

Functor (1A)

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Based on

<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

Haskell in 5 steps

https://wiki.haskell.org/Haskell_in_5_steps

Typeclasses

Typeclasses are like *interfaces*

defines some *behavior*
comparing for *equality*
comparing for *ordering*
enumeration

Instances of that typeclass
types possessing such *behavior*

Such *behavior* is defined by

- **function definition**
- **type declaration**

a **type** is an **instance** of a **typeclass** implies
the *functions* defined (implemented)
by the typeclass with that type can be used

No relation with classes in Java or C++

`(==) :: a -> a -> Bool` - a type declaration
`x == y = not (x /= y)` - a function definition

<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

Car Type Example

the **Eq** typeclass

defines the functions **==** and **/=**

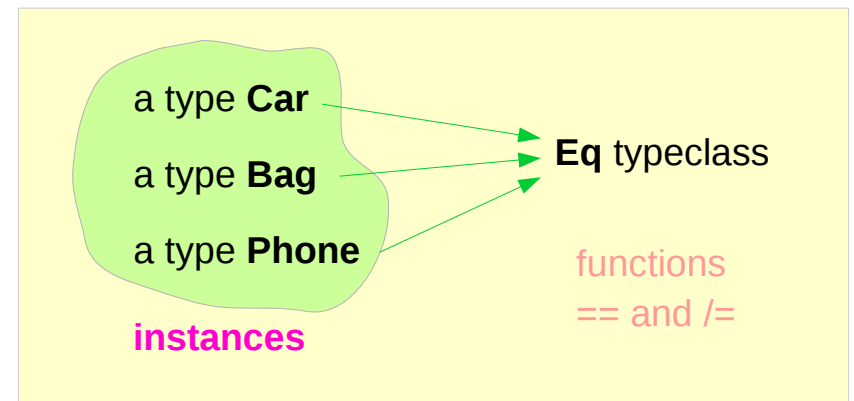
a type **Car**

comparing two cars **c1** and **c2** with the equality function **==**

The **Car** type is an **instance** of **Eq** typeclass

Instances : various types

Typeclass : a group or a class of these similar types



<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

TrafficLight Type Example (1)

```
class Eq a where
```

```
(==) :: a -> a -> Bool
```

```
(/=) :: a -> a -> Bool
```

```
x == y = not (x /= y)
```

```
x /= y = not (x == y)
```

- a type declaration

- a type declaration

- a function definition

- a function definition

```
data TrafficLight = Red | Yellow | Green
```

```
instance Eq TrafficLight where
```

```
Red == Red = True
```

```
Green == Green = True
```

```
Yellow == Yellow = True
```

```
_ == _ = False
```

```
ghci> Red == Red
```

```
True
```

```
ghci> Red == Yellow
```

```
False
```

```
ghci> Red `elem` [Red, Yellow, Green]
```

```
True
```

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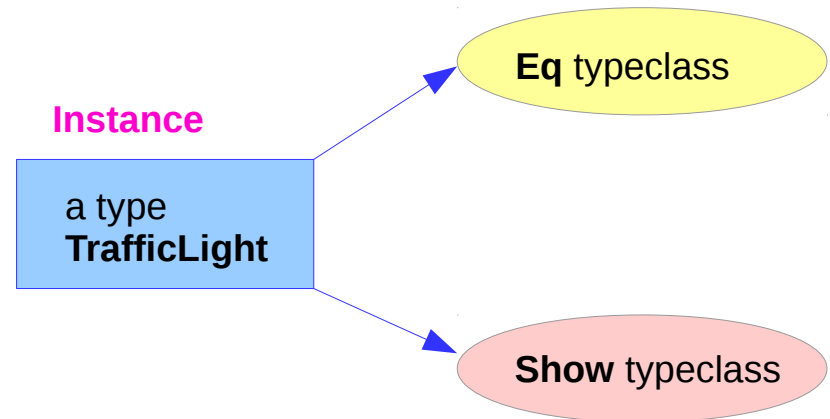
TrafficLight Type Example (2)

```
class Show a where  
  show :: a -> String  
  * * *
```

- a type declaration

```
data TrafficLight = Red | Yellow | Green
```

```
instance Show TrafficLight where  
  show Red = "Red light"  
  show Yellow = "Yellow light"  
  show Green = "Green light"
```



```
ghci> [Red, Yellow, Green]  
[Red light, Yellow light, Green light]
```

<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

Class Constraints

class (Eq a) => Num a where

...

class Num a where

...

class constraint on a class declaration

an instance of **Eq**

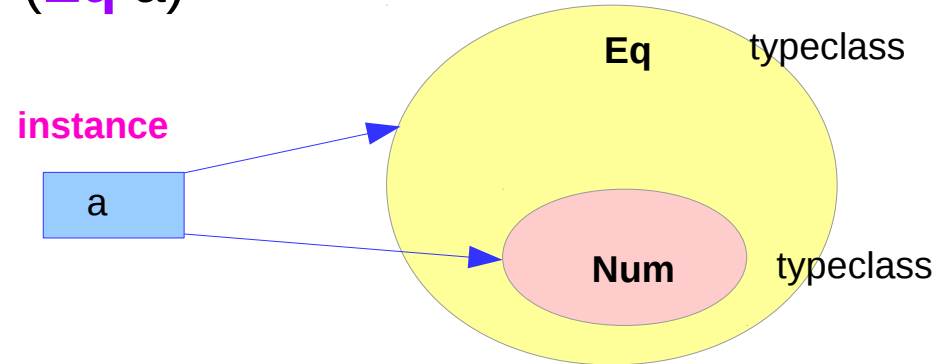
before being an instance of **Num**

the required function bodies can be defined in

- the **class declaration**
- an **instance declarations**,

we can safely use == because a is a part of **Eq**

(Eq a) =>



Num : a subclass of **Eq**

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Class Constraints : in class & instance declaration

class constraints in **class declarations**

to make a typeclass a **subclass** of another typeclass

```
class (Eq a) => Num a where
```

class constraints in **instance declarations**

to express **requirements** about the contents of some type.

```
instance (Eq x, Eq y) => Eq (Pair x y) where
```

```
Pair x0 y0 == Pair x1 y1 = x0 == x1 && y0 == y1
```

Derived instance

<http://cmsc-16100.cs.uchicago.edu/2016/Lectures/07-type-classes.php>

<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

Show Typeclass Example

the **a** : a concrete type

Maybe : not a concrete type
: a type constructor that takes one parameter
produces a concrete type.

