

Characteristics of Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles, Jr. and B. Shi

Outline

1 Joint Guassian Random Variables

Bivariate Gaussian Density

two random variables

Definition

The two random variables X and Y are said to be jointly Gaussian, if their joint density function is

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}} \cdot$$

$$\exp \left\{ \frac{-1}{2(1-\rho^2)} \cdot \left[\frac{(x-\bar{X})^2}{\sigma_X^2} - \frac{2\rho(x-\bar{X})(y-\bar{Y})}{\sigma_X\sigma_Y} + \frac{(y-\bar{Y})^2}{\sigma_Y^2} \right] \right\}$$

$$\bar{X} = E[X], \quad \bar{Y} = E[Y], \quad \sigma_X^2 = E[(X-\bar{X})^2], \quad \sigma_Y^2 = E[(Y-\bar{Y})^2],$$

$$\rho = E[(X-\bar{X})(Y-\bar{Y})]/\sigma_X\sigma_Y$$

Bivariate Gaussian Density - Maximum value

two random variables

$$f_{X,Y}(x,y) \leq f_{X,Y}(\bar{X}, \bar{Y}) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}.$$

Bivariate Gaussian Density - Uncorrelated

two random variables

when $\rho = 0$

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y} \cdot \exp\left\{-\frac{1}{2} \cdot \left[\frac{(x-\bar{X})^2}{\sigma_X^2} + \frac{(y-\bar{Y})^2}{\sigma_Y^2}\right]\right\}$$

$$f_{X,Y}(x,y) = f_X(x)f_Y(x)$$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma_X} \cdot \exp\left\{-\frac{(x-\bar{X})^2}{2\sigma_X^2}\right\}$$

$$f_Y(y) = \frac{1}{\sqrt{2\pi}\sigma_Y} \cdot \exp\left\{-\frac{(y-\bar{Y})^2}{2\sigma_Y^2}\right\}$$

