

Angle Recoding 2. Wu

4. EEAS

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③ Extended EAS-based CORDIC

$$S_2 = \left\{ \tan^{-1} (\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*}) : \right. \\ \left. \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\}, \quad s_0^*, s_1^* \in \{0, 1, \dots, w-1\} \right\}$$

$$\theta_i = \tan^{-1} (\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)})$$

the angle quantization error

$$|\xi_{m, \text{EAS}}| \triangleq \left| \theta - \sum_{j=0}^{R_m-1} \tan^{-1} (\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)}) \right|$$

Generalized EEAS Scheme

$$S_d = \left\{ \tan^{-1} (\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*} + \dots + \alpha_{d-1}^* \cdot 2^{-s_{d-1}^*}) : \right. \\ \left. \alpha_0^*, \alpha_1^*, \dots, \alpha_{d-1}^* \in \{-1, 0, +1\}, \right. \\ \left. s_0^*, s_1^*, \dots, s_{d-1}^* \in \{0, 1, \dots, w-1\} \right\}$$

Extended EAS (EEAS) - Wu

more flexible way of decomposing the rotation angle

better

the number of iterations

the error performance

$$S_{EAS} = \{ (\bar{\Omega} \cdot \tan^{-1}(2^{-r})) : \bar{\Omega} \in \{+1, 0, -1\}, r \in \{1, 2, \dots, n-1\} \}$$

$$S_{EEAS} = \{ (\bar{\Omega}_1 \cdot \tan^{-1}(2^{-r_1}) + \bar{\Omega}_2 \cdot \tan^{-1}(2^{-r_2})) : \\ \bar{\Omega}_1, \bar{\Omega}_2 \in \{+1, 0, -1\}, r_1, r_2 \in \{1, 2, \dots, n-1\} \}$$

The pseudo-rotation
for i-th micro rotations

$$\begin{aligned} \tilde{x}_{i+1} &= x_i - [\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)}] y_i \\ \tilde{y}_{i+1} &= y_i + [\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)}] x_i \end{aligned}$$

The pseudo-rotated vector $[x_{R_m}, y_{R_m}]$
after R_m (the required number of micro-rotations)

Needs to be scaled by a factor $K = \prod K_i$

$$K_i = \left[1 + (\sigma_1(i) \cdot 2^{-r_1(i)} + \sigma_2(i) \cdot 2^{-r_2(i)})^2 \right]^{-\frac{1}{2}}$$

$$\begin{aligned} \tilde{x}_{i+1} &= \tilde{x}_i - [k_1(i) \cdot 2^{-s_1(i)} + k_2(i) \cdot 2^{-s_2(i)}] \tilde{y}_i \\ \tilde{y}_{i+1} &= \tilde{y}_i + [k_1(i) \cdot 2^{-s_1(i)} + k_2(i) \cdot 2^{-s_2(i)}] \tilde{x}_i \end{aligned}$$

$$\tilde{x}_0 = x_{R_m}$$

$$\tilde{y}_0 = y_{R_m}$$

$$k_1, k_2 \in \{-1, 0, 1\}$$

$$s_1, s_2 \in \{1, 2, \dots, n-1\}$$

- [21] C.-S. Wu, A.-Y. Wu, and C.-H. Lin, "A high-performance/low-latency vector rotational CORDIC architecture based on extended elementary angle set and trellis-based searching schemes," *IEEE Trans. Circuits Syst. II: Anal. Digital Signal Process.*, vol. 50, no. 9, pp. 589–601, Sep. 2003.

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A Unified View for Vector Rotational CORDIC Algorithms and Architectures Based on Angle Quantization Approach

An-Yeu Wu and Cheng-Shing Wu

AR : to approximate θ
with the combination
of selected angle elements
from a pre-defined EAS
(Elementary Angle Set)

EAS : all possible values of $\theta(j)$

$$\text{EAS } S_1 = \left\{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\}, s^* \in \{0, 1, \dots, N-1\} \right\}$$

EAS S_1 consists of $\tan^{-1}(\text{Single signed power of two})$
 $\tan^{-1}(\text{single SPT})$
 $\tan^{-1}(\alpha^* \cdot 2^{-s^*})$

SPT-based digital filter design
 to increase the coefficient resolution
 → employ more SPT terms to represent filter coefficients

- [12] H. Samueli, "An improved search algorithm for the design of multiplierless FIR filters with power-of-two coefficients," *IEEE Trans. Circuits Syst.*, vol. 36, pp. 1044–1047, July 1989.
- [13] Y. C. Lim, R. Yang, D. Li, and J. Song, "Signed power-of-two term allocation scheme for the design of digital filters," *IEEE Trans. Circuits Syst. II*, vol. 46, pp. 577–584, May 1999.

