

Angle Recoding CORDIC

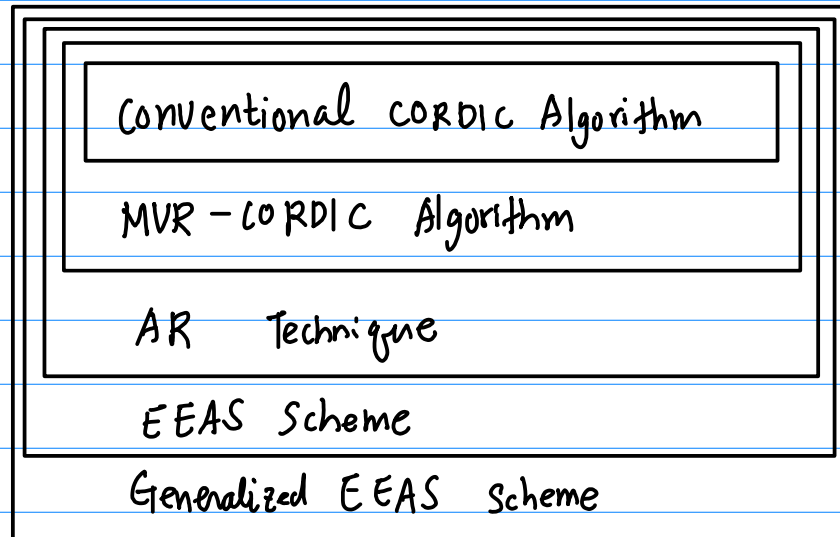
2. Wu

20180604

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Vector Rotational CORDIC



Vector Rotation Alg	Selection of Rotation Sequence	Elementary Angle Set	Micro Rotation	Angle Quantization	
				θ_i	N_A
Conventional CORDIC	$\mu = \{-1, +1\}$	EAS S	complete	$\mu(i) a(i)$	W Fixed
Angle Recoding	$\mu = \{-1, 0, +1\}$	EAS S_1	selective	$\tan^{-1}(\alpha(i) \cdot 2^{-s(i)})$	N' Variable
MVR-CORDIC	$\alpha = \{-1, 0, +1\}$	EAS S_1	selective	$\tan^{-1}(\alpha(i) \cdot 2^{-s(i)})$	R_m fixed
EAS	$\alpha_1, \alpha_2 = \{-1, 0, +1\}$	EEAS S_2	selective	$\tan^{-1}(\alpha_0(i) \cdot 2^{-s_0(i)} + \alpha_1(i) \cdot 2^{-s_1(i)})$	R_m fixed
Generalized EAS	$\alpha_1, \alpha_2, \dots, \alpha_{d-1} = \{-1, 0, +1\}$	EEAS S_d $d \geq 3$	selective	$\tan^{-1}(\alpha_0(i) \cdot 2^{-s_0(i)} + \alpha_{d-1}(i) \cdot 2^{-s_{d-1}(i)})$	R_m fixed

Extended EAS (EEAS) - Wu

more flexible way of decomposing the rotation angle

better

the number of iterations
the error performance

$$S_{EAS} = \{ (\sigma \cdot \tan^{-1}(2^{-r})) : \sigma \in \{+1, 0, -1\}, r \in \{1, 2, \dots, n-1\} \}$$

$$S_{EEAS} = \{ (\sigma_1 \cdot \tan^{-1}(2^{-r_1}) + \sigma_2 \cdot \tan^{-1}(2^{-r_2})) : \\ \sigma_1, \sigma_2 \in \{+1, 0, -1\}, r_1, r_2 \in \{1, 2, \dots, n-1\} \}$$

The pre do-rotation
for i -th micro rotations

$$\begin{aligned}x_{i+1} &= x_i - [\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)}] y_i \\y_{i+1} &= y_i + [\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)}] x_i\end{aligned}$$

