BandPass (4A)

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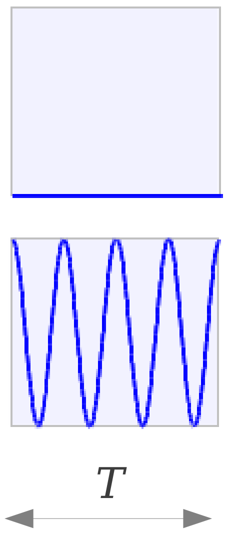
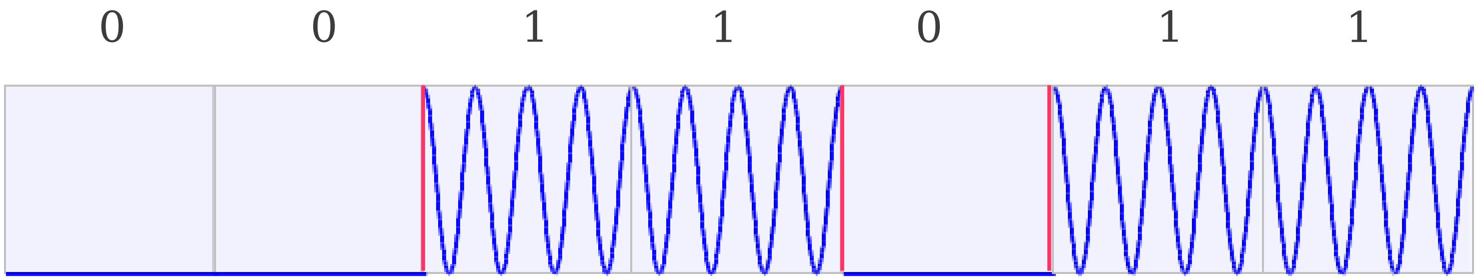
Binary ASK (Amplitude Shift Keying)

$$s_0(t) = \sqrt{\frac{2E_0(t)}{T}} \cos(\omega_0 t + \Phi)$$

$$s_1(t) = \sqrt{\frac{2E_1(t)}{T}} \cos(\omega_0 t + \Phi)$$

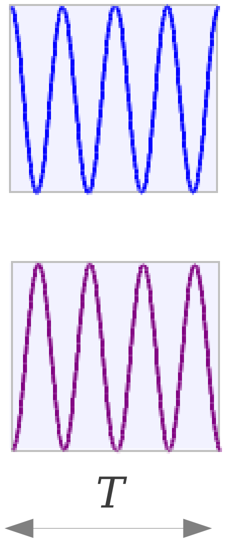


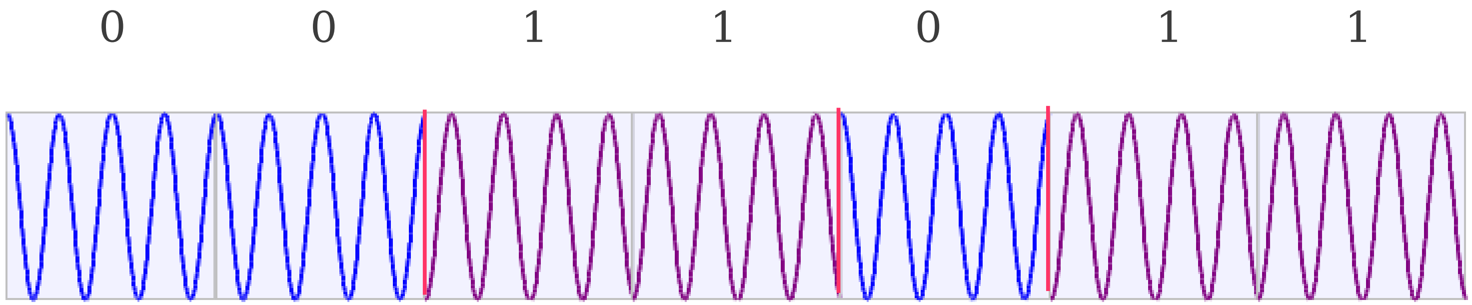


Binary PSK (Phase Shift Keying)

$$s_0(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \frac{2\pi \cdot 0}{2})$$

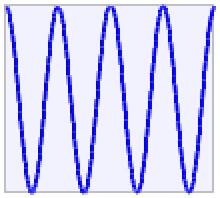
$$s_1(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \frac{2\pi \cdot 1}{2})$$



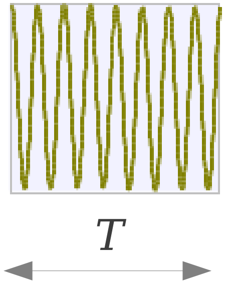


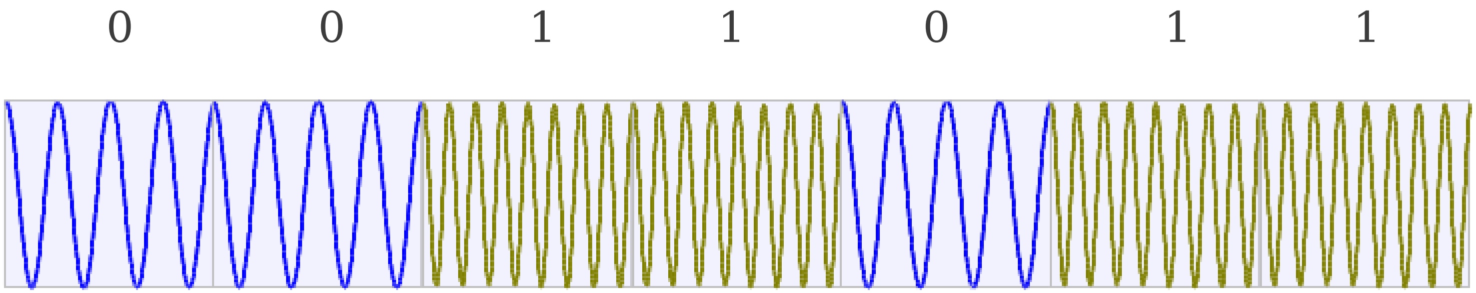
Binary FSK (Frequency Shift Keying)

$$s_0(t) = \sqrt{\frac{2E}{T}} \cos(\omega_0 t + \Phi)$$



$$s_1(t) = \sqrt{\frac{2E}{T}} \cos(\omega_1 t + \Phi)$$





Coherent and Non-Coherent Detection

Coherent Detection

The Sinusoidal Reference Signal
is synchronous in phase
with the carrier wave used in the modulator
Phase Locked Loop

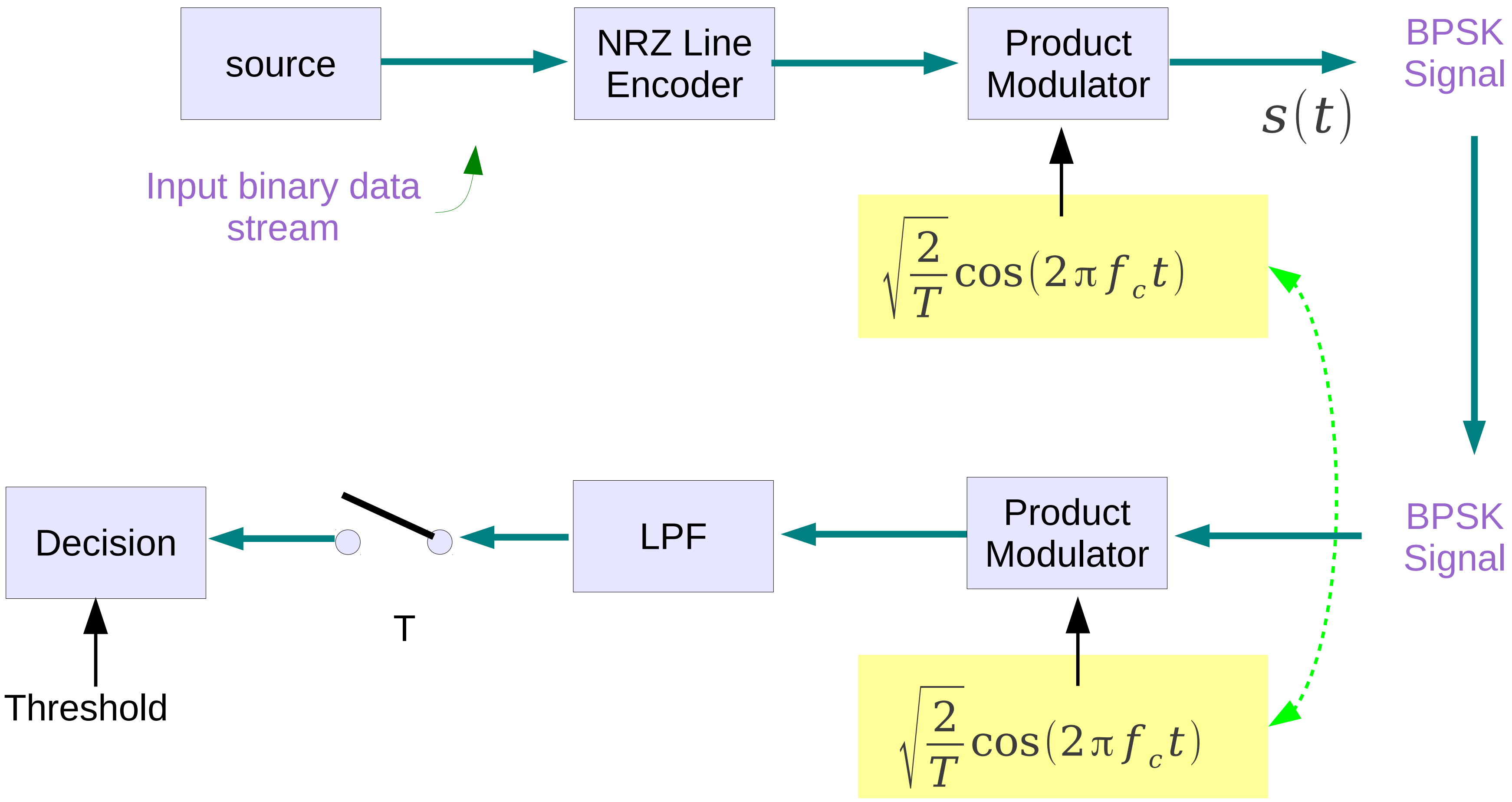
Synchronous Phase
Symbol Interval

Cost

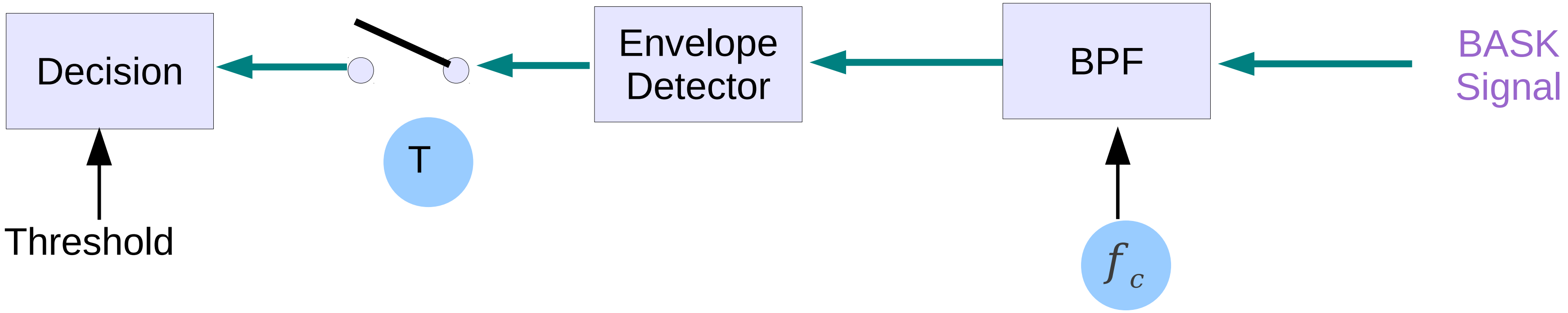
Non-Coherent Detection

Abandon phase synchronization

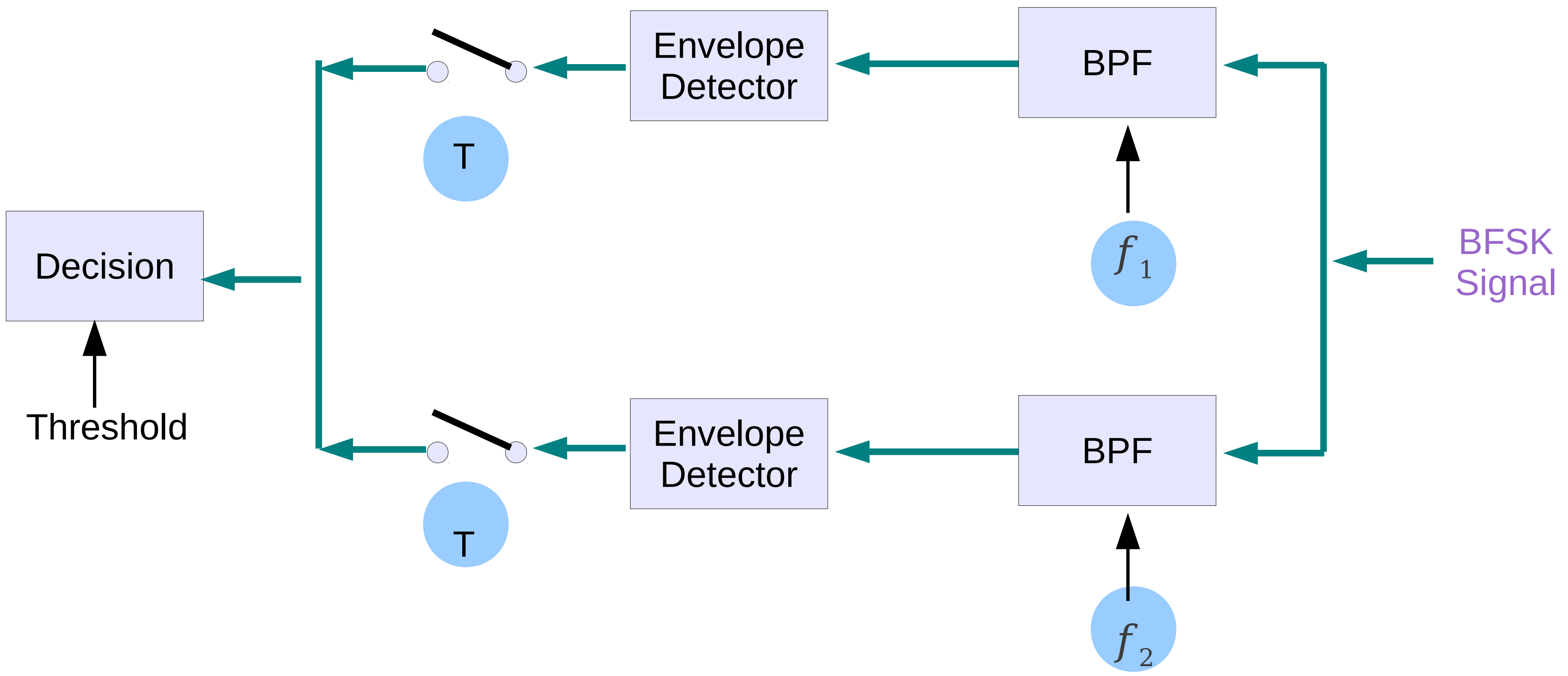
Coherent Detection of BPSK signals



Non-Coherent Detection of BASK signals

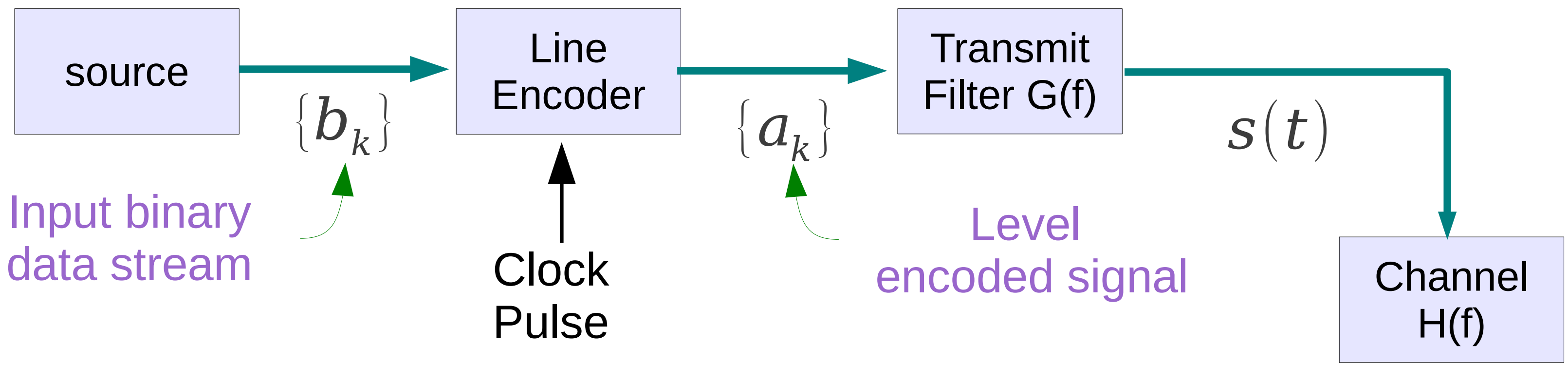


Non-Coherent Detection of BFSK signals



M-ary PAM

The amplitude of transmitted pulses is varied in a discrete manner in accordance with an input stream of digital data



M-ary PAM Bit Rate

$$T = T_b \log_2 M$$

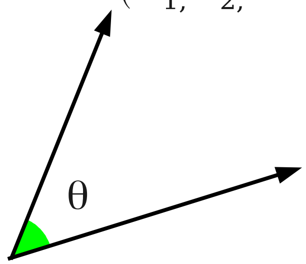
M possible amplitude level (M>2)
M symbols
Transmits sequence of symbols

T: Symbol duration
1/T : Symbol rate

Binary PAM
Tb : Bit duration
1/Tb : Bit rate

Inner Product of Vectors

$\vec{b} = (b_1, b_2, \cdots, b_n)$

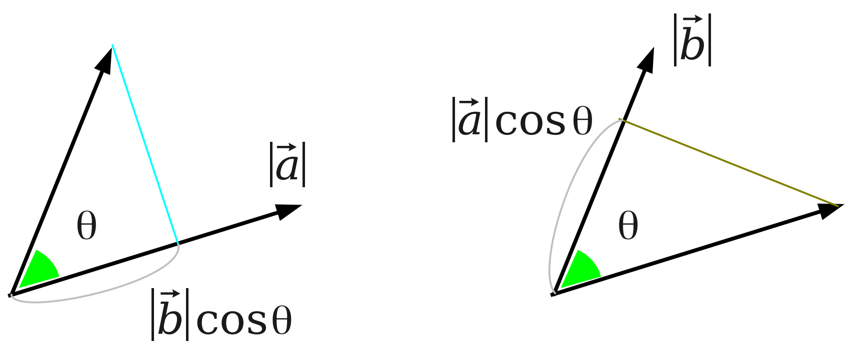


$\vec{a} = (a_1, a_2, \cdots, a_n)$

$$\vec{a} = (a_1, a_2, \cdots, a_n)$$

$$\vec{b} = (b_1, b_2, \cdots, b_n)$$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$



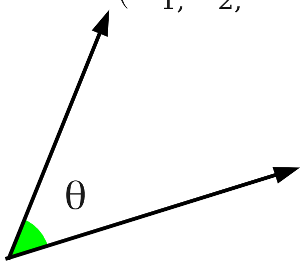
$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

orthogonal $\cos(\pm\pi/2) = 0$

$$\vec{a} \cdot \vec{b} = 0$$

Inner Product of Functions

$\vec{b} = (b_{1,} b_{2,} \cdots , b_n)$



$\vec{a} = (a_{1,} a_{2,} \cdots , a_n)$

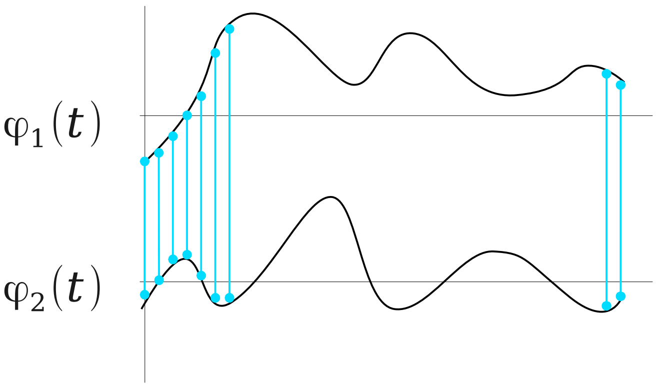
$\vec{a} = (\; a_{1,} \quad a_{2,} \quad \cdots \; , \quad a_n)$

$\vec{b} = (\; b_{1,} \quad b_{2,} \quad \cdots \; , \quad b_n)$

$$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

$$\varphi_1(t) = \cos(\omega t)$$

$$\varphi_1(t) = \sin(\omega t)$$



$$\langle \varphi_1(t), \varphi_2(t) \rangle = \frac{1}{T} \int_0^T \varphi_1(t) \varphi_2(t) \, dt$$

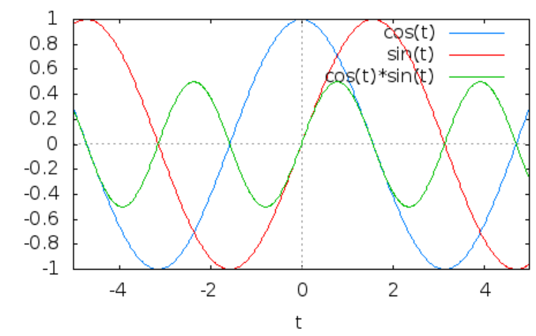
Inner Product of Trigonometric Functions

$$\varphi_1(t) = \cos(\omega t)$$

$$\varphi_2(t) = \sin(\omega t)$$

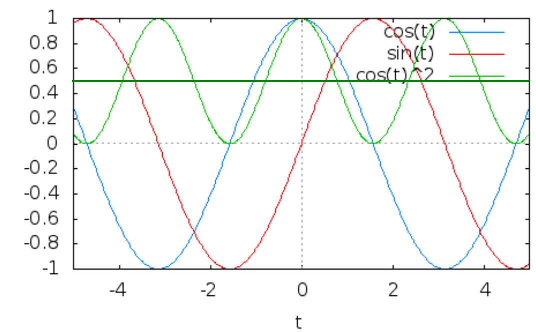
$$\langle \varphi_1(t),\ \varphi_2(t) \rangle = \int_0^T \cos(\omega t)\sin(\omega t)\, dt$$

$$= \int_0^T \frac{\sin(2\omega t)}{2}\, dt = 0$$



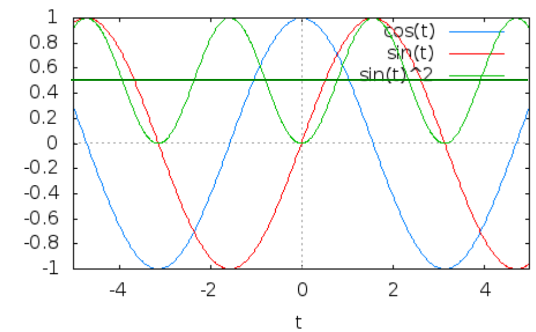
$$\langle \varphi_1(t),\ \varphi_1(t) \rangle = \int_0^T \cos^2(\omega t)\, dt$$

$$= \int_0^T \frac{1+\cos(2\omega t)}{2}\, dt = \frac{1}{2}$$

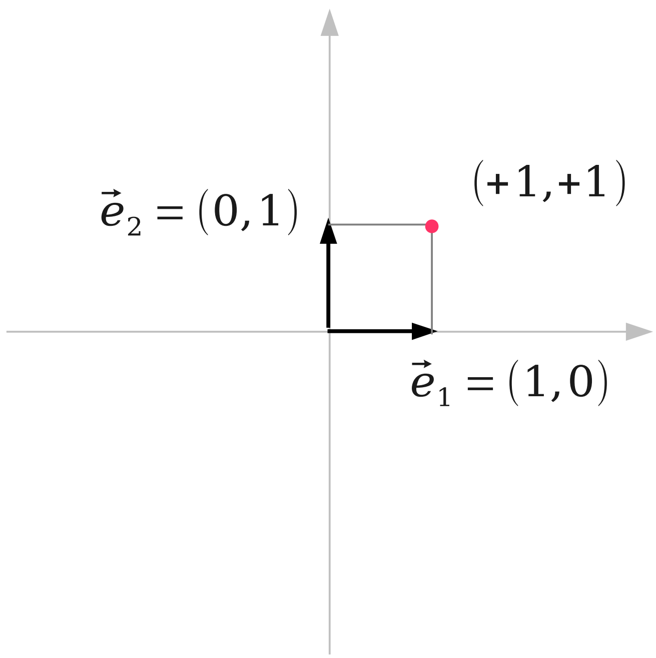


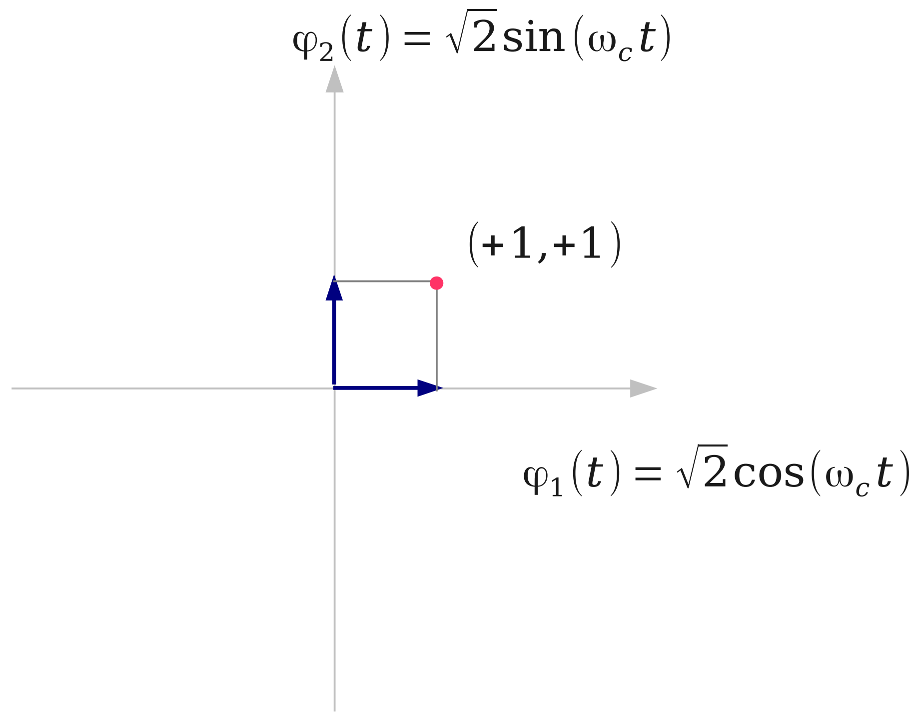
$$\langle \varphi_2(t),\ \varphi_2(t) \rangle = \int_0^T \sin^2(\omega t)\, dt$$

$$= \int_0^T \frac{1-\cos(2\omega t)}{2}\, dt = \frac{1}{2}$$



Orthonormal Basis (1)





$$\vec{e}_1 \cdot \vec{e}_2 = 0$$
$$\vec{e}_1 \cdot \vec{e}_1 = 1$$
$$\vec{e}_2 \cdot \vec{e}_2 = 1$$

$$\langle \varphi_1(t), \ \varphi_2(t) \rangle = \int_0^T \varphi_1(t)\varphi_2(t)\, dt = 0$$

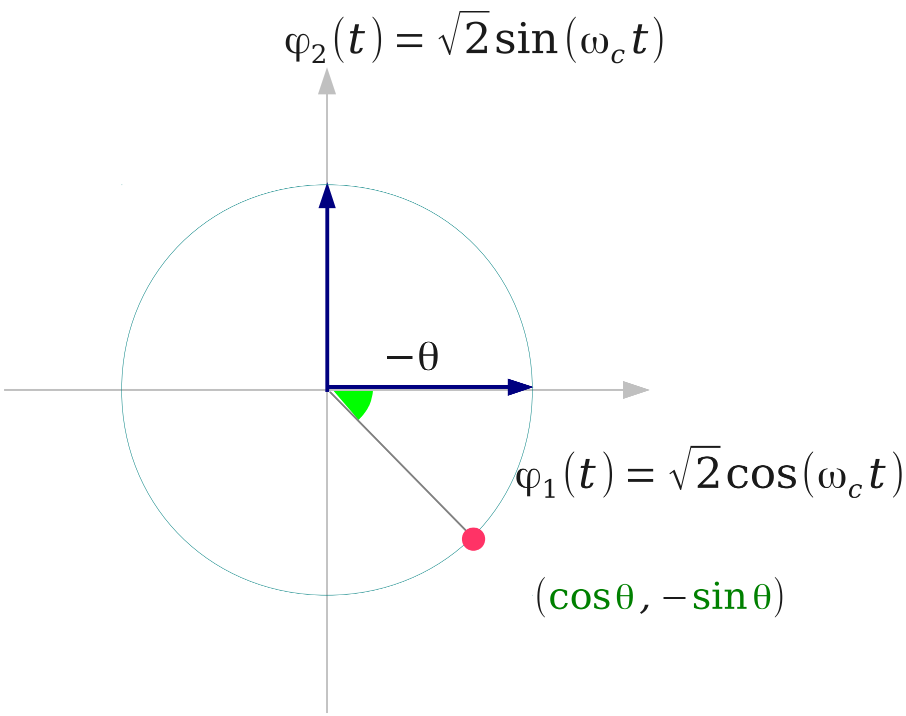
$$\langle \varphi_1(t), \ \varphi_1(t) \rangle = \int_0^T \varphi_1^2(t)\, dt = 1$$

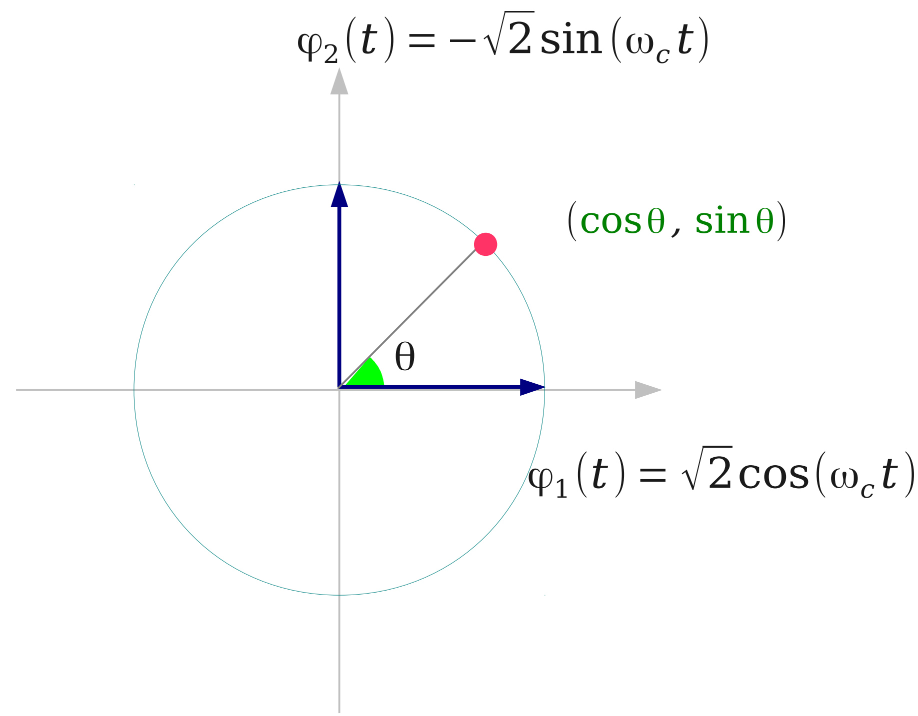
$$\langle \varphi_2(t), \ \varphi_2(t) \rangle = \int_0^T \varphi_2^2(t)\, dt = 1$$

Orthonormal Basis (2)

$$\cos(\omega_c t)\cos(\theta) + \sin(\omega_c t)\sin(\theta)$$
$$= \cos(\omega_c t - \theta)$$

$$\cos(\omega_c t)\cos(\theta) - \sin(\omega_c t)\sin(\theta)$$
$$= \cos(\omega_c t + \theta)$$



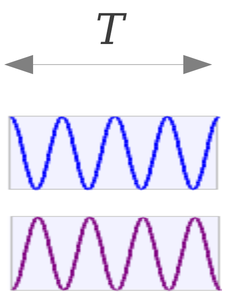


Amplitude

$$v(t) = A\cos(\omega_c t + \theta)$$

$$= \sqrt{2}\,A_{rms}\cos(\omega_c t + \theta)$$

Symbol Time in BPSK



$$P(t) = \frac{v^2(t)}{R}$$

Unit Resistance $R = 1$

$$P = \frac{1}{T}\int_0^T v^2(t)\,dt$$ **Average Power**

$$E = PT$$ **Average Energy**

$$= \frac{A^2}{T}\int_0^T \cos^2(\omega t + \theta)\,dt$$

$$= \frac{A^2}{T}\int_0^T \frac{1 + \cos(2\omega t + 2\theta)}{2}\,dt$$

$$= \frac{A^2}{2} = A_{rms}^2$$

$$A = \sqrt{2P}$$

$$A = \sqrt{\frac{2E}{T}}$$

Signal Space

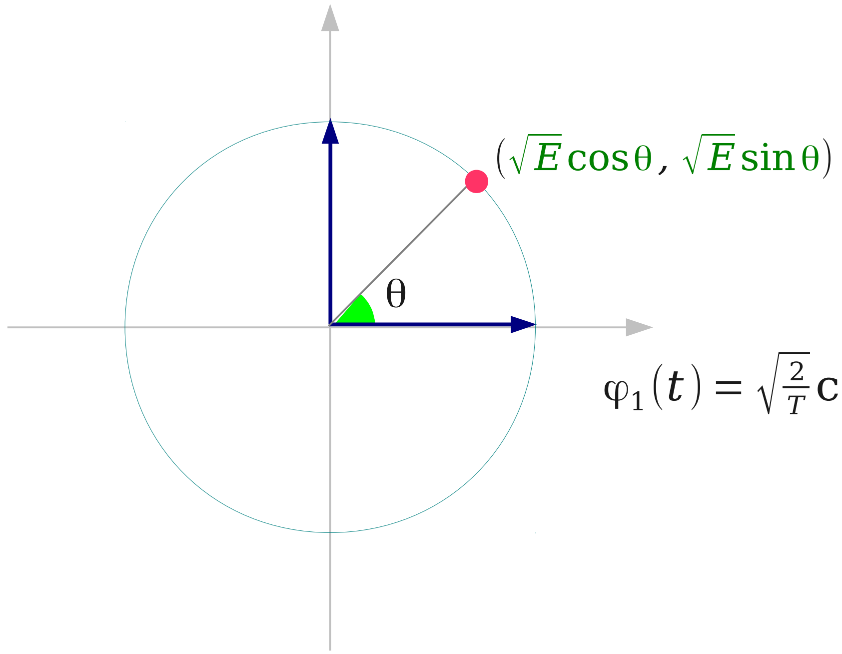
$$\sqrt{\frac{2E}{T}}\cos(\omega_c t + \theta) =$$

$$\sqrt{\frac{2}{T}}\cos(\omega_c t)\sqrt{E}\cos(\theta) - \sqrt{\frac{2}{T}}\sin(\omega_c t)\sqrt{E}\sin(\theta)$$

$$\varphi_1(t) = +\sqrt{\frac{2}{T}}\cos(\omega_c t)$$

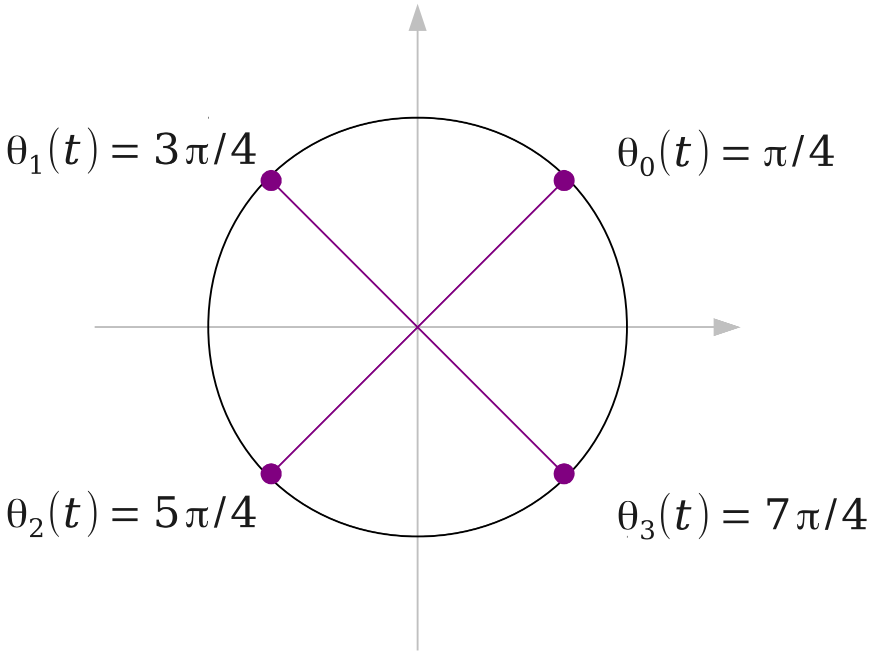
$$\varphi_2(t) = -\sqrt{\frac{2}{T}}\sin(\omega_c t)$$

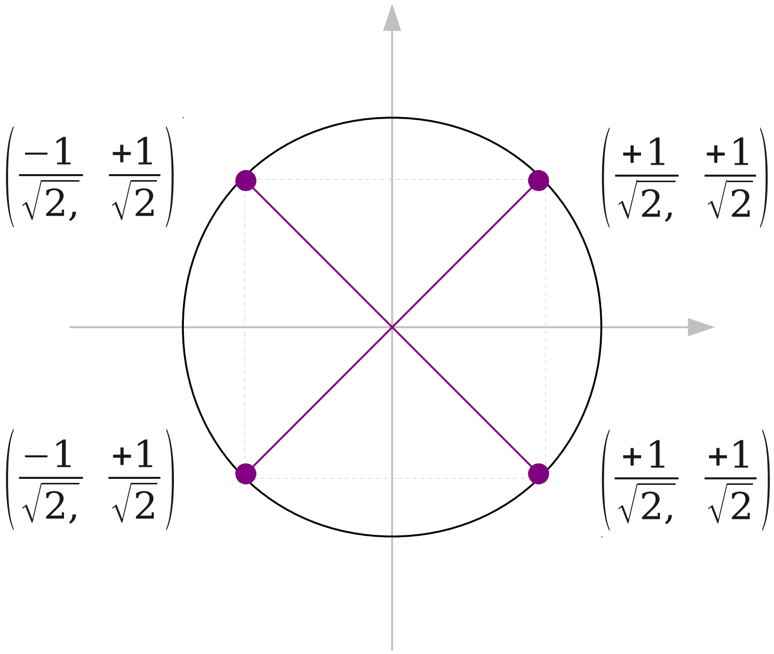
$$\varphi_2(t) = -\sqrt{\frac{2}{T}}\sin(\omega_c t)$$



$$\varphi_1(t) = \sqrt{\frac{2}{T}}\cos(\omega_c t)$$

Signal Space – QPSK (1)



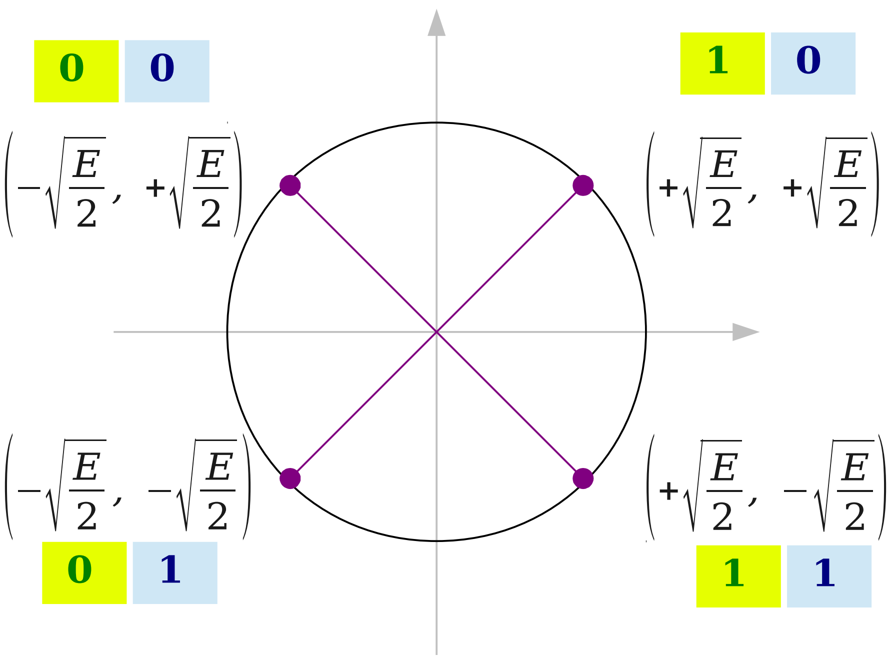


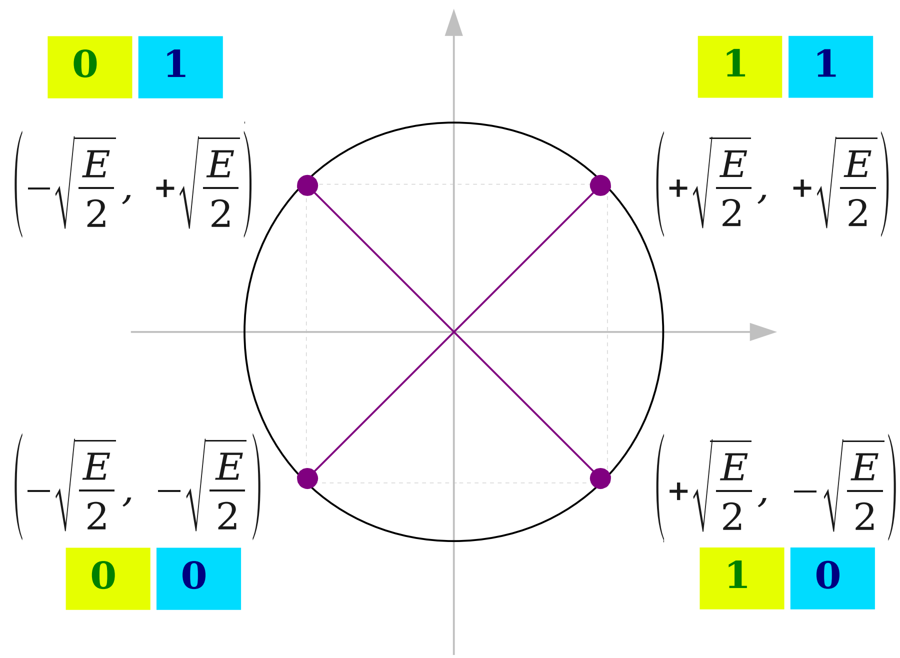
$$s_0(t) = \sqrt{\tfrac{2E}{T}}\cos(\omega_c t + \theta_0) = \sqrt{\tfrac{2E}{T}}\cos(\omega_c t + \pi/4) = \sqrt{\tfrac{2E}{T}}[\cos(\omega_c t)\cos(\pi/4) - \sin(\omega_c t)\sin(\pi/4)]$$

$$s_1(t) = \sqrt{\tfrac{2E}{T}}\cos(\omega_c t + \theta_1) = \sqrt{\tfrac{2E}{T}}\cos(\omega_c t + 3\pi/4) = \sqrt{\tfrac{2E}{T}}[\cos(\omega_c t)\cos(3\pi/4) - \sin(\omega_c t)\sin(3\pi/4)]$$

$$s_2(t) = \sqrt{\tfrac{2E}{T}}\cos(\omega_c t + \theta_2) = \sqrt{\tfrac{2E}{T}}\cos(\omega_c t + 5\pi/4) = \sqrt{\tfrac{2E}{T}}[\cos(\omega_c t)\cos(5\pi/4) - \sin(\omega_c t)\sin(5\pi/4)]$$

$$s_3(t) = \sqrt{\tfrac{2E}{T}}\cos(\omega_c t + \theta_3) = \sqrt{\tfrac{2E}{T}}\cos(\omega_c t + 7\pi/4) = \sqrt{\tfrac{2E}{T}}[\cos(\omega_c t)\cos(7\pi/4) - \sin(\omega_c t)\sin(7\pi/4)]$$

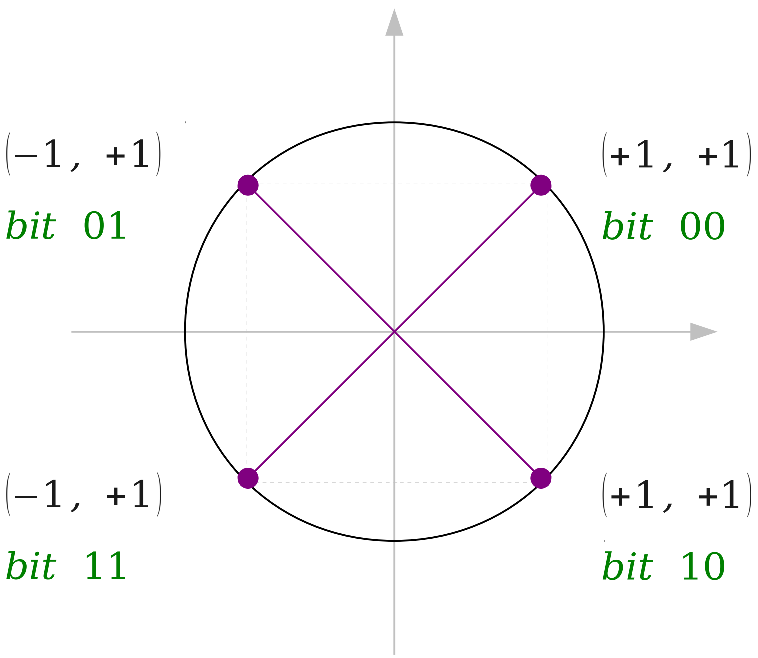




$s_0(t) = +\sqrt{\frac{E}{2}}\sqrt{\frac{2}{T}}\cos(\omega_c t) - \sqrt{\frac{E}{2}}\sqrt{\frac{2}{T}}\sin(\omega_c t)$	1	0	1	1
$s_1(t) = -\sqrt{\frac{E}{2}}\sqrt{\frac{2}{T}}\cos(\omega_c t) - \sqrt{\frac{E}{2}}\sqrt{\frac{2}{T}}\sin(\omega_c t)$	0	0	0	1
$s_2(t) = -\sqrt{\frac{E}{2}}\sqrt{\frac{2}{T}}\cos(\omega_c t) + \sqrt{\frac{E}{2}}\sqrt{\frac{2}{T}}\sin(\omega_c t)$	0	1	0	0
$s_3(t) = +\sqrt{\frac{E}{2}}\sqrt{\frac{2}{T}}\cos(\omega_c t) + \sqrt{\frac{E}{2}}\sqrt{\frac{2}{T}}\sin(\omega_c t)$	1	1	1	0

Gray Code

Signal Space – QPSK (3)

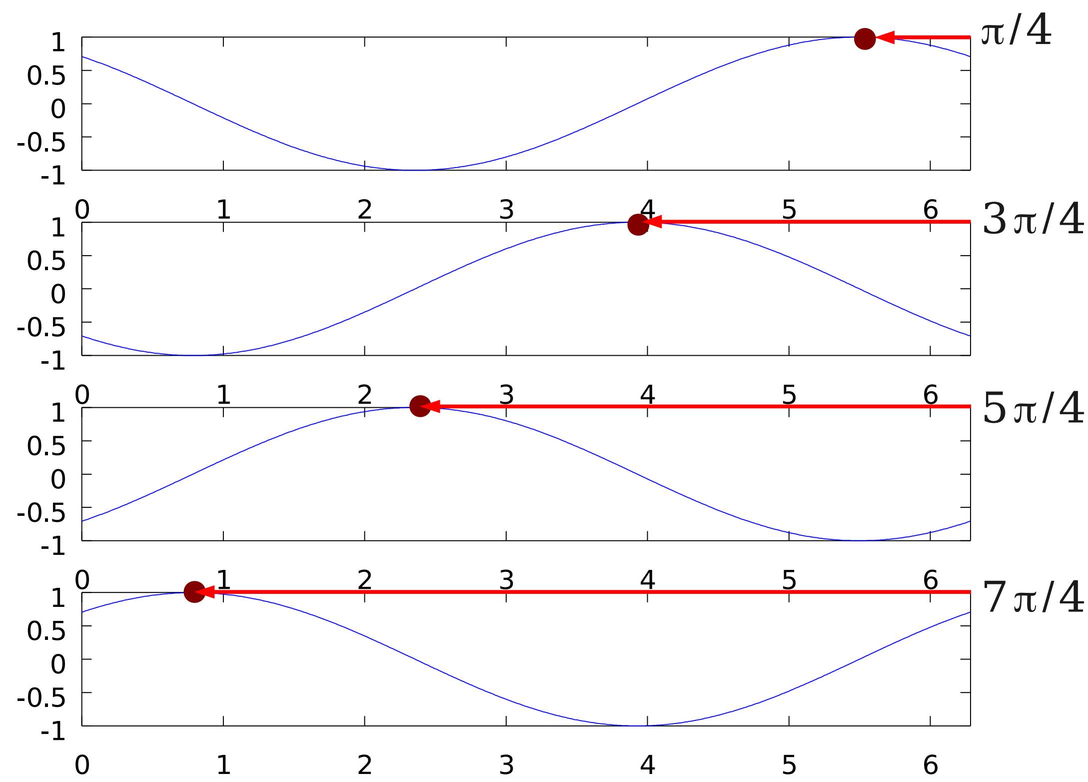


$$s_0(t) = \sqrt{\tfrac{E}{T}}[\cos(\omega_c t)\cdot(+1) - \sin(\omega_c t)\cdot(+1)]$$

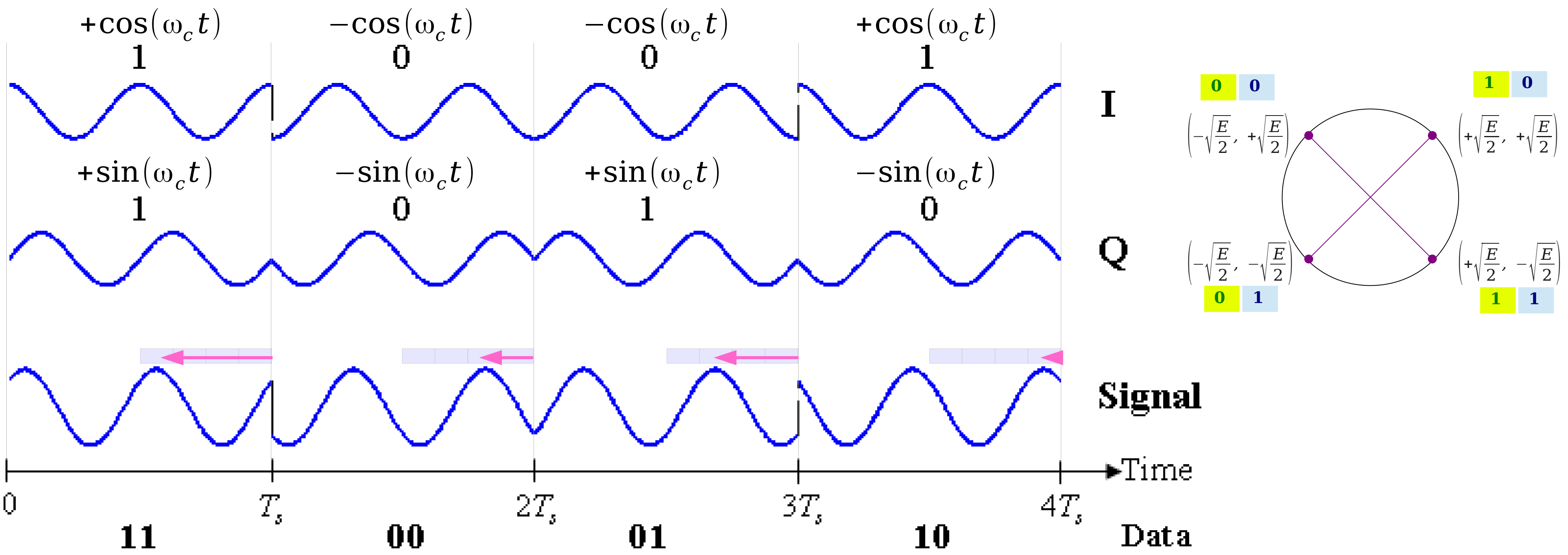
$$s_1(t) = \sqrt{\tfrac{E}{T}}[\cos(\omega_c t)\cdot(-1) - \sin(\omega_c t)\cdot(+1)]$$

$$s_2(t) = \sqrt{\tfrac{E}{T}}[\cos(\omega_c t)\cdot(-1) - \sin(\omega_c t)\cdot(-1)]$$

$$s_3(t) = \sqrt{\tfrac{E}{T}}[\cos(\omega_c t)\cdot(+1) - \sin(\omega_c t)\cdot(-1)]$$

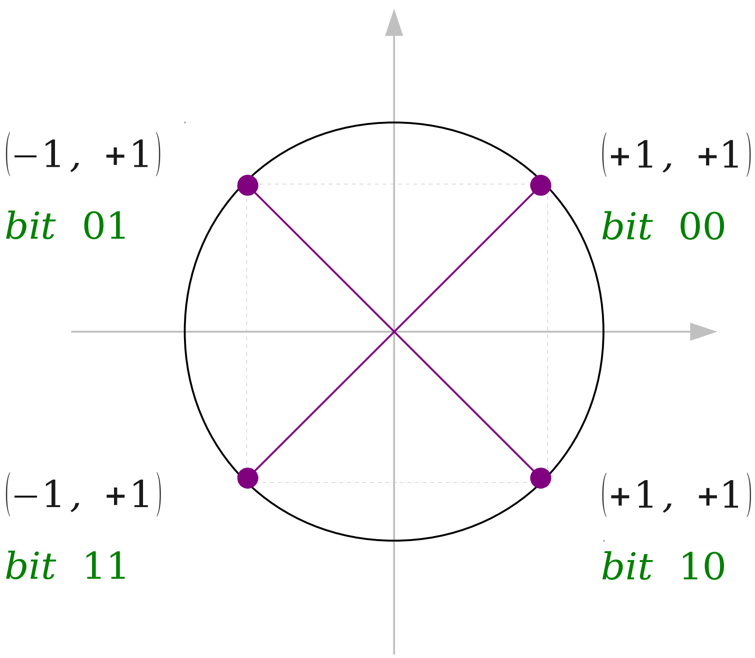


ISI



$\sqrt{2}\cos(\omega_c t + \frac{7\pi}{4})$ $\sqrt{2}\cos(\omega_c t + \frac{3\pi}{4})$ $\sqrt{2}\cos(\omega_c t + \frac{5\pi}{4})$ $\sqrt{2}\cos(\omega_c t + \frac{\pi}{4})$

Signal Space – QPSK (3)

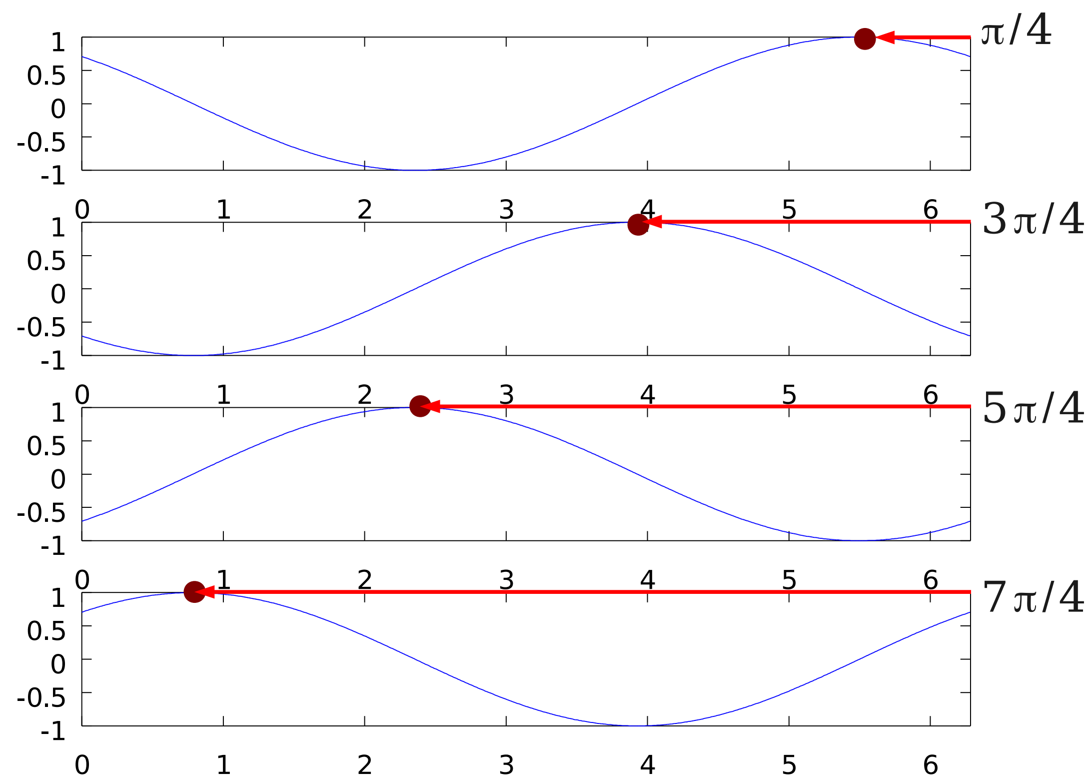


$$s_0(t) = \sqrt{\tfrac{E}{T}}[\cos(\omega_c t)\cdot(+1) - \sin(\omega_c t)\cdot(+1)]$$

$$s_1(t) = \sqrt{\tfrac{E}{T}}[\cos(\omega_c t)\cdot(-1) - \sin(\omega_c t)\cdot(+1)]$$

$$s_2(t) = \sqrt{\tfrac{E}{T}}[\cos(\omega_c t)\cdot(-1) - \sin(\omega_c t)\cdot(-1)]$$

$$s_3(t) = \sqrt{\tfrac{E}{T}}[\cos(\omega_c t)\cdot(+1) - \sin(\omega_c t)\cdot(-1)]$$



Autocorrelation of Random and Power Signals

Time Averaging and Ergodicity

Autocorrelation of Random and Power Signals

Time Averaging and Ergodicity

References

[1] http://en.wikipedia.org/
[2] http://planetmath.org/
[3] B. Sklar, "Digital Communications: Fundamentals and Applications"