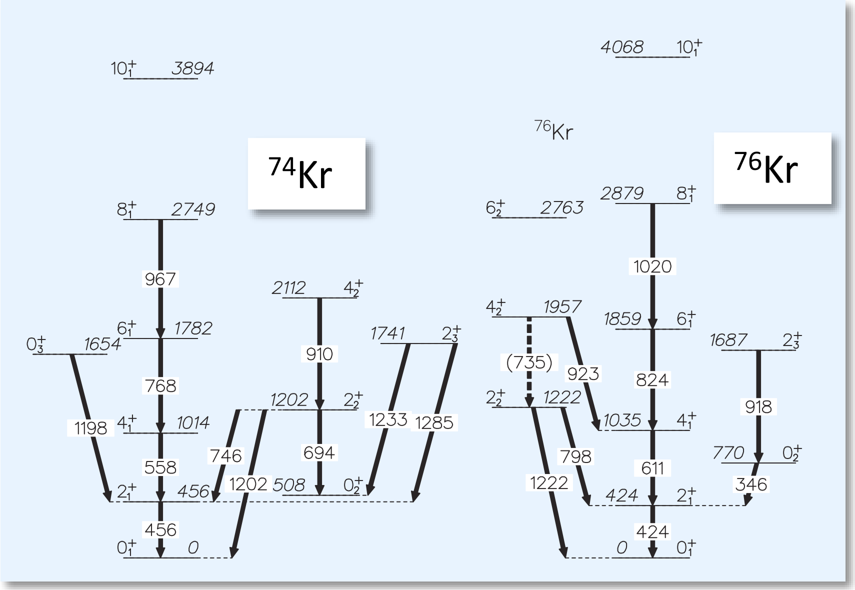
E. Clément et al.,
PR C75 054313 2007

Multistep Coulomb excitation of $^{74,76}Kr$ using radioactive beams of Kr on a ^{208}Pb target

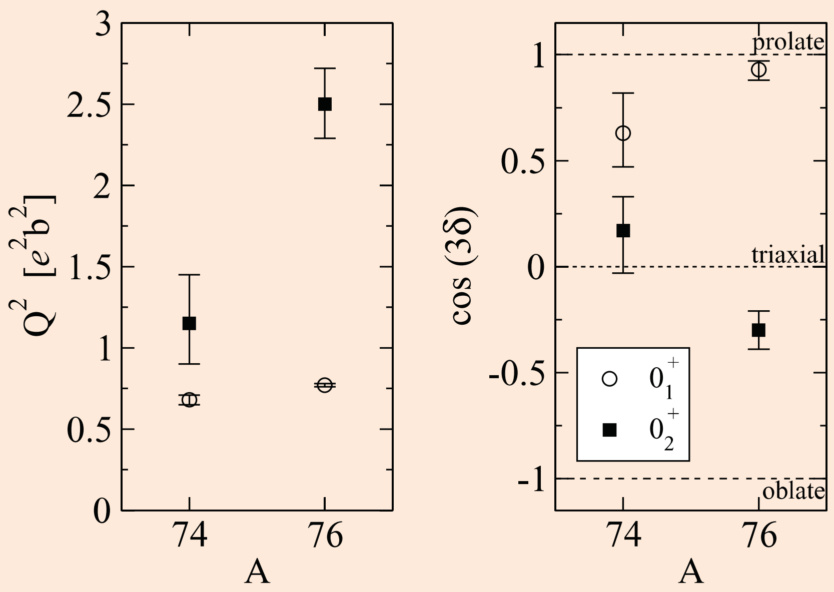
E. Clément et al.,
PR C75 054313 2007







Quadrupole shape invariants constructed from E2 matrix elements for ^{74,76}Kr



E. Clément et al.,
PR C75 054313 2007

$$\langle q^2 \rangle \equiv \langle 0_1^+ \| \hat{Q} \| 2_1^+ \rangle \langle 2_1^+ \| \hat{Q} \| 0_1^+ \rangle + \langle 0_1^+ \| \hat{Q} \| 2_2^+ \rangle \langle 2_2^+ \| \hat{Q} \| 0_1^+ \rangle$$

for the ground state

$$\langle q^3 \cos 3\delta \rangle \equiv \sum_{r,s=1,2} \langle 0_1^+ \| \hat{Q} \| 2_r^+ \rangle \langle 2_r^+ \| \hat{Q} \| 2_s^+ \rangle \langle 2_s^+ \| \hat{Q} \| 0_1^+ \rangle .$$

CONCLUSIONS: E0 TRANSITIONS

1). They give a unique perspective on shape coexistence in nuclei

2). They probe the proton and neutron configurations that occur in nuclei

3). They probe K quantum numbers through their $\Delta K = 0$ selection rule

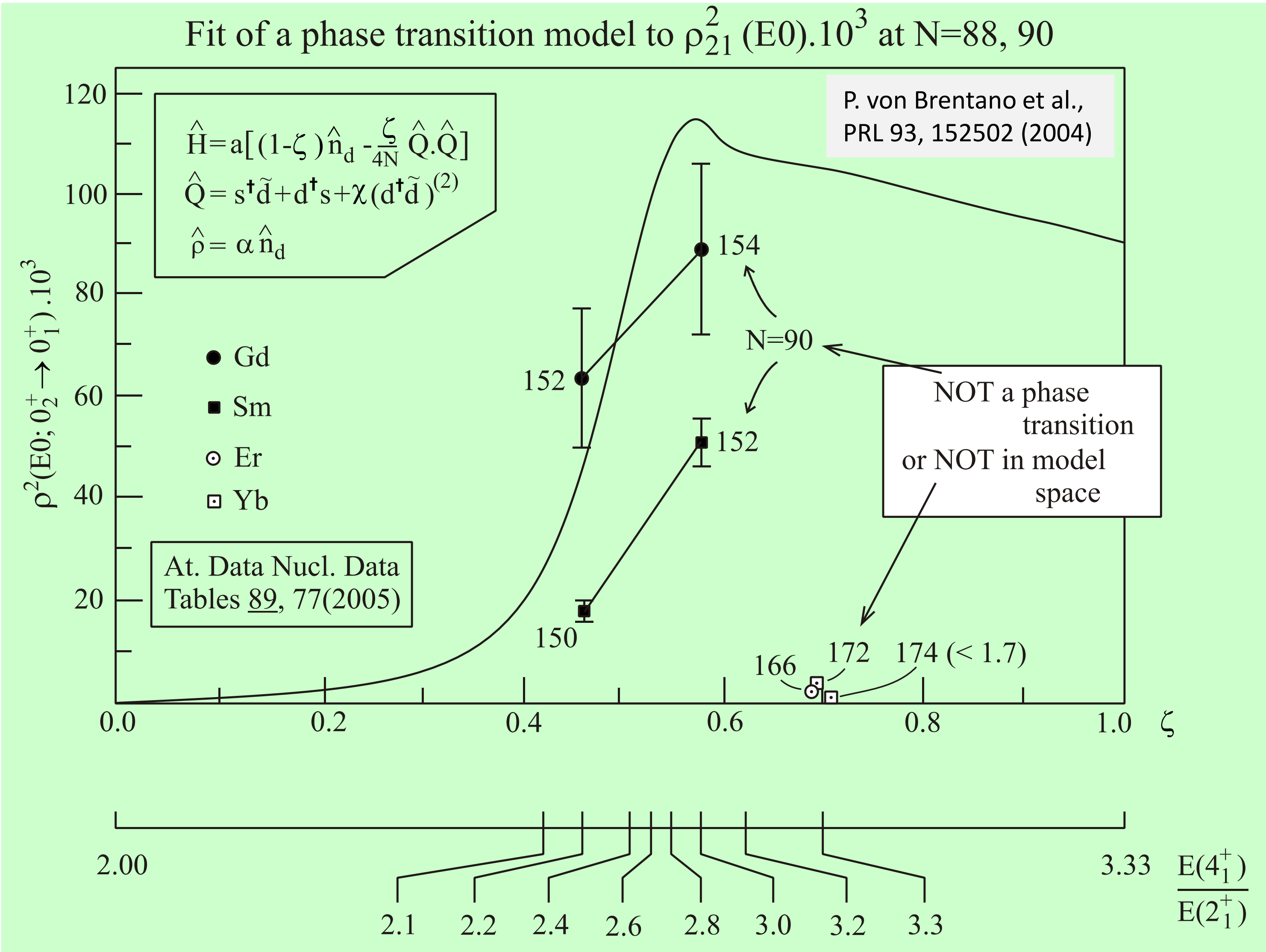
We need more data for:

$T_{1/2}(0^+)$ [and $T_{1/2}(2^+)$, $T_{1/2}(4^+)$, $T_{1/2}(3^+)$]