Maybe a : a concrete type

instance (Eq m) => Eq (Maybe m) where
Just x == **Just** y = x == y
Nothing == **Nothing** = True
_ == _ = False

<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

Functor typeclass

the Functor typeclass is basically for things that can be *mapped over*

ex) mapping over lists

the list type is part of the Functor typeclass

<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

Functor typeclass

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

The Functor typeclass

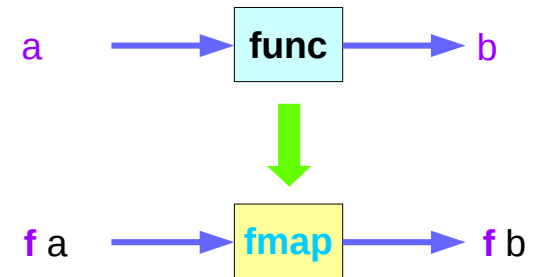
defines one function, **fmap**,
no default implementation

the **type variable f**

not a concrete type (a concrete type can hold a value)
a **type constructor** taking one **type parameter**

Maybe Int : a concrete type

Maybe : a type constructor that takes one type as the parameter



```
function fmap
function func
type constructor f
```

<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

Function `map` & `fmap`

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

`fmap` takes

- a **function** from one type to another (`a -> b`)
- a **Functor** `f` applied with **one type** (`f a`)

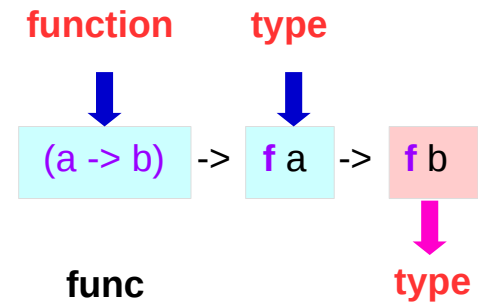
`fmap` returns

- a **Functor** `f` applied with **another type** (`f b`)

```
map :: (a -> b) -> [a] -> [b]
```

`map` takes

- a function from one type to another
- take a list of one type
- returns a list of another type



```
(* 2)
[ 1, 2, 3 ]
[ 2, 4, 6 ]
```

<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

List : an instance of Functor typeclass

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

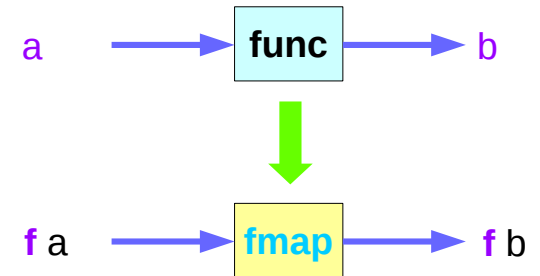
```
map :: (a -> b) -> [a] -> [b]
```

`map` is just a `fmap` that works only on **lists**

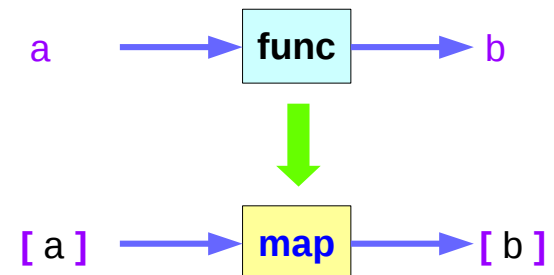
a list is an **instance** of the Functor typeclass.

```
instance Functor [ ] where
  fmap = map
```

`f` : a type constructor that takes one type
`[]` : a type constructor that takes one type
`[a]` : a concrete type (`[Int]`, `[String]` or `[[String]]`)



```
function fmap
function func
type constructor f
```



<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

List Examples

```
class Functor f where  
  fmap :: (a -> b) -> f a -> f b
```

```
map :: (a -> b) -> [a] -> [b]
```

```
instance Functor [ ] where  
  fmap = map
```

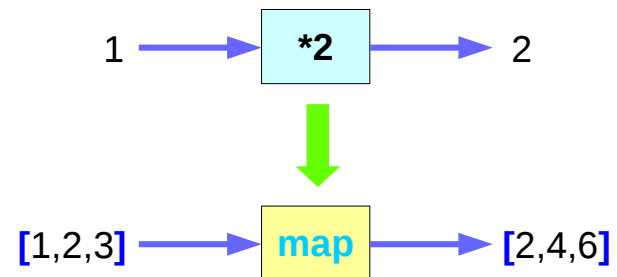
```
map :: (a -> b) -> [a] -> [b]
```

```
ghci> fmap (*2) [1..3]
```

```
[2,4,6]
```

```
ghci> map (*2) [1..3]
```

```
[2,4,6]
```



