Characteristics of Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

Bivariate Gaussian Density

two random variables

Definition

The two random variables X and Y are said to be jointly Gaussian, if their joint density function is

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \cdot$$

$$exp\left\{ \frac{-1}{2(1-\rho^2)} \cdot \left[\frac{(x-\overline{X})^2}{\sigma_X^2} - \frac{2\rho(x-\overline{X})(y-\overline{Y})}{\sigma_X\sigma_Y} + \frac{(y-\overline{Y})^2}{\sigma_Y^2} \right] \right\}$$

$\overline{X} = E[X]$, $Y = E[Y]$, $\sigma_X^2 = E[(X-\overline{X})^2]$, $\sigma_Y^2 = E[(Y-\overline{Y})^2]$,
$\rho = E[(X-\overline{X})(Y-\overline{Y})]/\sigma_X\sigma_Y$

Bivariate Gaussian Density - Maximum value
two random variables

$$f_{X,Y}(x,y) \le f_{X,Y}(\overline{X},\overline{Y}) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}.$$

Bivariate Gaussian Density - Uncorrelated
two random variables

when $\rho = 0$

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y} \cdot exp\left\{-\frac{1}{2}\cdot\left[\frac{(x-\overline{X})^2}{\sigma_X^2} + \frac{(y-\overline{Y})^2}{\sigma_Y^2}\right]\right\}$$

$$f_{X,Y}(x,y) = f_X(x)f_Y(x)$$

$$f_X(x) = \frac{1}{2\pi\sigma_X} \cdot exp\left\{-\frac{(x-\overline{X})^2}{2\sigma_X^2}\right\}$$

$$f_Y(y) = \frac{1}{2\pi\sigma_Y} \cdot exp\left\{-\frac{(y-\overline{Y})^2}{2\sigma_Y^2}\right\}$$