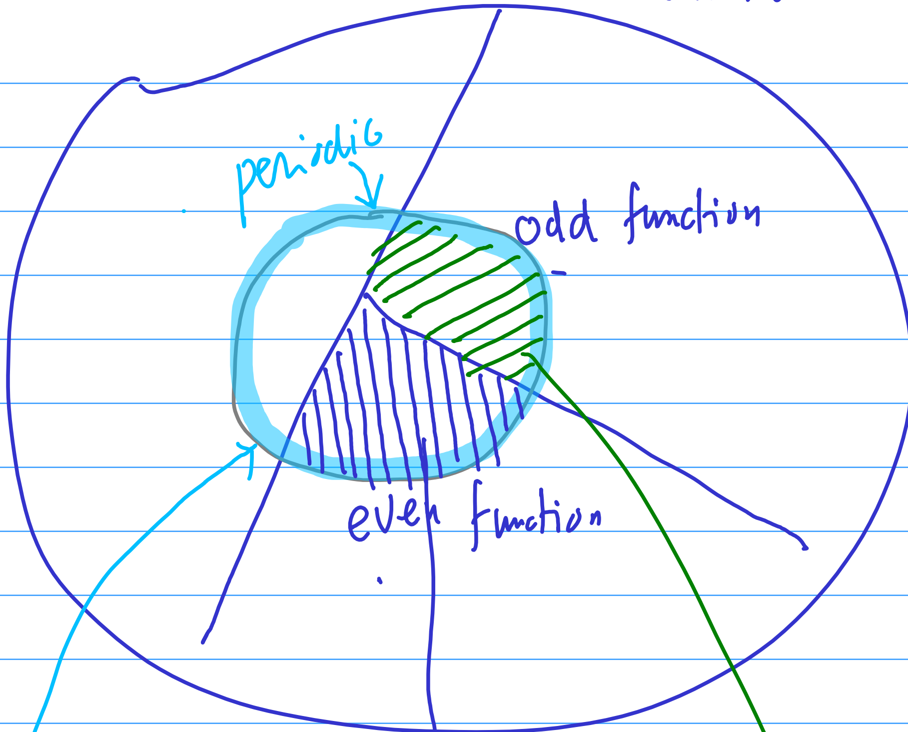
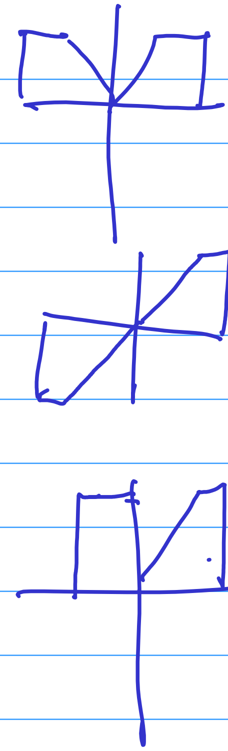
Cos & Sin Series (H.1)

20151224

b

all functions





Fourier series

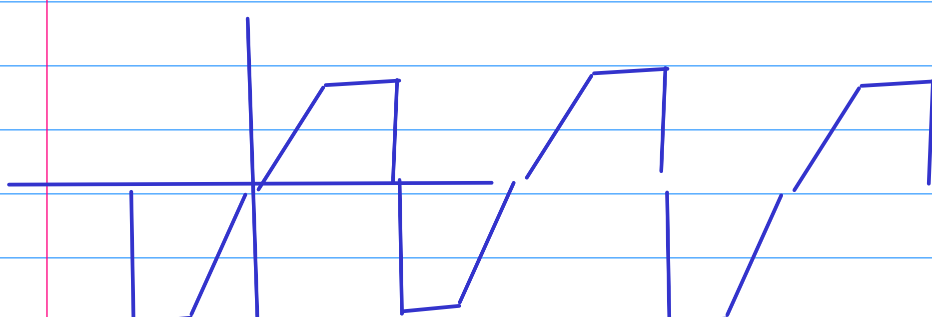
$\left\{ \begin{matrix} \cos mx \\ \sin nx \end{matrix} \right\}$

cosine series

$\{\cos mx\}$

sine series

$\{\sin mx\}$



continuous function

periodic

discrete set of values

a_n

b_n

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos(nx) + b_n \sin(nx)\right)}$$

$\left.\begin{matrix} a_0, & a_1, & a_2, & a_3, & \cdots \\ & b_1, & b_2, & b_3 & \cdots \end{matrix}\right)$ 알 때

$f(x)$ 를 만들수 있다.

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\cos(nx)\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{+\pi} f(x)\sin(nx)\,dx$$

원래 함수 $f(x)$로부터 계수구하기.

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)}$$

$\left.\begin{array}{l} a_0,\ \ a_1,\ a_2,\ a_3,\ \cdots\cdots \\ \quad\ \ \ b_1,\ b_2,\ b_3\ \cdots \end{array}\right)$ 알 때

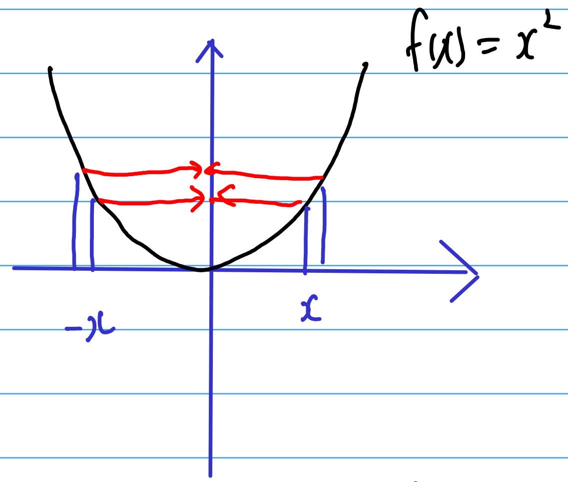
$f(x)$ 를 만들수 있다.

$$\boxed{\begin{aligned} a_0 &= \frac{1}{p}\int_{-p}^{+p} f(x)\,dx \\ a_n &= \frac{1}{p}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx \\ b_n &= \frac{1}{p}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx \end{aligned}}$$

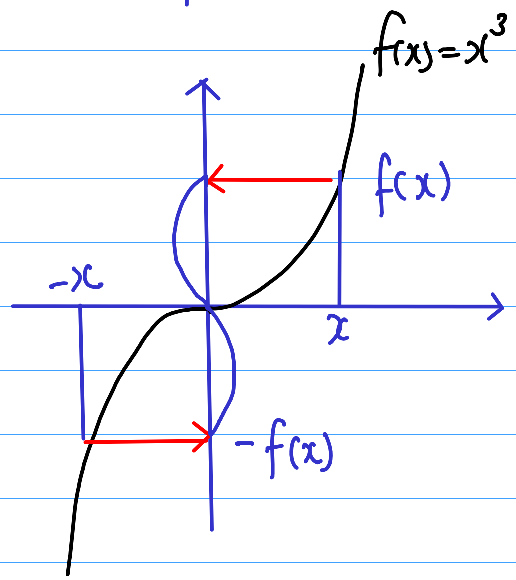
원래 함수 $f(x)$로부터 계수구하기.

even function $f(-x) = f(x)$

odd function $f(-x) = -f(x)$



y축 대칭 $f(-x) = f(x)$



원점 대칭

$x^0, x^2, x^4, x^6 \cdots\cdots$ even

$x^1, x^3, x^5, x^7, \cdots\cdots$ odd

odd odd

even

$x^1 \cdot x^3 = x^4$ $\qquad x^{1+3}$

odd even odd

$x^1 \cdot x^2 = x^3$ $\qquad x^{1+2}$

$x^2 \cdot x^4 = x^6$ $\qquad x^{2+4}$

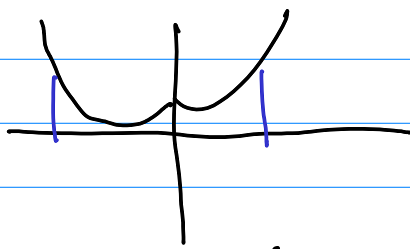
(a) $x^2 \cdot x^4 = x^6$ $\qquad e \times e = e$

(b) $x^1 \cdot x^3 = x^4$ $\qquad o \times o = e$

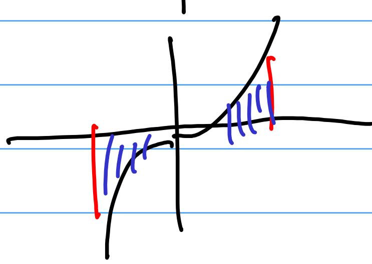
(c) $x^2 \cdot x^1 = x^3$ $\qquad e \times o = o$

(d) $x^2 + x^2 = 2x^2$ $\qquad e + e = e$

(e) $x^1 + x^1 = 2x^1$ $\qquad o + o = o$

(f) 

$$\int_{-a}^{+a} f_e(x)\,dx = 2\int_{0}^{+a} f_e(x)\,dx$$

(g) 

$$\int_{-a}^{+a} f_o(x)\,dx = 0.$$

Cosine series ..... $f(x)$ even인 경우

Sine series ... $f(x)$ odd인 경우

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)}$$

$$a_0 = \frac{1}{p}\int_{-p}^{+p} f(x)\,dx$$

$$a_n = \frac{1}{p}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx$$

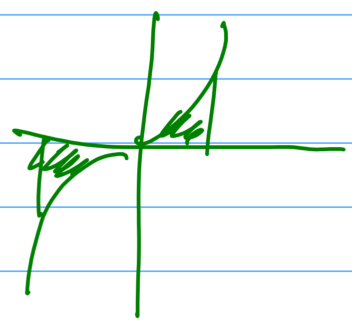
$$b_n = \frac{1}{p}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx$$

＊ f(x) 가 even 이면 Cosine series

$$a_0 = \frac{1}{p}\int_{-p}^{+p} \underbrace{f(x)}_{\text{even}}\,dx = \frac{2}{p}\int_0^{p} f(x)\,dx$$

$$a_n = \frac{1}{p}\int_{-p}^{+p} \underbrace{\underbrace{f(x)}_{\text{even}}\ \underbrace{\cos\left(\frac{n\pi}{p}x\right)}_{\text{even}}}_{\text{even}}\,dx = \frac{2}{p}\int_0^{p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx$$

$$b_n = \frac{1}{p}\int_{-p}^{+p} \underbrace{\underbrace{f(x)}_{\text{even}}\ \underbrace{\sin\left(\frac{n\pi}{p}x\right)}_{\text{odd}}}_{\text{odd}}\,dx = 0$$



$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{p}x\right) \quad \text{Cosine series}}$$

even 함수일 때

Sine Series

* $f(x)$ 가 odd 이면

$$a_0 = \frac{1}{p}\int_{-p}^{+p} \underbrace{f(x)}_{odd}\,dx = 0$$

$$a_n = \frac{1}{p}\int_{-p}^{+p} \underbrace{\underbrace{f(x)}_{odd}\ \underbrace{\cos\left(\frac{n\pi}{p}x\right)}_{even}}_{odd}\,dx = 0$$

$$b_n = \frac{1}{p}\int_{-p}^{+p} \underbrace{\underbrace{f(x)}_{odd}\ \underbrace{\sin\left(\frac{n\pi}{p}x\right)}_{odd}}_{even}\,dx = \frac{2}{p}\int_{0}^{p} f(x)\ \sin\left(\frac{n\pi}{p}x\right)dx$$

$$\boxed{f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{p}x\right) \quad \text{sine series}}$$

→ odd 함수일 때

* $f(x)$: even $[-p, +p]$ Cosine Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{p} x\right)$$

$$a_0 = \frac{2}{p} \int_0^p f(x)\, dx$$

$$a_n = \frac{2}{p} \int_0^p f(x) \cos\left(\frac{n\pi}{p} x\right) dx$$

$f(x)$: odd $[-p, +p]$ Sine Series

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{p} x\right)$$

$$b_n = \frac{2}{p} \int_0^p f(x) \sin\left(\frac{n\pi}{p} x\right) dx$$

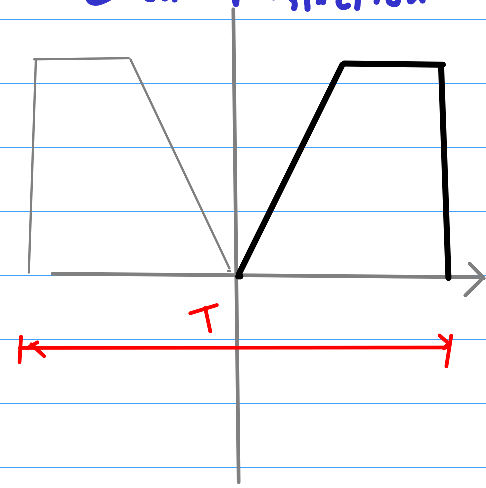
Half - Range Expansion

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{p}x\right)}$$

$$\boxed{a_0 = \frac{2}{p}\int_0^p f(x)\,dx}$$

$$\boxed{a_n = \frac{2}{p}\int_0^p f(x)\cos\left(\frac{n\pi}{p}x\right)dx}$$

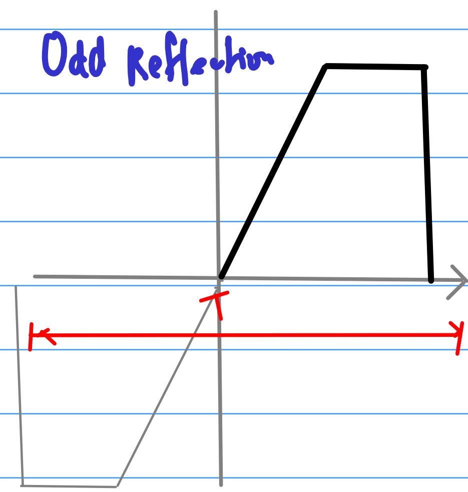
Even Reflection



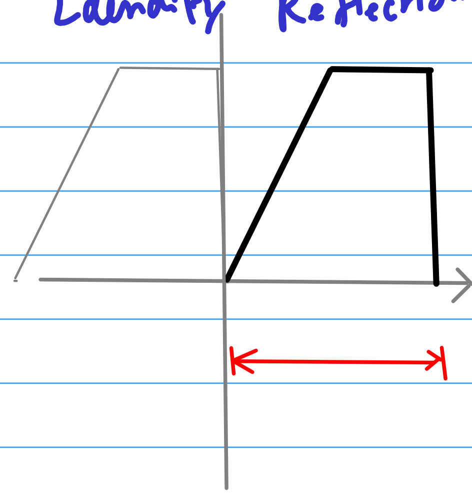
$$\boxed{f(x) = \sum_{n=1}^{\infty}\left(b_n \sin\left(\frac{n\pi}{p}x\right)\right)}$$