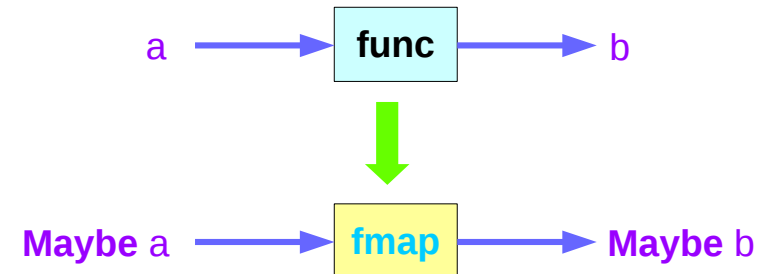
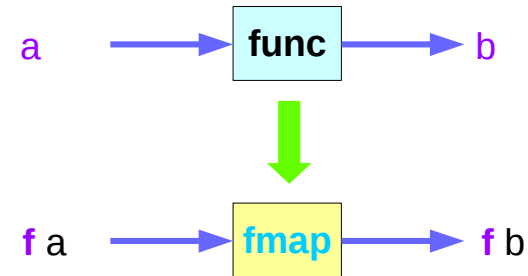
<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

Maybe : an instance of Functor typeclass

```
class Functor f where  
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where  
  fmap func (Just x) = Just (func x)  
  fmap func Nothing = Nothing
```

f	↔	Maybe
f a	↔	Maybe a
f b	↔	Maybe b



<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

Maybe : a type constructor

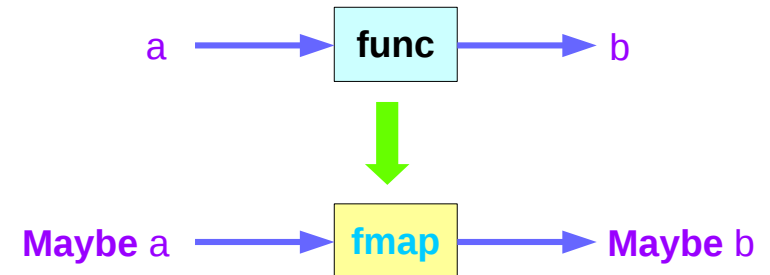
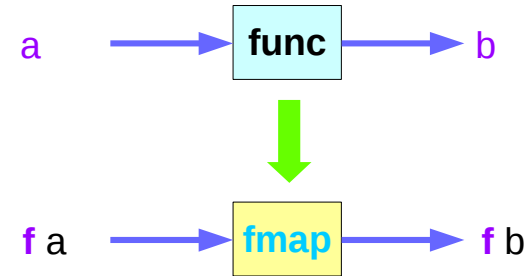
```
class Functor f where  
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where  
  fmap func (Just x) = Just (func x)  
  fmap func Nothing = Nothing
```

f : a type variable

f : a type constructor taking one type parameter

Maybe : an instance of **Functor** typeclass



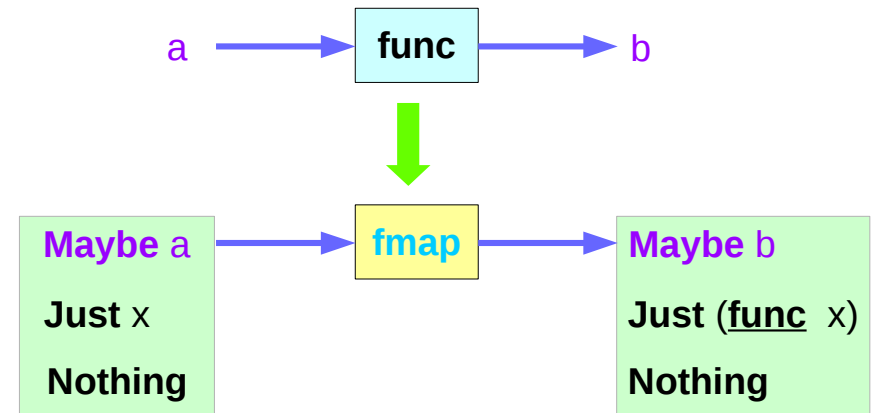
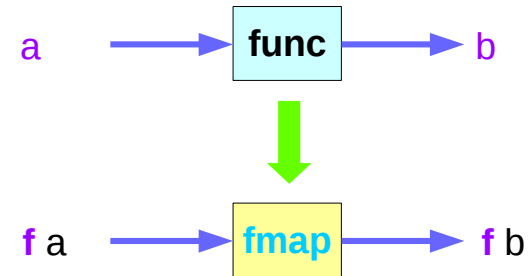
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Maybe : an argument to `fmap`, together with a

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap func (Just x) = Just (func x)
  fmap func Nothing = Nothing
```

```
fmap :: (a -> b) -> f a -> f b
fmap func (Just x) = Just (func x)
fmap func Nothing = Nothing
```



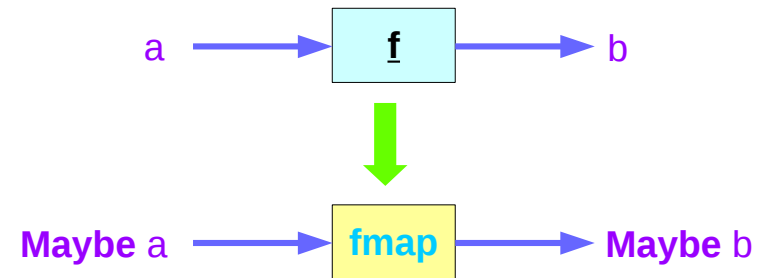
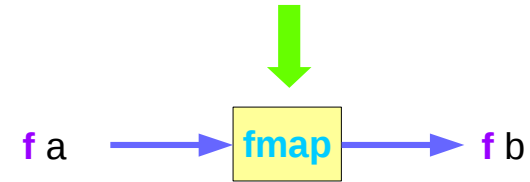
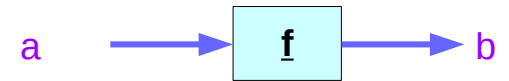
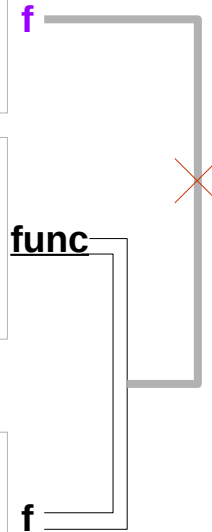
<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

Maybe : fmap takes a function

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap func (Just x) = Just (func x)
  fmap func Nothing = Nothing
```

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```



`f` is different from the type constructor `f`

`func` : `a -> b`

`f` : `a -> b`

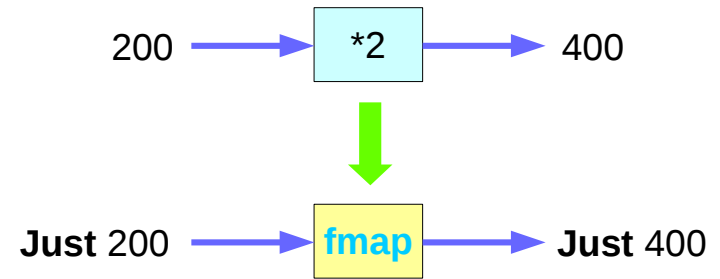
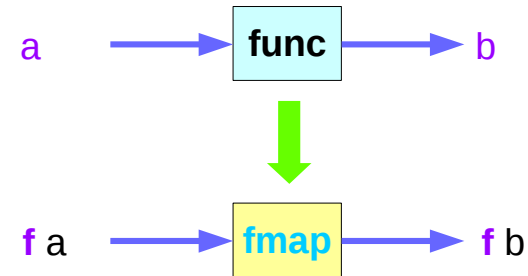
<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

Maybe Examples (1)

```
class Functor f where  
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where  
  fmap f (Just x) = Just (f x)  
  fmap f Nothing = Nothing
```

```
ghci> fmap (*2) (Just 200)  
Just 400  
ghci> fmap (*2) Nothing  
Nothing
```



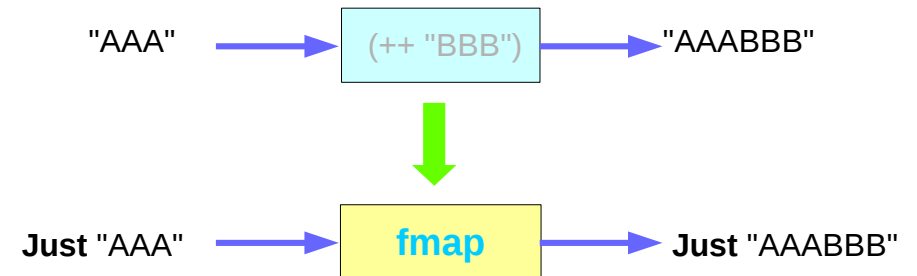
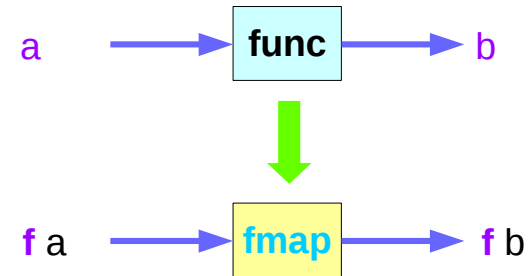
<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

Maybe Examples (2)

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```

```
ghci> fmap (++ "BBB") (Just "AAA")
Just "AAABBB"
ghci> fmap (++ "BBB") Nothing
Nothing
```



<http://learnyouahaskell.com/making-our-own-types-and-typeclasses#the-functor-typeclass>

Maybe as a functor

Functor typeclass:

- transforming one type to another
- transforming operations of one type to those of another

Maybe is an instance of a **functor** type class

Functor provides **fmap** method

maps functions of the *base type* (such as *Integer*)
to *functions* of the *lifted type* (such as *Maybe Integer*).

<https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell>

Maybe as a functor

A *function* `f` transformed with `fmap` can work on a `Maybe` value

case maybeVal of

```
Nothing -> Nothing    -- there is nothing, so just return Nothing
Just val -> Just (f val) -- there is a value, so apply the function to it
```

```
father :: Person -> Maybe Person
mother :: Person -> Maybe Person
```

```
f :: Int -> Int
fmap f :: Maybe Integer -> Maybe Integer
```

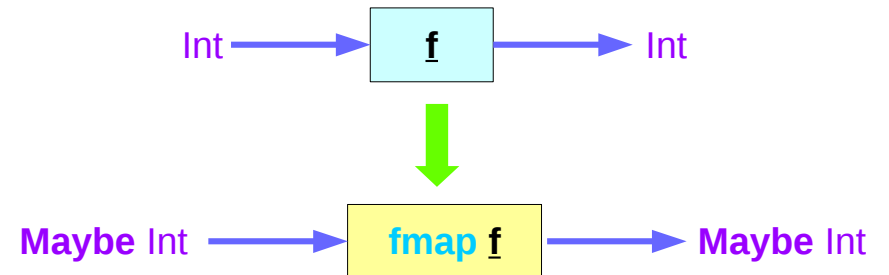
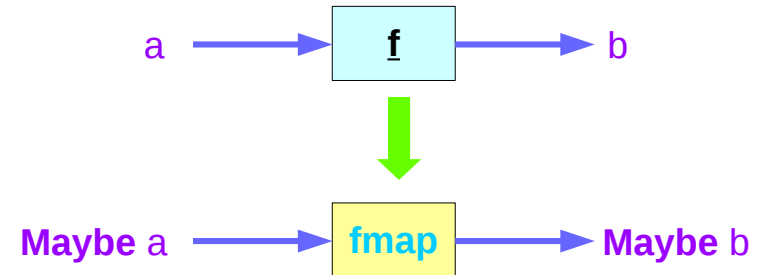
a `Maybe Integer` value: `m_x`

```
fmap f m_x
```

<https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell>

Transforming operations

Functor provides `fmap` method
maps functions of the *base type* (such as `Integer`)
to *functions* of the *lifted type* (such as `Maybe Integer`).