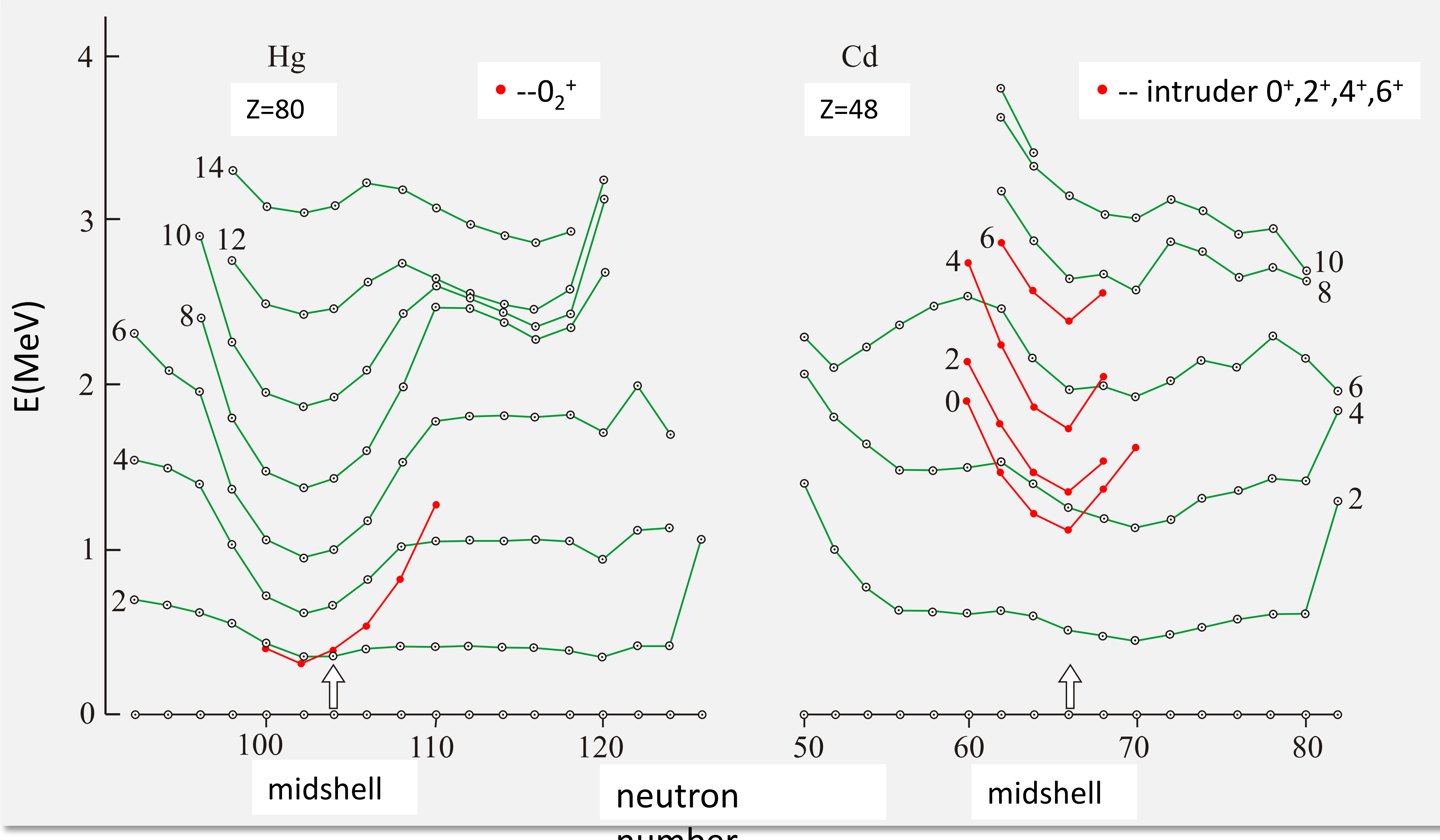
conversion electron intensities

E2 / M1 mixing ratios—to extract E2 + M1 + E0

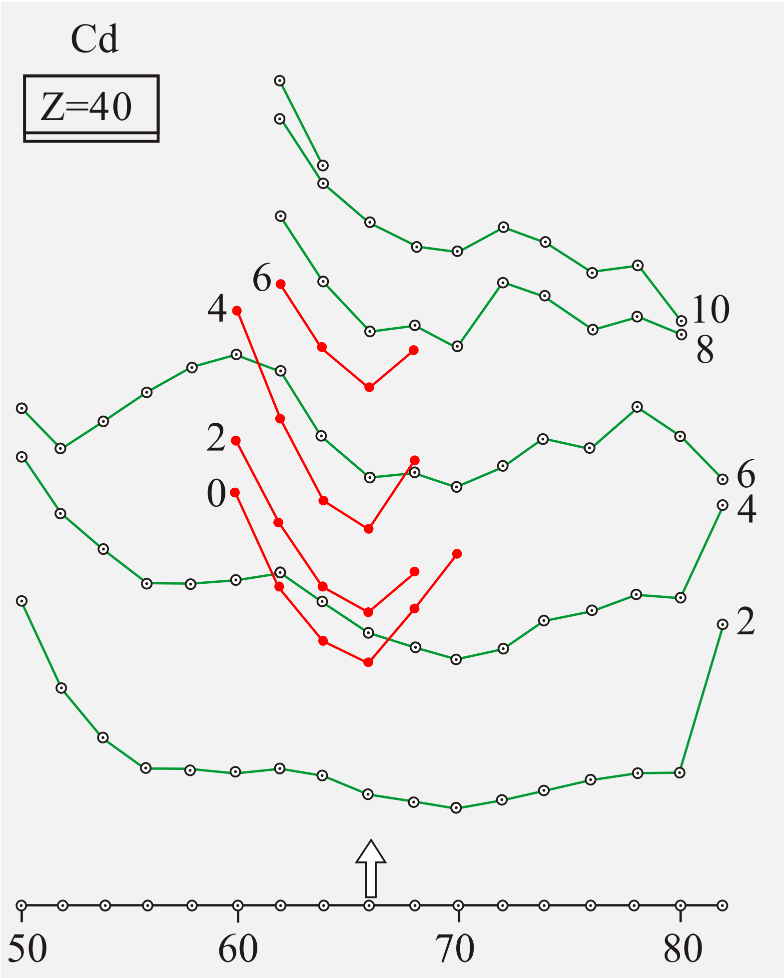
Electric monopole transition strengths: critical test of phase transition models



Shape coexistence in the Hg and Cd isotopes



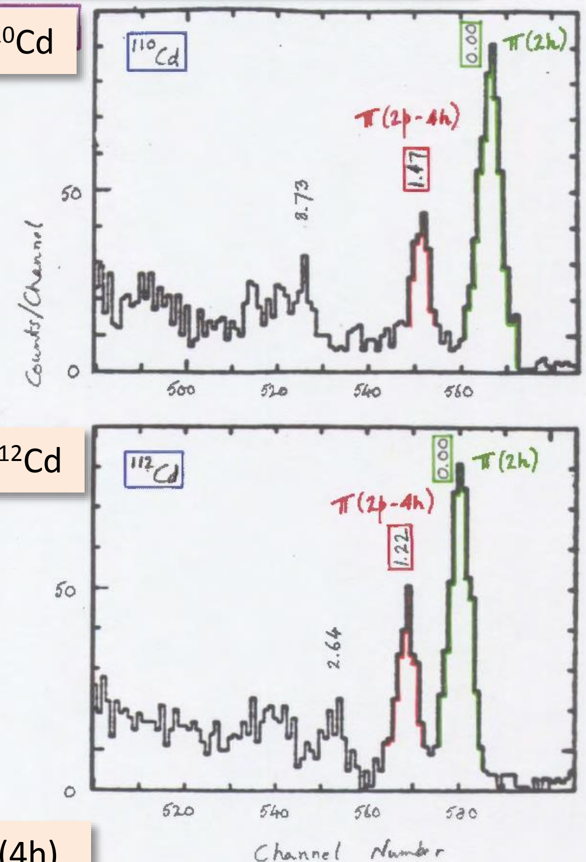
Shape coexistence in the Cd isotopes



Fielding NP A281 389 1977

^{108}Pd(^{3}He,n)^{110}Cd

^{110}Pd(^{3}He,n)^{112}Cd

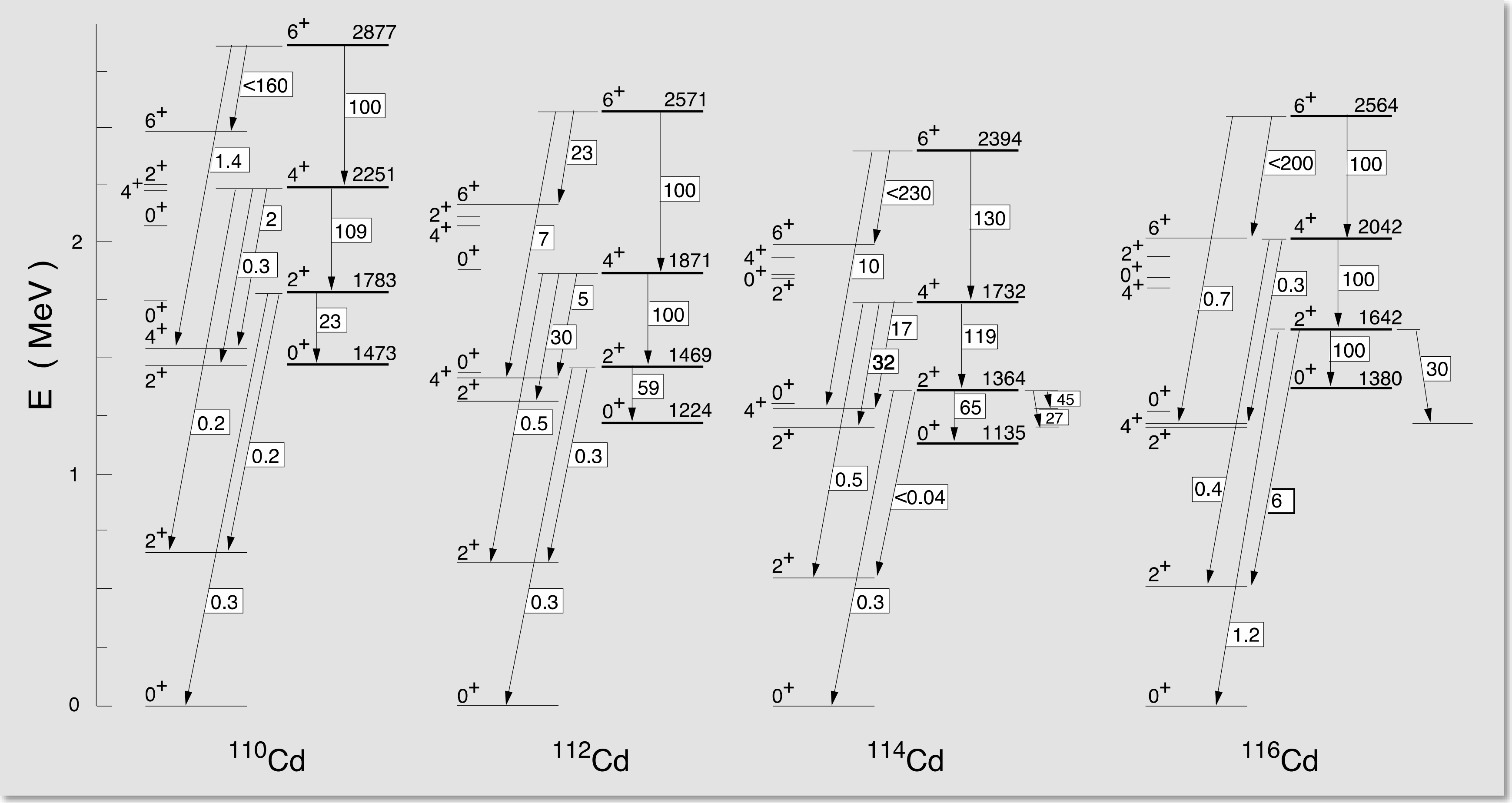


Pd targets π(4h)

