

# Applicative Methods (3B)

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# The definition of Applicative

```
class (Functor f) => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

$f (a \rightarrow b) ::$  a **function** wrapped in f

$f a ::$  a **value** wrapped in f

The class has a two methods :

**pure** brings arbitrary values into the functor

**(<\*>)** takes a **function** wrapped in a functor f  
and a **value** wrapped in a functor f  
and returns the result of the application  
which is also wrapped in a functor f

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

# The Maybe instance of Applicative

```
instance Applicative Maybe where
  pure          = Just
  (Just f) <*> (Just x) = Just (f x)
  _          <*> _      = Nothing
```

`pure` wraps the value with `Just`;

`<*>` applies

the function wrapped in `Just`

to the value wrapped in `Just` if both exist,

and results in `Nothing` otherwise.

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

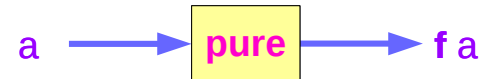
# An Instance of the Applicative Typeclass

```
class (Functor f) => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

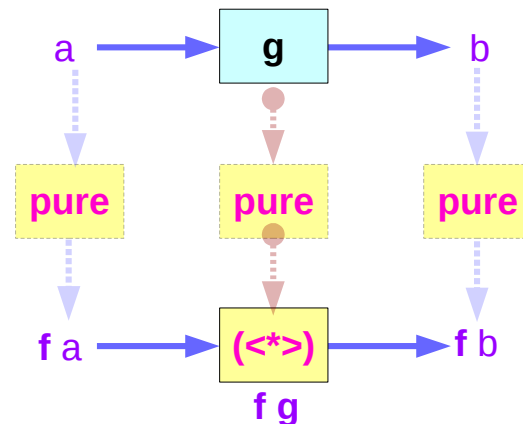
**f** : Functor, Applicative

```
instance Applicative Maybe where
  pure = Just
  Nothing <*> _ = Nothing
  (Just f) <*> something = fmap f something
```

**f** : function in a context



(Functor f) => Applicative f



(Functor f) => Applicative f

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

# Left associative `<*>`, `fmap`, and `<$>`

```
pure g <*> x <*> y <*> z
```

```
g :: f (a -> b -> c -> d)
x :: f a
y :: f b
z :: f c
```

```
fmap g x <*> y <*> z
```

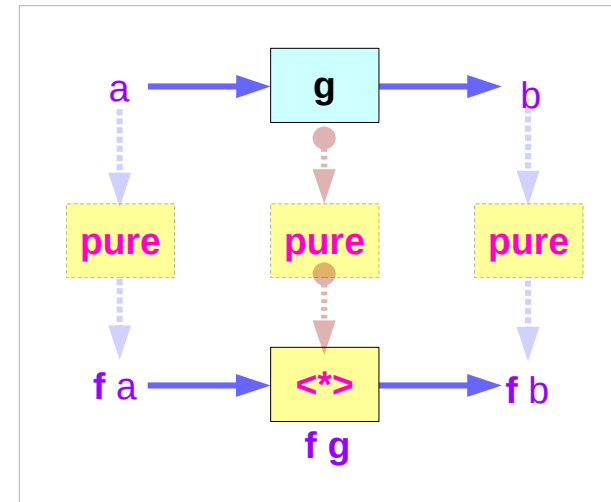
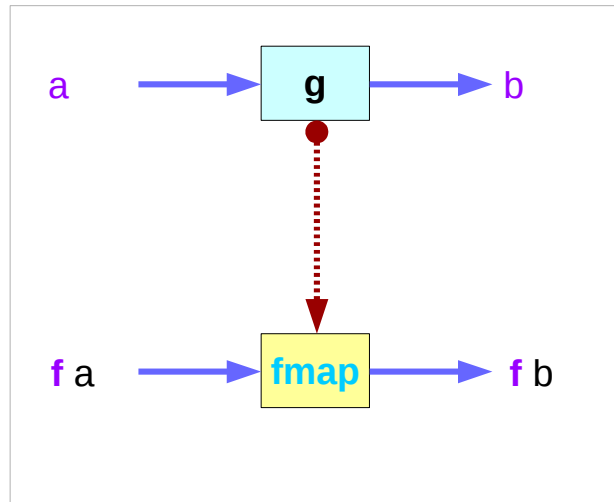


```
g <$> x <*> y <*> z
```

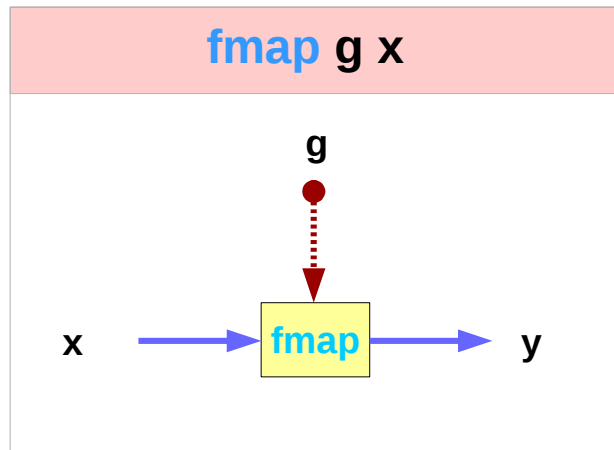
infix operator `<$>`

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

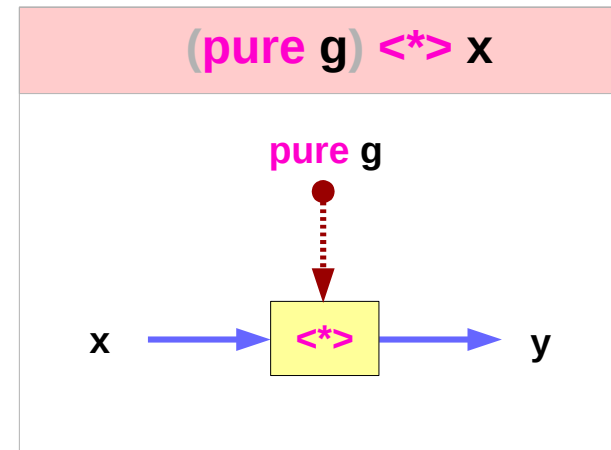
$$\text{fmap } g \ x = (\text{pure } g) \langle * \rangle x$$



pure = f

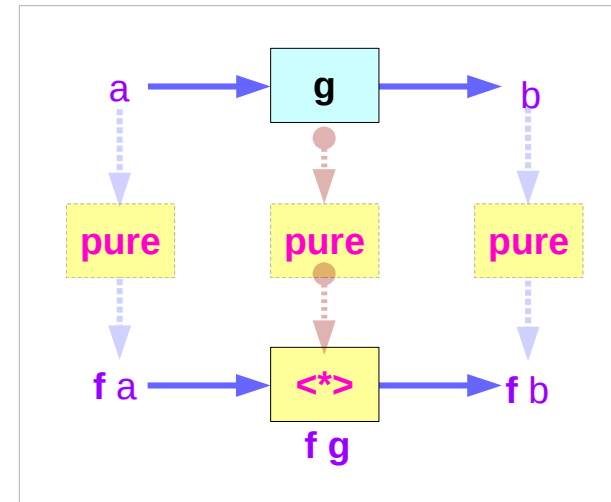
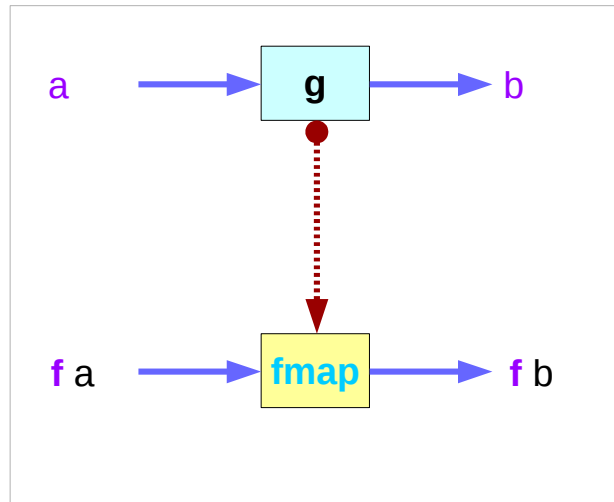


$x :: f \ a$   
 $y :: f \ b$

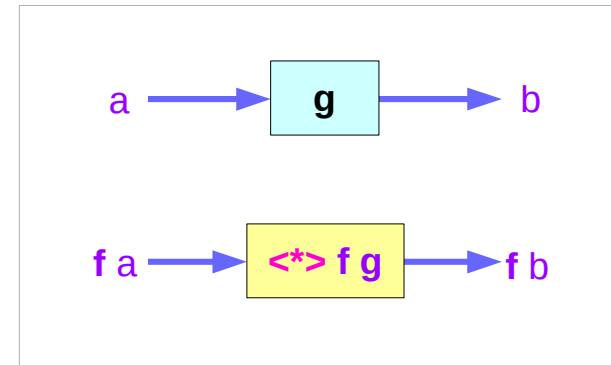


<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

# $f a \langle * \rangle f g$



$pure = f$



<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>



# Left associative <\*> examples

```
ghci> pure (+) <*> Just 3 <*> Just 5  
Just 8
```

```
pure (+) <*> Just 3 <*> Just 5
```

```
Just (+3) <*> Just 5
```

```
Just 8
```

```
ghci> pure (+) <*> Just 3 <*> Nothing  
Nothing
```