<https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell>

Maybe as a functor

`m_x` : a Maybe Integer value (`Just 101`, `Nothing`, ...)
`f` :: Int -> Int

you can do `fmap f m_x`
to apply the function `f` directly to the `Maybe` Integer
without worrying whether it is `Nothing` or not

|

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```

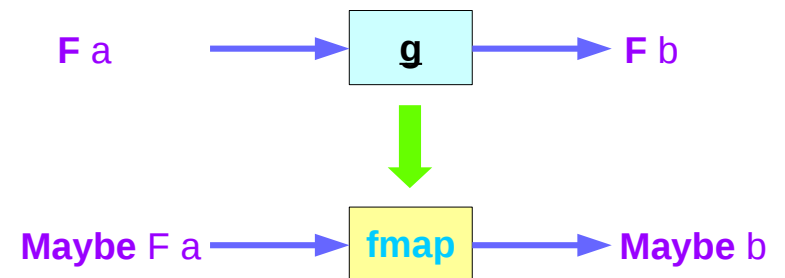
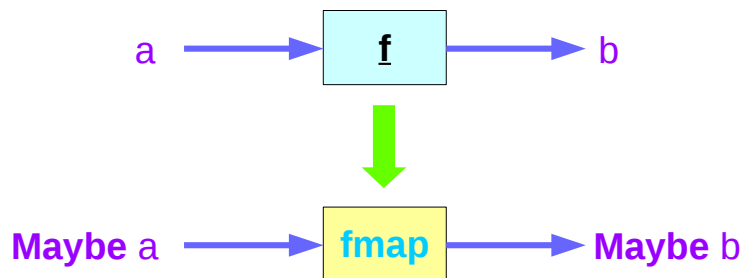
<https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell>

Maybe as a functor

Can apply a whole chain of
lifted `Integer -> Integer` functions
to `Maybe Integer` values
and only have to worry about explicitly checking for `Nothing`
once when you're finished.

```
class Functor f where
  fmap :: (a -> b) -> f a -> f b
```

```
instance Functor Maybe where
  fmap f (Just x) = Just (f x)
  fmap f Nothing = Nothing
```



<https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell>

Maybe class

The Maybe type definition

```
data Maybe a = Just a | Nothing
  deriving (Eq, Ord)
```

Maybe is

- an instance of **Eq** and **Ord** (as a base type)

- an instance of **Functor**

- an instance of **Monad**

For Functor, the fmap function f

- moves inside the Just constructor and
- is identity on the Nothing constructor.

For Monad,

- the bind operation passes through Just, while
- Nothing will force the result to always be Nothing.

<https://wiki.haskell.org/Maybe>

Maybe as a Monad

```
f::Int -> Maybe Int  
f 0 = Nothing  
f x = Just x
```

if x==0 then Nothing else Just x

```
g::Int -> Maybe Int  
g 100 = Nothing  
g x   = Just x
```

if x==100 then Nothing else Just x

```
h::Int -> Maybe Int  
h x = case f x of  
    Just n -> g n  
    Nothing -> Nothing
```

if **f** x==Nothing then Nothing else **g** n

```
h'::Int -> Maybe Int  
h' x = do n <- f x  
        g n
```

g (**f** x)

h & **h'** give the same results

h 0 = **h'** 0 = **h** 100 = **h'** 100 = Nothing;

h x = **h'** x = Just x

<https://wiki.haskell.org/Maybe>

Maybe as a Library Function

When the module is imported `import Data.Maybe`

`maybe :: b -> (a -> b) -> Maybe a -> b`

Applies the second argument (a->b) to the third Maybe a, when it is Just x, otherwise returns the first argument (b).

`isJust, isNothing`

Test the argument, returning a Bool based on the constructor.

`ListToMaybe, maybeToList`

Convert to/from a one element or empty list.

`mapMaybe`

A different way to filter a list.

<https://wiki.haskell.org/Maybe>

Maybe as Monad

maybe :: b->(a->b) -> Maybe a -> b

The maybe function takes

a default value (b),

a function (a->b), and

a Maybe value (Maybe a).

If the Maybe value is Nothing,

the function returns the default value.

Otherwise, it applies the function to the value inside the Just and returns the result.

```
>>> maybe False odd (Just 3)
```

```
True
```

```
>>> maybe False odd Nothing
```

```
False
```

<https://hackage.haskell.org/package/base-4.10.0.0/docs/Data-Maybe.html>

Then Operator (>>) and **do** Statements

```
putStr "Hello" >>  
putStr " " >>  
putStr "world!" >>  
putStr "\n"
```

```
do { putStr "Hello"  
    ; putStr " "  
    ; putStr "world!"  
    ; putStr "\n" }
```

https://en.wikibooks.org/wiki/Haskell/do_notation

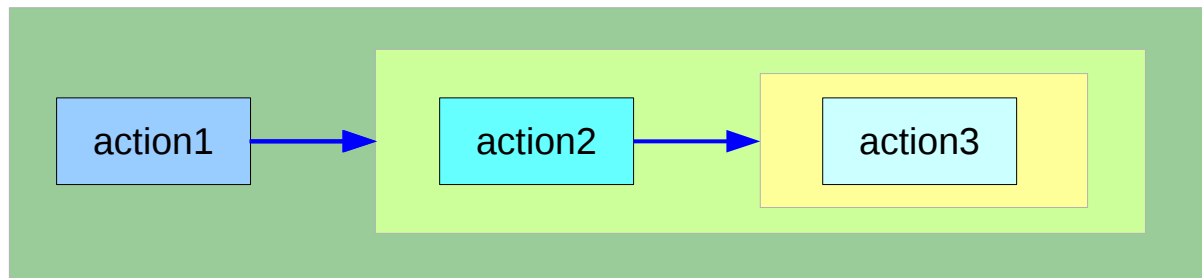
Translating in **do** notation

```
do { action1  
  ; action2  
  ; action3 }
```

```
action1 >>  
do { action2  
  ; action3 }
```

```
do { action1  
  ; do { action2  
        ; action3 } }
```

```
do { action1  
  ; do { action2  
        ; do { action3 } } }
```



can **chain** any actions
as long as all of them are
in **the same monad**

https://en.wikibooks.org/wiki/Haskell/do_notation

Bind Operator (>=) and do statements

The bind operator (>=)

passes a value (the result of an action or function),
downstream in the binding sequence.

```
action1 >= (\ x1 ->  
  action2 >= (\ x2 ->  
    mk_action3 x1 x2 ))
```

anonymous function
(lambda expression)
is used

do notation assigns a variable name
to the passed value using the <-

```
do { x1 <- action1  
  ; x2 <- action2  
  ; mk_action3 x1 x2 }
```

https://en.wikibooks.org/wiki/Haskell/do_notation

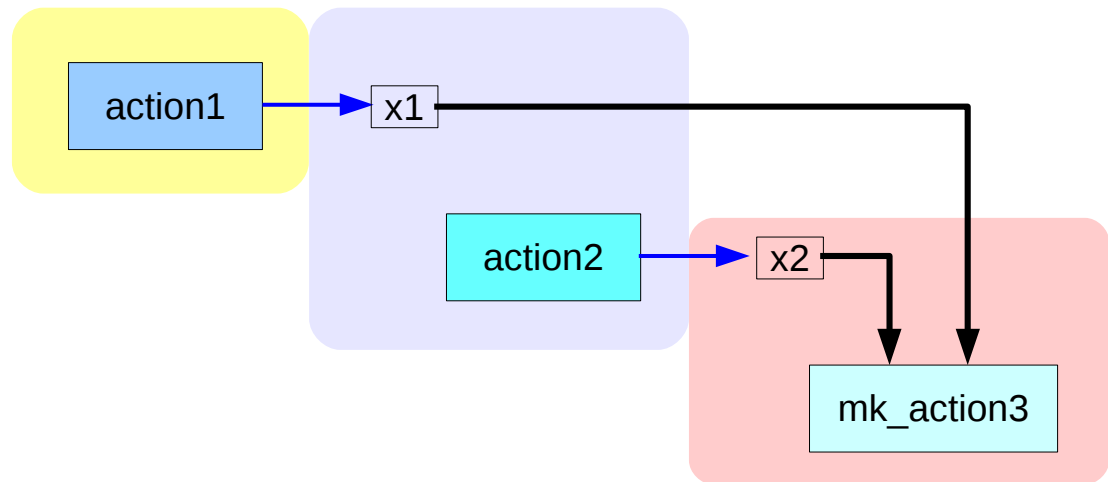
Translation using the bind operator (>>=)

```
do { x1 <- action1
    ; x2 <- action2
    ; mk_action3 x1 x2 }
```

```
action1 >>= (\ x1 -> action2 >>= (\ x2 -> mk_action3 x1 x2 ))
```

```
action1
>>=
(\ x1 -> action2
>>=
(\ x2 -> mk_action3 x1 x2 ))
```

```
action1 >>= (\ x1 ->
action2 >>= (\ x2 ->
mk_action3 x1 x2 ))
```



https://en.wikibooks.org/wiki/Haskell/do_notation

Anonymous Function

```
\x -> x + 1
```

```
(\x -> x + 1) 4
```

```
5 :: Integer
```

```
(\x y -> x + y) 3 5
```

```
8 :: Integer
```

```
addOne = \x -> x + 1
```

Lambda Expression

https://wiki.haskell.org/Anonymous_function

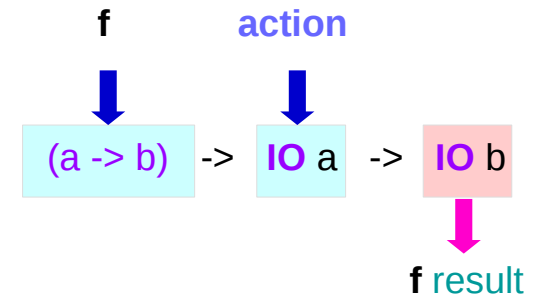
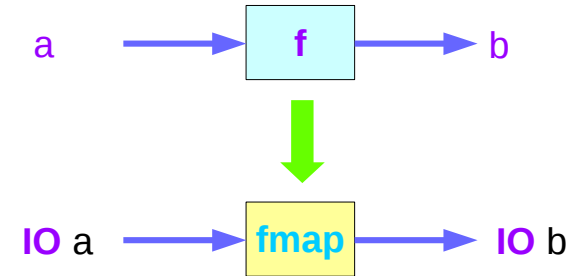
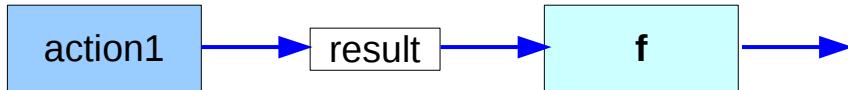
Functor Typeclass

instance Functor IO where

fmap f action = do

result <- action

return (f result)



instance Functor Maybe where

fmap func (Just x) = Just (func x)

fmap func Nothing = Nothing

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Functor Typeclass

```
main = do line <- getLine
        let line' = reverse line
            putStrLn $ "You said " ++ line' ++ " backwards!"
            putStrLn $ "Yes, you really said" ++ line' ++ " backwards!"
```

```
main = do line <- fmap reverse getLine
        putStrLn $ "You said " ++ line ++ " backwards!"
        putStrLn $ "Yes, you really said" ++ line ++ " backwards!"
```

instance Functor IO where

```
fmap f action = do
  result <- action
  return (f result)
```

```
fmap reverse getLine = do
  result <- getLine
  return (reverse result)
```

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

\$ Operator

\$ operator to avoid parentheses

Anything appearing after \$

will take precedence over anything that comes before.

```
putStrLn (show (1 + 1))
```

```
putStrLn (show $ 1 + 1)
```

```
putStrLn $ show (1 + 1)
```

```
putStrLn $ show $ 1 + 1
```

<https://stackoverflow.com/questions/940382/haskell-difference-between-dot-and-dollar-sign>

. Operator

. operator to chain functions

`putStrLn (show (1 + 1))`

`(1 + 1)` is not a function, so the `.` operator cannot be applied

`show` can take an `Int` and return a `String`.

`putStrLn` can take a `String` and return an `IO()`.

`(putStrLn . show) (1 + 1)`



`putStrLn . show $ 1 + 1`

<https://stackoverflow.com/questions/940382/haskell-difference-between-dot-and-dollar-sign>

Functor Typeclass

instance Functor **((->) r)** where

fmap **f g** = (\x -> **f** (**g** x))

A function takes any thing and returns any thing

g :: **a** -> **b**

g :: **r** -> **a**

fmap :: (**a** -> **b**) -> **f** **a** -> **f** **b**

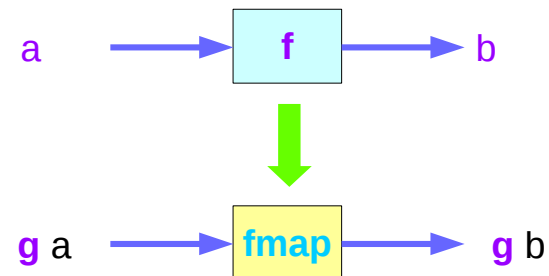
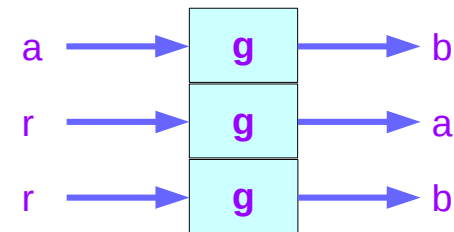
fmap :: (**a** -> **b**) -> ((->) **r** **a**) -> ((->) **r** **b**)

fmap :: (**a** -> **b**) -> (**r** -> **a**) -> (**r** -> **b**)

instance Functor **Maybe** where

fmap **f** (**Just** x) = **Just** (**f** x)

fmap **f** **Nothing** = **Nothing**



<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

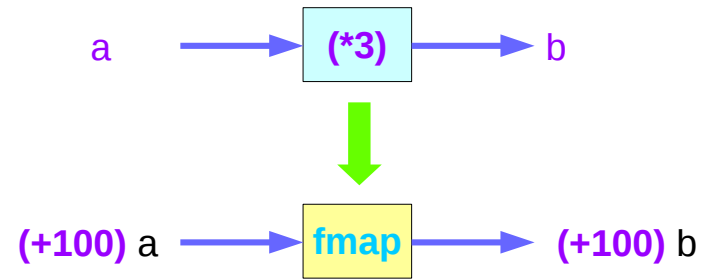
Functor Typeclass

instance Functor ((->) r) where
`fmap f g = (\x -> f (g x))`

instance Functor ((->) r) where
`fmap = (.)`

```
ghci> :t fmap (*3) (+100)
fmap (*3) (+100) :: (Num a) => a -> a
ghci> fmap (*3) (+100) 1
303
ghci> (*3) `fmap` (+100) $ 1
303
ghci> (*3) . (+100) $ 1
303
ghci> fmap (show . (*3)) (*100) 1
"300"
```

instance Functor Maybe where
`fmap f (Just x) = Just (f x)`
`fmap f Nothing = Nothing`

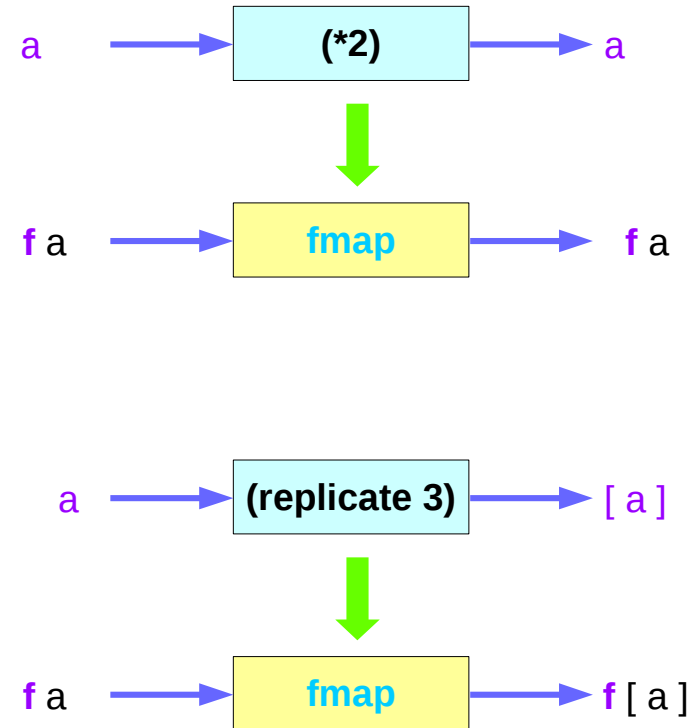


<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Functor Typeclass

```
ghci> :t fmap (*2)
fmap (*2) :: (Num a, Functor f) => f a -> f a
```

```
ghci> :t fmap (replicate 3)
fmap (replicate 3) :: (Functor f) => f a -> f [a]
```



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Functor Typeclass

```
ghci> fmap (replicate 3) [1,2,3,4]
[[1,1,1],[2,2,2],[3,3,3],[4,4,4]]
```

```
ghci> fmap (replicate 3) (Just 4)
Just [4,4,4]
```

```
ghci> fmap (replicate 3) (Right "blah")
Right ["blah","blah","blah"]
```

```
ghci> fmap (replicate 3) Nothing
Nothing
```

```
ghci> fmap (replicate 3) (Left "foo")
Left "foo"
```

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Functor Laws

`fmap id = id`

`id :: a -> a`

`id x = x`

instance Functor Maybe where

`fmap func (Just x) = Just (func x)`

`fmap func Nothing = Nothing`

instance Functor Maybe where

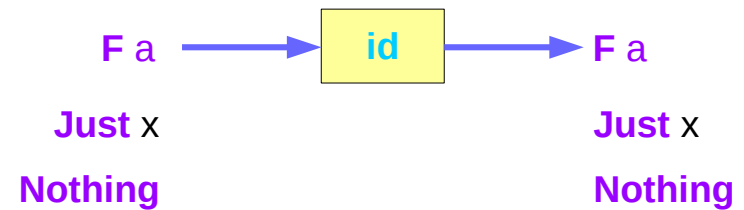
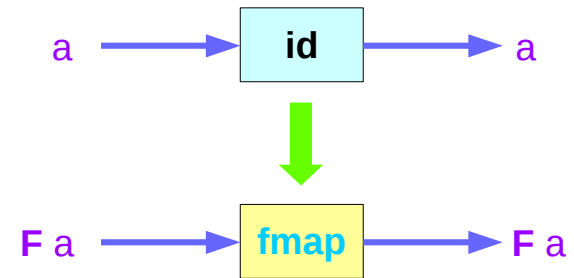
`fmap f (Just x) = Just (f x)`

`fmap f Nothing = Nothing`

instance Functor Maybe where

`fmap id (Just x) = Just (id x)`

`fmap id Nothing = Nothing`



<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Functor Typeclass

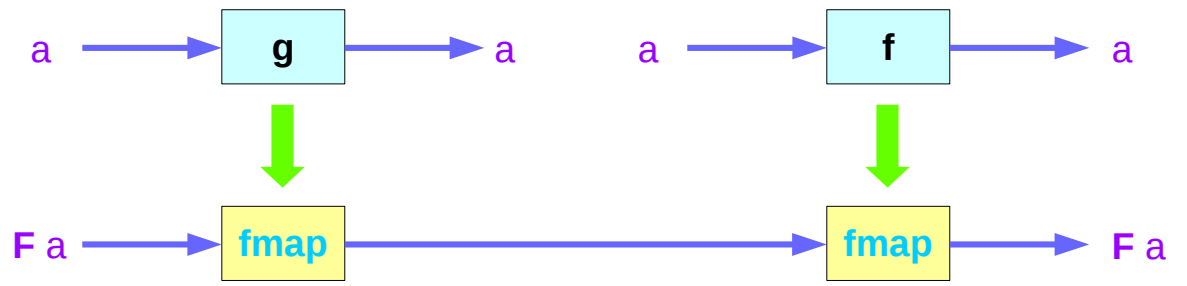
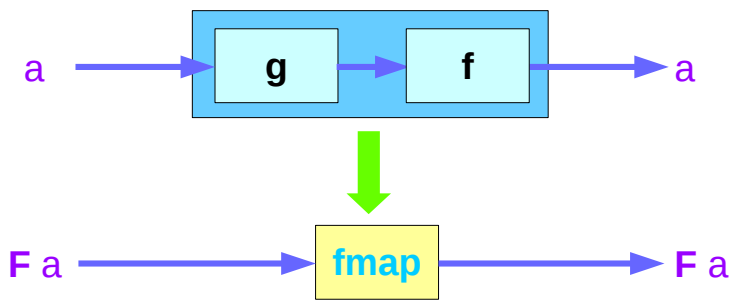
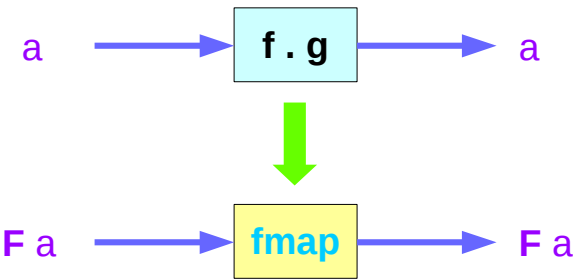
```
ghci> fmap id (Just 3)
Just 3
ghci> id (Just 3)
Just 3
ghci> fmap id [1..5]
[1,2,3,4,5]
ghci> id [1..5]
[1,2,3,4,5]
ghci> fmap id []
[]
ghci> fmap id Nothing
Nothing
```

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Functor Laws

$$\text{fmap } (f . g) = \text{fmap } f . \text{fmap } g$$

$$\text{fmap } (f . g) F = \text{fmap } f (\text{fmap } g F)$$



<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

Functor Laws

`fmap (f . g) = fmap f . fmap g`

`fmap (f . g) F = fmap f (fmap g F)`

instance Functor Maybe where

`fmap f (Just x) = Just (f x)`

`fmap f Nothing = Nothing`

`fmap (f . g) Nothing = Nothing`

`fmap f (fmap g Nothing) = Nothing`

`fmap (f . g) (Just x) = Just ((f . g) x) = Just (f (g x))`

`fmap f (fmap g (Just x)) = fmap f (Just (g x)) = Just (f (g x))`

<http://learnyouahaskell.com/functors-applicative-functors-and-monoids>

References

- [1] <ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf>
- [2] <https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf>