

# Characteristics of Multiple Random Variables

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Based on  
Probability, Random Variables and Random Signal Principles,  
P.Z. Peebles, Jr. and B. Shi

# Outline

## 1 Simulation of Multiple Random Variables

# Estimate of mean

## $N$ Gaussian random variables

### Definition

$$\hat{\bar{X}}_N = \frac{1}{N} \sum_{n=1}^N x_n$$

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# Mean of the estimate of mean

$N$  Gaussian random variables

## Definition

$$E \left[ \hat{X}_N \right] = E \left[ \frac{1}{N} \sum_{n=1}^N X_n \right] = \frac{1}{N} \sum_{n=1}^N E[X_n] = \bar{X}$$

## Variance of the estimate of mean (1)

 $N$  Gaussian random variables

## Definition

$$\begin{aligned} E \left[ \left( \hat{X}_N - \bar{X} \right)^2 \right] &= \sigma_{\hat{X}_N}^2 = E \left[ \hat{X}_N^2 - 2\bar{X}\hat{X}_N + \bar{X}^2 \right] \\ &= E \left[ \hat{X}_N^2 \right] - \bar{X}^2 = -\bar{X}^2 + E \left[ \frac{1}{N} \sum_{m=1}^N X_m \frac{1}{N} \sum_{n=1}^N X_n \right] \\ &= -\bar{X}^2 + \frac{1}{N^2} \sum_{m=1}^N \sum_{n=1}^N E[X_m X_n] \end{aligned}$$

## Variance of the estimate of mean (2)

 $N$  Gaussian random variables

## Definition

$$E[X_m X_n] = E[X^2] \quad (n = m)$$

$$E[X_m X_n] = \bar{X}^2 \quad (n \neq m)$$

$$\begin{aligned}\sigma_{X_N}^2 &= -\bar{X}^2 + \frac{1}{N^2} \left[ NE[X^2] + (N^2 - N)\bar{X}^2 \right] \\ &= \frac{1}{N} \left[ E[X^2] - \bar{X}^2 \right] = \sigma_X^2 / N\end{aligned}$$

## Variance of the estimate of mean (3)

 $N$  Gaussian random variables

## Definition

$$P \left\{ \left| \widehat{X}_N - \bar{X} \right| < \varepsilon \right\} \geq 1 - (\sigma_{X_N}^2 / \varepsilon^2) = 1 - \frac{\sigma_X^2}{N\varepsilon^2}$$

$$\widehat{X}_N^2 = \frac{1}{N} \sum_{n=1}^N X_n^2$$

$$\widehat{\sigma}_N^2 = \frac{1}{N-1} \sum_{n=1}^N (X_n - \widehat{X}_N)^2$$



# Weak Law of Large Numbers

$N$  Gaussian random variables

## Definition

$$\lim_{N \rightarrow \infty} P \left\{ \left| \hat{X}_N - \bar{X} \right| < \varepsilon \right\} = 1$$

# Strong Law of Large Numbers

$N$  Gaussian random variables

## Definition

$$P \left\{ \lim_{N \rightarrow \infty} \hat{X}_N = \bar{X} \right\} = 1$$