Deformed bands in $^{110-116}$Cd

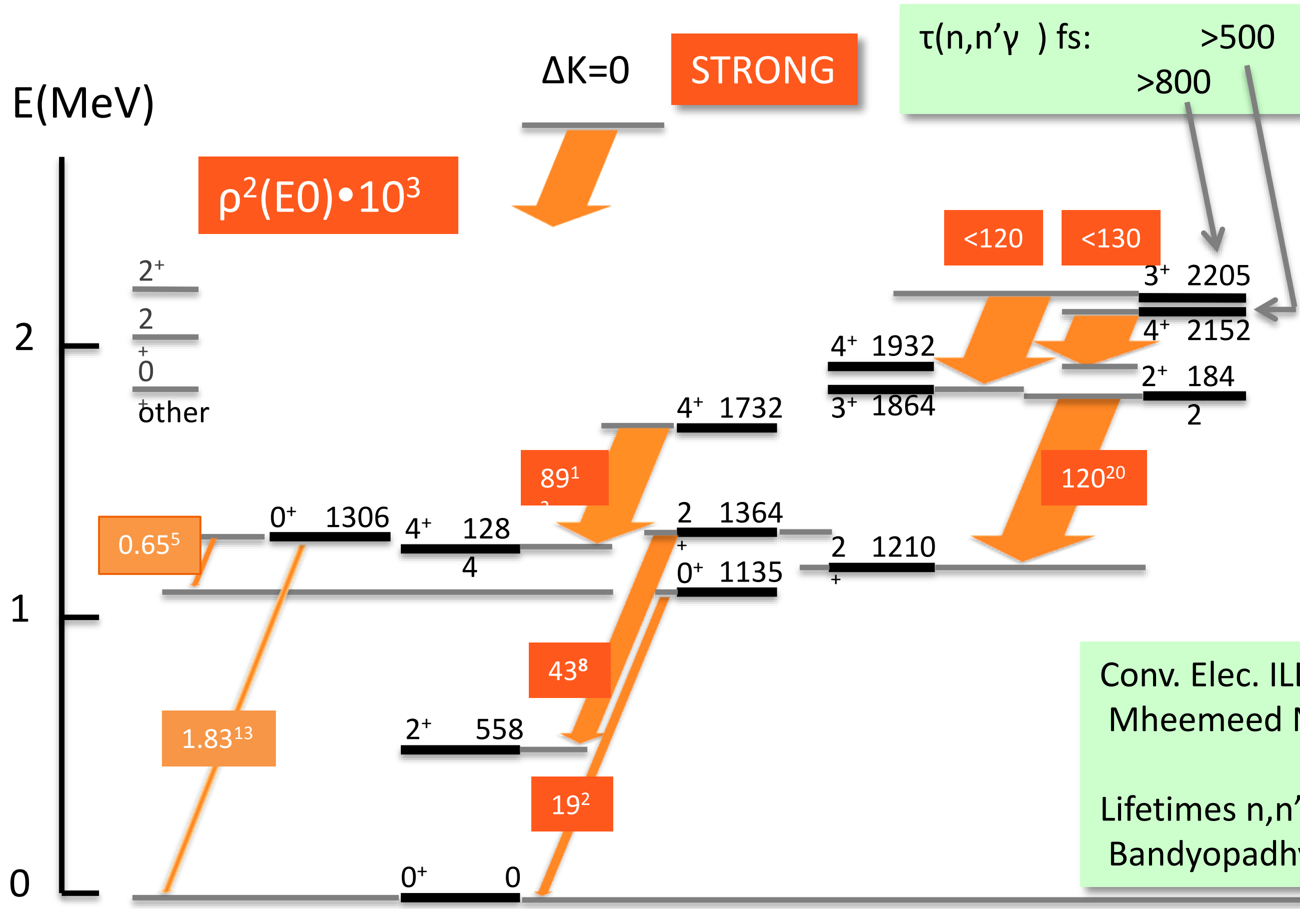
Figure from Rowe & Wood

B(E2)'s in W.u. [100 = rel. value]



The spectroscopy of mixing in the Cd isotopes: ρ^2 (E0) values in ^{114}Cd

^{114}Cd



Based on a figure in Wood
NP A651 323 1999

Conv. Elec. ILL BILL/GAMS
Mheemeed NP A412 113 1984

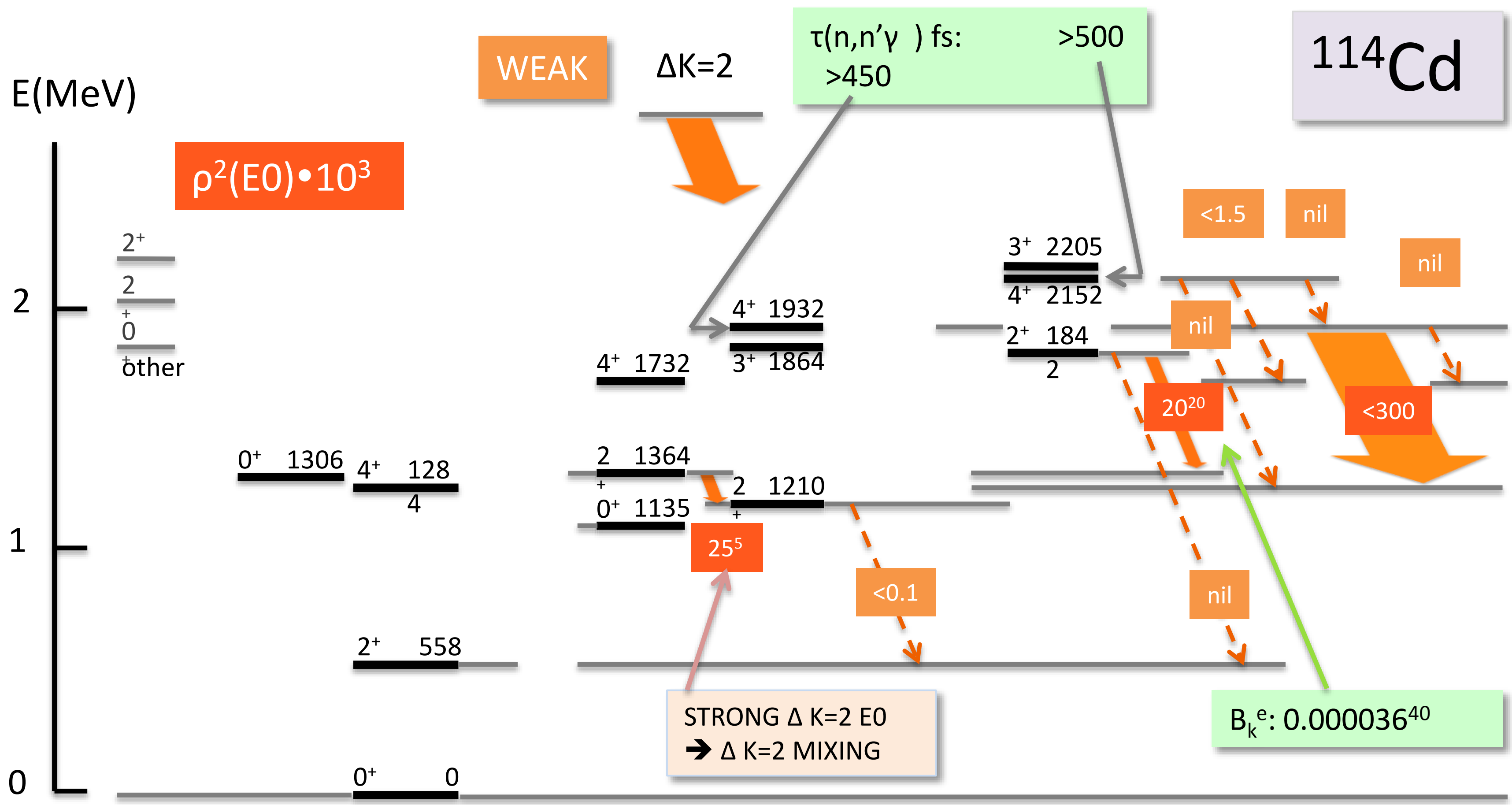
Lifetimes n,n'γ Doppler U. Kentucky
Bandyopadhyay PR C76 054308 2007

$$\rho^2 \cdot 10^3 = \alpha^2\beta^2\left(\Delta\langle r^2\rangle\right)^2 \cdot 10^3\,\frac{Z^2}{R_0^4}$$

→ *E0* strength is a function of mixing.

$$R_0 = 1.2A^{1/3}\ \text{fm}$$

E0 transition strengths in ^{114}Cd support the existence of good K quantum numbers



Spectroscopy of mixing in the Cd isotopes: ρ^2 (E0)$\cdot 10^3$ values in ^{114}Cd

$\rho^2(E0) \cdot 10^3$ max ~ 100 (obs.)

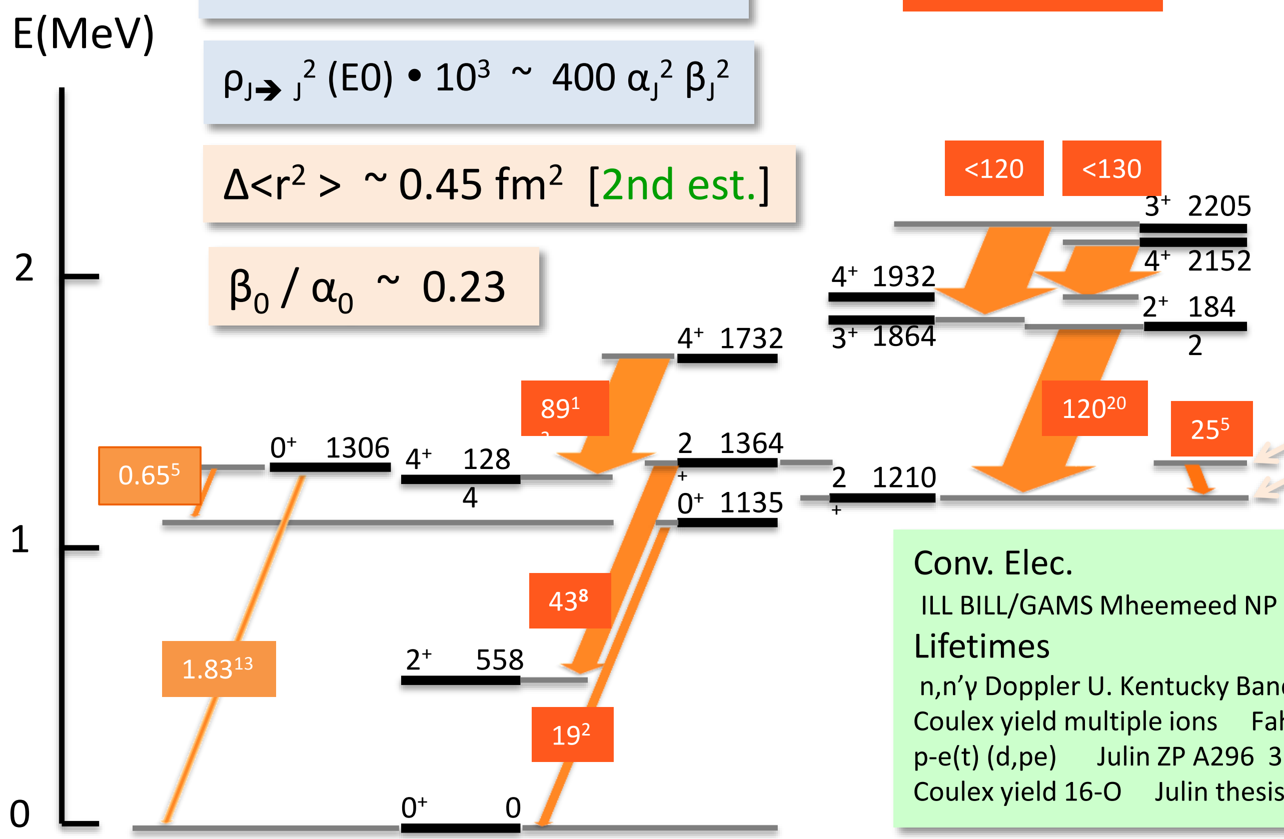
$\rho_{J \rightarrow J}^2 (E0) \cdot 10^3 \sim 400\, \alpha_J^2\, \beta_J^2$

$\Delta\langle r^2 \rangle \sim 0.45\ fm^2$ [2nd est.]