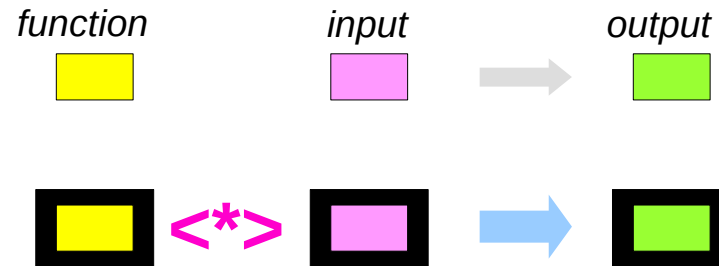
```
ghci> pure (+) <*> Nothing <*> Just 5  
Nothing
```

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

# Infix Operators $\langle * \rangle$ vs $\langle \$ \rangle$ - overview

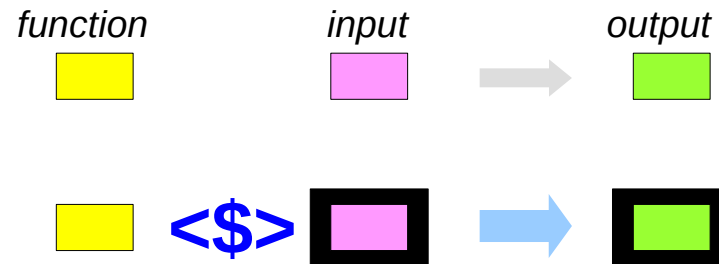
**h**  $\langle * \rangle$  **x**  $\langle * \rangle$  **y**

$h :: f (a \rightarrow b \rightarrow c)$   
 $x :: f a$   
 $y :: f b$



**g**  $\langle \$ \rangle$  **x**  $\langle * \rangle$  **y**

$g :: (a \rightarrow b \rightarrow c)$   
 $x :: f a$   
 $y :: f b$



<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

# Infix Operators $\langle * \rangle$ vs $\langle \$ \rangle$ - a type view

**h**  $\langle * \rangle$  **x**  $\langle * \rangle$  **y**

$h :: f (a \rightarrow b \rightarrow c)$   
 $x :: f a$   
 $y :: f b$

$h :: f (a \rightarrow b \rightarrow c)$

$x :: f a$

$h \langle * \rangle x :: f (b \rightarrow c)$

$h :: f (a \rightarrow b \rightarrow c)$

$x :: f a$

$h \langle * \rangle x :: f (b \rightarrow c)$

$y :: f b$

$h \langle * \rangle x \langle * \rangle y :: f c$

**g**  $\langle \$ \rangle$  **x**  $\langle * \rangle$  **y**

$g :: (a \rightarrow b \rightarrow c)$   
 $x :: f a$   
 $y :: f b$

$g :: (a \rightarrow b \rightarrow c)$

$x :: f a$

$g \langle \$ \rangle x :: f (b \rightarrow c)$

$g :: (a \rightarrow b \rightarrow c)$

$x :: f a$

$g \langle \$ \rangle x :: f (b \rightarrow c)$

$y :: f b$

$g \langle \$ \rangle x \langle * \rangle y :: f c$

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

# Infix Operators $\langle * \rangle$ vs $\langle \$ \rangle$ - a curried function view

$h \langle * \rangle x \langle * \rangle y$

$h :: f (a \rightarrow b \rightarrow c)$

$x :: f a$

$y :: f b$

$g \langle \$ \rangle x \langle * \rangle y$

$g :: (a \rightarrow b \rightarrow c)$

$x :: f a$

$y :: f b$

$h \langle * \rangle x$

$h :: f (a \rightarrow b \rightarrow c)$

$x :: f a \rightarrow \langle * \rangle \rightarrow h \langle * \rangle x$

$g \langle \$ \rangle x$

$g :: (a \rightarrow b \rightarrow c)$

$x :: f a \rightarrow \langle \$ \rangle \rightarrow g \langle \$ \rangle x$

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

# Infix Operators $\langle * \rangle$ vs $\langle \$ \rangle$ examples

**h**  $\langle * \rangle$  **x**  $\langle * \rangle$  **y**

Just (+)  $\langle * \rangle$  Just 3  $\langle * \rangle$  Just 2  
Just (+3)  $\langle * \rangle$  Just 2  
Just 5

**g**  $\langle \$ \rangle$  **x**  $\langle * \rangle$  **y**

(+)  $\langle \$ \rangle$  Just 3  $\langle * \rangle$  Just 2  
Just (+3)  $\langle * \rangle$  Just 2  
Just 5

**h**  $\langle * \rangle$  **x**

Just (+)

Just 3  $\rightarrow$   $\langle * \rangle$   $\rightarrow$  Just (3+)

**g**  $\langle \$ \rangle$  **x**

(+)

Just 3  $\rightarrow$   $\langle \$ \rangle$   $\rightarrow$  Just (3+)

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

# the minimal complete definition

```
class (Functor f) => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

*the minimal complete definition*

```
(<$>) :: (Functor f) => (a -> b) -> f a -> f b
g <$> x = fmap g x
```

*Not in the minimal complete definition*

$g :: a \rightarrow b, \quad x :: f a$

```
instance Applicative Maybe where
  pure = Just
  Nothing <*> _ = Nothing
  (Just g) <*> something = fmap g something
```

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

# The Applicative Typeclass

**Applicative** is a superclass of **Monad**.

every **Monad** is also a **Functor** and an **Applicative**

**fmap**, **pure**, **(<\*>)** can all be used with **monads**.

a **Monad** instance

requires **Functor** and **Applicative** instances.

defines the types and roles of **return** and **(>>)**

**fmap** : defined in **Functors**

**pure**, **(<\*>)** : defined in **Applicatives**

**return**, **(>>)** : defined in **Monads**

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

# (<\$>) VS (\$)

(<\$>) infix operator

(<\$>) :: (Functor f) => (a -> b) -> f a -> f b

g <\$> x = fmap g x

The \$ operator is for avoiding parentheses

putStrLn (show (1 + 1))

putStrLn \$ show (1 + 1)

putStrLn \$ show \$ 1 + 1           – right associative

(\$) calls the function which is its left-hand argument of \$

on the value which is its right-hand argument of \$



# The Applicative Laws

The identity law:  $\text{pure id} \langle * \rangle v = v$        $\text{id} :: a \rightarrow a$        $v :: f\ a$

Homomorphism:  $\text{pure g} \langle * \rangle \text{pure x} = \text{pure (g x)}$        $g :: a \rightarrow b$        $x :: a$

Interchange:  $u \langle * \rangle \text{pure y} = \text{pure (\$ y)} \langle * \rangle u$        $u :: f\ (a \rightarrow b)$        $y :: a$

Composition:  $u \langle * \rangle (v \langle * \rangle w) = \text{pure (.)} \langle * \rangle u \langle * \rangle v \langle * \rangle w$        $w :: f\ a$        $v :: f\ (a \rightarrow b)$        $u :: f\ (b \rightarrow c)$

Left associative       $u \langle * \rangle v \langle * \rangle w = (u \langle * \rangle v) \langle * \rangle w$

$u :: f\ (c \rightarrow b \rightarrow a)$   
 $v :: f\ c$   
 $u \langle * \rangle v :: f\ (b \rightarrow a)$   
 $w :: f\ b$   
 $u \langle * \rangle v \langle * \rangle w = f\ a$

# The Identity Law

The identity law

**pure id** <\*> v = v

id :: a -> a

v :: f a

**pure** to inject values into the functor  
in a *default, featureless* way,  
so that the result is as close as possible  
to the plain value.

applying the **pure id** morphism does nothing,  
exactly like with the plain **id** function.

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

# The Homomorphism Law

The homomorphism law

`pure g <*> pure x = pure (g x)`

`g :: a -> b`

`x :: a`

applying a "**pure**" function to a "**pure**" value is the same as applying the function to the value in the *ordinary way* and then using **pure** on the result.  
means **pure** preserves function application.

**applying** a non-effectful function **g** to a non-effectful argument **x** in an effectful context **pure** is the same as just **applying** the function **g** to the argument **x** and then injecting the result (**f x**) into the effectual context with **pure**.

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

# The Interchange Law

The interchange law

$$u \langle * \rangle \text{pure } y = \text{pure } (\$ y) \langle * \rangle u$$
$$u :: f (a \rightarrow b)$$
$$y :: a$$

$(\$ y)$  is the *function* that supplies  $y$   
as argument to another function  
– a higher order function

Function  $\$$  Argument

$$\$ y$$

$(y)$  as a single argument

applying a morphism  $u$  to a "pure" value  $\text{pure } y$   
is the same as applying  $\text{pure } (\$ y)$  to the morphism  $u$

$$\text{Just } (+3) \langle * \rangle \text{Just } 2$$
$$\text{Just } (\$ 2) \langle * \rangle \text{Just } (+3)$$

when evaluating the application of  
an effectful function ( $u$ ) to a pure argument ( $\text{pure } y$ ),  
the order doesn't matter – commutative.

$$u :: f (a \rightarrow b)$$
$$y :: a$$
$$u \langle * \rangle \text{pure } y :: f b$$
$$\text{pure } y :: f a$$
$$\text{pure } (\$ y) \langle * \rangle u :: f b$$
$$\text{pure } (\$ y) :: f (a)$$

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

# The Composition Law

The composition law  $\text{pure } (.) \langle * \rangle u \langle * \rangle v \langle * \rangle w = u \langle * \rangle (v \langle * \rangle w)$   $w :: f a$   $v :: f (a \rightarrow b)$   $u :: f (b \rightarrow c)$

$\text{pure } (.)$  composes morphisms similarly

to how  $(.)$  composes functions:

applying the composed morphism

$\text{pure } (.) \langle * \rangle u \langle * \rangle v$  to  $w$

gives the same result  $(u \langle * \rangle (v \langle * \rangle w))$

as applying  $u$  to the result  $(v \langle * \rangle w)$

of applying  $v$  to  $w$

it is expressing a sort of associativity property of  $\langle * \rangle$ .

