Characteristics of Multiple Random Variables

Young W Lim

June 25, 2020

Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

Estimate of mean
N Gaussian random variables

Definition

$$\hat{\bar{x}}_N = \frac{1}{N}\sum_{n=1}^{N} x_n$$

$$\hat{\bar{X}}_N = \frac{1}{N}\sum_{n=1}^{N} X_n$$

Mean of the estimate of mean

N Gaussian random variables

Definition

$$E\left[\hat{\overline{X}}_N\right] = E\left[\frac{1}{N}\sum_{n=1}^{N} X_n\right] = \frac{1}{N}\sum_{n=1}^{N} E\left[X_n\right] = \overline{X}$$

Variance of the estimate of mean (1)

N Gaussian random variables

Definition

$$E\left[\left(\hat{\overline{X}}_N - \overline{X}\right)^2\right] = \sigma^2_{X_N} = E\left[\hat{\overline{X}}^2_N - 2\overline{X}\hat{\overline{X}}_N + \overline{X}^2\right]$$

$$= E\left[\hat{\overline{X}}^2_N\right] - \overline{X}^2 = -\overline{X}^2 + E\left[\frac{1}{N}\sum_{m=1}^{N} X_m \frac{1}{N}\sum_{n=1}^{N} X_n\right]$$

$$= -\overline{X}^2 + \frac{1}{N^2}\sum_{m=1}^{N}\sum_{n=1}^{N} E\left[X_m X_n\right]$$

Variance of the estimate of mean (2)

N Gaussian random variables

Definition

$$E\left[X_m X_n\right] = E\left[X^2\right] \qquad (n = m)$$

$$E\left[X_m X_n\right] = \overline{X}^2 \qquad (n \neq m)$$

$$\sigma_{X_N}^2 = -\overline{X}^2 + \frac{1}{N^2}\left[NE\left[X^2\right] + (N^2 - N)\overline{X}^2\right]$$

$$= \frac{1}{N}\left[E\left[X^2\right] - \overline{X}^2\right] = \sigma_X^2 / N$$

Variance of the estimate of mean (3)

N Gaussian random variables

Definition

$$P\left\{\left|\hat{\bar{X}}_N - \bar{X}\right| < \varepsilon\right\} \geq 1 - (\sigma^2_{X_N}/\varepsilon^2) = 1 - \frac{\sigma^2_X}{N\varepsilon^2}$$

$$\widehat{X^2_N} = \frac{1}{N}\sum_{n=1}^{N} X_n^2$$

$$\widehat{\sigma^2_N} = \frac{1}{N-1}\sum_{n=1}^{N}(X_n - \hat{\bar{X}}_N)^2$$

Weak Law of Large Numbers
N Gaussian random variables

Definition

$$\lim_{N \to \infty} P\left\{\left|\hat{\overline{X}}_N - \overline{X}\right| < \varepsilon\right\} = 1$$

Strong Law of Large Numbers
N Gaussian random variables

Definition

$$P\left\{\lim_{N\to\infty}\hat{\overline{X}}_N=\overline{X}\right\}=1$$