$\beta_0 / \alpha_0 \sim 0.23$

$\rho^2(E0) \cdot 10^3$

^{114}Cd



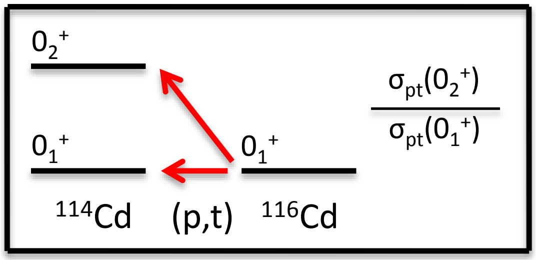
NOTE:
despite the proximity of these two 2^+ states, the ρ^2 (E0) value is not large

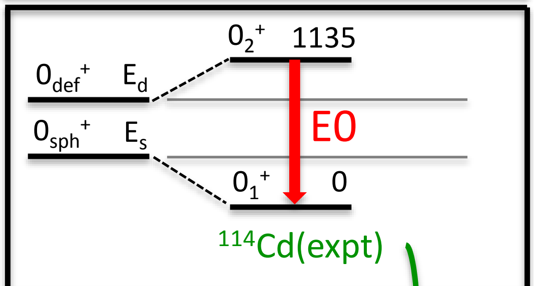
Conv. Elec.
ILL BILL/GAMS Mheemeed NP A412 113 1984

Lifetimes
n,n'γ Doppler U. Kentucky Bandyopadhyay PR C76 054308 2007
Coulex yield multiple ions Fahlander NP A485 327 1988
p-e(t) (d,pe) Julin ZP A296 315 1980
Coulex yield 16-O Julin thesis Univ. Jyvaskyla 1979

Spectroscopy of mixing in the Cd isotopes: ^{116}Cd (p,t) ^{114}Cd and ρ^2 (E0) • 10^3

•





Fortune PR C35 2318 1987

$|V_{J=0}|$ ~ 330 keV

β_0 / α_0 ~ 0.28 $\quad \alpha_0^2 + \beta_0^2 = 1$

$\rho_{0\rightarrow 0}^2$ (E0) ~ $[\Delta\langle r^2\rangle]^2 \alpha_0^2 \beta_0^2$

$\Delta\langle r^2\rangle = \langle r^2\rangle_{def} - \langle r^2\rangle_{sph}$ [unknown]

$$\rho_{0\rightarrow 0}^2 (E0) \cdot 10^3 = 19 = \frac{48^2 [\Delta\langle r^2\rangle]^2 \; 10^3 \; [0.28 \times 0.96]^2}{[1.2 \times 114^{1/3}]^4}$$

$\Delta\langle r^2\rangle$ ~ 0.4 fm^2 [first estimate]

$\rho_{J\rightarrow J}^2$ (E0) • 10^3 ~ 300 $\alpha_J^2 \beta_J^2$

Wood et al.
NP A651 323 1999

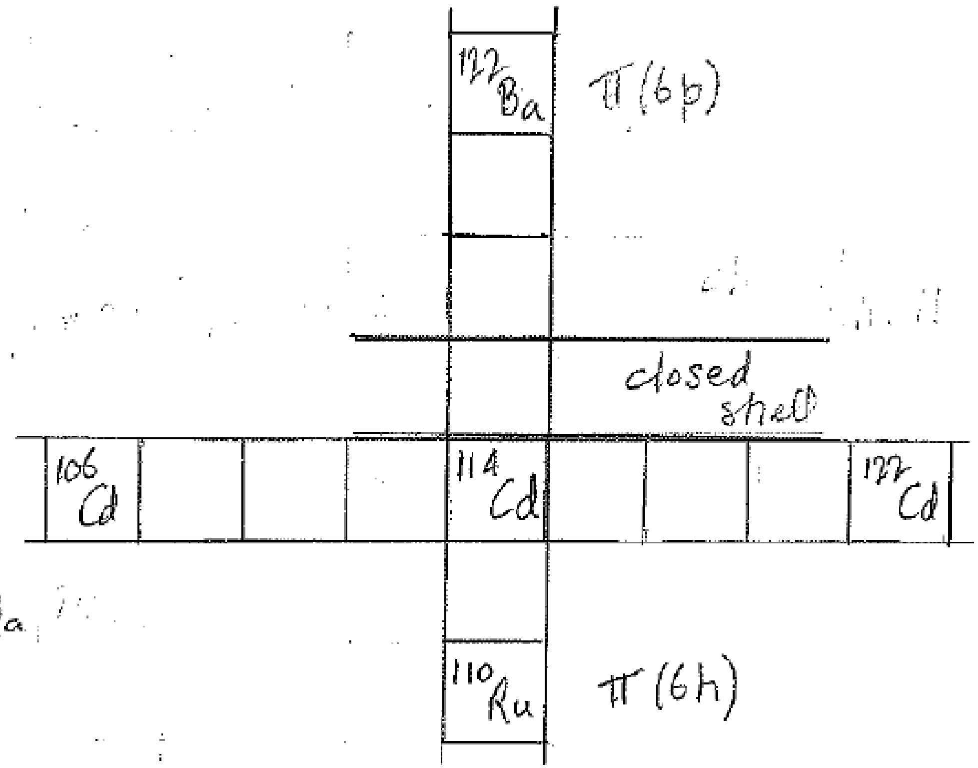
The spectroscopy of mixing in the Cd isotopes: ^{114}Cd unmixed energies

^{114}Cd : unmixed energies

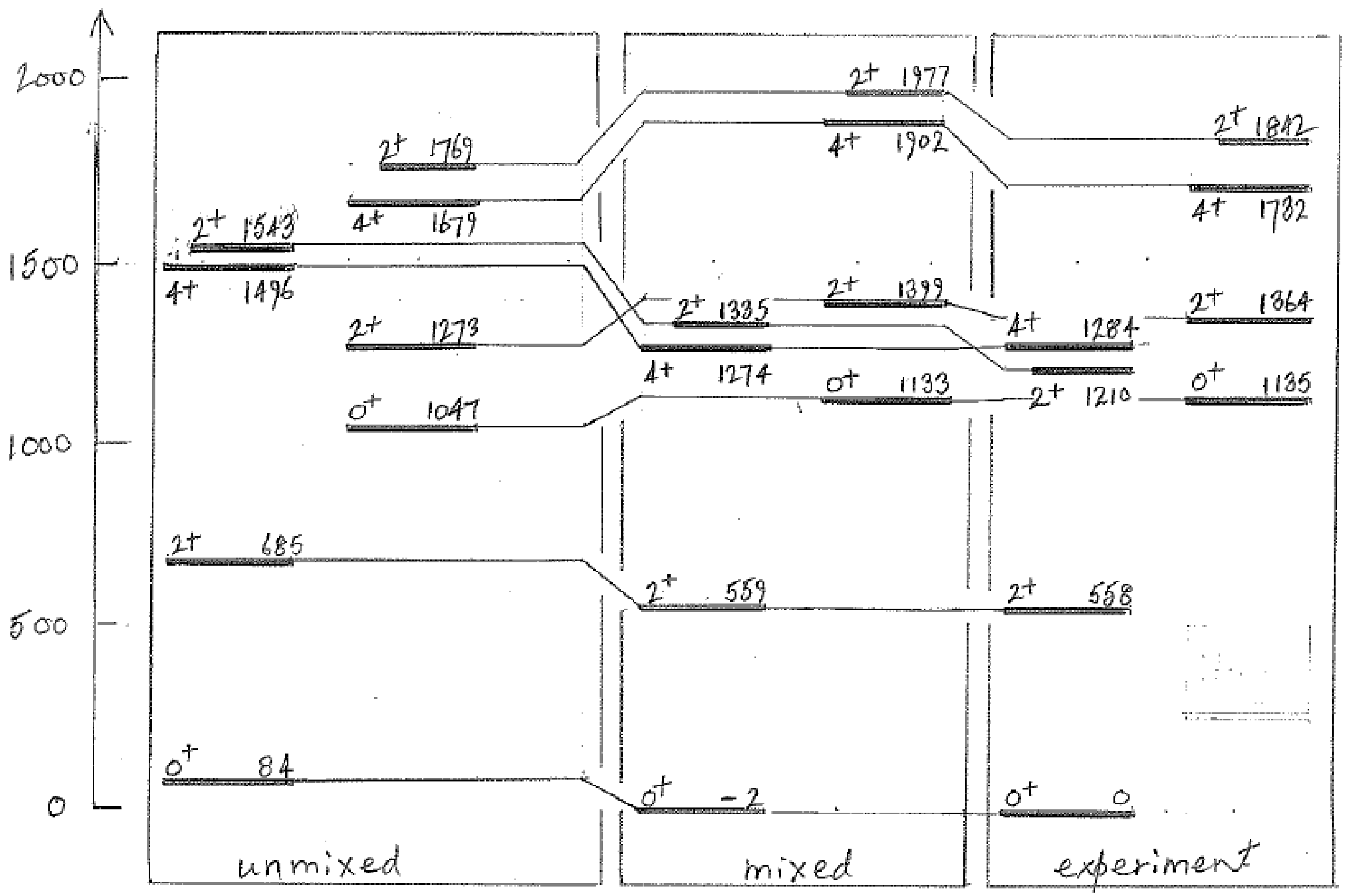
^{114}Cd $\pi(2h)$	avg.	^{106}Cd	^{122}Cd
2_1	601	688	569
4_1	1412	1494	1329
2_2	1543	1717	1368

$\pi(2p\text{-}4h)$	"avg."	^{110}Ru	^{122}Ba
2_1	226	241	196
4_1	632	663	569
2_2	722	613	940

$$\text{"avg."} \equiv \frac{2}{3} \times {}^{110}\text{Ru} + \frac{1}{3} \times {}^{122}\text{Ba}$$



The spectroscopy of mixing in the Cd isotopes: ^{114}Cd energies



The spectroscopy of mixing in the Cd isotopes: ρ^2 (E0) values in ^{114}Cd

J_i	α_J	β_J	$\rho^2_{J\to J}(E0) = 228\,\alpha_J^2\beta_J^2$	$\rho^2_{J\to J}(E0)$ expt.
0_1	0.9613	0.2755	16	~~16±1~~ 19
2_1	0.9220	0.3872	29	~~36±5~~ 43
4_1	0.8038	0.5948	52	~~67±10~~ 89
2_2	0.8218	0.5698	50	~~95±19~~ 122

The spectroscopy of mixing in the Cd isotopes: M(E2) and B(E2) values in ^{114}Cd

★ B(E2) properties:

Grodzins' rule: $[E(2_1^+)\,\mathrm{keV}][B(E2; 0_1^+ \to 2_1^+)\,e^2 \cdot b^2]\,\frac{A}{Z^2} \approx 16.0$

$$M_{20} = \sqrt{B(E2; 0_1^+ \to 2_1^+)}\ e \cdot b$$

$E(2_1^+)^a$ 601 keV $\Rightarrow$ $M_{20}^a = 0.73\ e \cdot b$

$E(2_1^+)^b$ 226 keV $\Rightarrow$ $M_{20}^b = 1.20\ e \cdot b$

	calc.	expt.
$M_{2_1 0_1} = \alpha_0 \alpha_2 M_{20}^a + \beta_0 \beta_2 M_{20}^b$	$= 0.775$	0.714^{21}
$M_{4_1 2_1} = (\alpha_2 \alpha_4 M_{20}^a + \beta_2 \beta_4 M_{20}^b)(1.604)$	$= 1.311$	1.35^{4}
$M_{0_2 2_1} = -\alpha_2 \beta_0 M_{20}^a + \alpha_0 \beta_2 M_{20}^b$	$= 0.261$	0.300^{+7}_{-9}
α_2 -1.5% $\Rightarrow$	0.300	
† $M_{4_2 2_3} = (\beta_2 \beta_4 M_{20}^a + \alpha_2 \alpha_4 M_{20}^b)(1.604)$	$= 1.696$	1.85^{+10}_{-6}
$M_{2_3 0_2} = \beta_0 \beta_2 M_{20}^a + \alpha_0 \alpha_2 M_{20}^b$	$= 1.142$	0.51^{3}

α_2 -1.5% $\Rightarrow$ $\rho^2(E0)_{2\to2}$: $29 \to 33$ (36^5)

$B(E2; 2_1 \to 0_1) = \frac{M_{2_1 0_1}^2}{5}\ e^2 \cdot b^2 \times 302.3\ \frac{\mathrm{W.u.}}{e^2 \cdot b^2}$: 36 cf. 33^2 Raman

$B(E2; 0_2 \to 2_1) = M_{0_2 2_1}^2$: 21 cf. 27.2^{14} (α_2 -1.5% → 27)

$B(E2; 4_1 \to 2_1) = \frac{M_{4_1 2_1}^2}{9}$: 58 cf. 61^4

$B(E2; 4_2 \to 2_3) = \frac{M_{4_2 2_3}^2}{9}$: 97 cf. 115^{+13}_{-8}

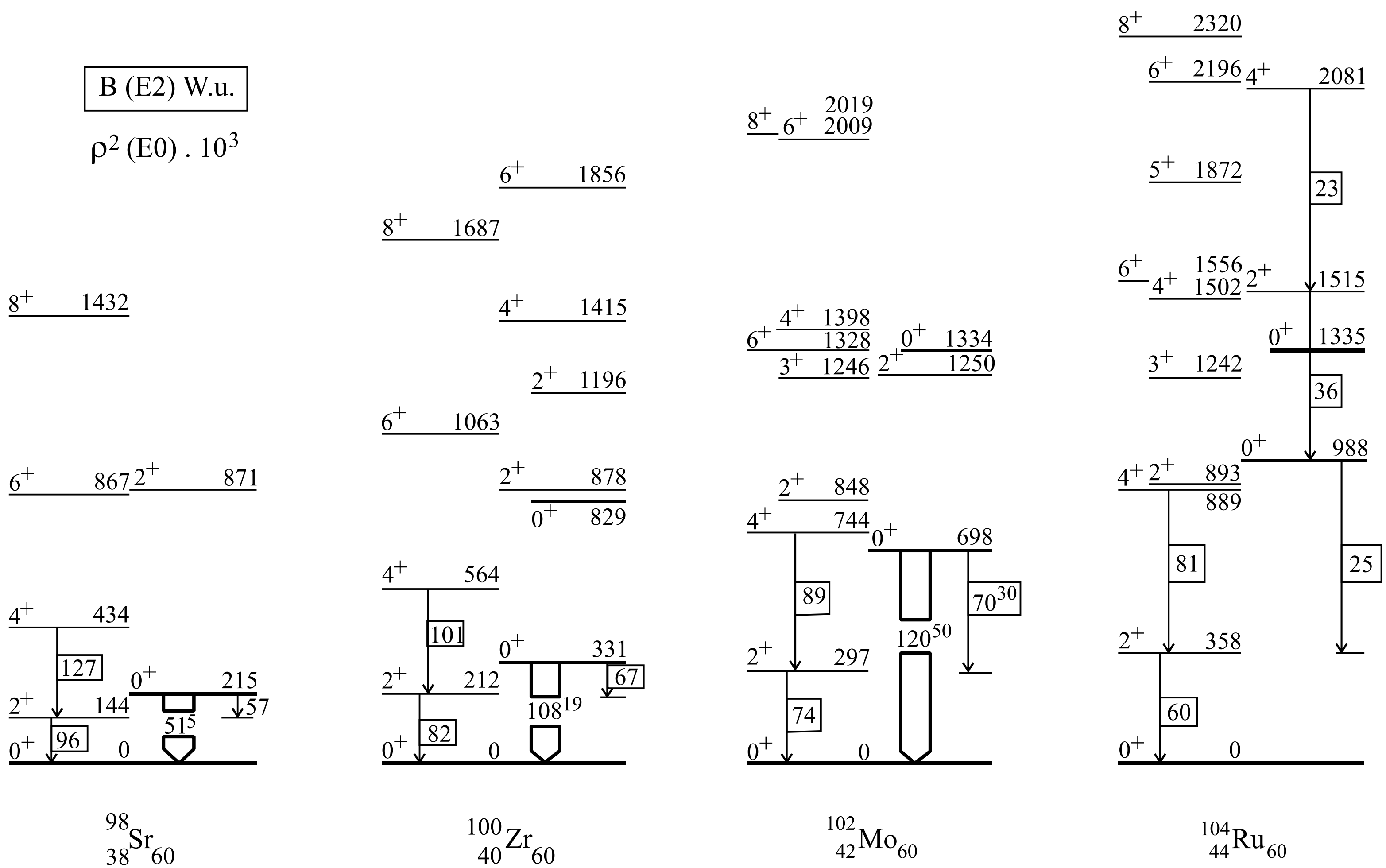
$B(E2; 2_3 \to 0_2) = M_{2_3 0_2}^2/5$: 79 cf. 65^9 / but see Fahlander

$M_{2_1 2_1} = (\alpha_2^2 M_{20}^a + \beta_2^2 M_{20}^b)(-1.195) = -0.957\ e \cdot b$ cf. $-0.36^{+1}_{-3}\ e \cdot b$

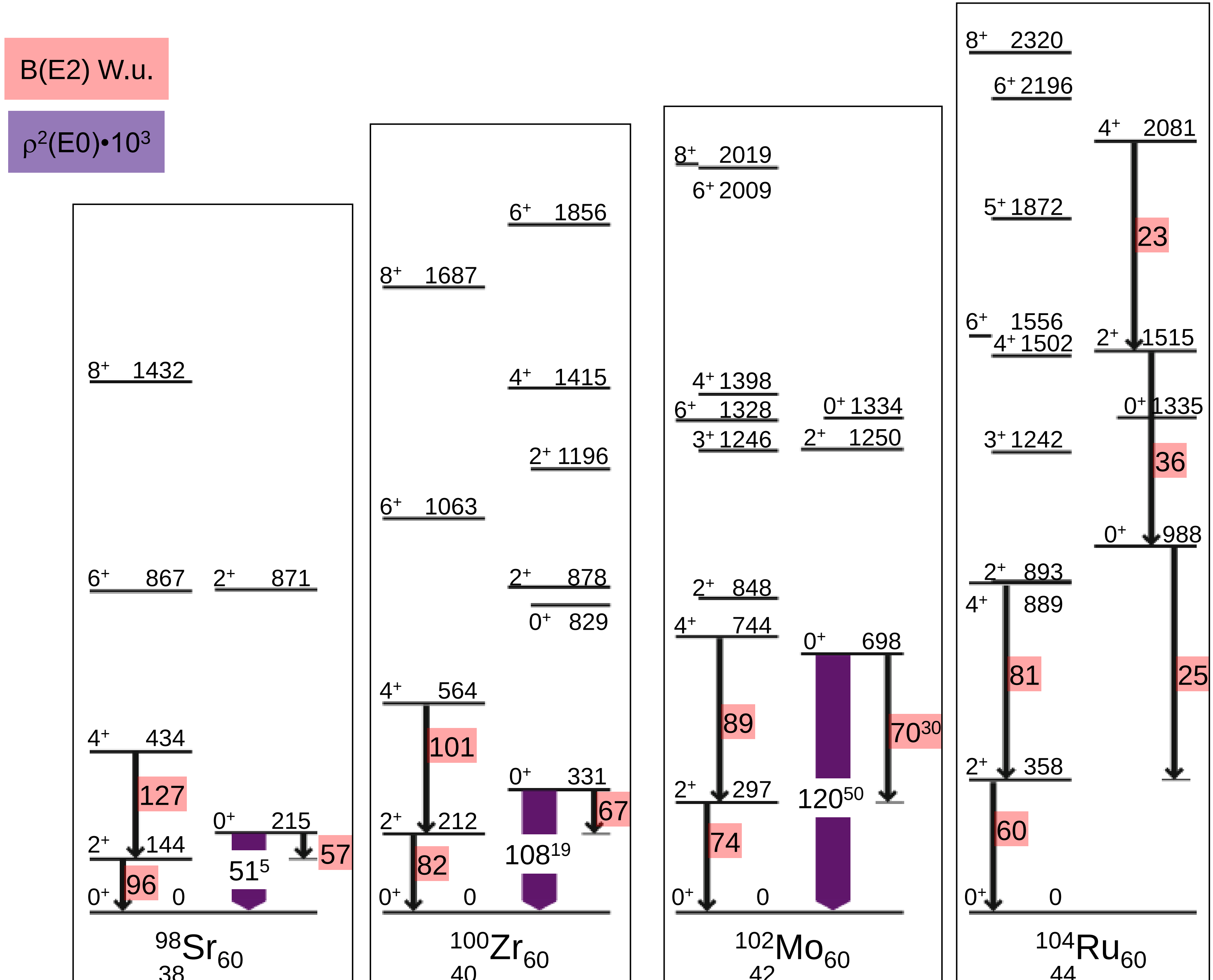
need 2_1–2_2 mixing

† α_2 decreases by 1.5% for $\lambda_2^{(1)}$: 559 → 547 keV

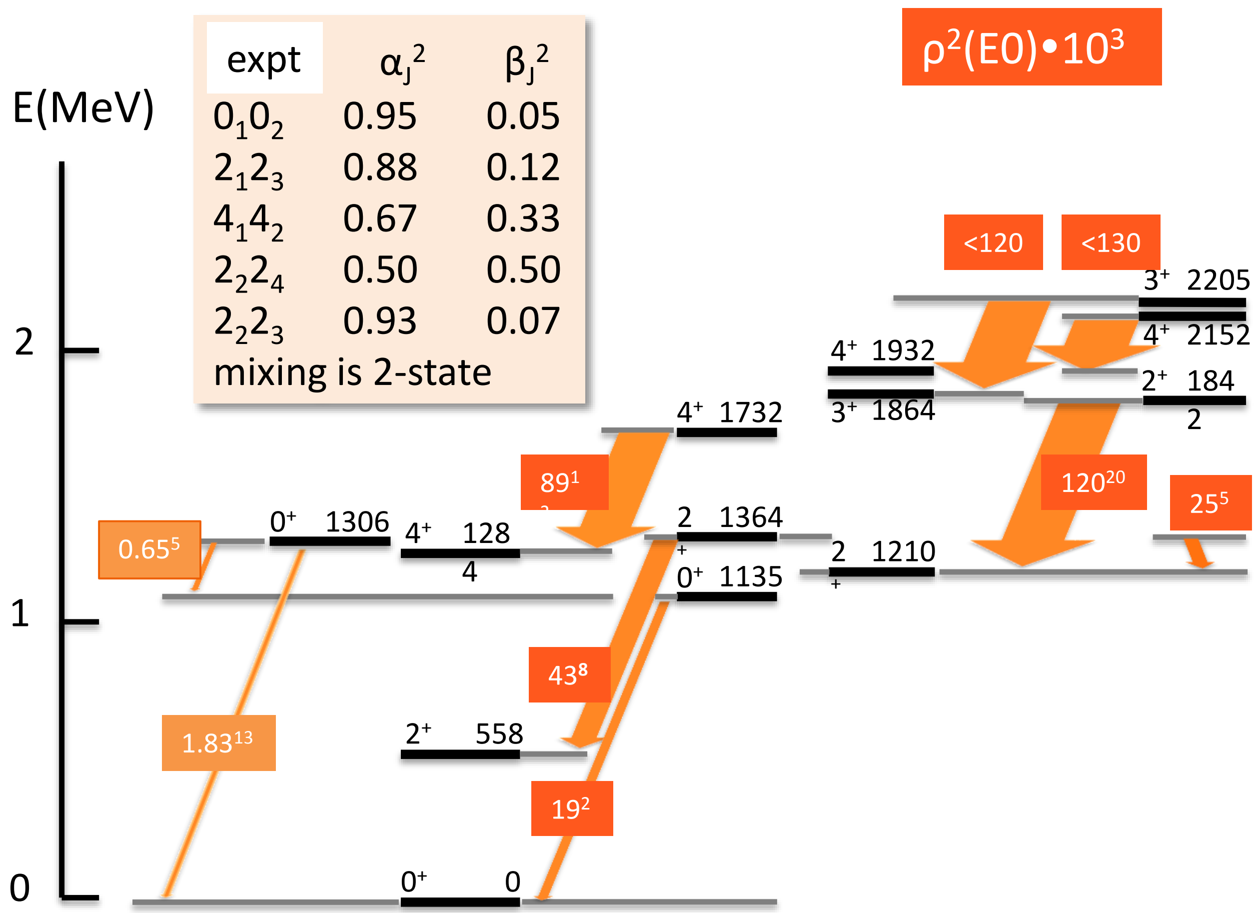
Electric monopole transition strengths in the N = 60 isotones



Systematics of low-lying collective states in N=60 isotones



Spectroscopy of mixing in the Cd isotopes: ρ^2 (E0)•10^3 values in ^{114}Cd



expt	α_J^2	β_J^2
$0_1 0_2$	0.95	0.05
$2_1 2_3$	0.88	0.12
$4_1 4_2$	0.67	0.33
$2_2 2_4$	0.50	0.50
$2_2 2_3$	0.93	0.07

mixing is 2-state

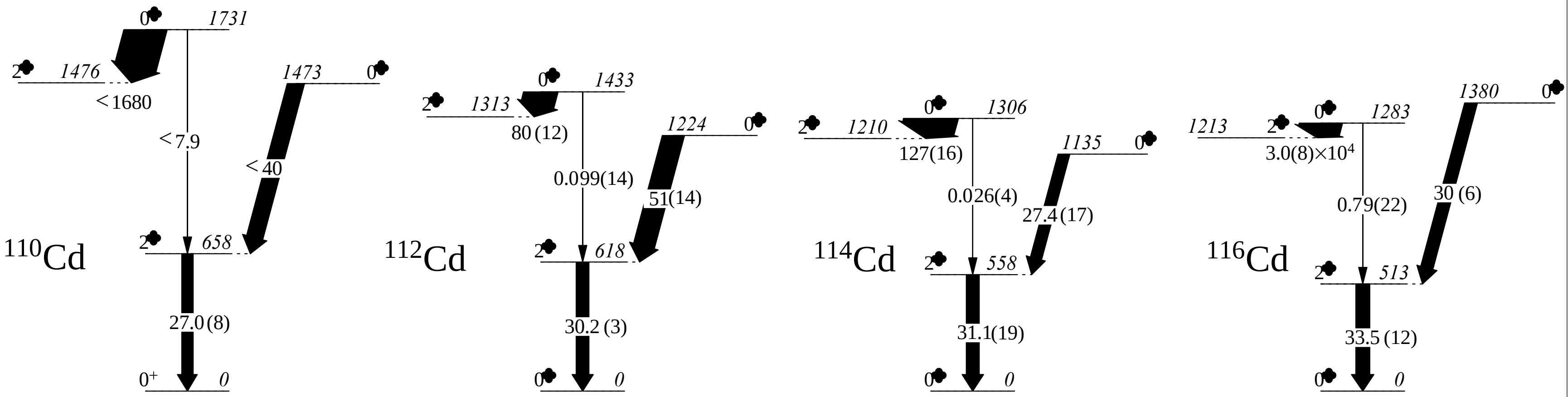
^{114}Cd

th	α_J^2	β_J^2
$0_1 0_2$	0.99	0.01
$2_1 2_3$	0.97	0.02
$4_1 4_2$	0.94	0.05
$2_2 2_4$	0.87	0.04
$2_2 2_3$	0.87	0.09

Garrett, Green, Wood
PR C78 044307 2008
To fit B(E2)'s

IBM2-MIX
mixing is multi-state

Excited 0^+ decays in the Cd isotopes

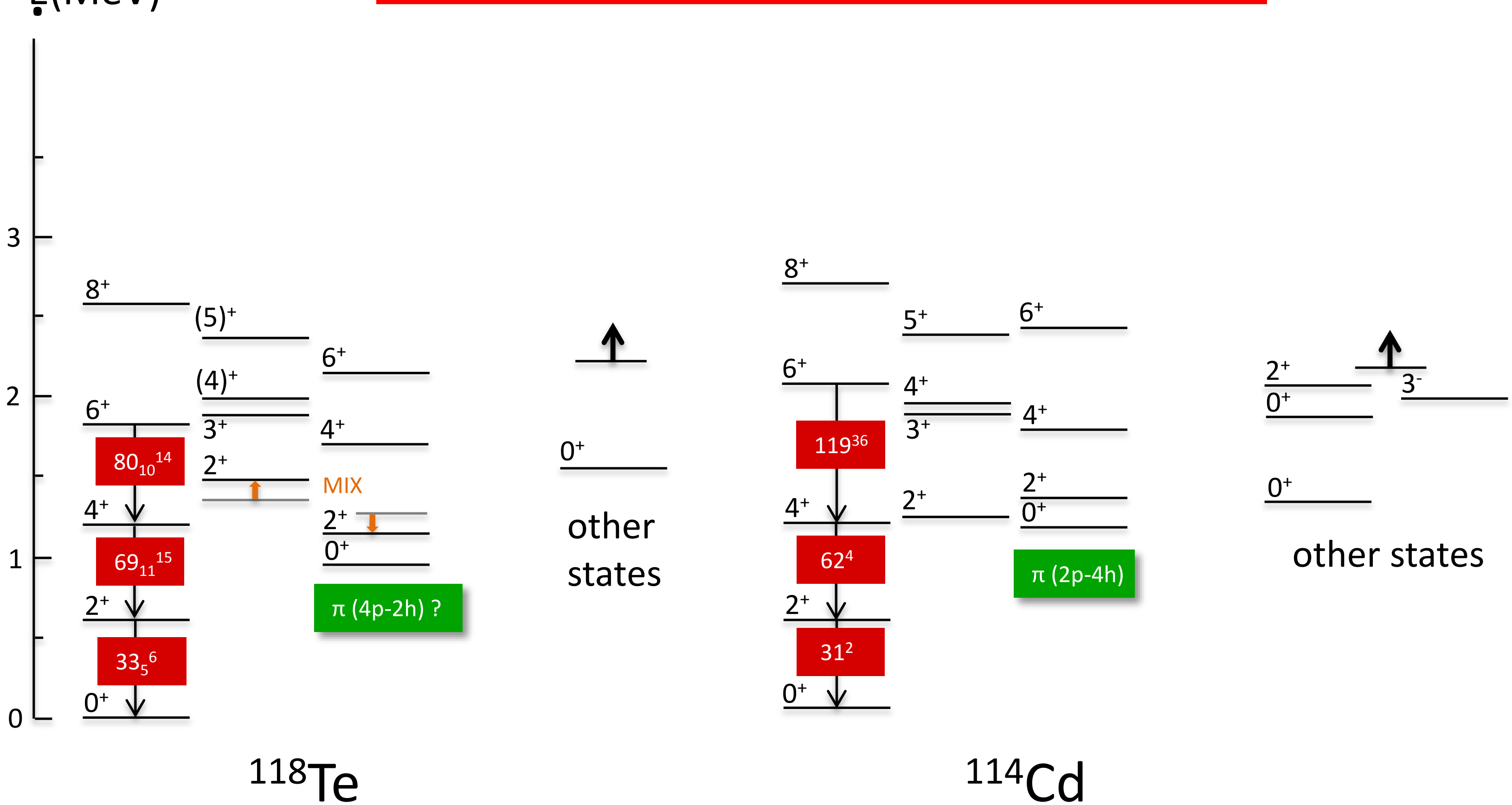


Deformed band head 0^+ states: strong E2 decay to "one-phonon" 2^+ states

"Two-phonon" 0^+ states: very weak E2 decay to "one-phonon" 2^+ states; but strong E2 decay to "two-phonon" 2^+ states

Introduction to mid-shell collectivity in Z = 48, 52 (N = 66) isotones

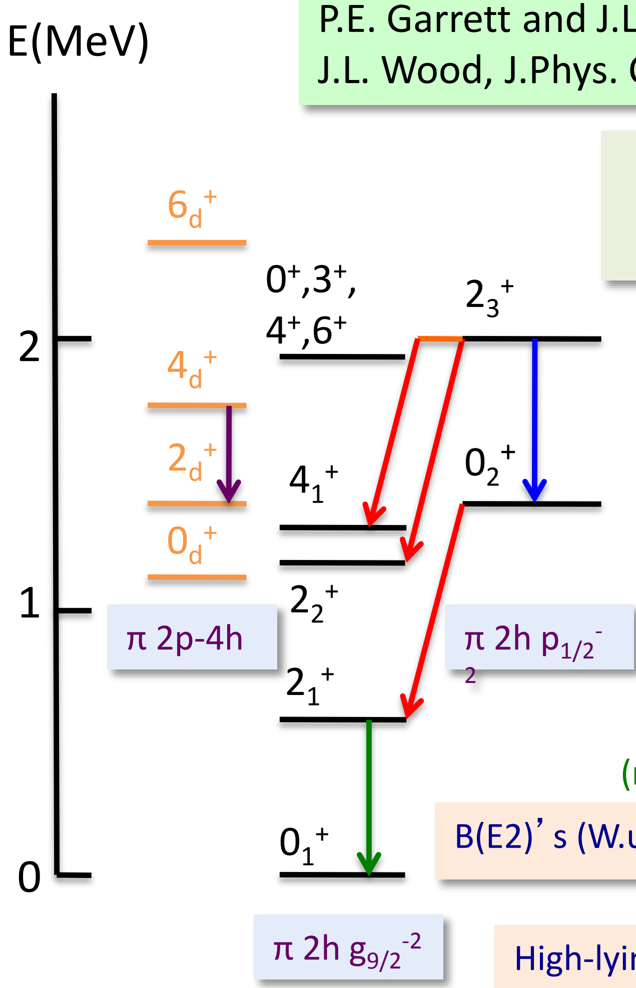
Differences exist for 0_3^+ and higher-lying states



Demise of quadrupole vibrations in $^{110-116}$Cd:

low-energy 0^+ states are shell and subshell excitations

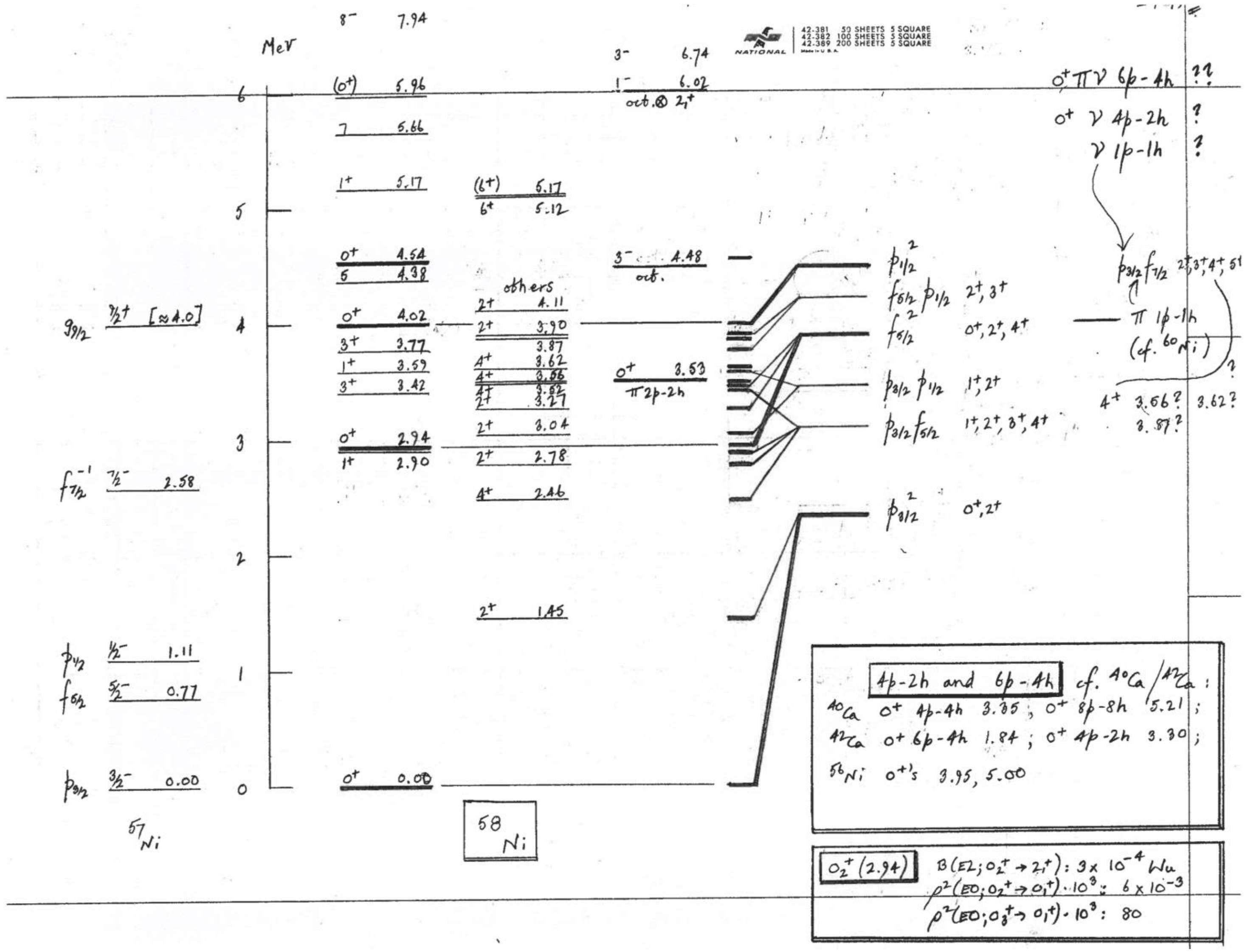
P.E. Garrett and J.L. Wood, J.Phys. G37, 064028 (2010)
J.L. Wood, J.Phys. Conf. Ser. 403, 012011 (2012)



trans.	^{110}Cd	^{112}Cd	^{114}Cd	^{116}Cd	harm. vib.
4_d2_d	115^{35}		119^{12}		
2_30_2	24^{2}	27^{8}	17^{5}	35^{10}	42
2_34_1	< 5	< 0.4	< 0.3	< 7	31
2_32_2	$< 0.7^{6}$	< 2	2.8^{4}	2.0^{6}	17
0_22_1	< 7.9	0.0099^{44}	0.0026^{4}	0.55^{4}	60
2_10_1 (norm)	27	30	31	34	30

B(E2)'s (W.u.) from lifetime measurements @ Univ. of Kentucky using (n,n'γ).

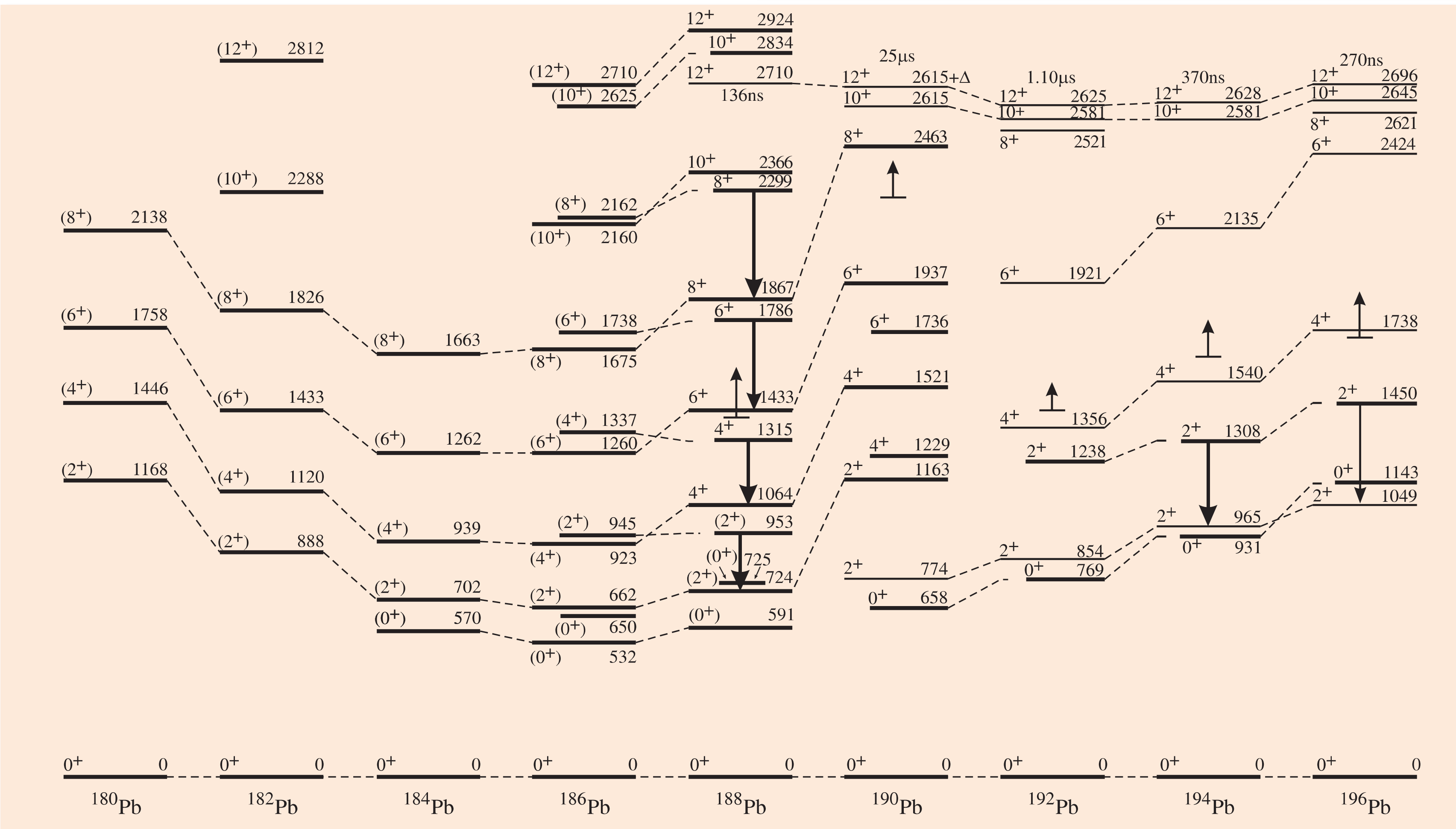
High-lying, low-energy transitions are difficult to observe: used 8Pi array @ TRIUMF-ISAC and ultrahigh statistics β-decay scheme studies.