```
w :: f a           -- value
v :: f (a -> b)   -- func1
u :: f (b -> c)   -- func2
```

```
v <*> w :: f b
u <*> (v <*> w) :: f c
```

```
pure (.) <*> u <*> v :: f (a -> c)
pure (.) <*> u <*> v <*> w :: f c
```

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

# The Composition Law and Left Associativity

The composition law

$$\text{pure } (.) \langle * \rangle u \langle * \rangle v \langle * \rangle w = \text{pure } (.) \langle * \rangle (v \langle * \rangle w)$$

$$w :: f \ a \quad v :: f \ (a \rightarrow b) \quad u :: f \ (b \rightarrow c)$$

$$\text{pure } (.) \langle * \rangle \text{pure } g \langle * \rangle \text{pure } h \langle * \rangle \text{pure } x$$

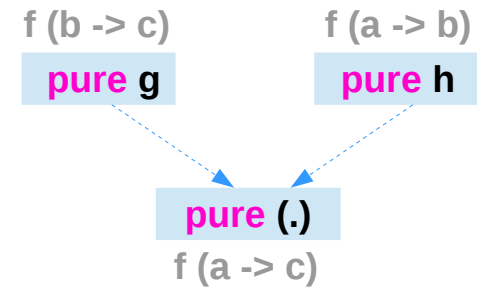
$$(g \cdot h) \ x$$

$$((\text{pure } (.) \langle * \rangle \text{pure } g) \langle * \rangle \text{pure } h) \langle * \rangle \text{pure } x$$



$$= \text{pure } g \langle * \rangle (\text{pure } h \langle * \rangle \text{pure } x)$$

$$g \ (h \ x)$$



$$u = \text{pure } g :: f \ (b \rightarrow c)$$

$$g :: (b \rightarrow c)$$

$$v = \text{pure } h :: f \ (a \rightarrow b)$$

$$h :: (a \rightarrow b)$$

$$w = \text{pure } x :: f \ a$$

$$x :: a$$

Left associative

$$u \langle * \rangle v \langle * \rangle w = (u \langle * \rangle v) \langle * \rangle w$$

$$u :: f \ (c \rightarrow b \rightarrow a) \quad v :: f \ c \quad w :: f \ b$$

$$u :: f \ (c \rightarrow b \rightarrow a)$$

$$v :: f \ c$$

$$u \langle * \rangle v :: f \ (b \rightarrow a)$$

$$w :: f \ b$$

$$u \langle * \rangle v \langle * \rangle w = f \ a$$

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

# liftA2

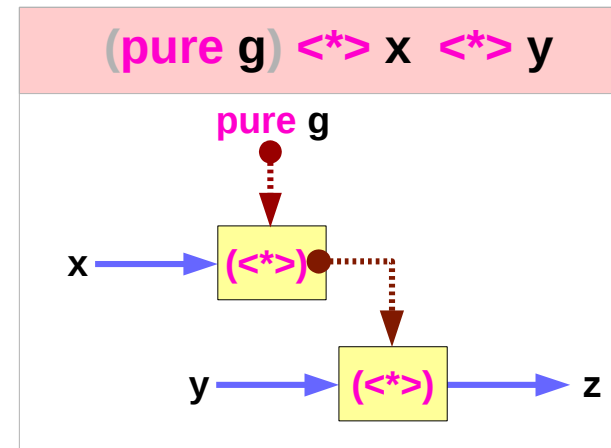
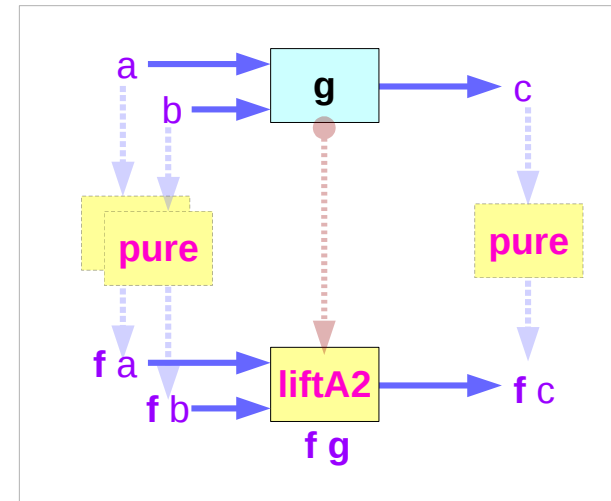
**liftA2** :: (a -> b -> c) -> f a -> f b -> f c

lift a binary function (a->b->c) to actions.

Some functors support an implementation of **liftA2** that is more efficient than the default one.

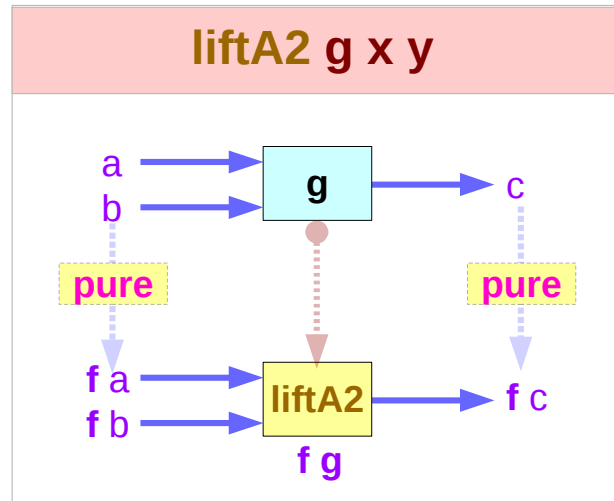
**liftA2** may have an efficient implementation whereas **fmap** is an expensive operation,

sometimes better to use **liftA2** than to use **fmap** over the structure and then use <\*>.