EAS $\$_1$ consists of \tan^{-1} (Single signed power of two)
 \tan^{-1} (Single SPT)
 $\tan^{-1}(\alpha^* \cdot 2^{-s^*})$

EAS $\$_2$ consists of \tan^{-1} (two signed power of two)
 \tan^{-1} (two SPT)
 $\tan^{-1}(\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s^*})$

Two Signed - Power - of - Two terms

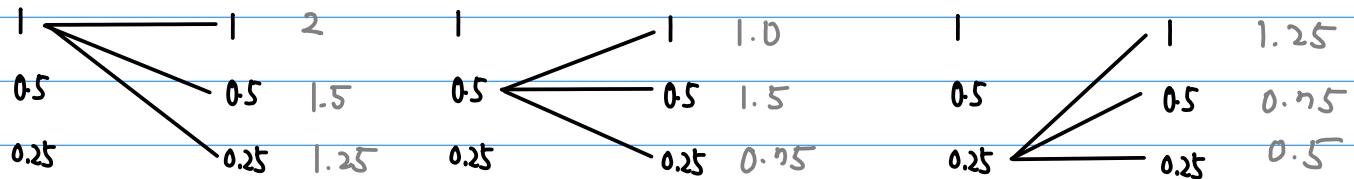
$$S_2 = \{ \tan^+ (\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*}) : \\ \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\} \\ s_0^*, s_1^* \in \{0, 1, \dots, w-1\} \}$$

S_1

$$\begin{array}{ll} 1 & 1 = 2^0 \quad \tan^{-1}(2^0) \\ 0.5 & \frac{1}{2} = 2^{-1} \quad \tan^{-1}(2^{-1}) \\ 0.25 & \frac{1}{4} = 2^{-2} \quad \tan^{-1}(2^{-2}) \end{array}$$

S_2

$$\begin{array}{ll} 2 & 1+1 = 2^0 + 2^0 \quad \pm \tan^{-1}(2^0 + 2^0) \\ 1.5 & 1+\frac{1}{2} = 2^0 + 2^1 \quad \pm \tan^{-1}(2^0 + 2^1) \\ 1.25 & 1+\frac{1}{4} = 2^0 + 2^2 \quad \pm \tan^{-1}(2^0 + 2^2) \\ 1.0 & 1 = 2^0 \quad \pm \tan^{-1}(2^0) \\ 0.75 & \frac{1}{2} + \frac{1}{4} = 2^1 + 2^2 \quad \pm \tan^{-1}(2^1 + 2^2) \\ 0.5 & \frac{1}{2} = 2^1 \quad \pm \tan^{-1}(2^1) \\ 0.25 & \frac{1}{4} = 2^2 \quad \pm \tan^{-1}(2^2) \end{array}$$



$$2^0, 2^1, 2^2$$

$$\{0, 1, 2\} = \{0, 1, w-1\}$$

$$w=3$$

$$S_0^*, S_1^* \in \{0, 1, 2\}$$

$$2^{S_0^*}, 2^{S_1^*} \in \{2^0, 2^1, 2^2\}$$

as the word size w increases,
the size of the set S_2 increases exponentially

$$\theta_i = \tan^{-1} (\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)})$$

R_m : the number of the subangle N_A

$$S_2 = \{ \theta_i \mid i = 0, 1, \dots, R_m \}$$

$$S_2 = \{ \tan^{-1} (\alpha_0^* \cdot 2^{-s_0^*} + \alpha_1^* \cdot 2^{-s_1^*}) : \\ \alpha_0^*, \alpha_1^* \in \{-1, 0, +1\} \\ s_0^*, s_1^* \in \{0, 1, \dots, w-1\} \}$$

the optimization problem of the EEAS-based CORDIC algorithm

given θ and R_m

find $\alpha_0(j)$, $\alpha_1(j)$, $s_0(j)$, and $s_1(j)$

the combination of elementary angles
from EEAS S_2

minimize the angle quantization error

$$|\xi_{m, \text{EEAS}}| \triangleq \theta - \sum_{j=0}^{R_m-1} \tan^{-1} (\alpha_0(j) 2^{-s_0(j)} + \alpha_1(j) 2^{-s_1(j)})$$