MeV
6
5
4
3
2
1
0
$g_{9/2}$ 9/2+ [≈4.0]
$f_{7/2}^{-1}$ 7/2− 2.58
$p_{1/2}$ 1/2− 1.11
$f_{5/2}$ 5/2− 0.77
$p_{3/2}$ 3/2− 0.00
57Ni
8− 7.94
(0+) 5.96
7 5.66
1+ 5.17
0+ 4.54
5 4.38
0+ 4.02
3+ 3.77
1+ 3.59
3+ 3.42
0+ 2.94
1+ 2.90
0+ 0.00
(6+) 5.17
6+ 5.12
others
2+ 4.11
2+ 3.90
3.87
4+ 3.62
4+ 3.56
4+ 3.52
2+ 3.27
2+ 3.04
2+ 2.78
4+ 2.46
2+ 1.45
58Ni
3− 6.74
1− 6.02
oct.⊗ 2_1^+
3− 4.48
oct.
0+ 3.53
π 2p-2h
$p_{1/2}^2$
$f_{5/2}p_{1/2}$ 2+, 3+
$f_{5/2}^2$ 0+, 2+, 4+
$p_{3/2}p_{1/2}$ 1+, 2+
$p_{3/2}f_{5/2}$ 1+, 2+, 3+, 4+
$p_{3/2}^2$ 0+, 2+
0+ πν 6p-4h ??
0+ ν 4p-2h ?
ν 1p-1h ?
$p_{3/2}f_{7/2}$ 2+, 3+, 4+, 5+
π 1p-1h
(cf. 60Ni)
?
4+ 3.56? 3.62?
3.87?
4p-2h and 6p-4h cf. 40Ca / 42Ca :
40Ca 0+ 4p-4h 3.35 ; 0+ 8p-8h 5.21 ;
42Ca 0+ 6p-4h 1.84 ; 0+ 4p-2h 3.30 ;
56Ni 0+'s 3.95, 5.00
0_2^+ (2.94) B(E2; $0_2^+ \to 2_1^+$): 3 × 10^{-4} Wu
ρ^2(E0; $0_2^+ \to 0_1^+$)·10^3: 6 × 10^{-3}
ρ^2(E0; $0_3^+ \to 0_1^+$)·10^3: 80

Coexisting deformed bands in the even-mass Pb isotopes

Figure from Heyde & Wood

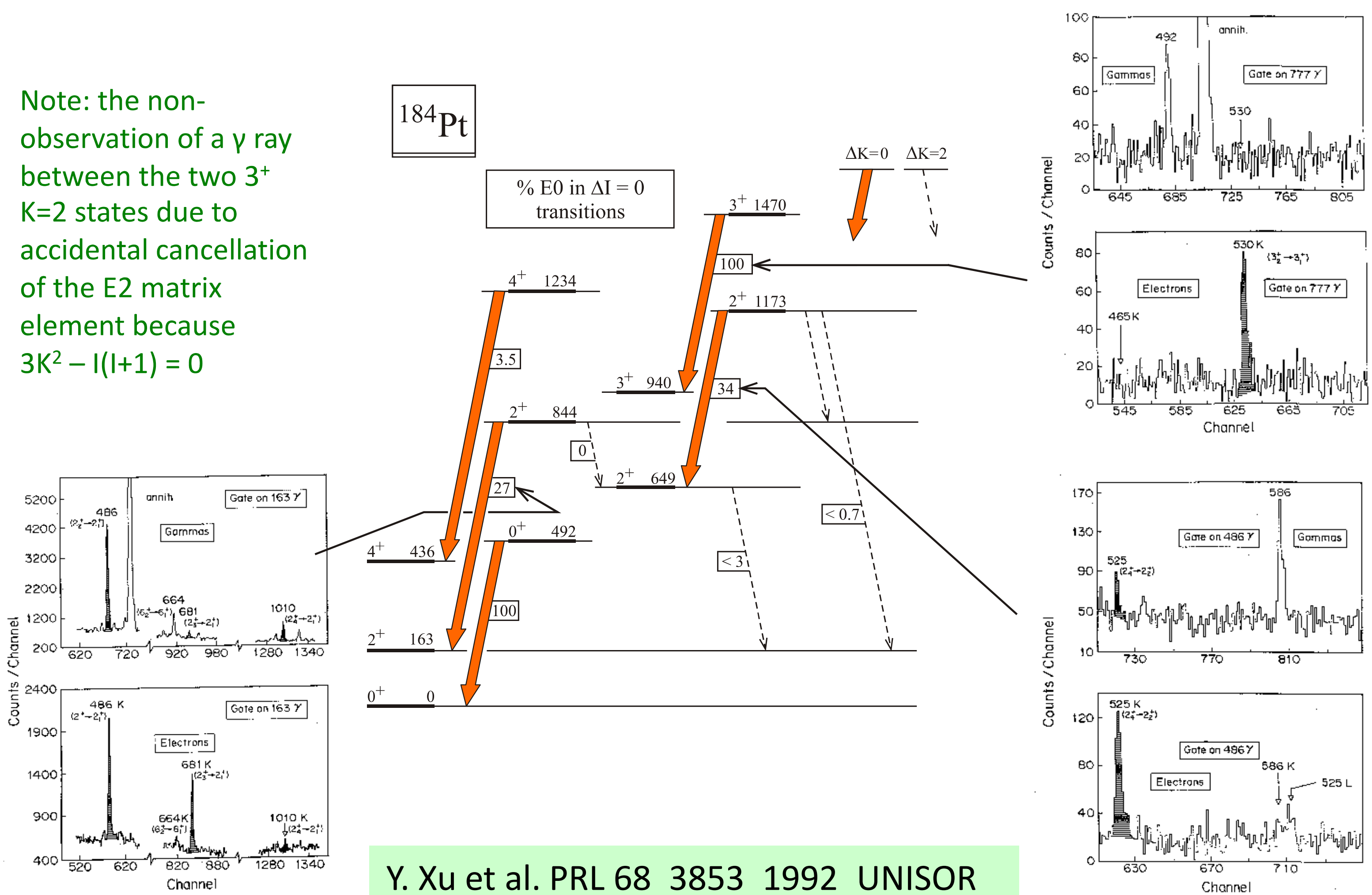
Heavy arrows indicate E0+M1+E2 transitions:
G.D. Dracoulis et al., PR C67 R 051301 2003



Shape coexistence in ^{184}Pt:

revealed by E0 transitions

Note: the non-observation of a γ ray between the two 3^+ K=2 states due to accidental cancellation of the E2 matrix element because $3K^2 - I(I+1) = 0$



Y. Xu et al. PRL 68 3853 1992 UNISOR

Zirconium isotopes have excited 0^+ states that are strongly populated in two- and four-nucleon transfer reactions

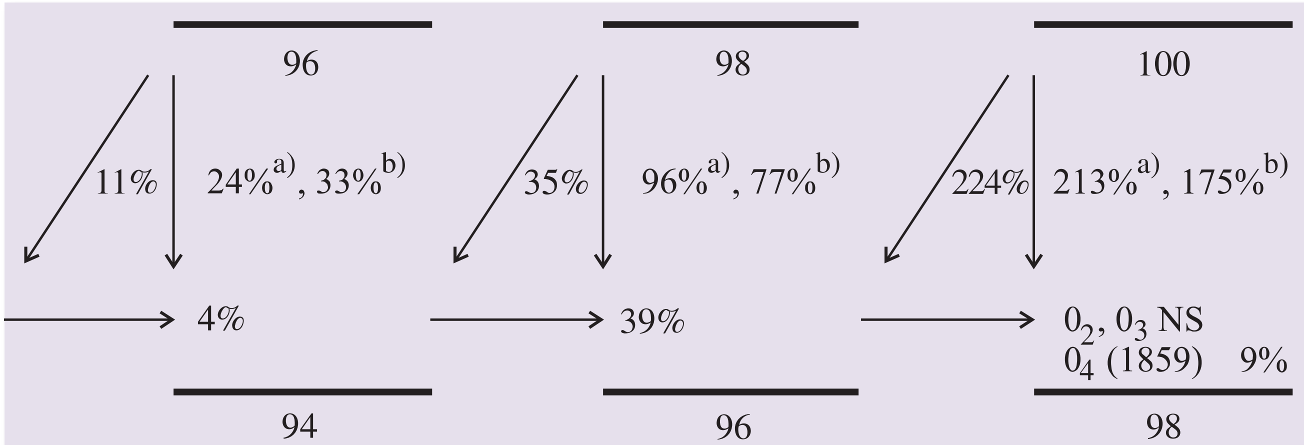


Figure: Heyde & Wood

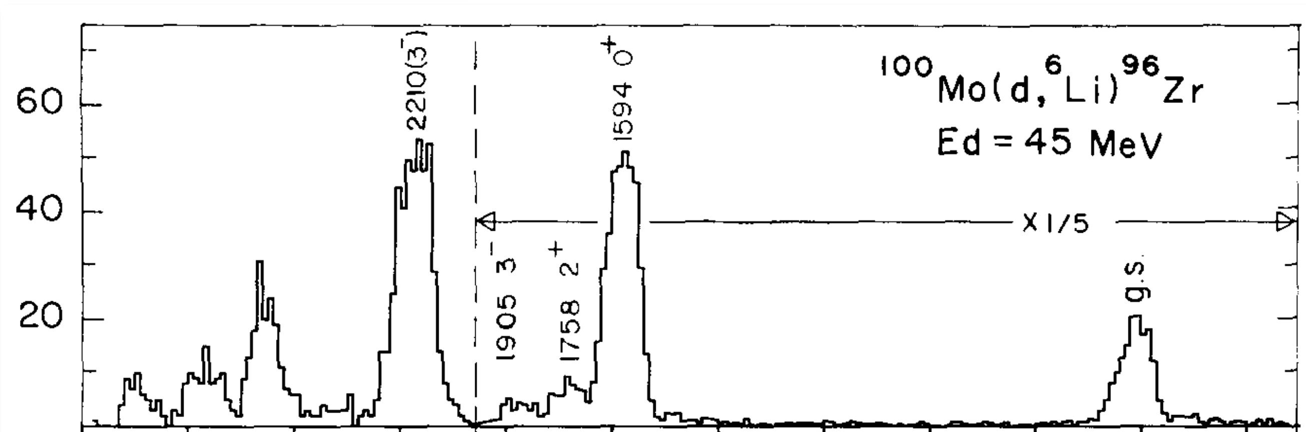


Figure from A. Saha et al. PL B82 208 1979

(^{6}Li,^{8}B)--a)

(^{14}C,^{16}O)--b)

(d,^{6}Li)

(t,p)

gs → 0_2^+ %

gs → gs 100%

gs = 0_1^+