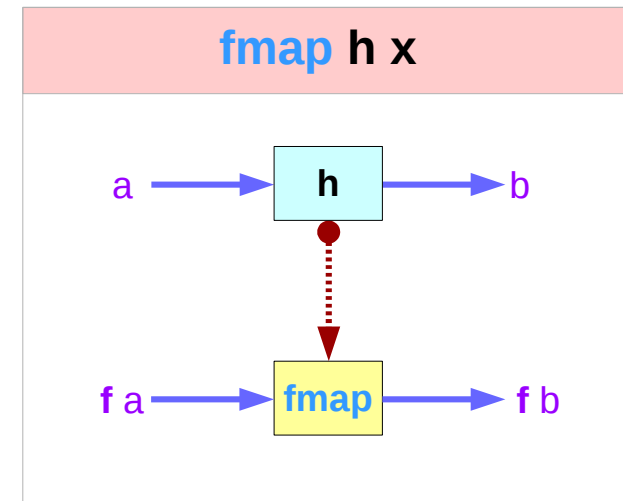
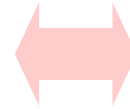
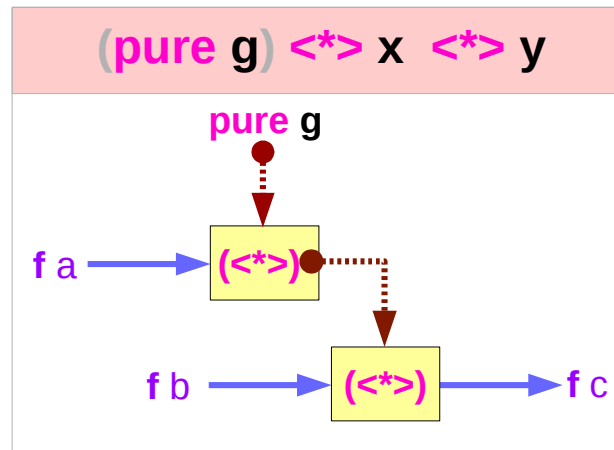


<http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2>

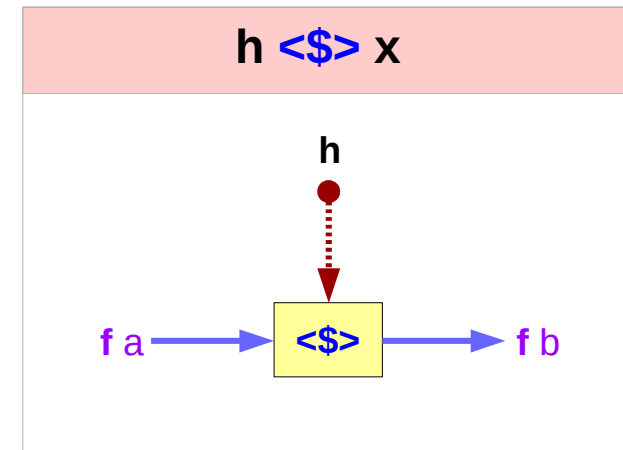
# liftA2, <\*>, fmap, <\$>



`g :: a -> b -> c`

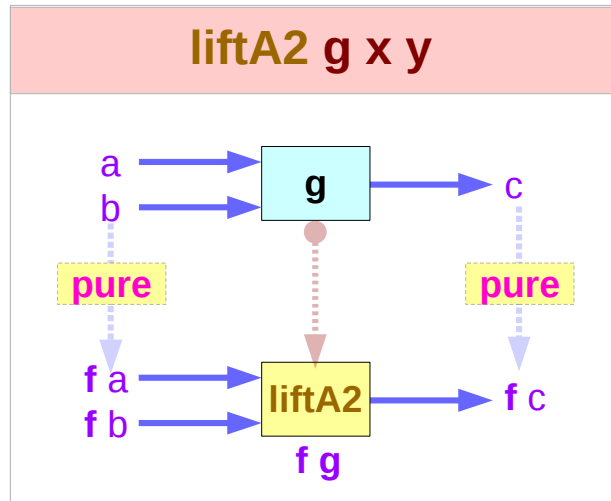


`h :: a -> b`





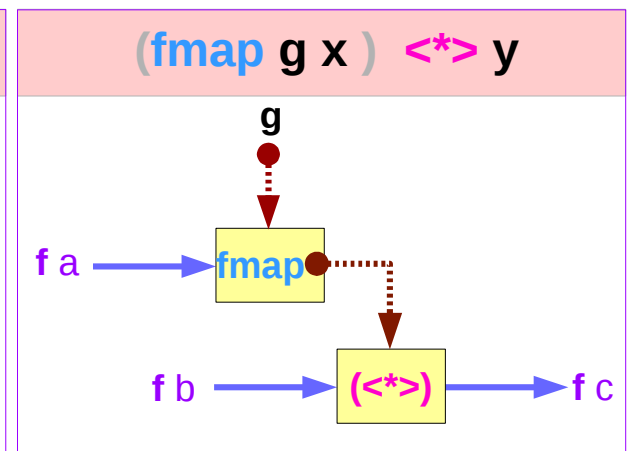
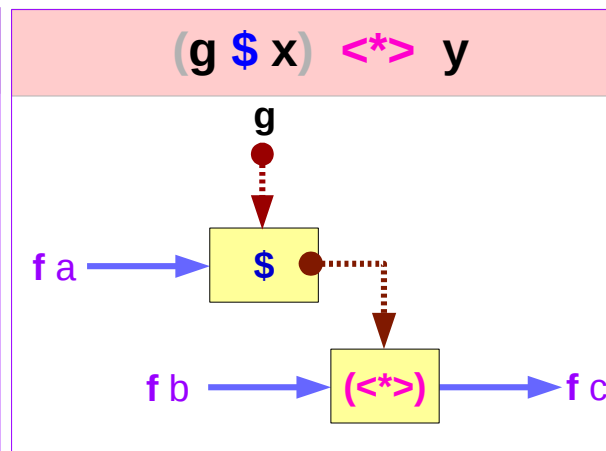
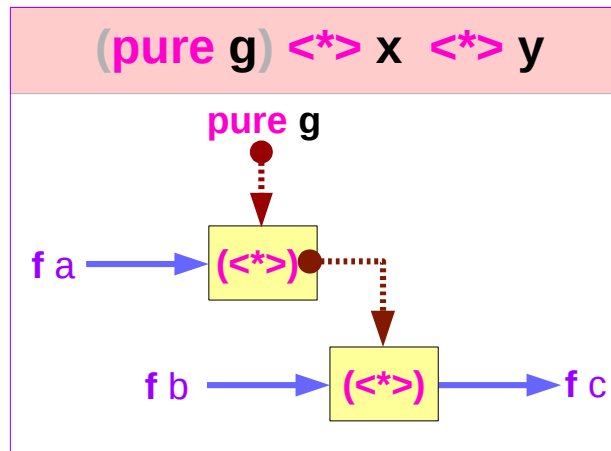
# pure g <\*> x <\*> y equivalent



$g :: a \rightarrow b \rightarrow c$

$g :: a \rightarrow b \rightarrow c$

$g :: a \rightarrow b \rightarrow c$



# liftA2

`liftA2 g x y`

`liftA2 :: (a -> b -> c) -> f a -> f b -> f c`

`g :: a -> b -> c`

`x :: f a`

`y :: f b`

`liftA2 g x y :: f c`

`pure g <*> x <*> y`

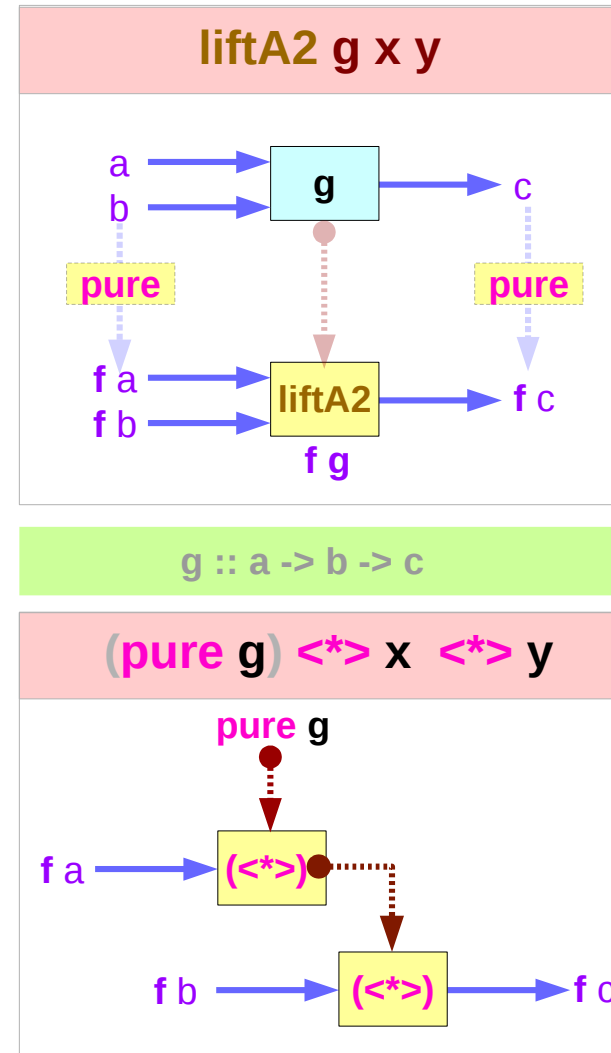
`g :: a -> b -> c`

`x :: f a`

`y :: f b`

`z :: f c`

`pure g <*> x <*> y :: f c`



[https://wiki.haskell.org/Applicative\\_functor](https://wiki.haskell.org/Applicative_functor)

# Limitations of Functors

$(a \rightarrow b \rightarrow c) \rightarrow (f a \rightarrow f b \rightarrow f c)$  – let `fmap2` Functor as an extension of `fmap`

`fmap` ::  $(a \rightarrow b) \rightarrow (f a \rightarrow f b)$

`fmap2` :: Functor `f` =>  $(a \rightarrow b \rightarrow c) \rightarrow (f a \rightarrow f b \rightarrow f c)$

`fmap2` `h` `fa` `fb` = undefined

`h` ::  $a \rightarrow b \rightarrow c$

`fa` :: `f a`

`fb` :: `f b`

`h` ::  $a \rightarrow (b \rightarrow c)$

`fmap` `h` ::  $f a \rightarrow f (b \rightarrow c)$

`fmap` `h` `fa` ::  $f (b \rightarrow c)$  – now `f (b -> c)` must be applied to `f b`

`fmap` gives us a way to apply functions  $(a \rightarrow b)$  to values `(f a)` inside a Functor context, but `fmap` cannot be used to apply a functions  $f (b \rightarrow c)$  which are themselves in a Functor context to values `f b` in a Functor context.

<http://www.openhaskell.com/lectures/applicative.html>

# pure, fmap, and liftA2

```
class Functor f => Applicative f where
  pure  :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b
```

```
pure    :: a -> f a           - fmap0 → pure
fmap    :: (a -> b) -> f a -> f b - fmap1 → fmap
fmap2   :: (a -> b -> c) -> f a -> f b -> f c - fmap2 → liftA2
```

```
liftA2 :: Applicative f => (a -> b -> c) -> f a -> f b -> f c
```

```
liftA2 h fa fb = (h `fmap` fa) <*> fb
```

```
liftA2 h fa fb = h <$> fa <*> fb
```

```
(<$>) :: Functor f => (a -> b) -> f a -> f b
```

```
(<$>) = fmap
```

```
liftA2 :: Applicative f => (a -> b -> c -> d) -> f a -> f b -> f c -> f d
```

```
liftA3 h fa fb fc = ((h <$> fa) <*> fb) <*> fc
```

<http://www.openhaskell.com/lectures/applicative.html>

# liftA2 examples

**liftA2** :: Applicative f => (a -> b -> c) -> f a -> f b -> f c

**liftA2 (+) (Just 5) (Just 6) = Just 11**

**liftA2 h fa fb = (h `fmap` fa) <\*> fb**

**liftA2 h fa fb = h <\$> fa <\*> fb**

**fmap (+) (Just 5) = Just (+5)**

**(+) <\$> (Just 5) = Just (+5)**

**<\*> :: Applicative f => f (a -> b) -> f a -> f b**

**(Just (+5)) <\*> (Just 6) = Just 11**

let v1 = IO (Just (+5))

let v2 = IO (Just 6)

liftA2 (<\*>) v1 v2 = IO (Just 11)

<https://blog.ssanj.net/posts/2014-08-10-boosting-liftA2.html>

# <\*> or liftA2 implementations

**liftA2** :: (a -> b -> c) -> f a -> f b -> f c

A minimal complete definition :

either one of the two

- 1) pure and <\*>
- 2) pure and liftA2

- 1) pure g <\*> x <\*> y
- 2) liftA2 g x y

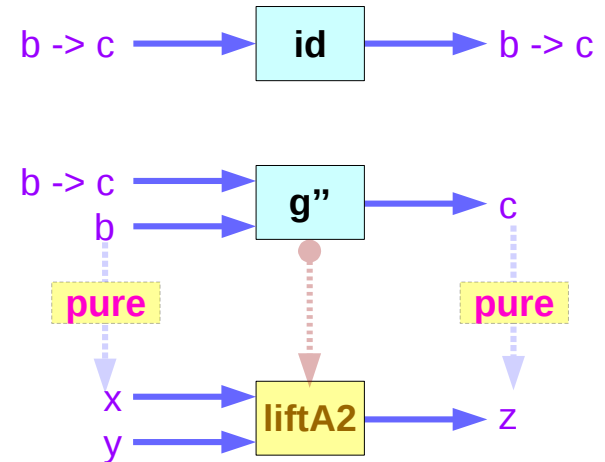
If it defines both, then they must behave the same as their default definitions:

<http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2>

# $\langle * \rangle = \text{liftA2 id}$

$\text{liftA2 id } x \ y = \text{id } \langle \$ \rangle \ x \ \langle * \rangle \ y = x \ \langle * \rangle \ y$

$\text{liftA2 id } x \ y = x \ \langle * \rangle \ y$



$\text{liftA2 } g \ x \ y = g \ \langle \$ \rangle \ x \ \langle * \rangle \ y$

$g :: a \rightarrow b \rightarrow c$

$x :: f \ a$

$y :: f \ b$

$\text{liftA2 } g'' \ x \ y = g'' \ \langle \$ \rangle \ x \ \langle * \rangle \ y$

$g'' :: (b \rightarrow c) \rightarrow b \rightarrow c$

$x :: f \ (b \rightarrow c)$

$y :: f \ b$

$\text{liftA2 id } x \ y = \text{id } \langle \$ \rangle \ x \ \langle * \rangle \ y = x \ \langle * \rangle \ y$

$\text{id} :: (b \rightarrow c) \rightarrow (b \rightarrow c)$

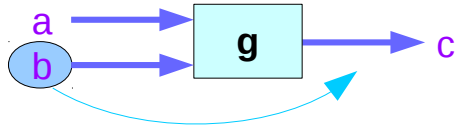
$x :: f \ (b \rightarrow c)$

$y :: f \ b$

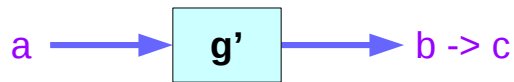
$\langle * \rangle = \text{liftA2 id}$

<http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2>

# $g'' :: (b \rightarrow c) \rightarrow b \rightarrow c$



$g :: a \rightarrow b \rightarrow c$

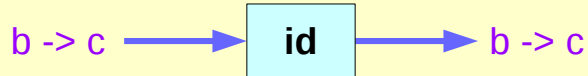


$g' :: a \rightarrow (b \rightarrow c)$

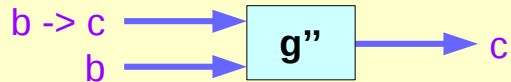
view the function  
as having one input only

consider the case  
when  $a$  is  $(b \rightarrow c)$

Then  $g''$  is the same as  $id$



$id :: (b \rightarrow c) \rightarrow (b \rightarrow c)$



$g'' :: (b \rightarrow c) \rightarrow (b \rightarrow c)$



# Results and effects in a scope

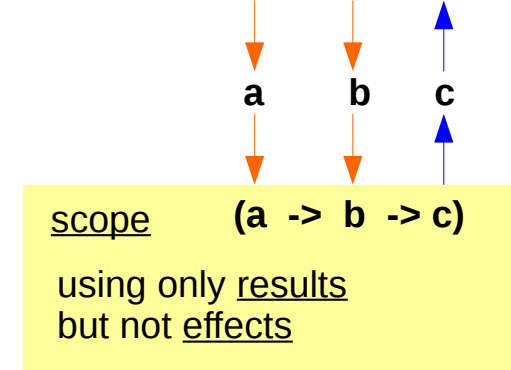
Actually, using the **liftA** commands  
we can pull results of applicative functors  
into a scope where we can talk

exclusively about functor results **c**  
and not about effects. **f c**

Note that functor results can also be functions. **c**

This scope is simply a function,  
which contains the code that we used in the non-functorial setting.

**liftA2** :: (a -> b -> c) -> f a -> f b -> f c

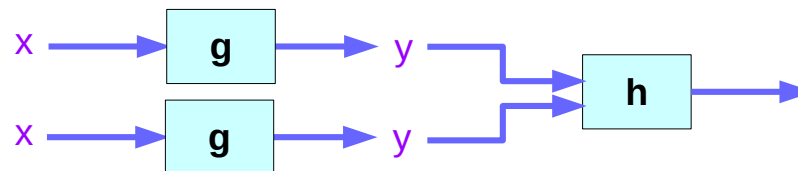


<http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2>

# liftA3 – a non-functorial expression

Consider the non-functorial expression:

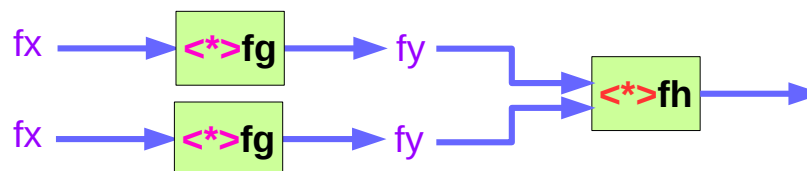
```
x :: x
g :: x -> y
h :: y -> y -> z
```



```
let y = g x
in h y y
```

generalization

```
fx :: f x
fg :: f (x -> y)
fh :: f (y -> y -> z)
```

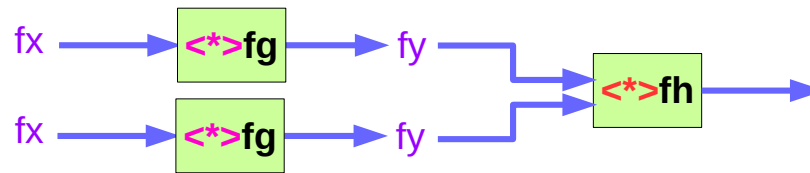


[https://wiki.haskell.org/Applicative\\_functor](https://wiki.haskell.org/Applicative_functor)

# liftA3 – using <\*> only

```
let fy = fg <*> fx
  in fh <*> fy <*> fy
```

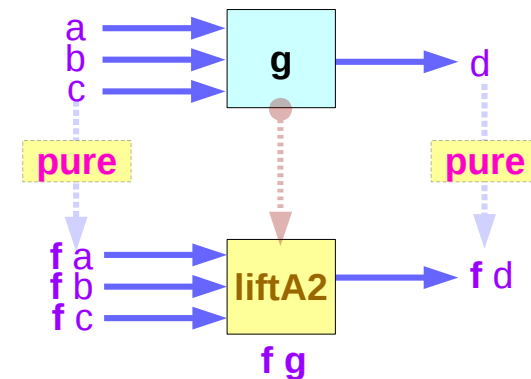
if **fy** writes something to the terminal  
then **fh <\*> fy <\*> fy** writes twice.



this runs the effect of **fy** twice.

How the effect is run only once and the result is used twice?

→ utilize **liftA3**



[https://wiki.haskell.org/Applicative\\_functor](https://wiki.haskell.org/Applicative_functor)

# liftA3 – using three input function

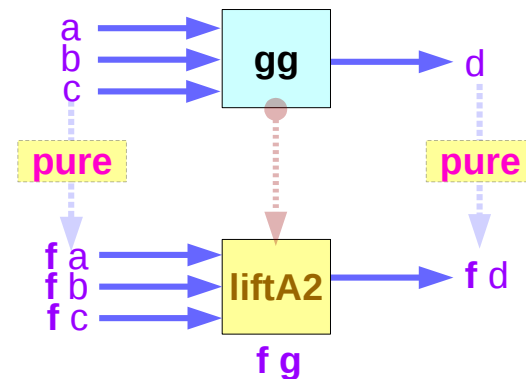
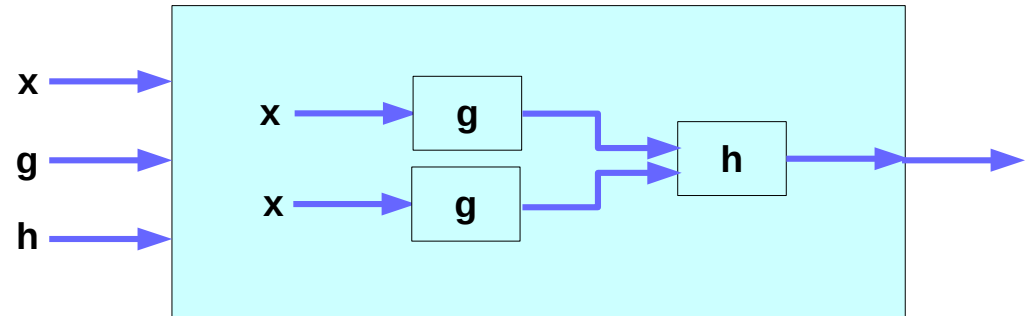
**liftA3** :: Applicative f => (a -> b -> c -> d) -> f a -> f b -> f c -> f d

**liftA3** h fa fb fc = ((h <\$> fa) <\*> fb) <\*> fc

**liftA3**

```
(\x g h -> let y = g x in h y y)
```

fx fg fh



<http://hackage.haskell.org/package/base-4.10.1.0/docs/Control-Applicative.html#v:liftA2>

# liftA3 – effects, results and scopes

Actually, using the **liftA** commands

we can pull results of applicative functors

**y** from **fy**

into a scope where we can talk

**y -> y -> z**

exclusively about functor results

**y**

and not about effects.

**fy**

Note that functor results can also be functions.

**y**

This scope is simply a function,

**y -> y -> z**

which contains the code that we used in the non-functorial setting.

## liftA3

```
(\x g h -> let y = g x in h y y)
```

```
fx fg fh
```

The order of effects is entirely determined by the order of arguments to liftA3

[https://wiki.haskell.org/Applicative\\_functor](https://wiki.haskell.org/Applicative_functor)

# liftA2 (<\*>)

10

down vote

accepted

The wiki article says that **liftA2 (<\*>)** can be used to compose applicative functors.

It's easy to see how to use it from its type:

**o** :: (Applicative **f**, Applicative **f1**) =>

**f** (**f1** (a -> b)) -> **f** (**f1** a) -> **f** (**f1** b)

**o** = liftA2 (<\*>)

<https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers>

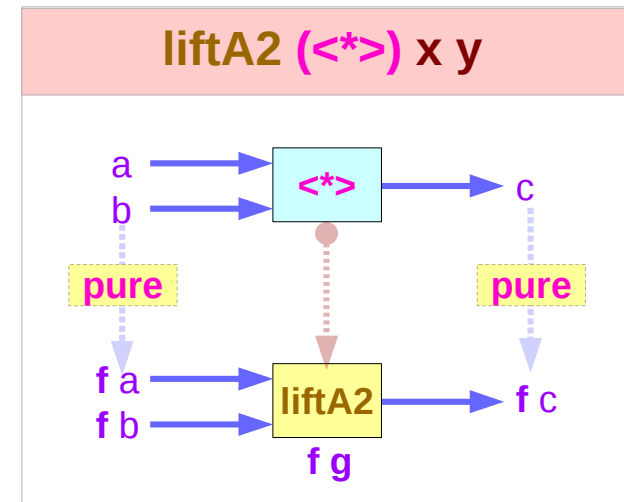
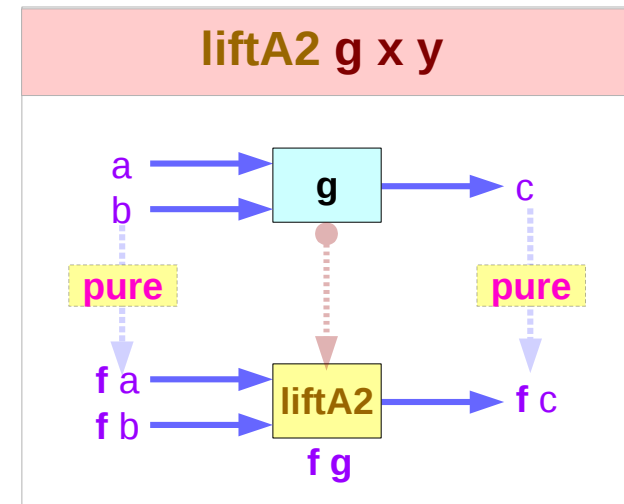
# liftA2 (<\*>) for composite applicative functors

$o :: (\text{Applicative } f, \text{Applicative } f1) \Rightarrow$   
 $f (f1 (a \rightarrow b)) \rightarrow f (f1 a) \rightarrow f (f1 b)$

$o = \text{liftA2 } (<*>)$

$f1 (a \rightarrow b) <*> f1 a <*> f1 b$

liftA2 (<\*>) can be used to compose applicative functors.



<https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers>

# liftA2 (<\*>) Examples (1)

if `f` is `Maybe` and `f1` is `[]` we get:

```
Just [(+1),(+6)] `o` Just [1, 6]
Just [2,7,7,12]
```

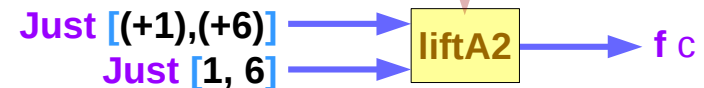
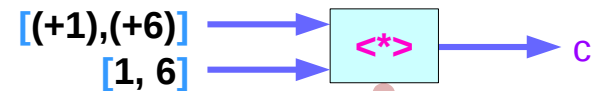
```
liftA2 (<*>) Just [(+1),(+6)] Just [1, 6]
Just ( [(+1),(+6)] <*> [1, 6] )
Just [2, 7, 7, 12]
```

```
[(+1), (+6)]    [1, 6]
```

```
(+1) [1, 6]
```

```
(+6) [1, 6]
```

`liftA2 (<*>) Just [(+1),(+6)] Just [1, 6]`



<https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers>



# liftA2 (<\*>) Examples (2)

if `f` is `Maybe` and `f1` is `[]` we get:

`[Just (+1),Just (+6)]`

`[Just 1, Just 6]`

`[Just (+1),Just (+6)] `o` [Just 1, Just 6]`

`Just (+1) [Just 1, Just 6]`

`[Just 2, Just 7, Just 7, Just 12]`

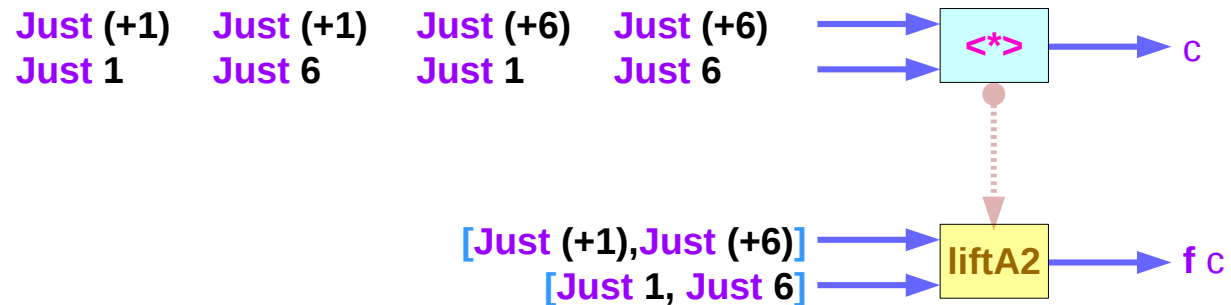
`Just (+6) [Just 1, Just 6]`

`liftA2 (<*>) [Just (+1),Just (+6)] [Just 1, Just 6]`

`[Just (+1) <*> Just 1, Just (+1) <*> Just 6, Just (+6) <*> Just 1, Just (+6) <*> Just 6]`

`[Just 2, Just 7, Just 7, Just 12]`

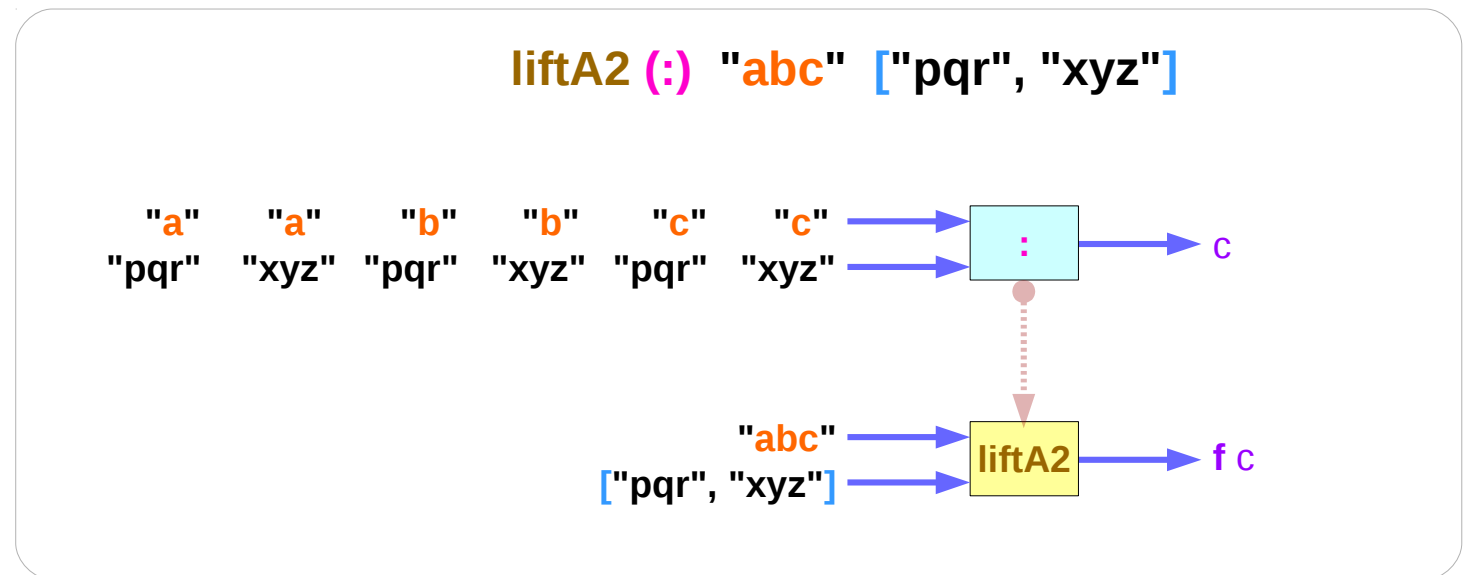
`liftA2 (<*>) [Just (+1),Just (+6)] [Just 1, Just 6]`



<https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers>

# liftA2 (:)

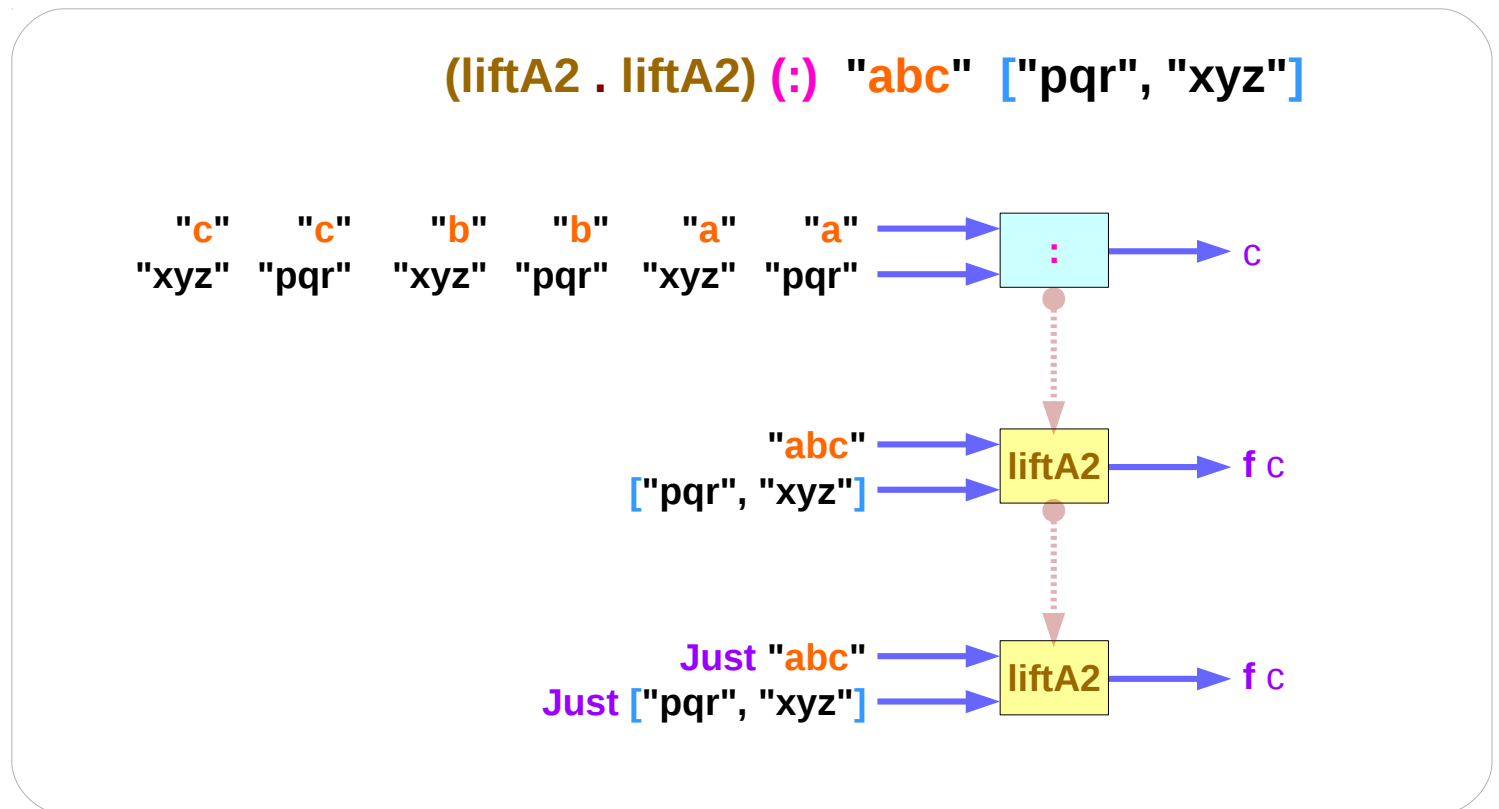
```
liftA2 (:) "abc" ["pqr", "xyz"]  
["apqr", "axyz", "bpqr", "bxyz", "cpqr", "cxyz"]
```



<https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers>

# liftA2 (:)