The pseudo-rotated vector $[x_{R_m}, y_{R_m}]$
after R_m (the required number of micro-rotations)

needs to be scaled by a factor $K = \prod K_i$

$$K_i = \left[1 + \left(\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)} \right)^2 \right]^{-\frac{1}{2}}$$

$$\begin{aligned}\tilde{x}_{i+1} &= \tilde{x}_i - [k_1(i) \cdot 2^{-s_1(i)} + k_2(i) \cdot 2^{-s_2(i)}] \tilde{y}_i \\ \tilde{y}_{i+1} &= \tilde{y}_i + [k_1(i) \cdot 2^{-s_1(i)} + k_2(i) \cdot 2^{-s_2(i)}] \tilde{x}_i\end{aligned}$$

$$\tilde{x}_0 = x_{R_m}$$

$$\tilde{y}_0 = y_{R_m}$$

$$k_1, k_2 \in \{-1, 0, 1\}$$

$$s_1, s_2 \in \{1, 2, \dots, n-1\}$$

- [21] C.-S. Wu, A.-Y. Wu, and C.-H. Lin, "A high-performance/low-latency vector rotational CORDIC architecture based on extended elementary angle set and trellis-based searching schemes," *IEEE Trans. Circuits Syst. II: Anal. Digital Signal Process.*, vol. 50, no. 9, pp. 589–601, Sep. 2003.

A Unified View for Vector Rotational CORDIC Algorithms and Architectures Based on Angle Quantization Approach

An-Yeu Wu and Cheng-Shing Wu

AR : to approximate θ
with the combination
of selected angle elements
from a pre-defined EAS
(Elementary Angle Set)

EAS : all possible values of $\theta(j)$

$$\text{EAS } \hat{S}_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\}, \\ s^* \in \{0, 1, \dots, N-1\} \}$$

EAS \hat{S}_1 consists of $\tan^{-1}(\text{Single signed power of two})$
 $\tan^{-1}(\text{Single SPT})$
 $\tan^{-1}(\alpha^* \cdot 2^{-s^*})$

SPT-based digital filter design

to increase the coefficient resolution

→ employ more SPT terms to represent filter coefficients

[12] H. Samueli, "An improved search algorithm for the design of multiplierless FIR filters with power-of-two coefficients," *IEEE Trans. Circuits Syst.*, vol. 36, pp. 1044–1047, July 1989.

[13] Y. C. Lim, R. Yang, D. Li, and J. Song, "Signed power-of-two term allocation scheme for the design of digital filters," *IEEE Trans. Circuits Syst. II*, vol. 46, pp. 577–584, May 1999.

EAS \hat{S}_1 consists of $\tan^{-1}(\text{Single signed power of two})$
 $\tan^{-1}(\text{Single SPT})$
 $\tan^{-1}(\alpha^* \cdot 2^{-s^*})$

EAS \hat{S}_2 consists of $\tan^{-1}(\text{two signed power of two})$
 $\tan^{-1}(\text{two SPT})$
 $\tan^{-1}(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*})$

Two Signed - Power - of - Two terms

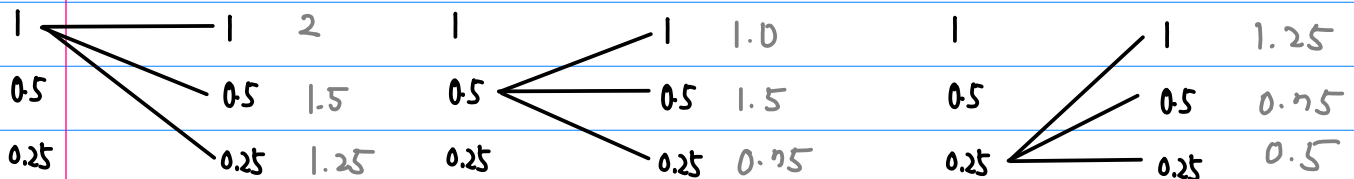
$$S_2 = \left\{ \tan^{-1} \left(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*} \right) : \right. \\ \left. \begin{array}{l} \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\} \\ s_0^*, s_1^* \in \{0, 1, \dots, W-1\} \end{array} \right\}$$

S_1

1	$1 = 2^{-0}$	$\tan^{-1}(2^{-0})$
0.5	$\frac{1}{2} = 2^{-1}$	$\tan^{-1}(2^{-1})$
0.25	$\frac{1}{4} = 2^{-2}$	$\tan^{-1}(2^{-2})$

S_2

2	$1+1 = 2^0 + 2^{-0}$	$\pm \tan^{-1}(2^0 + 2^{-0})$
1.5	$1+\frac{1}{2} = 2^0 + 2^{-1}$	$\pm \tan^{-1}(2^0 + 2^{-1})$
1.25	$1+\frac{1}{4} = 2^0 + 2^{-2}$	$\pm \tan^{-1}(2^0 + 2^{-2})$
1.0	$1 = 2^{-0}$	$\pm \tan^{-1}(2^{-0})$
0.75	$\frac{1}{2}+\frac{1}{4} = 2^{-1} + 2^{-2}$	$\pm \tan^{-1}(2^{-1} + 2^{-2})$
0.5	$\frac{1}{2} = 2^{-1}$	$\pm \tan^{-1}(2^{-1})$
0.25	$\frac{1}{4} = 2^{-2}$	$\pm \tan^{-1}(2^{-2})$



$$2^{-0}, 2^{-1}, 2^{-2}$$

$$\{0, 1, 2\} = \{0, 1, w-1\}$$

$$w=3$$

$$s_0^*, s_i^* \in \{0, 1, 2\}$$

$$2^{s_0^*}, 2^{s_i^*} \in \{2^{-0}, 2^{-1}, 2^{-2}\}$$

as the wordsize w increases,
the size of the set S_2 increases exponentially

$$\theta_i = \tan^{-1} (\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)})$$

R_m : the number of the subangle N_A

$$S_2 = \{ \theta_i \mid i = 0, 1, \dots, R_m \}$$

$$S_2 = \left\{ \tan^{-1} (\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*}) : \right. \\ \left. \begin{array}{l} \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\} \\ s_0^*, s_1^* \in \{0, 1, \dots, w-1\} \end{array} \right\}$$

the optimization problem of the EEAS-based CORDIC algorithm

given θ and R_m

find $\alpha_0(j)$, $\alpha_1(j)$, $s_0(j)$, and $s_1(j)$

the combination of elementary angles
from EEAS S_2

Minimize the angle quantization error

$$\left| \sum_{m, EEAS} \right| \triangleq \theta - \sum_{j=0}^{R_m-1} \tan^{-1} (\alpha_0(j) 2^{-s_0(j)} + \alpha_1(j) 2^{-s_1(j)})$$

given $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix}$

$$\begin{bmatrix} x(j+1) \\ y(j+1) \end{bmatrix} = \begin{bmatrix} 1 & \alpha_0(j) 2^{-s_0(j)} + \alpha_1(j) 2^{-s_1(j)} \\ \alpha_0(j) 2^{-s_0(j)} + \alpha_1(j) 2^{-s_1(j)} & 1 \end{bmatrix} \begin{bmatrix} x(j) \\ y(j) \end{bmatrix}$$

$$\begin{bmatrix} x_f \\ y_f \end{bmatrix} = P \begin{bmatrix} x(R_m) \\ y(R_m) \end{bmatrix} = \frac{1}{\prod_{j=0}^{R_m-1} \sqrt{1 + [\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)}]^2}} \begin{bmatrix} x(R_m) \\ y(R_m) \end{bmatrix}$$

Micro rotation procedure
the scaling operation

↳ additions

increased hardware
reduced iteration steps

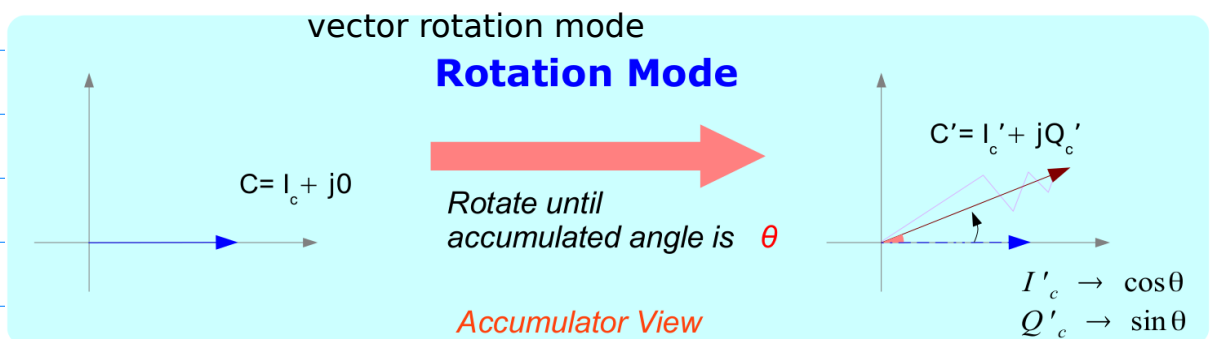
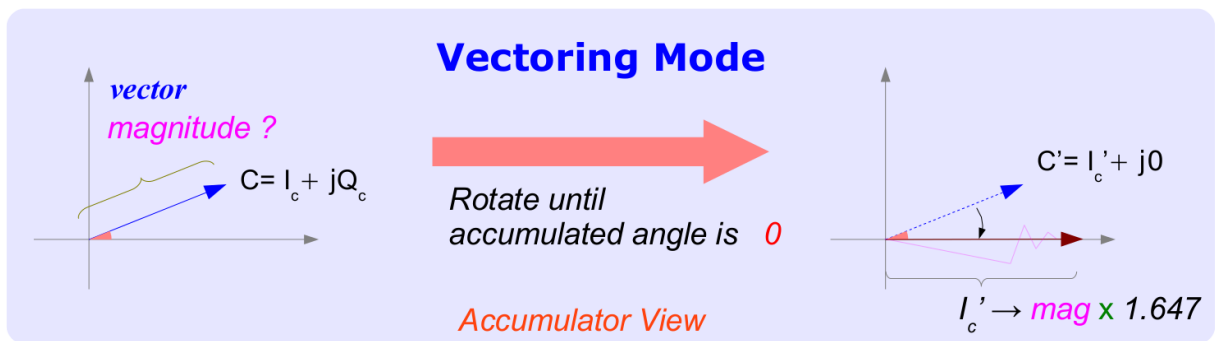
MVR (Modified Vector Rotation)

1) Repeat of Elementary Angles θ_i, θ_i

2) fixed total micro-rotation Number R_m

* Vector Rotation Mode

* and the rotation angles are known in advance



Modified Vector Rotational MVR CORDIC

- reduce the iteration number
- maintaining the SQNR performance
- modifying the basic microrotation procedure

Three Searching Algorithm

- ① the selective prerotation
- ② the selective scaling
- ③ iteration-tradeoff scheme

Angle Quantization

Quantization process on the rotational angle θ

decompose θ into several subangles θ_i 's

the angle quantization error

$$\xi_m \triangleq \theta - \sum_{i=0}^{N_A-1} \theta_i$$

N_A the number of subangles
 $\theta_0, \theta_1, \dots, \theta_{N_A-1}$

$$\theta = \theta_0 + \theta_1 + \dots + \theta_{N_A-1} + \xi_m$$

data : W -bit word length

the iteration number : N $N \leq W$

the restricted iteration number : R_m $R_m \ll W$

AQ Process : 2 Design Issues

① need to determine the sub-angles θ_i

② select / combine sub angles
to minimize the angle quantization angle ξ_m

CSD (Canonical Signed Digit) Quantization

digital filter designs

coefficients are recoded

in terms of SPT (Signed Power of Two) terms

multiplication can be easily realized
with shift-and-add operations

$$h_2 = (-0.156249)_{10} \Rightarrow (0.0\bar{1}011)_2$$

$W=8$, 3 non-zero digits

- ① CSD quantization decomposes coefficients into several SPT terms (sub-coefficients)
- ② the multiplication of a coefficient can be reformed through the combination of the non-zero SPT sub-coefficients

Quantize the rotation angle θ

decompose the rotation angle θ
into several sub-angles θ_i 's

the rotational operation of each θ_i
should be easily realized

If each θ_i can be realized
using only shift-and-add operations

the rotation of θ can be performed
through successive applications of
sub-angle rotations
in a cost-effective way

Approximation target	Coefficient r_i	Rotation angle θ
Basic Element	Non-zero digit 2^{-i}	Sub-angle $\alpha(i) = \tan^{-1}(2^{-i})$
Basic Operation	shift-and-add operation	2 shift-and-add operations
Approximation Equation	$r_i \approx \sum_{j=0}^{N_D-1} g_j \cdot 2^{-d_j}$	$\theta \approx \sum_{j=0}^{N_A-1} \alpha(j) \cdot a(s_j)$
	$g_j \in \{-1, 0, +1\}$ $d_j \in \{0, 1, \dots, w-1\}$ <p>$N_D =$ the number of non-zero digits</p>	<p>$N_A =$ the number of sub-angles</p>

Vector Rotation CORDIC Family

① Conventional CORDIC

② AR

③ MVR

④ EEAS

① Conventional CORDIC

elementary angle $a(i) = \tan^{-1}(2^{-i})$

the number of elementary angles N

the rotation sequence $\mu(i) = \{-1, +1\}$
 $+1, -1, -1, +1, +1, \dots$

the i -th rotation angle $a(i)$

the w -bit word length

the iteration number $N \leq w$

the angle quantization error

$$\xi_{m, \text{CORDIC}} \equiv \theta - \sum_{i=0}^{M-1} \mu(i) a(i)$$

① AR [Hu]

skip certain micro rotations

the rotation sequence $\mu(i) = \{-1, 0, +1\}$

$\mu(i) = 0 \rightarrow$ skip

desire to minimize

$$\sum_{i=0}^N |\mu(i)|$$

so that the total number of CORDIC iterations can be minimized

Angle Recoding \leftarrow Multiplier Recoding

angle recoding method for efficient implementation of the CORDIC algorithm
Hu & Naganathan, ISCAS 89

Greedy algorithm

$$\theta(0) = \theta, \{\mu(i) = 0, 0 \leq i \leq N-1\}, k=0$$

repeat until $|\theta(k)| < a(N-1)$ Do

choose $i_k, 0 \leq i_k \leq N-1$

$$| |\theta(k)| - a(i_k) | = \text{Min}_{0 \leq i \leq N-1} | |\theta(k)| - a(i) |$$

$$\theta(k+1) = \theta(k) - \mu(i_k) a(i_k)$$

$$\mu(i_k) = \text{Sign}(\theta(k))$$

try to approach the target rotation angle θ
step by step

decisions are made in each step
by choosing the best combination of $\alpha(i)$ $a(s(i))$

So as to minimize $|\xi_m|$

$\alpha(i)$, $a(i)$ are determined such that
the error function is minimized

$$J(i) = |\theta(i) - \alpha(i)a(s(i))|$$

$$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m) a(s(m))$$

terminated if no further improvement can be found

$$J(i) \geq J(i-1)$$

or $\alpha(R_m-1)$ and $s(R_m-1)$
are determined at the end

$$\xi_{m, \text{CORDIC}} \equiv \theta - \sum_{i=0}^M \mu(i) \alpha(i) \quad \mu(i) = \{-1, 0, +1\}$$

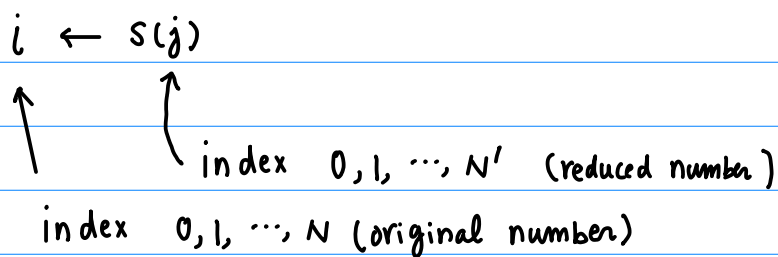
$$= \theta - \sum_{j=0}^{N'} \tilde{\theta}(j)$$

$$N' \equiv \sum_{i=0}^{N-1} |\mu(i)| \quad \{+1, 0, +1\}$$

the effective iteration number N'

$S(j)$ the rotational sequence

determines the micro-rotation angle in the j -th iteration



$$\mu(S(j)) \leftarrow \alpha(j)$$

$$\downarrow \quad \uparrow$$

$$\{-1, +1\}$$

$$\mu(i) = \begin{cases} \mu(S(j)) & i = S(j) \\ 0 & i \neq S(j) \text{ --- reduced index} \end{cases}$$

er

$$\begin{aligned}
 i &= 0, \overset{\text{see}}{\boxed{1, 2}}, 3, \dots, N-1 \\
 s(j) &= 0, \boxed{1, 2}, 3, \dots, N-1 && \text{rotational sequence} \\
 \alpha(j) &= -1, \boxed{0, 0}, +1, \dots, -1 && \text{directional sequence} \\
 j &= 0, -, -, 1, \dots, N'-1 && \text{effective iteration number} \\
 N' &= N-2
 \end{aligned}$$

the j -th micro-rotation of $a(s(j))$

elementary angle

$$\begin{aligned}
 a(i) &= \tan^{-1}(2^{-i}) \\
 a(s(j)) &= \tan^{-1}(2^{-s(j)})
 \end{aligned}$$

$$\alpha(j) a(s(j)) = \alpha(j) \tan^{-1}(2^{-s(j)}) \quad \alpha(j) \in \{-1, +1\}$$

$$\Leftrightarrow \mu(i) a(i) \quad \mu(i) \in \{-1, 0, +1\}$$

$$\sum_{m, \text{CORDIC}} \equiv \theta - \sum_{i=0}^{N-1} \mu(i) a(i) \quad \mu(i) \in \{-1, 0, +1\}$$

$$= \theta - \left[\sum_{j=0}^{N'} \tilde{\theta}(j) \right]$$

$$= \theta - \left[\sum_{j=0}^{N'} \tan^{-1}(\alpha(j) \cdot 2^{-s(j)}) \right] \quad \alpha(j) \in \{-1, +1\}$$

$$\tilde{\theta}(j) = \alpha(j) \tan^{-1}(2^{-s(j)})$$

$$= \tan^{-1}(\alpha(j) \cdot 2^{-s(j)})$$

$$S_i = \{ \tan^{-1}(\alpha \cdot 2^{-s}) \mid \alpha \in \{-1, 0, +1\}, s \in \{0, 1, 2, \dots, N-1\} \}$$

② MVR (Modified Vector Rotational)

two modifications

① **repetition** of elementary angles

each micro-rotation of elementary angle
can be performed repeatedly

- more possible combinations
- smaller ξ_m

② **confinement** of total micro-rotation number

confine the iteration number
in the micro-rotation phase
to **R_m** ($R_m \ll W$)

The role of **R_m** is quite similar
to the **number of non-zero digit**
 N_D in CSD recoding scheme

$$\sum_{m, \text{MVR}}^x \triangleq \theta - \sum_{i=0}^{R_m-1} \alpha(i) a(s(i))$$

the rotational sequence

$$s(i) \in \{0, 1, \dots, W-1\}$$

the micro-rotation angle

in the i -th iteration

the directional sequence

$$\alpha(i) \in \{-1, 0, +1\}$$

the direction of the i -th

micro-rotation of $a(s(i))$

$$\alpha(i) a(s(i)) = \tilde{\theta}(j)$$

$$\sum_{m, MVR} \cong \theta - \sum_{j=0}^{R_m-1} \alpha(j) a(s(j))$$

the rotational sequence $s(j)$

$$j = 0, 1, 2, \dots, R_m-1$$



$$s(j) \in \{0, 1, \dots, W-1\}$$

determines the micro-rotation angle $a(s(j))$
in the j -th iteration

the directional sequence $\alpha(j)$

$$\alpha(j) \in \{-1, 0, +1\}$$

controls the direction of the j -th
micro-rotation of $a(s(j))$

$$\alpha(j) a(s(j)) = \tilde{\theta}(j)$$

$i = 0, 1, 2, 3, \dots, W-1$	
$s(j) = 0, 1, 2, 3, \dots, W-1$	rotational sequence
$\alpha(j) = -1, 0, 0, +1, \dots, -1$	directional sequence
$j = 0, \dots, R_m-1$	effective iteration number
$R_m \ll W$	

sub-angle $(\alpha(j) a(s(j))) \sim \tilde{\theta}(j)$

$$\begin{aligned}\xi_{m,AR} &= \theta - \left[\sum_{j=0}^{N'-1} \tan^{-1}(\alpha(j) \cdot 2^{-s(j)}) \right] \\ &= \theta - \left[\sum_{j=0}^{N'-1} \tilde{\theta}(j) \right], \quad \tilde{\theta}(j) = \tan^{-1}(\alpha(j) \cdot 2^{-s(j)})\end{aligned}$$

$$N' \triangleq \sum_{j=0}^{N-1} |\mu(j)| \quad \text{the effective iteration number}$$

EAS formed by MVR-CORDIC
is the same as AR
also performs AQ

The major difference

1) the total number of sub-angles N_A

the total iteration number

in the micro-rotation phase

is kept fixed to a pre-defined value of R_m

$$N_A = R_m$$

2) the sub-angle θ_j corresponds to $\alpha(j) a(s(j))$

$$\theta_j = \alpha(j) a(s(j)) = \tilde{\theta}_j$$

Optimization Problem

EAS point of view

Given θ , find the combination of R_m elementary angles from EAS S_i , such that the angle quantization error $|\xi_{m, \text{MUR}}|$ is minimized.

Semi-greedy algorithm

trade offs between computational complexities and performance

key issue in the MVR-CORDIC

is to find the best sequences of

$s(i)$ and $\alpha(i)$ to minimize $|\xi_m|$

subject to the constraint that

the total iteration number is confined to R_m

1) Greedy Algorithm

2) Exhaustive Algorithm

3) Semigreedy Algorithm

1) Greedy Algorithm

try to approach the target rotation angle, θ , step by step in each step, decisions are made on $\alpha(i)$ and $s(i)$ by choosing the best combination of $\alpha(i)$ and $s(i)$ so as to minimize $|\xi_m|$

$\alpha(i)$ and $s(i)$ are determined such that the error function $J(i) = |\theta(i) - \alpha(i) a(s(i))|$ is minimized

$\theta(i)$: the residue angle in the i -th step

$$\theta(i) = \theta - \sum_{m=0}^{i-1} \alpha(m) a(s(m))$$

the searching is terminated

if no further improvements can be found

$$J(i) \geq J(i-1)$$

$\alpha(R_m-1)$ and $s(R_m-1)$ are determined

at the end of the searching

the greedy algorithm terminates

Only when the residue angle error cannot be further reduced.

Initialization:

given θ , w , R_m

$$\theta^{(i)} = \theta - \sum_{m=0}^{i-1} \alpha^{(m)} a(s^{(m)})$$

Select $\alpha^{(i)} \in \{-1, 0, +1\}$
 $s^{(i)} \in \{0, 1, 2, \dots, w-1\}$
to minimize $J^{(i)} = \theta^{(i)} - \alpha^{(i)} a(s^{(i)})$

N
 $J^{(i)} < J^{(i-1)}$

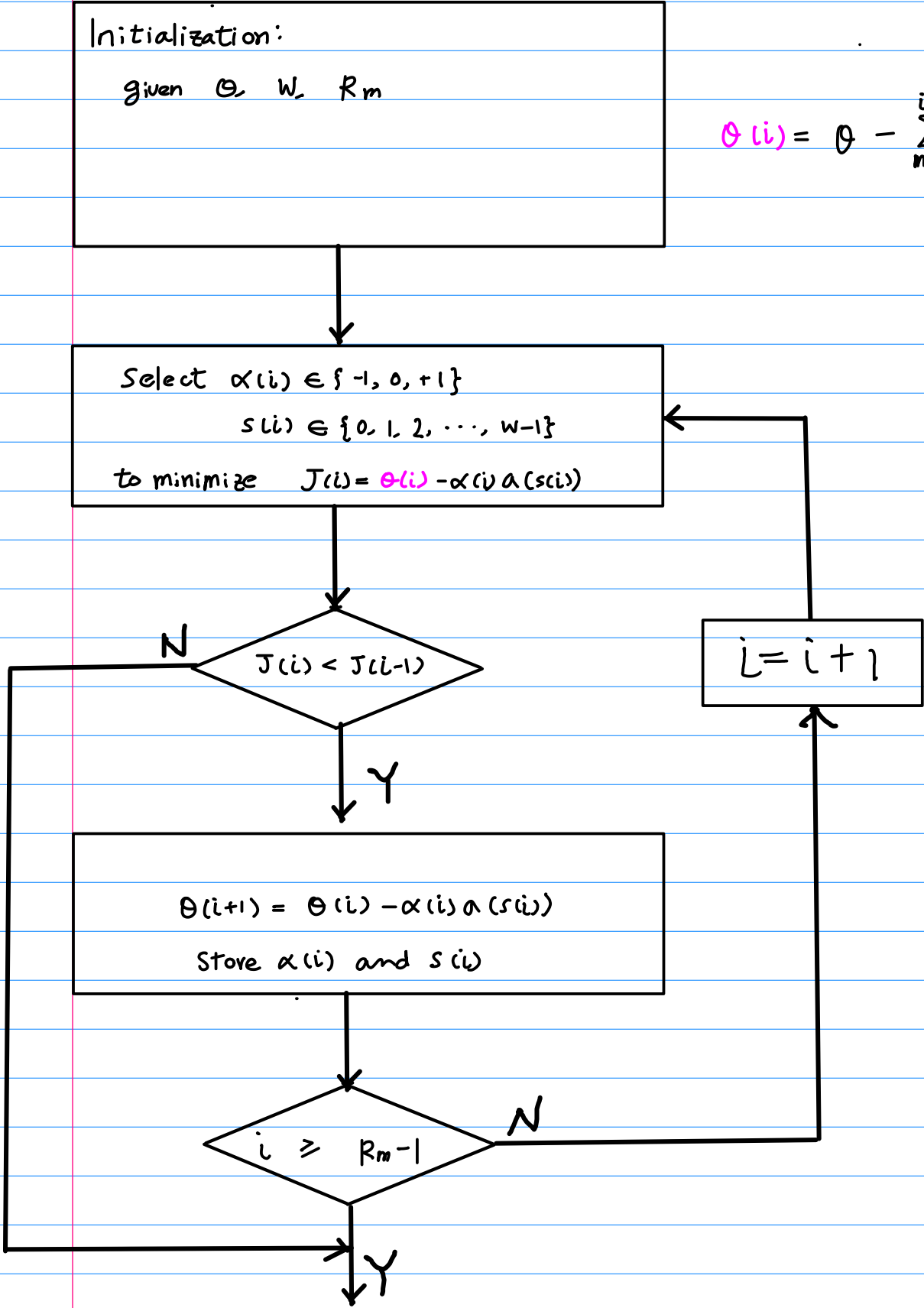
Y

$\theta^{(i+1)} = \theta^{(i)} - \alpha^{(i)} a(s^{(i)})$
Store $\alpha^{(i)}$ and $s^{(i)}$

$i \geq R_m - 1$
N

Y

$i = i + 1$



2) Exhaustive Algorithm

search for the entire solution space

all possible combinations of

$$\sum_{i=0}^{R_m-1} \alpha(i) a(s(i))$$

in a single step

decisions for $\alpha(i)$ and $s(i)$, $0 \leq i \leq R_m-1$

by minimizing the error function

$$J = \left| 0 - \sum_{i=0}^{R_m-1} \alpha(i) a(s(i)) \right|$$

global optimal solution

Initialization:

given Θ, W, R_m

let $\Theta(0) = \Theta,$

$i = 0$

$J(-1) = \infty$

Select $\alpha(i) \in \{-1, 0, +1\}$

$s(i) \in \{0, 1, 2, \dots, W-1\}$

for $0 \leq i \leq R_m - 1$

to minimize $J(i) = \Theta - \sum_{l=0}^{R_m-1} \alpha(l) a(s(i))$

$$(3 \cdot W) \cdot (3 \cdot W) \dots (3 \cdot W) \\ = 3^{R_m} \cdot W^{R_m}$$

Store $\alpha(i)$ and $s(i)$

for $0 \leq i \leq R_m - 1$

in the i -th block

decision of $\alpha(k)$ and $s(k)$ for $iD \leq k \leq (i+1)D-1$

$$\text{minimizes } J = \left| \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k)) \right|$$

$$\text{where } \theta(i) = \theta - \sum_{m=0}^{i-1} \left[\sum_{k=mD}^{(m+1)D-1} \alpha(k) a(s(k)) \right]$$

the residue angle in the i -th step

$$s = \left\lceil \frac{R_m}{D} \right\rceil$$

$$\theta(i) = \theta - \left[\sum_{k=0D}^{1D-1} \alpha(k) a(s(k)) + \sum_{k=1D}^{2D-1} \alpha(k) a(s(k)) + \dots + \sum_{k=(i-1)D}^{iD-1} \alpha(k) a(s(k)) \right]$$

Initialization:

given θ, W, R_m

let $\theta(0) = \theta,$

$i = 0$

$J(-1) = \infty$

Select $\alpha(k) \in \{-1, 0, +1\}$

$s(k) \in \{0, 1, 2, \dots, W-1\}$

for $iD \leq k \leq (i+1)D - 1$

to minimize $J(i) = \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k))$

N
 $J(i) < J(i-1)$

$\theta(i+1) = \theta(i) - \sum_{k=iD}^{(i+1)D-1} \alpha(k) a(s(k))$

Store $\alpha(k), s(k)$

$i \geq \lceil \frac{R_m}{D} \rceil - 1$
N

Y

$i = i + 1$

③ Extended EAS-based CORDIC

$$\mathcal{S}_2 = \left\{ \tan^{-1} \left(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*} \right) : \right. \\ \left. \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\}, s_0^*, s_1^* \in \{0, 1, \dots, W-1\} \right\}$$

$$\theta_i = \tan^{-1} \left(\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)} \right)$$

$$\left| \xi_{m, \text{EAS}} \right| \triangleq$$

$$\left| \theta - \sum_{j=0}^{R_m-1} \tan^{-1} \left(\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)} \right) \right|$$

Generalized EEAS Scheme

$$S_d = \left\{ \tan^{-1} \left(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*} + \dots + \alpha_{d-1}^* \cdot 2^{-s_{d-1}^*} \right) : \right. \\ \left. \begin{array}{l} \alpha_0^*, \alpha_1^*, \dots, \alpha_{d-1}^* \in \{-1, 0, +1\}, \\ s_0^*, s_1^*, \dots, s_{d-1}^* \in \{0, 1, \dots, W-1\} \end{array} \right\}$$