EAS S₁

elementary angle

r(1) = atan(-2^{\{-0\}})
r(2) = atan(-2^{\{-1\}})
r(3) = atan(-2^{\{-2\}})
r(4) = atan(0)
r(5) = atan(2^{\{-2\}})
r(6) = atan(2^{\{-1\}})
r(7) = atan(2^{\{0\}})



EAS S₂

r(1) = atan(-2^{\{-0\}} - 2^{\{-0\}})
r(2) = atan(-2^{\{-0\}} - 2^{\{-1\}})
r(3) = atan(-2^{\{-0\}} - 2^{\{-2\}})
r(4) = atan(-2^{\{-0\}})
r(5) = atan(-2^{\{-1\}} - 2^{\{-2\}})
r(6) = atan(-2^{\{-1\}})
r(7) = atan(-2^{\{-2\}})
r(8) = atan(0)
r(9) = atan(2^{\{-2\}})
r(10) = atan(2^{\{-1\}})
r(11) = atan(2^{\{-1\}} + 2^{\{-2\}})
r(12) = atan(2^{\{-0\}})
r(13) = atan(2^{\{-0\}} + 2^{\{-2\}})
r(15) = atan(2^{\{-0\}} + 2^{\{-1\}})
r(16) = atan(2^{\{-0\}} + 2^{\{0\}})

given $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix}$

$$\begin{bmatrix} x(j+1) \\ y(j+1) \end{bmatrix} = \begin{bmatrix} 1 & \alpha_0(j) 2^{-s_0(j)} + \alpha_1(j) 2^{-s_1(j)} \\ \alpha_0(j) 2^{-s_0(j)} + \alpha_1(j) 2^{-s_1(j)} & 1 \end{bmatrix} \begin{bmatrix} x(j) \\ y(j) \end{bmatrix}$$

$$\begin{bmatrix} x_f \\ y_f \end{bmatrix} = P \begin{bmatrix} x(R_m) \\ y(R_m) \end{bmatrix} = \frac{1}{\prod_{j=0}^{n-1} \sqrt{1 + [\alpha_0(j) \cdot 2^{-s_0(j)} + \alpha_1(j) \cdot 2^{-s_1(j)}]^2}} \begin{bmatrix} x(R_m) \\ y(R_m) \end{bmatrix}$$

Micro rotation procedure
the scaling operation

4 additions

increased hardware
reduced iteration steps

$W = 16$

elementary angle

$r(1) = \text{atan}(2^{-1})$
 $r(2) = \text{atan}(2^{-2})$
 $r(3) = \text{atan}(2^{-3})$
 $r(4) = \text{atan}(2^{-4})$
 $r(5) = \text{atan}(2^{-5})$
 $r(6) = \text{atan}(2^{-6})$
 $r(7) = \text{atan}(2^{-7})$
 $r(8) = \text{atan}(2^{-8})$
 $r(9) = \text{atan}(2^{-9})$
 $r(10) = \text{atan}(2^{-10})$
 $r(11) = \text{atan}(2^{-11})$
 $r(12) = \text{atan}(2^{-12})$
 $r(13) = \text{atan}(2^{-13})$
 $r(14) = \text{atan}(2^{-14})$
 $r(15) = \text{atan}(2^{-15})$
 $r(16) = \text{atan}(0)$
 $r(17) = \text{atan}(2^{-15})$
 $r(18) = \text{atan}(2^{-14})$
 $r(19) = \text{atan}(2^{-13})$
 $r(20) = \text{atan}(2^{-12})$
 $r(21) = \text{atan}(2^{-11})$
 $r(22) = \text{atan}(2^{-10})$
 $r(23) = \text{atan}(2^{-9})$
 $r(24) = \text{atan}(2^{-8})$
 $r(26) = \text{atan}(2^{-7})$
 $r(27) = \text{atan}(2^{-6})$
 $r(28) = \text{atan}(2^{-4})$
 $r(29) = \text{atan}(2^{-4})$
 $r(30) = \text{atan}(2^{-3})$
 $r(31) = \text{atan}(2^{-2})$
 $r(32) = \text{atan}(2^{-1})$