```
(liftA2 . liftA2) (:) (Just "abc") (Just ["pqr", "xyz"])  
Just ["apqr", "axyz", "bpqr", "bxyz", "cpqr", "cxyz"]
```



<https://stackoverflow.com/questions/12587195/examples-of-haskell-applicative-transformers>

# <\$> related operators

Functor map <\$>

**<\$>** :: Functor **f** => (a -> b) -> **f a** -> **f b**

**<\$** :: Functor **f** => a -> **f b** -> **f a**

**\$>** :: Functor **f** => **f a** -> b -> **f b**

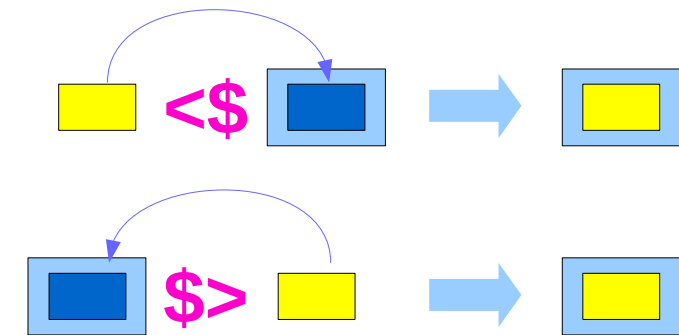
replace **b** in **f b** with **a** ... **f a**

replace **a** in **f a** with **b** ... **f b**

The **<\$>** operator is just a synonym  
for the **fmap** function in the Functor typeclass.

**fmap** generalizes **map** for **lists**  
to other data types : **Maybe**, **IO**, **Map**.

Replacing the core



<https://haskell-lang.org/tutorial/operators>

# <\$, <\$>, \$> operators

there are two additional operators provided  
which replace a **value** inside a Functor  
instead of applying a function.

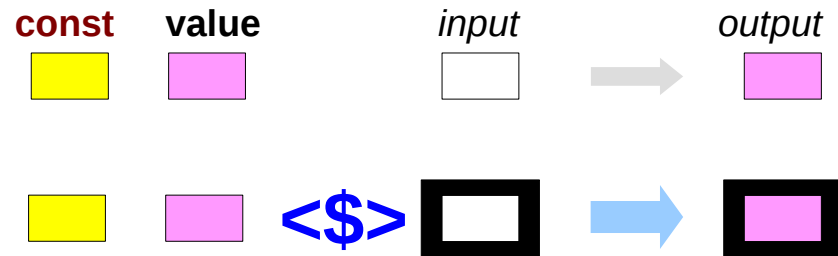
This can be both more convenient in some cases,  
as well as for some Functors be more efficient.

**value** <\$ **functor** = **const value** <\$> **functor**

**functor** \$> **value** = **const value** <\$> **functor**

$x \text{ <\$ } y = y \text{ \$> } x$        $y :: \text{functor}$

$x \text{ \$> } y = y \text{ <\$ } x$        $x :: \text{functor}$



<https://haskell-lang.org/tutorial/operators>

# <\$, <\$>, \$> operators examples

```
import Data.Functor
```

```
Prelude> Just 1 $> 2
```

```
Just 2
```

```
Prelude> Just 2 $> 1
```

```
Just 1
```

```
Prelude> 1 <$ Just 3
```

```
Just 1
```

```
Prelude> 3 <$ Just 1
```

```
Just 3
```

```
Prelude> 1 <$ Just 3
```

```
Just 1
```

```
Prelude> 3 <$ Just 1
```

```
Just 3
```

```
import Data.Functor
```

```
Prelude> (+1) <$> Just 2
```

```
Just 3
```

```
Prelude> (+1) <$> Just 3
```

```
Just 4
```

```
Prelude> (+1) <$> Nothing
```

```
Nothing
```

```
Prelude> const 2 <$> Just 111
```

```
Just 2
```

<https://www.schoolofhaskell.com/school/to-infinity-and-beyond/pick-of-the-week/Simple%20examples>

# <\$> examples

```
#!/usr/bin/env stack
-- stack --resolver ghc-7.10.3 runghc
import Data.Monoid ((<>))

main :: IO ()
main = do
  putStrLn "Enter your year of birth"
  year <- read <$> getLine
  let age :: Int
      age = 2020 - year
  putStrLn $ "Age in 2020: " <> show age
```

**getLine :: IO String**

**Input: read "12"::Double**

**Output: 12.0**

-- this infix synonym for mappend is found in Data.Monoid  
**x <\$> y = mappend x y**  
**infixr 6 <\$>**

<https://haskell-lang.org/tutorial/operators>

# <\*> related operators

Applicative function application <\*>

<\*> :: Applicative f => f (a -> b) -> f a -> f b

(\*>) :: Applicative f => f a -> f b -> f b

<\*> :: Applicative f => f a -> f b -> f a

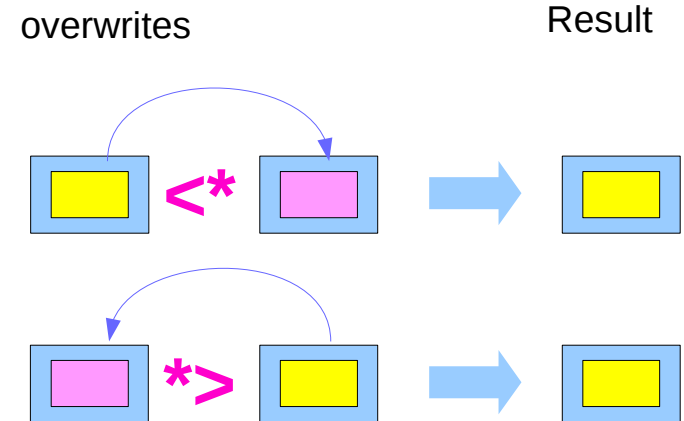
<\*> is an operator that applies  
a wrapped function to a wrapped value.

<\*> is a part of the Applicative typeclass,

<\*> is very often used as follows

```
foo <$> bar <*> baz
```

```
faa <*> bar <*> baz
```



<https://haskell-lang.org/tutorial/operators>



# \*> operator

two helper operators

\*> ignores the value from the first argument.

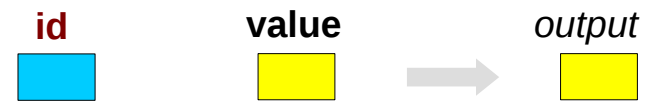
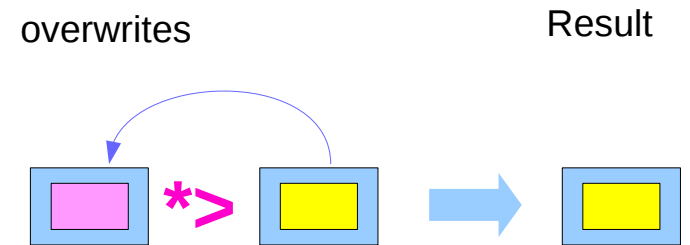
\*> is completely equivalent to >> in Monad

$a1 *> a2 = (id \lt \$ a1) \lt * \gt a2$

$a1 *> a2 = do$

`_ <- a1`

`a2`



$(id \lt \$ a1)$



$(id \lt \$ a1) \lt * \gt a2$

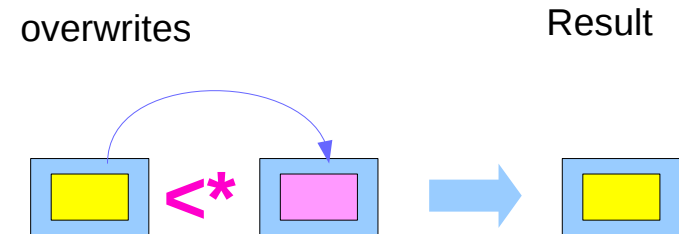


<https://haskell-lang.org/tutorial/operators>

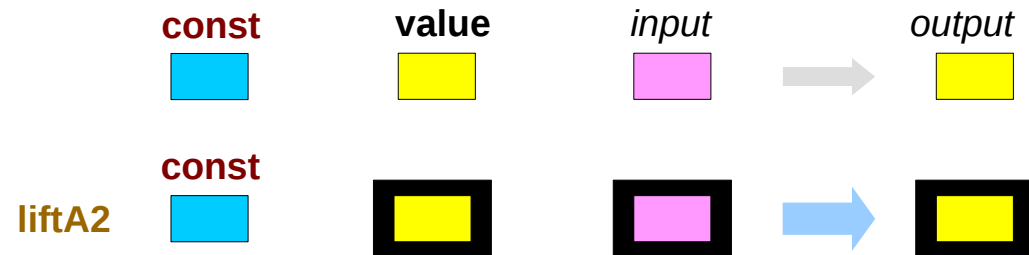
# <\* operator

<\* is the same thing in reverse: perform the first action then the second,  
but only take the value from the first action.

(<\*) = liftA2 const



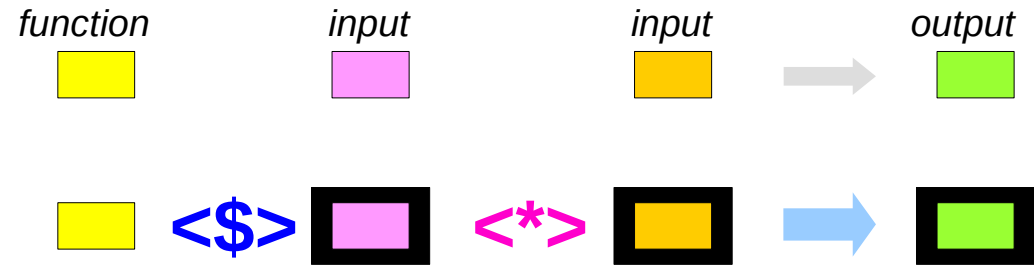
```
a1 <* a2 = do
  res <- a1
  _ <- a2
  return res
```



<https://haskell-lang.org/tutorial/operators>

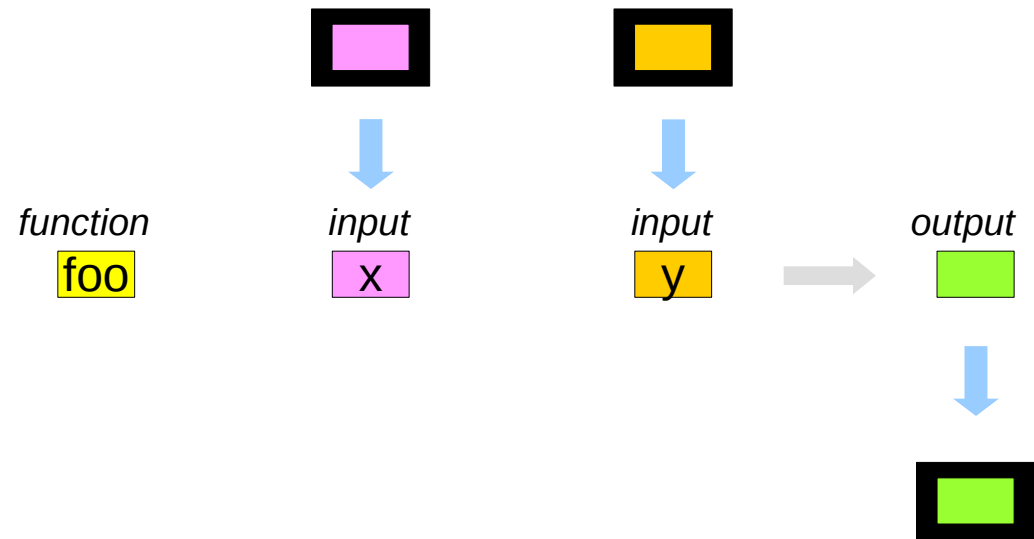
# <\*> examples

foo <\$> bar <\*> baz



With a Monad, this is equivalent to:

```
do x <- bar
  y <- baz
  return (foo x y)
```



<https://haskell-lang.org/tutorial/operators>

# <\*> examples

examples including parsers and serialization libraries.

using the **aeson** package: (handling **JSON** data)

```
data Person = Person { name :: Text, age :: Int } deriving Show
```

```
-- We expect a JSON object, so we fail at any non-Object value.
```

```
instance FromJSON Person where
```

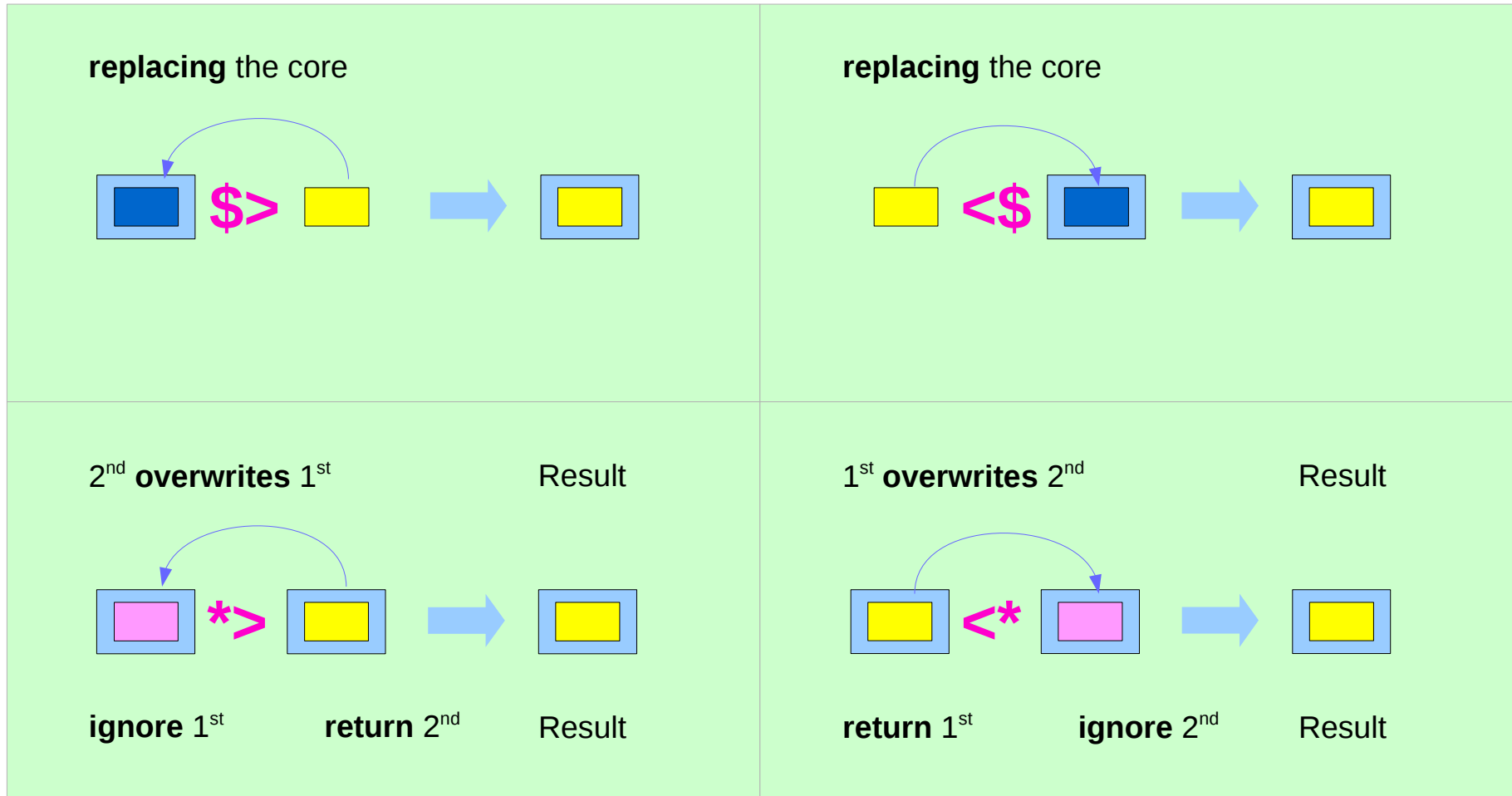
```
  parseJSON (Object v) = Person <$> v .: "name" <*> v .: "age"
```

```
  parseJSON _ = empty
```

- . append-head operator (cons)
- . function composition operators
- . name qualifier

<https://haskell-lang.org/tutorial/operators>

# (\$> v.s. <\$) and (\*> v.s. <\*)



[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

# (\*> v.s. >>) and (pure v.s. return)

(\*>) :: Applicative f => f a -> f b -> f b

(>>) :: Monad m => m a -> m b -> m b

pure :: Applicative f => a -> f a

return :: Monad m => a -> m a

the constraint changes from **Applicative** to **Monad**.

(\*>) in **Applicative**

(>>) in **Monad**

pure in **Applicative**

return in **Monad**

[https://en.wikibooks.org/wiki/Haskell/Applicative\\_functors](https://en.wikibooks.org/wiki/Haskell/Applicative_functors)

## References

- [1] <ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf>
- [2] <https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf>