

Sec.14

EGM 3520 Mechanics of Materials (MoM)

Beer et al. 2012, Mechanics of Materials, McGraw-Hill.

P4.18, p.239

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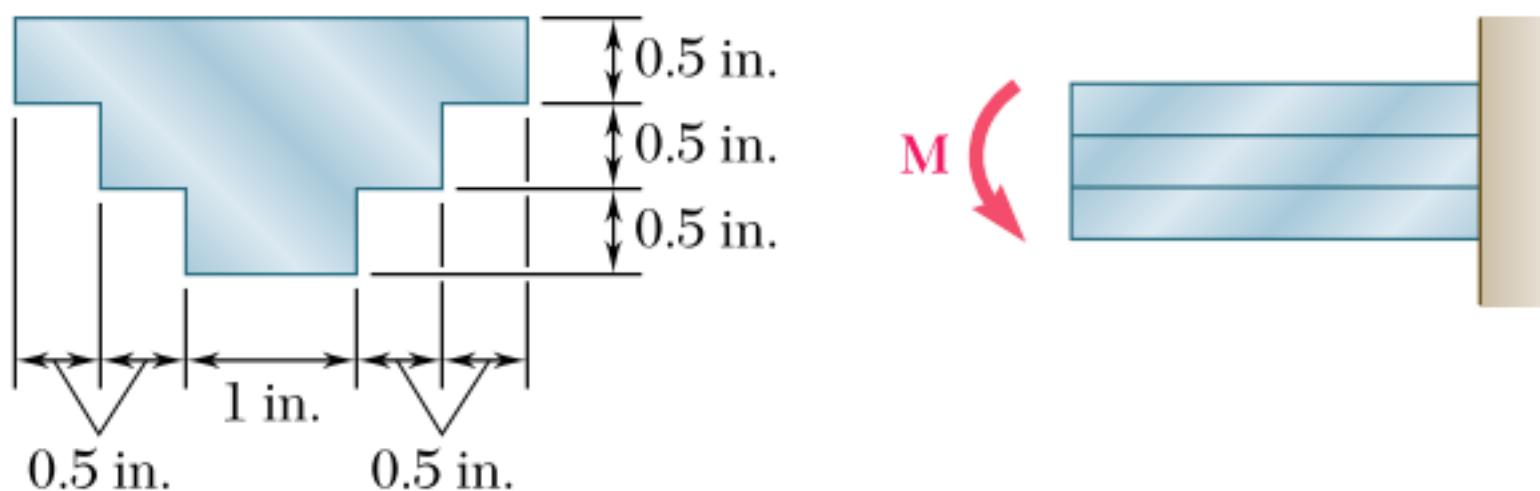


Fig. P4.18

Knowing that for the casting shown the allowable stress is 5 ksi in tension and 18 ksi in compression, determine the largest couple \mathbf{M} that can be applied.

Pause video NOW !

Work out the next step

→ **on your own first**

→ **discuss with teammates**

if you get stuck

then continue to watch the video

"Intelligence consists of this; that we recognize the similarity between different things, and the difference between similar things."

Baron de la Brède et de Montesquieu (1689-1755)

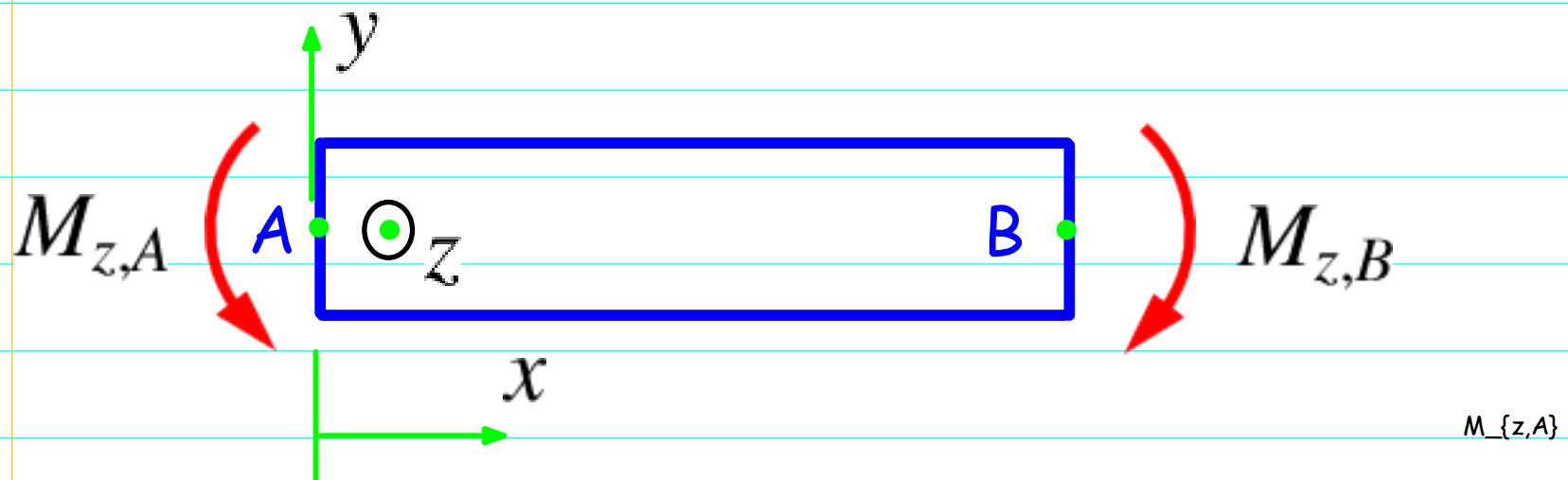
quoted in [Quantum field theory, E. Zeidler, 2008, p.175]

FBDs

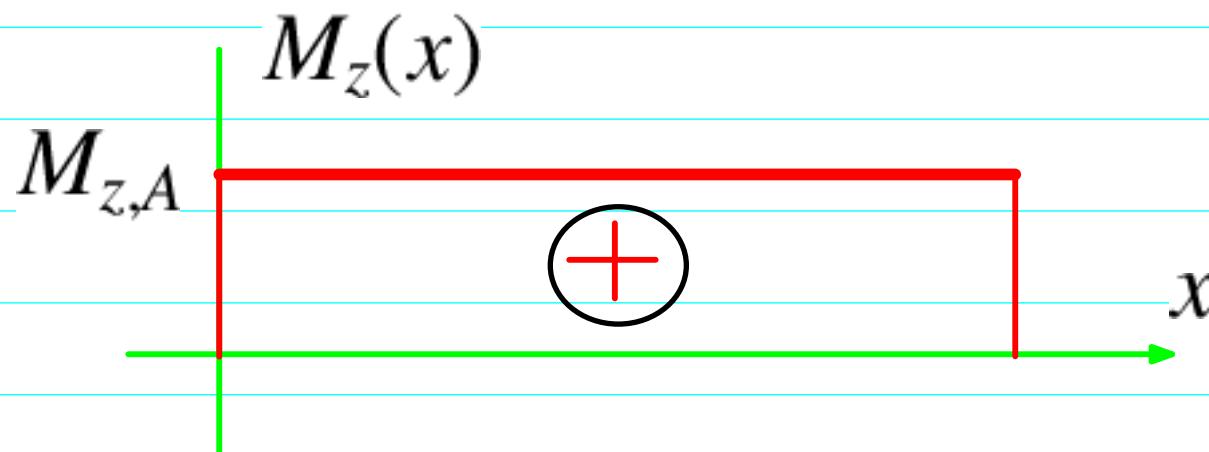


Method

FBD and bending moment diagram



Use negative curvature ("down") convention



Normal stress on beam cross section

$$\sigma_x = + \frac{M_z y}{I_z} \quad (1)$$

\sigma_x = \color{red}{+} \frac{M_z y}{I_z}

due to "down" convention

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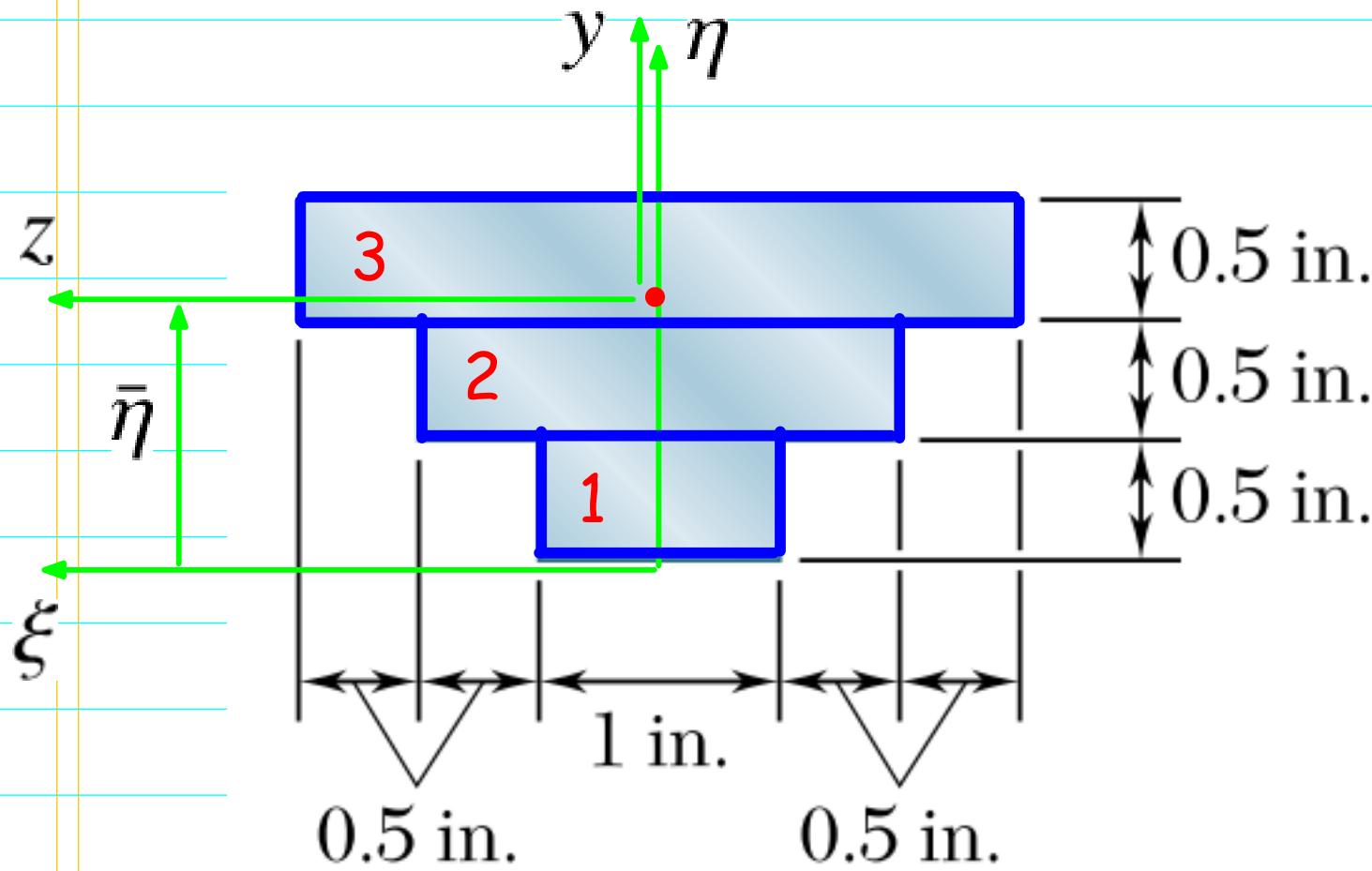
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Computation

Find the centroid of cross section



Area of cross section

$$A = A_1 + A_2 + A_3 \quad (1)$$

$(\bar{\xi}, \bar{\eta})$ coordinates of centroid

$$\bar{\xi} = 0 \quad (2)$$

$$\bar{\eta} = \frac{A_1 \bar{\eta}_1 + A_2 \bar{\eta}_2 + A_3 \bar{\eta}_3}{A} \quad (3)$$

$$\bar{\eta} = \frac{A_1 \bar{\eta}_1 + A_2 \bar{\eta}_2 + A_3 \bar{\eta}_3}{A}$$

$$A_1 = 0.5 \text{ in}^2, A_2 = 1 \text{ in}^2, A_3 = 1.5 \text{ in}^2 \quad (1)$$

$$A_1 = 0.5 \text{ in}^2, A_2 = 1 \text{ in}^2, A_3 = 1.5 \text{ in}^2$$

$$\bar{\eta}_1 = 0.25 \text{ in}, \bar{\eta}_2 = 0.75 \text{ in}, \bar{\eta}_3 = 1.25 \text{ in} \quad (2)$$

$$\bar{\eta}_1 = 0.25 \text{ in}, \bar{\eta}_2 = 0.75 \text{ in}, \bar{\eta}_3 = 1.25 \text{ in}$$

$$y = \eta - \bar{\eta} \quad (3)$$

$$y = \eta - \bar{\eta}$$

Ordinate of top surface from centroid

$$y_{top} = \eta_{top} - \bar{\eta} \quad (4)$$

$$y_{top} = \eta_{top} - \bar{\eta}$$

$$\eta_{top} = 1.5 \text{ in} \quad (5)$$

$$\eta_{top} = 1.5 \text{ in}$$

Ordinate of bottom surface from centroid

$$y_{bot} = \eta_{bot} - \bar{\eta} = 0 - \bar{\eta} = -\bar{\eta} \quad (6)$$

$$y_{bot} = \eta_{bot} - \bar{\eta} = 0 - \bar{\eta} = -\bar{\eta}$$

Normal stress at top surface (tension)

$$\sigma_{x,top} = +\frac{M_{z,A} y_{top}}{I_z} \leq \sigma_{U,tension} \quad (7)$$

$$\sigma_{x,top} = +\frac{M_{z,A} y_{top}}{I_z} \leq \sigma_{U,tension}$$

$$M_{z,A,1} = \frac{I_z \sigma_{U,tension}}{y_{top}} \quad (8)$$

$$M_{z,A,1} = \frac{I_z \sigma_{U,tension}}{y_{top}}$$

Normal stress at bottom surface (compression)

$$|\sigma_{x,bot}| = + \frac{M_{z,A} |y_{bot}|}{I_z} \leq \sigma_{U,comp} \quad (1)$$

$|\sigma_{x,bot}| = \color{red}{+} \frac{M_{z,A} |y_{bot}|}{I_z}$
 $\leq \sigma_{U,comp}$

$$M_{z,A,2} = \frac{I_z \sigma_{U,comp}}{|y_{bot}|} \quad (2)$$

$M_{z,A,2} = \frac{I_z \sigma_{U,comp}}{|y_{bot}|}$

Maximum allowable bending moment

$$M_{allow} = \min(M_{z,A,1}, M_{z,A,2}) \quad (3)$$

$M_{allow} = \min(M_{z,A,1}, M_{z,A,2})$

Find 2nd area moment of inertia

$$I_\xi = I_{\xi,1} + I_{\xi,2} + I_{\xi,3} \quad (4)$$

$I_{\xi,i} = I_{\xi,1} + I_{\xi,2} + I_{\xi,3}$

$$I_{\xi,i} = \frac{b_i (h_i)^3}{12} + A_i (\bar{\eta}_i)^2 \text{ for } i = 1, 2, 3 \quad (5)$$

$I_{\xi,i} = \frac{b_i (h_i)^3}{12} + A_i (\bar{\eta}_i)^2 \text{ for } i=1,2,3$

$$I_z = I_\xi - A (\bar{\eta})^2 \quad (6)$$

$I_z = I_\xi - A (\bar{\eta})^2$