$$\boxed{b_n = \frac{2}{p}\int_0^p f(x)\sin\left(\frac{n\pi}{p}x\right)dx}$$

Odd Reflection



Identity Reflection



$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)}$$

$$a_0 = \frac{1}{p}\int_{-p}^{+p} f(x)\,dx$$

$$a_n = \frac{1}{p}\int_{-p}^{+p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx$$

$$b_n = \frac{1}{p}\int_{-p}^{+p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx$$

$\int_{-p}^{+p} \bullet \, dx \Rightarrow f(x)$ is defined on $(-p, +p)$

What if $f(x)$ is defined on $(0, L)$

$\Downarrow$ expansion

$(-p, +p)$

1. even reflection
2. odd reflection
3. identity reflection

$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{p} x\right)}$$

Fourier Cosine Series

$$a_0 = \frac{2}{p} \int_0^p f(x)\, dx$$

$$a_n = \frac{2}{p} \int_0^p f(x) \cos\left(\frac{n\pi}{p} x\right) dx$$

$$\boxed{f(x) = \sum_{n=1}^{\infty} \left(b_n \sin\left(\frac{n\pi}{p} x\right)\right)}$$

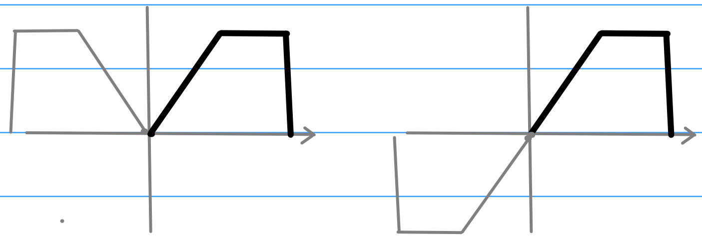
Fourier Sine Series

$$b_n = \frac{2}{p} \int_0^p f(x) \sin\left(\frac{n\pi}{p} x\right) dx$$

Cosine & Sine Series utilize $(0, p)$ portion of a function

$\Rightarrow$ No need to make physical reflection   Just identify $p = L$

Even Reflection   Odd Reflection



$a_0 \quad a_n$ b_n

Half range expansions

$$\left\{ \begin{matrix} 1, & \cos\frac{\pi}{p}x, & \cos\frac{2\pi}{p}x, & \cos\frac{3\pi}{p}x, & \cdots, \\ & \sin\frac{\pi}{p}x, & \sin\frac{2\pi}{p}x, & \sin\frac{3\pi}{p}x, & \cdots \end{matrix} \right\}$$

orthogonal on the interval $[-p, +p]$

orthogonal series expansion

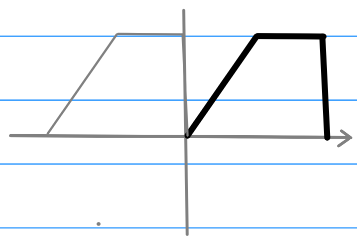
orthogonal on the interval $[a, a+2p]$ $\quad \int_a^{a+2p} \bullet \, dx$

$a=0 \quad [0, 2p] \quad \int_0^{2p} \bullet \, dx$

$a=-p \quad [-p, +p] \quad \int_{-p}^{p} \bullet \, dx$

$[0, 2p] \quad (0, L) \quad 2p = L \quad p = \frac{L}{2}$

Identity Reflection $\Rightarrow$ No need to make physical reflection



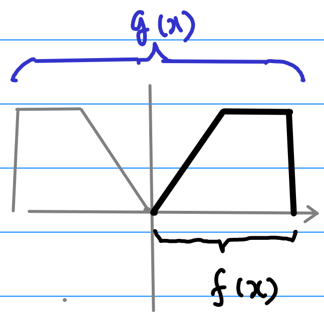
Just identify $p = \frac{L}{2}$

a_0, a_n, b_n

Half range expansions

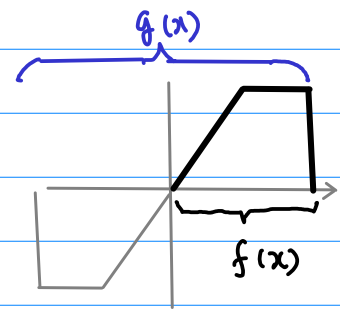
a_0, a_n

$$\boxed{g(x)} = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{p}x\right)$$



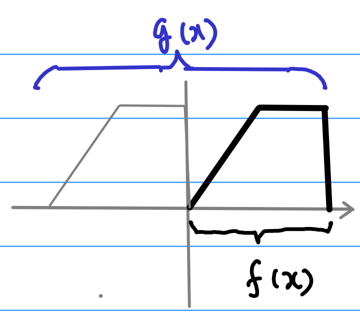
b_n

$$\boxed{g(x)} = \sum_{n=1}^{\infty}\left(b_n \sin\left(\frac{n\pi}{p}x\right)\right)$$



a_0, a_n, b_n

$$\boxed{g(x)} = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi}{p}x\right) + b_n \sin\left(\frac{n\pi}{p}x\right)\right)$$



$(-p, +p)$ $\qquad \int_{-p}^{+p} \cdot \, dx$

$(0, L)$ $\qquad \int_{0}^{L} \quad dx$

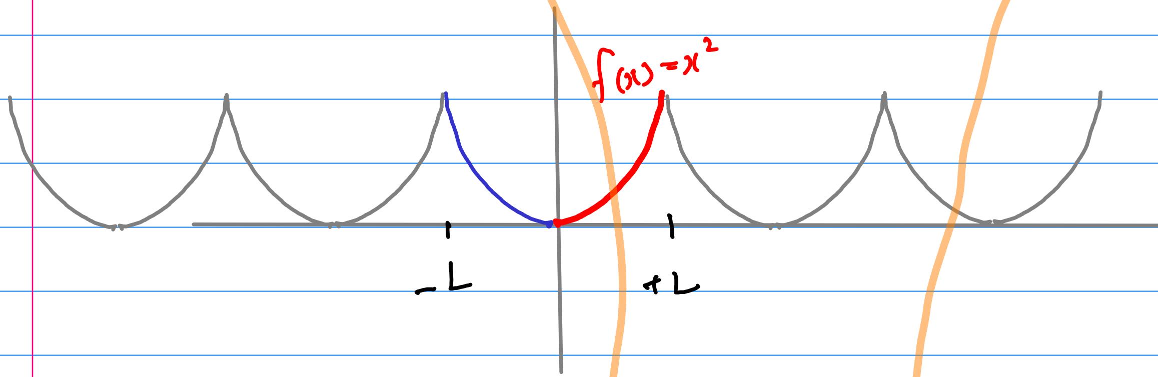
$$\boxed{f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{p}x\right)\right) \quad \text{Cosine series}}$$

even 함수일 때

$$\boxed{f(x) = \sum_{n=1}^{\infty} \left(b_n \sin\left(\frac{n\pi}{p}x\right)\right) \quad \text{sine series}}$$

odd 함수일 때

$a_0 = \frac{2}{L}\int_0^L \cdot \, dx$ $\quad$ $a_n = \frac{2}{L}\int_0^L \cdot \, dx$

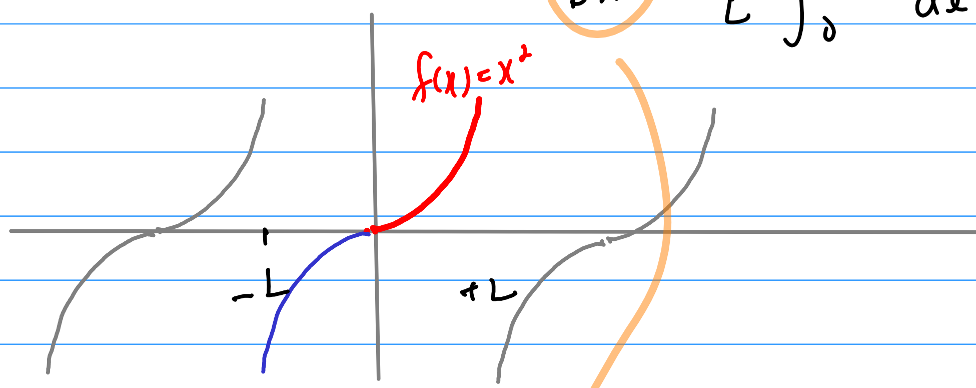


2L

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{p} x\right)$$

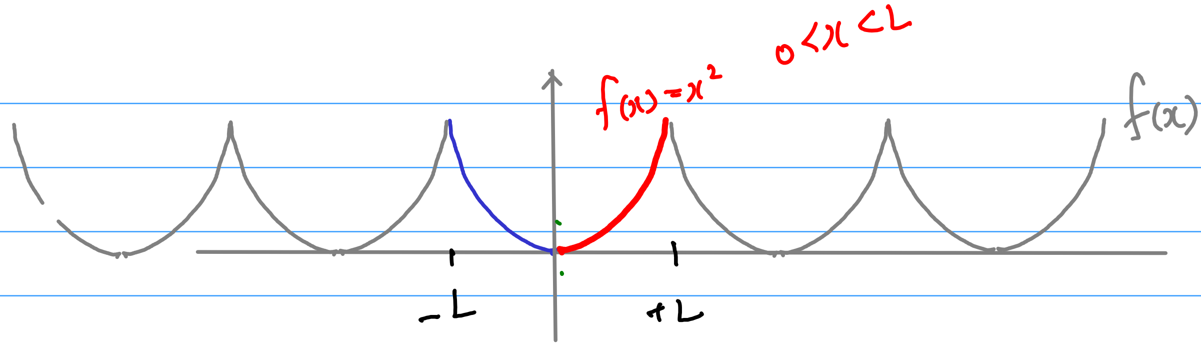
$-L < x < +L$

$b_n = \frac{2}{L}\int_0^L \cdot \, dx$



$-L < x < L$

$$f(x) = \sum_{n=1}^{\infty} \left(b_n \sin\left(\frac{n\pi}{p} x\right) \right.$$

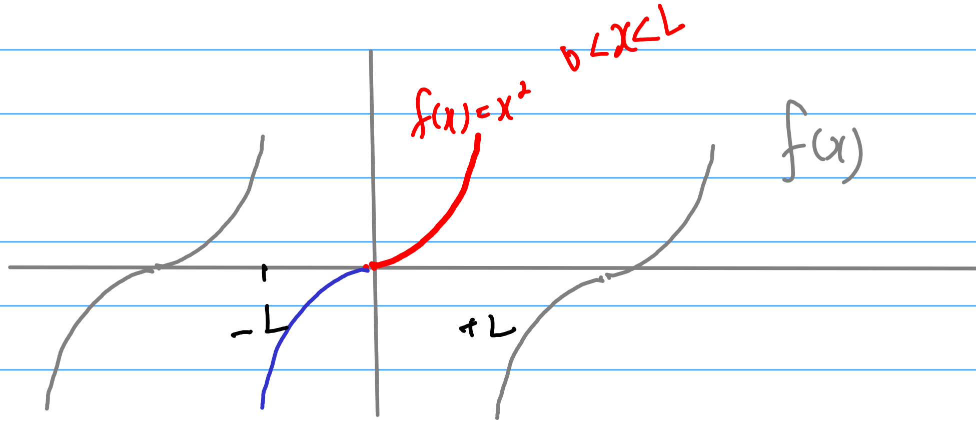


$0 < x < L, \; f(x) = x^2$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{p} x\right)$$

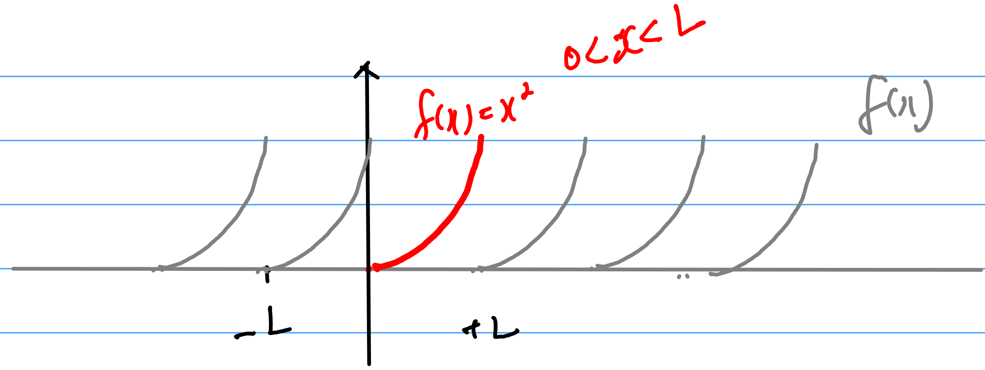
$$a_0 = \frac{2}{3} L^2$$

$$a_n = \frac{4 L^2 (-1)^n}{n^2 \pi^2}$$



$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{p} x\right)$$

$$b_n = \frac{2L^2(-1)^{n+1}}{n\pi} + \frac{4L^2}{n^3\pi^3}\left[(-1)^n - 1\right]$$



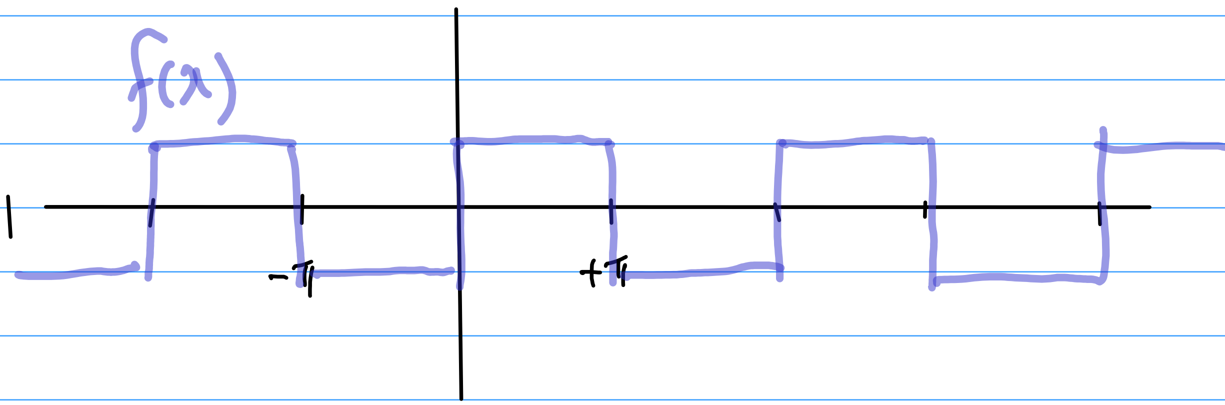
$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{p} x\right) + b_n \sin\left(\frac{n\pi}{p} x\right)\right)$$

$$a_0 = \frac{2}{3} L^2$$

$$a_n = \frac{L^2}{n^3 \pi^3}$$

$$b_n = -\frac{L^2}{n\pi}$$

07/12/11 2) p 18 Zill & Wright

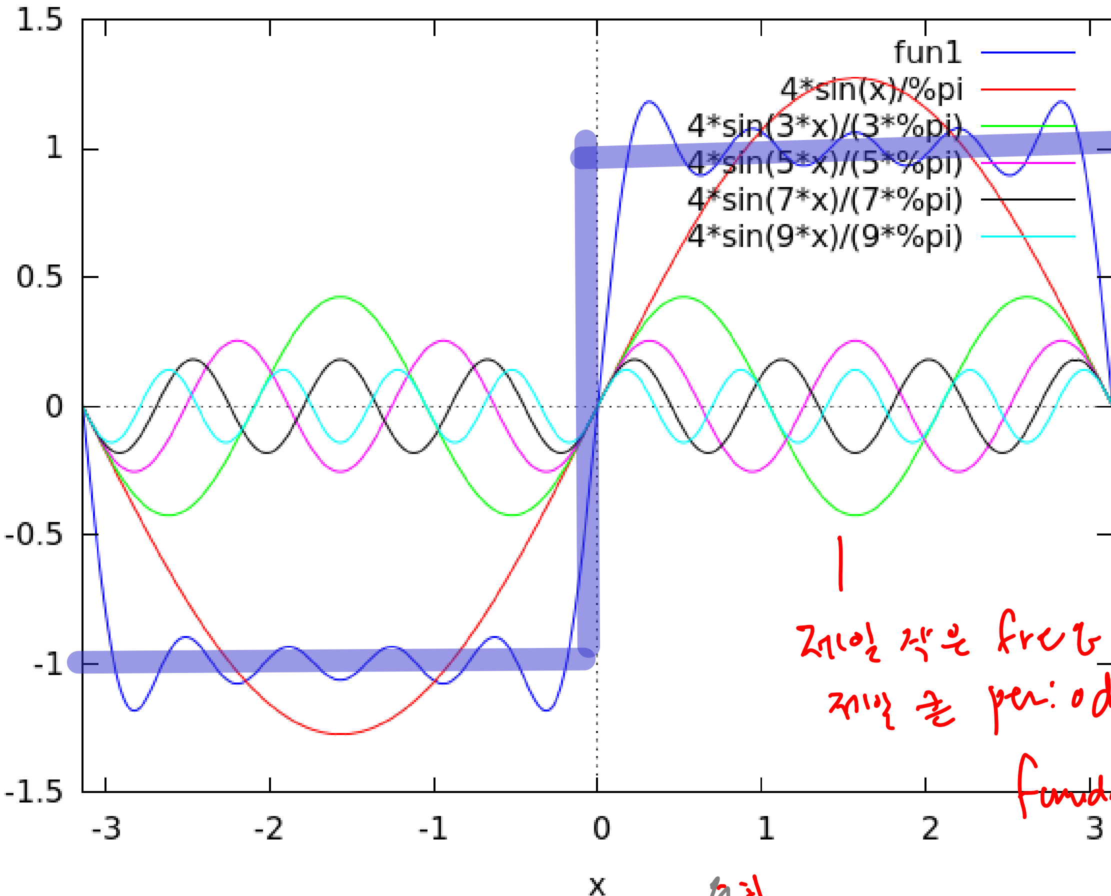


Fourier Sine Series

$$\boxed{b_n} = \frac{2}{\pi}\int_0^{\pi} f(x)\cdot \sin(nx)\, dx$$

$$= \frac{2}{\pi}\,\frac{1-(-1)^n}{n}$$

$$f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty} b_n \sin(nx)$$



제일 작은 freq
제일 큰 period → $n=1$

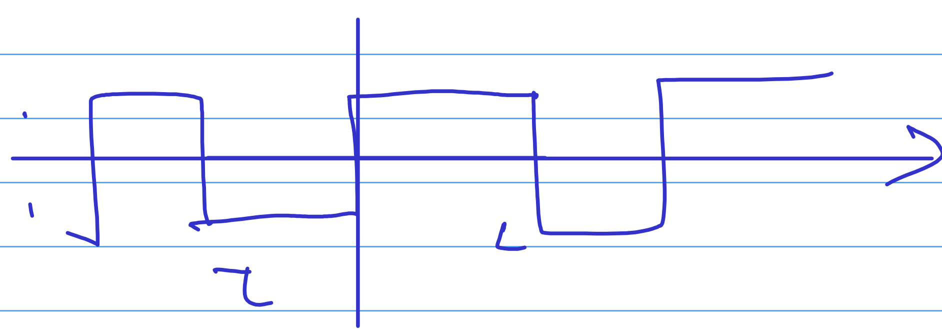
fundamental peri

가중치

Sine freq
1, 2, 3.

Sine Series

$$\boxed{f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{p}x\right) \qquad \boxed{b_n = \frac{2}{p}\int_0^p f(x)\sin\left(\frac{n\pi}{p}x\right)dx}}$$



```
(%i3) f(x, n) := b(n) * sin(n*x);
```
(%o3) $f(x,n):=b(n)\sin(n\,x)$

```
(%i2) b(n) := (2/%pi) * (1 - (-1)^n)/ n;
```
(%o2) $b(n):=\dfrac{\frac{2}{\pi}\left(1-(-1)^{n}\right)}{n}$

```
(%i4) f(x, 1);
```
(%o4) $\dfrac{4\sin(x)}{\pi}$

```
(%i5) f(x, 2);
```
(%o5) 0

```
(%i6) f(x, 3);
```
(%o6) $\dfrac{4\sin(3\,x)}{3\,\pi}$

```
(%i9) plot2d( [f(x,1)+f(x,3)+f(x,5)+f(x,7)+f(x,9), f(x,1),f(x,3),f(x,5),f(x,7),f(x,9)], [x, -%pi, %pi]);
```
(%o9) */home/young/maxout.gnuplot_pipes*

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{p} x\right)$$

$$a_0 = \frac{2}{p}\int_0^p f(x)\,dx$$

$$a_n = \frac{2}{p}\int_0^p f(x)\cos\left(\frac{n\pi}{p} x\right) dx$$

$$f(x) = \sum_{n=0}^{\infty} A_n \cos\left(\frac{n\pi}{p} x\right)$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{p} x\right)$$

$$\int_0^p f(x)\,dx = \int_0^p \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{p} x\right) dx$$

$$= \int_0^p \frac{a_0}{2}\,dx + \sum_{n=1}^{\infty} \int_0^p a_n \cos\left(\frac{n\pi}{p} x\right) dx$$

$$\int_0^p f(x)\,dx = \frac{p}{2} a_0$$

$$\frac{2}{p} \int_0^p f(x)\,dx = a_0$$

$$f(x) \cos\left(\frac{m\pi}{p} x\right) = \left[\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{p} x\right)\right] \cos\left(\frac{m\pi}{p} x\right)$$

$$\int_0^p f(x) \cos\left(\frac{m\pi}{p} x\right) dx = \underbrace{\int_0^p \frac{a_0}{2} \cos\left(\frac{m\pi}{p} x\right) dx}_{0}$$

$$+ \sum_{n=1}^{\infty} a_n \int_0^p \cos\left(\frac{n\pi}{p} x\right) \cos\left(\frac{m\pi}{p} x\right) dx$$

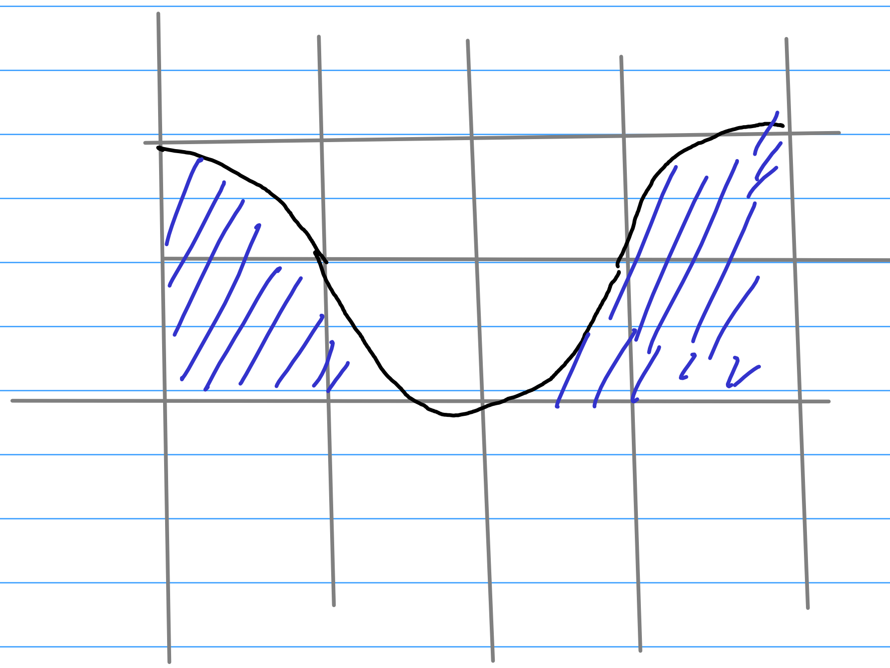
$$= a_m \frac{p}{2}$$

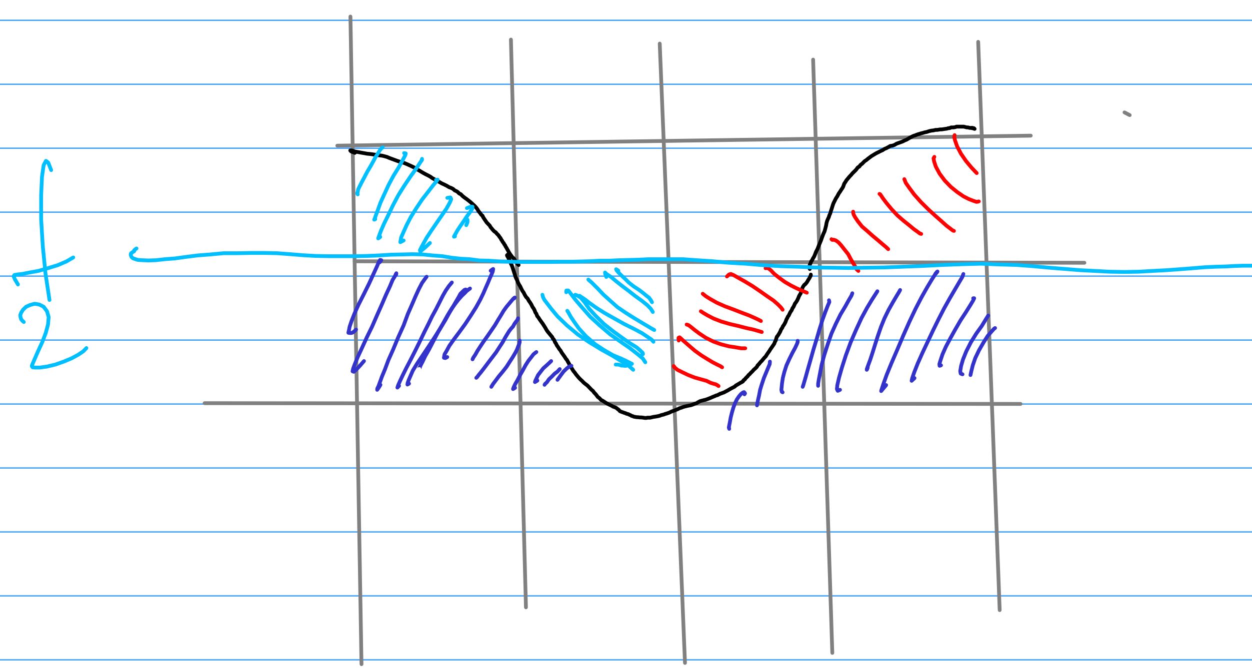
$$\frac{2}{p} \int_0^p f(x) \cos\left(\frac{m\pi}{p} x\right) dx = a_m$$

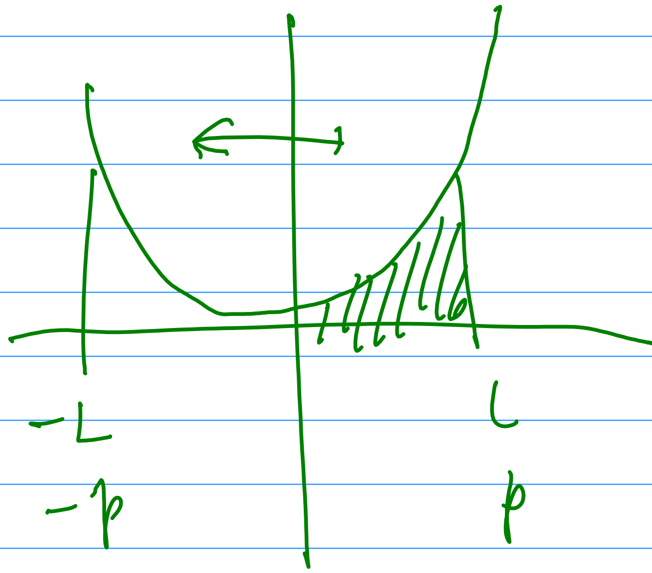
$$\cos\left(\frac{n\pi}{p}x\right) \qquad \underline{0 \le x \le p}$$

$$\cos(\theta) \qquad -\pi < \theta < \pi$$

$$\underline{0 < \theta < \pi}$$







$$\int_{-p}^{p} f(x)\,dx$$

$$= \int_{0}^{p} dx$$

$$\int_{-p}^{p} dx$$

책 예제 ① ㅏ Fourier series

$$A_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx = \frac{1}{2L}\int_{-L}^{L}(L-x)\,dx = \frac{1}{2L}\left[Lx - \frac{1}{2}x^2\right]_{-L}^{L}$$

$$= \frac{1}{2L}(2L^2) = L$$

$$A_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L}(L-x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L} \underline{L\cos\left(\frac{n\pi x}{L}\right)} - x\cos\left(\frac{n\pi x}{L}\right)dx$$

odd x even = odd

$$= \frac{1}{L}\int_{-L}^{L} -x\cos\left(\frac{n\pi x}{L}\right)dx = 0$$

$$B_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L}(L-x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$= \frac{1}{L}\int_{-L}^{L} \underline{L\sin\left(\frac{n\pi x}{L}\right)} - x\sin\left(\frac{n\pi x}{L}\right)dx$$

odd x odd = even

$$= \frac{1}{L}\int_{-L}^{L} -x\sin\left(\frac{n\pi x}{L}\right)dx = 0$$

$$\int x\sin(kx)\,dx = -\frac{1}{k}x\cos(kx) + \frac{1}{k^2}\sin(kx)$$

$$= \frac{1}{L}\int_{-L}^{L} -x\sin\left(\frac{n\pi x}{L}\right)dx = 0$$

$$= -\frac{1}{L}\int_{-L}^{L} x\sin\left(\frac{n\pi}{L}x\right)dx$$

$$= \frac{1}{L}\left[-\frac{L}{n\pi}x\cos\left(\frac{n\pi}{L}x\right) + \underbrace{\left(\frac{L}{n\pi}\right)^2\sin\left(\frac{n\pi}{L}x\right)}_{\to 0}\right]_{-L}^{L}$$

$$= -\frac{1}{L}\left[-\frac{L}{n\pi}L\cos\left(\frac{n\pi}{L}L\right) + \frac{L}{n\pi}(-L)\cos\left(\frac{n\pi}{L}(-L)\right)\right.$$

$$\left.\underbrace{\left(\frac{L}{n\pi}\right)^2\sin\left(\frac{n\pi}{L}L\right)}_{\to 0} - \underbrace{\left(\frac{L}{n\pi}\right)^2\sin\left(\frac{n\pi}{L}(-L)\right)}_{\to 0}\right]$$

$$= -\frac{1}{L}\left[-\frac{L}{n\pi}L\cos\left(\frac{n\pi}{L}L\right) + \frac{L}{n\pi}(-L)\cos\left(\frac{n\pi}{L}(-L)\right)\right]$$

$$= -\frac{1}{L}\left[-\frac{2L^2}{n\pi}(-1)^n\right] = \frac{2L}{n\pi}(-1)^n$$

$$\int f\, g'\, dx = f\, g - \int f'\, g\, dx$$

$$\int x \sin(kx)\, dx = x \left[\frac{-1}{k} \cos(kx) \right] - \int \left[\frac{-1}{k} \cos(kx) \right] dx$$

$$= -\frac{1}{k} x \cos(kx) + \frac{1}{k} \int \cos(kx)\, dx$$

$$= -\frac{1}{k} x \cos(kx) + \frac{1}{k} \left[\frac{1}{k} \sin(kx) \right]$$

$$= -\frac{1}{k} x \cos(kx) + \frac{1}{k^2} \sin(kx)$$

$$\int x \sin(kx)\, dx = -\frac{1}{k} x \cos(kx) + \frac{1}{k^2} \sin(kx)$$

$$\Big[F(x) \Big]_{-a}^{+a} = F(a) - F(-a$$

$$\int f g' \, dx = f g - \int f' g \, dx$$

$$\int x \cos(kx)\, dx = x \left[\frac{1}{k} \sin(kx) \right] - \int \left[\frac{1}{k} \sin(kx) \right] dx$$

$$= \frac{1}{k} x \sin(kx) - \frac{1}{k} \int \sin(kx)\, dx$$

$$= \frac{1}{k} x \sin(kx) + \frac{1}{k} \left[\frac{1}{k} \cos(kx) \right]$$

$$= \frac{1}{k} x \sin(kx) + \frac{1}{k^2} \cos(kx)$$

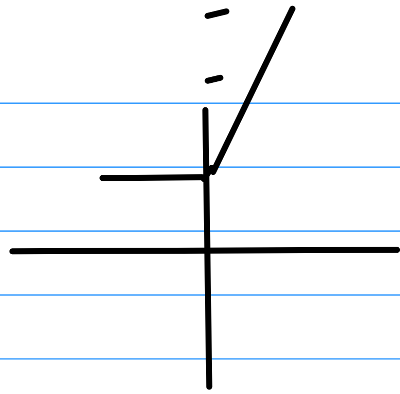
$$\int x \cos(kx)\, dx = \frac{1}{k} x \sin(kx) + \frac{1}{k^2} \cos(kx)$$

$$\int x \cos(kx)\, dx = \frac{1}{k} x \sin(kx) + \frac{1}{k^2} \cos(kx)$$

$$\int x \sin(kx)\, dx = -\frac{1}{k} x \cos(kx) + \frac{1}{k^2} \sin(kx)$$

$$\int x^2 \cos(kx)\, dx = ?$$

$$\int x^2 \sin(kx)\, dx = ?$$



$$A_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx = \frac{1}{2L}\left[\int_{-L}^{0} L\,dx + \int_0^L 2x\,dx\right]$$

$$= \frac{1}{2L}\left[L^2 + L^2\right] = \frac{2L^2}{2L} = L$$

$$A_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos(kx)\,dx = \frac{1}{L}\left[\int_{-L}^{0} L\cos(kx)\,dx + \int_0^L 2x\cos(kx)\,dx\right]$$

$$= \frac{1}{L}\left[\left[\frac{L}{k}\sin(kx)\right]_{-L}^{0} + 2\left[\frac{1}{k}x\sin(kx) + \frac{1}{k^2}\cos(kx)\right]_0^L\right]$$

$$= \frac{1}{L}\left[\underbrace{\left[\frac{L}{k}\sin\left(\frac{n\pi}{L}x\right)\right]_{-L}^{0}}_{0} + 2\left[\underbrace{\frac{L}{n\pi}\sin\left(\frac{n\pi}{L}x\right)}_{0} + \left(\frac{L}{n\pi}\right)^2\cos\left(\frac{n\pi}{L}x\right)\right]_0^L\right]$$

$$= \frac{1}{L}\left[2\left(\frac{L}{n\pi}\right)^2\left((-1)^n - 1\right)\right]$$

$\cos\left(\frac{n\pi}{L}x\right)$

$- \cos(0)$

$$= \frac{2L}{n^2\pi^2}\left((-1)^n - 1\right)$$

$$B_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin(kx)\,dx = \frac{1}{L}\left[\int_{-L}^{0} L\sin(kx)\,dx + \int_{0}^{L} 2x\sin(kx)\,dx\right]$$

$$= \frac{1}{L}\left[\left[-\frac{L}{k}\cos(kx)\right]_{-L}^{0} + 2\left[\frac{-1}{k}x\cos(kx) + \frac{1}{k^2}\sin(kx)\right]_{0}^{L}\right]$$

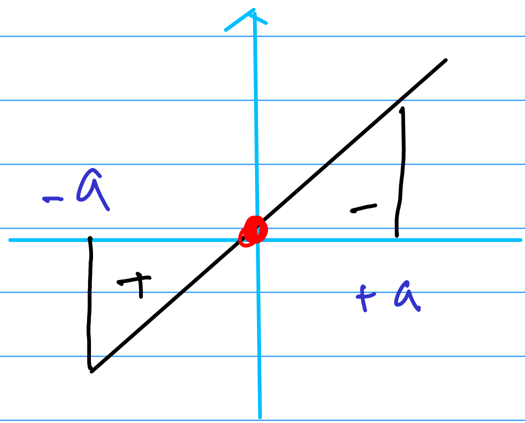
$$= \frac{1}{L}\left[\left[-\frac{L^2}{n\pi}\cos\left(\frac{n\pi}{L}x\right)\right]_{-L}^{0} + 2\left[-\frac{L}{n\pi}x\cos\left(\frac{n\pi}{L}x\right) + \underbrace{\left(\frac{L}{n\pi}\right)^2\sin\left(\frac{n\pi}{L}x\right)}_{\to 0}\right]_{0}^{L}\right]$$

$$= \frac{1}{L}\left[\left[-\frac{L^2}{n\pi}\left(1-(-1)^n\right) + 2\left[-\frac{L^2}{n\pi}(-1)^n\right]\right]\right]$$

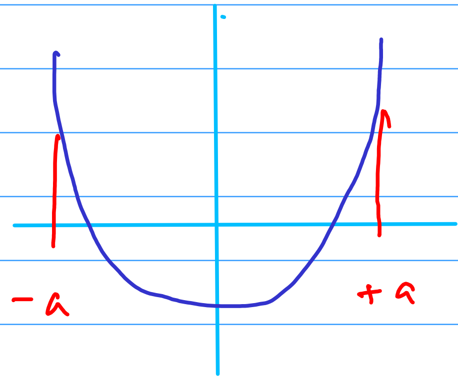
$$= \frac{1}{L}\left[\frac{L^2}{n\pi}\left(-1+(-1)^n\right) - \frac{2L^2}{n\pi}(-1)^n\right]$$

$$= \frac{1}{L}\left[\frac{L^2}{n\pi}\left(-1+(-1)^n - 2(-1)^n\right)\right]$$

$$= -\frac{L}{n\pi}\left(1+(-1)^n\right)$$



$$\int_{-a}^{+a} f_{odd}(x)\, dx = 0$$



$$\int_{-a}^{a} f_{even}(x)\, dx$$

$$= 2\int_{0}^{a} f_{even}(x)\, dx$$

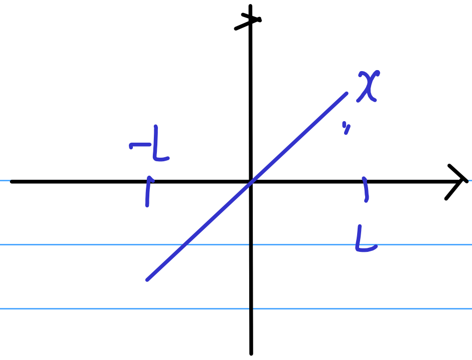
$$\cos(\pi) = -1$$

$$\cos(2\pi) = +1$$

$$\cos(3\pi) = -1$$

$$\cos(4\pi) = +1$$

$$\underline{\cos(n\pi)} = \underline{(-1)^n}$$



$$A_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx = \frac{1}{2L}\int_{-L}^{L} x\,dx = 0$$

$$A_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos(kx)\,dx = \frac{1}{L}\int_{-L}^{L} x\cdot\cos(kx)\,dx = 0$$

$$B_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin(kx)\,dx = \frac{1}{L}\int_{-L}^{L} x\sin(kx)\,dx$$

$$= \frac{1}{L}\left[-\frac{1}{k}x\cos(kx) + \frac{1}{k^2}\sin(kx)\right]_{-L}^{L}$$

$$= \frac{1}{L}\left[-\frac{L}{n\pi}x\cos\left(\frac{n\pi}{L}x\right) + \left(\frac{L}{n\pi}\right)^2\sin\left(\frac{n\pi}{L}x\right)\right]_{-L}^{L}$$

$$= \frac{1}{L}\left[-\frac{L}{n\pi}L\cos(n\pi) + \underbrace{\left(\frac{L}{n\pi}\right)^2\sin(n\pi)}_{\to 0} - \left(-\frac{L}{n\pi}(-L)\cos(-n\pi) + \underbrace{\left(\frac{L}{n\pi}\right)^2\sin(-n\pi)}_{\to 0}\right)\right]$$

$$= \frac{1}{L}\left(\frac{L^2}{n\pi}\right)\left(-(-1)^n - (-1)^n\right)$$

$$= -\frac{2L}{n\pi}(-1)^n$$