Capacitor in an AC circuit

Young Won Lim
08/02/2017

Copyright (c) 2011 – 2017 Young W. Lim.

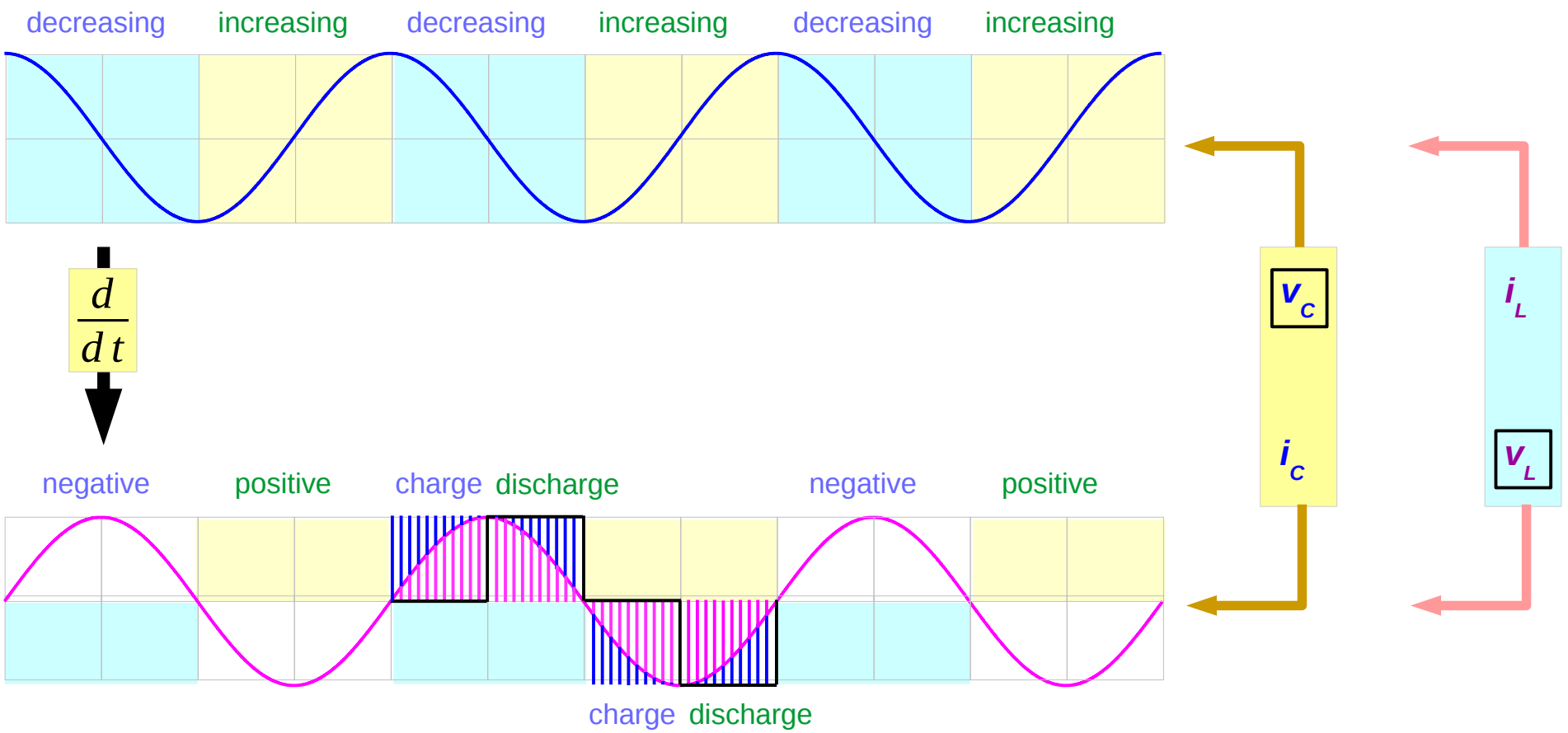
 Permission is granted to copy, distribute and/or modify this document under the terms of the GNU Free Documentation License, Version 1.2 or any later version published by the Free Software Foundation; with no Invariant Sections, no Front-Cover Texts, and no Back-Cover Texts. A copy of the license is included in the section entitled "GNU Free Documentation License".

Please send corrections (or suggestions) to youngwlim@hotmail.com.

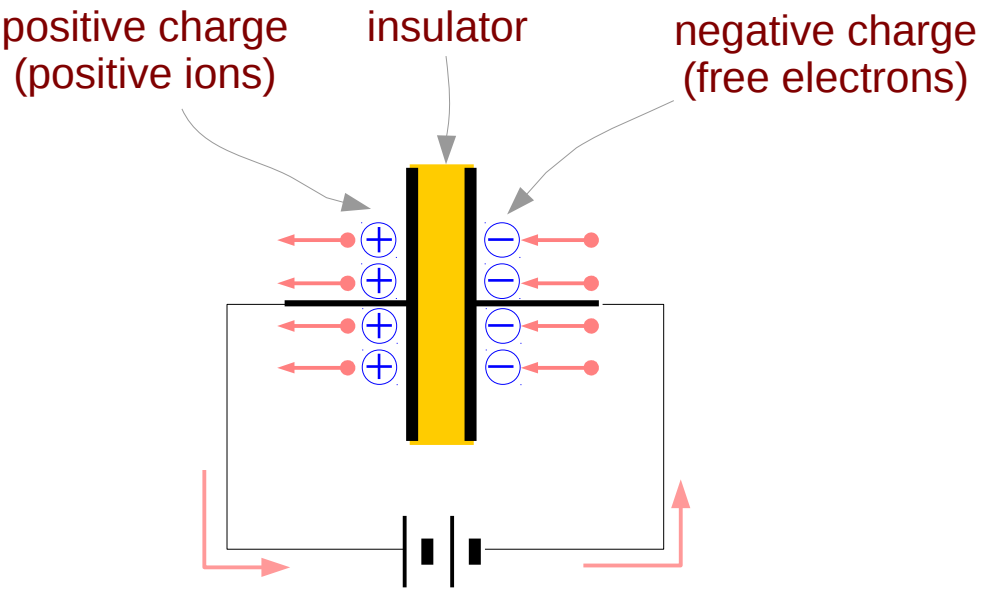
This document was produced by using OpenOffice and Octave.

Young Won Lim
08/02/2017

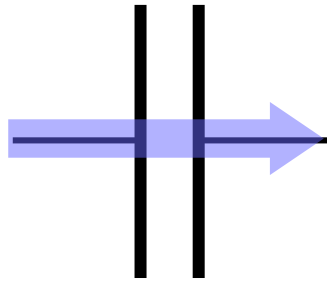
Everchanging signal pairs



Capacitor Current



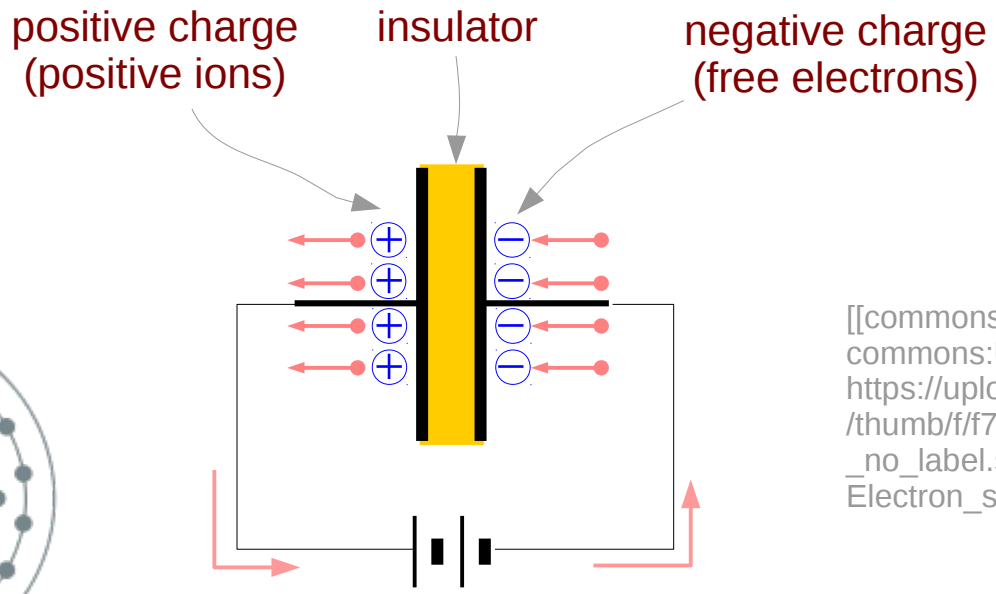
No actual electrons movement
across insulator materials

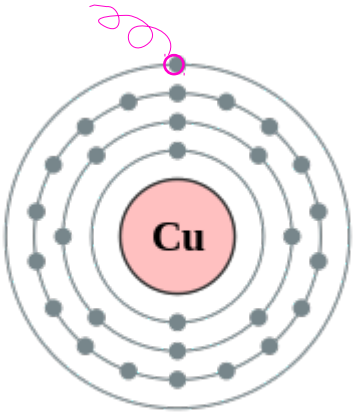


But, think as **Displacement Current**

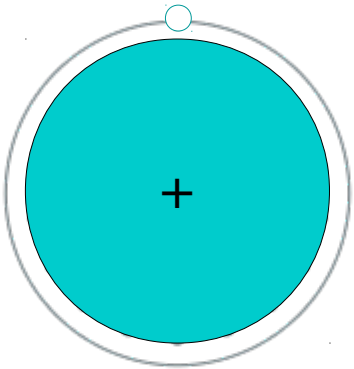
flows through the capacitor

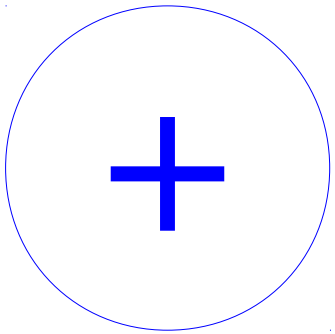
Positive ions and free electrons

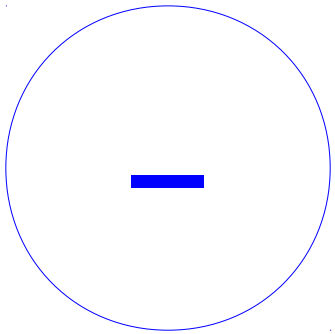




[[commons:User crap]] (original work by commons:User:Greg Robson) https://upload.wikimedia.org/wikipedia/commons/thumb/f/f7/Electron_shell_029_Copper_-_no_label.svg/200px-Electron_shell_029_Copper_-_no_label.svg.png





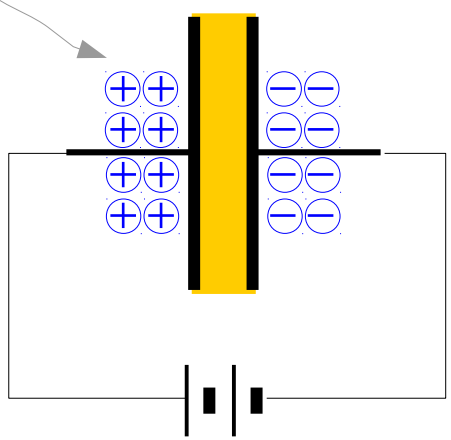


Three States

positive charge
(positive ions)

Positively Charged State

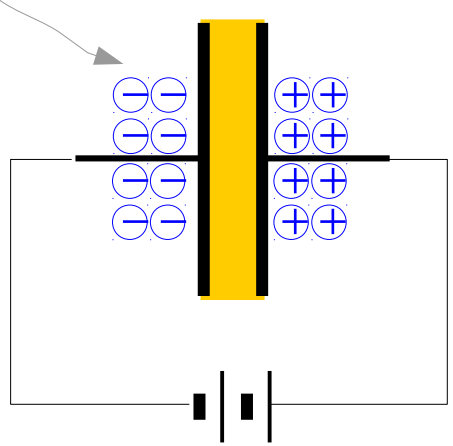
fully charged → no current

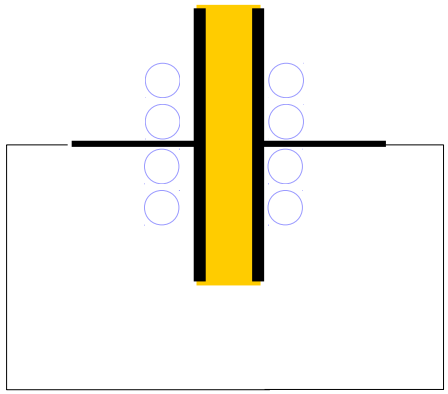


negative charge
(free electrons)

Negatively Charged State

fully charged → no current



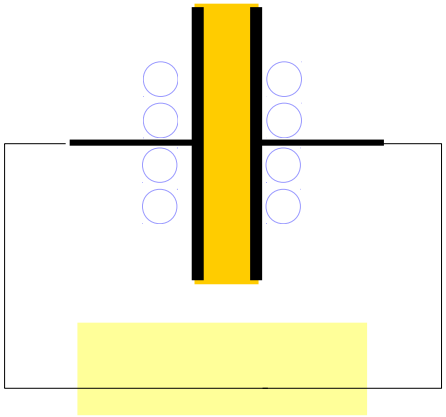


Fully Discharged State

possible large current

Currents in the Fully Discharged State

Initially no current

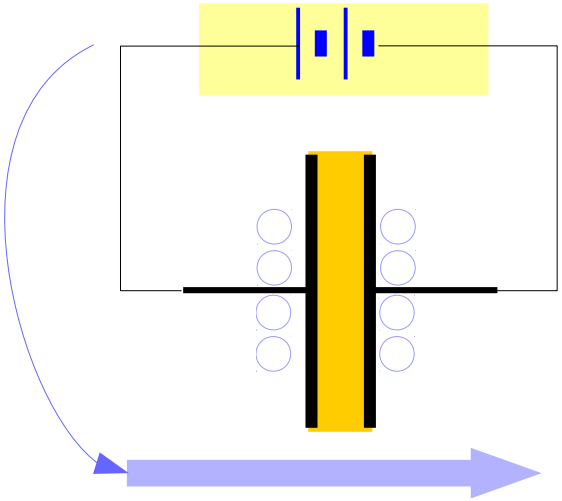


Fully Discharged State

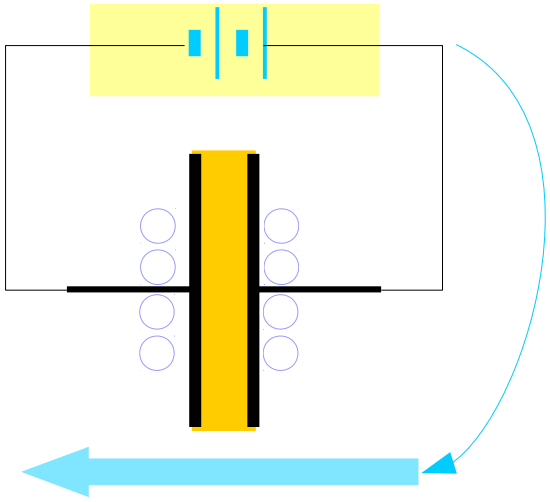
Fully Discharged State

This state can flow
large current
in either direction

Fully Discharged State



large current

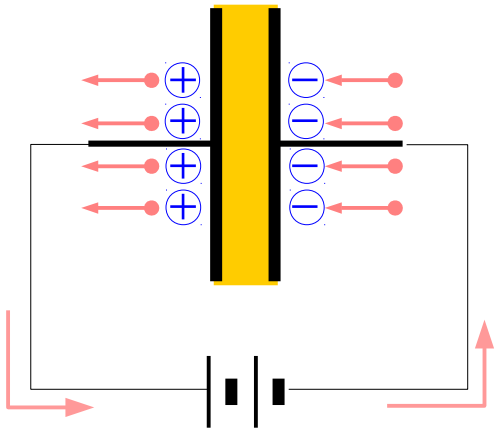


large current

Inter-State Current Flowing

Under Positively Charging

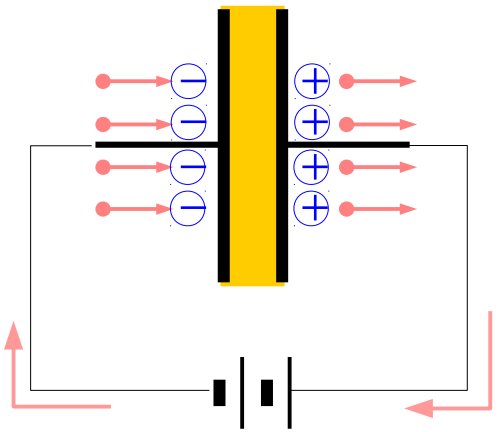
(+) current flow direction



electron flow direction

Under Negatively Charging

(–) current flow direction

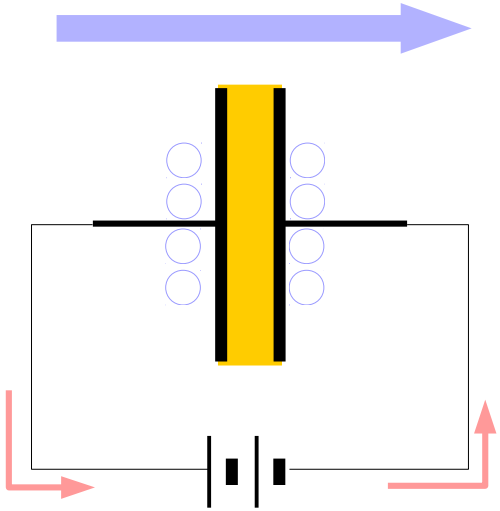


electron flow direction

Inter-State Current Flowing

Fully Discharged State

(+) current flow direction

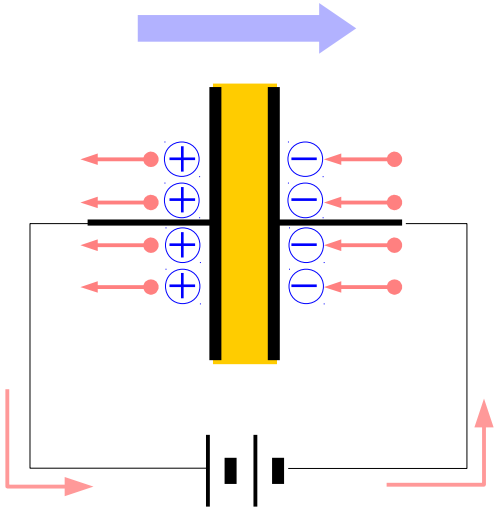


electron flow direction

Initial large current

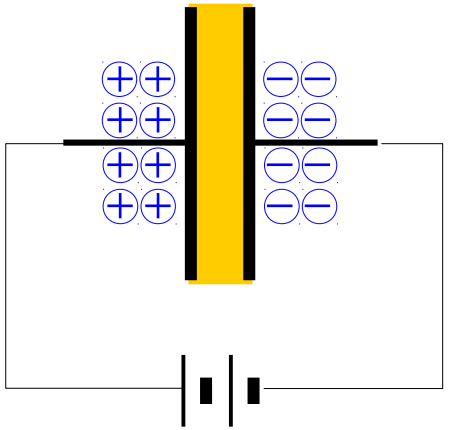
Under Positively Charging

(+) current flow direction



electron flow direction

Positively Charged State



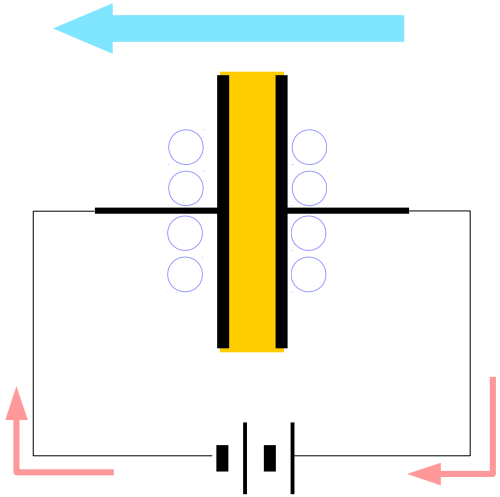
Crowded →
No more space

no current

Inter-State Current Flowing

Fully Discharged State

(–) current flow direction

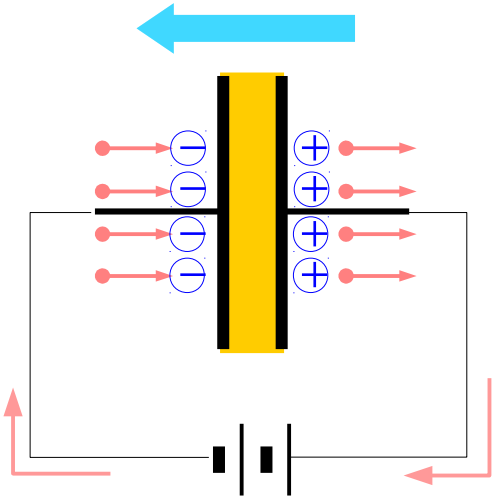


electron flow direction

Initial large current

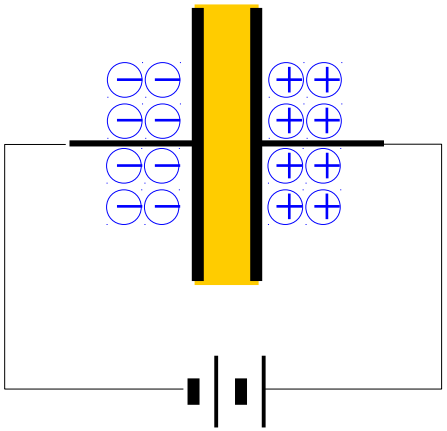
Under Negatively Charging

(–) current flow direction



electron flow direction

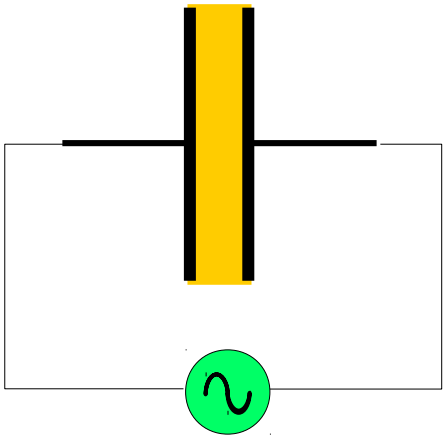
Negatively Charged State

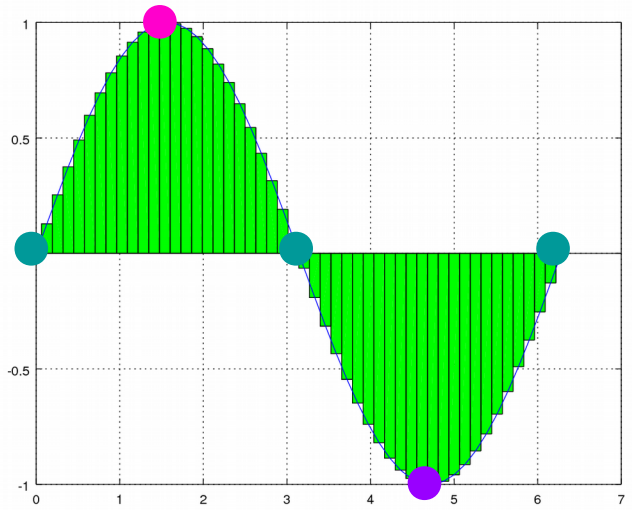


Crowded →
No more space

no current

An AC Voltage Source





An AC Voltage Source

Fully Discharged State

Under Positively Charging

Positively Charged State

Under Negatively Charging

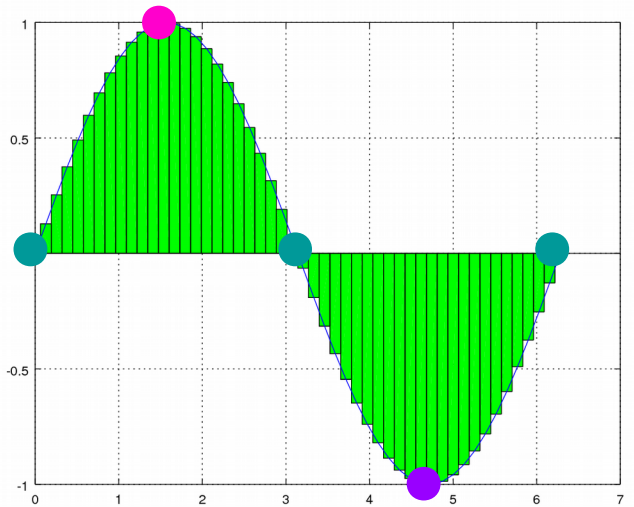
Fully Discharged State

Under Negatively Charging

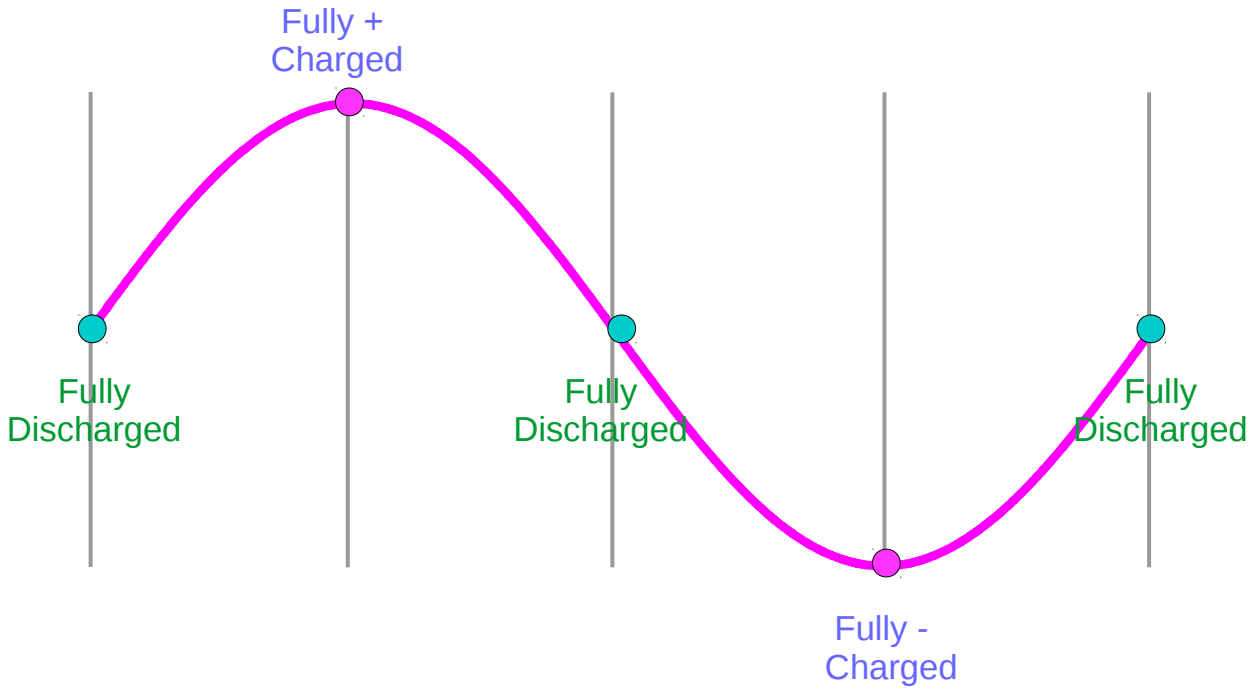
Negatively Charged State

Under Positively Charging

Fully Discharged State

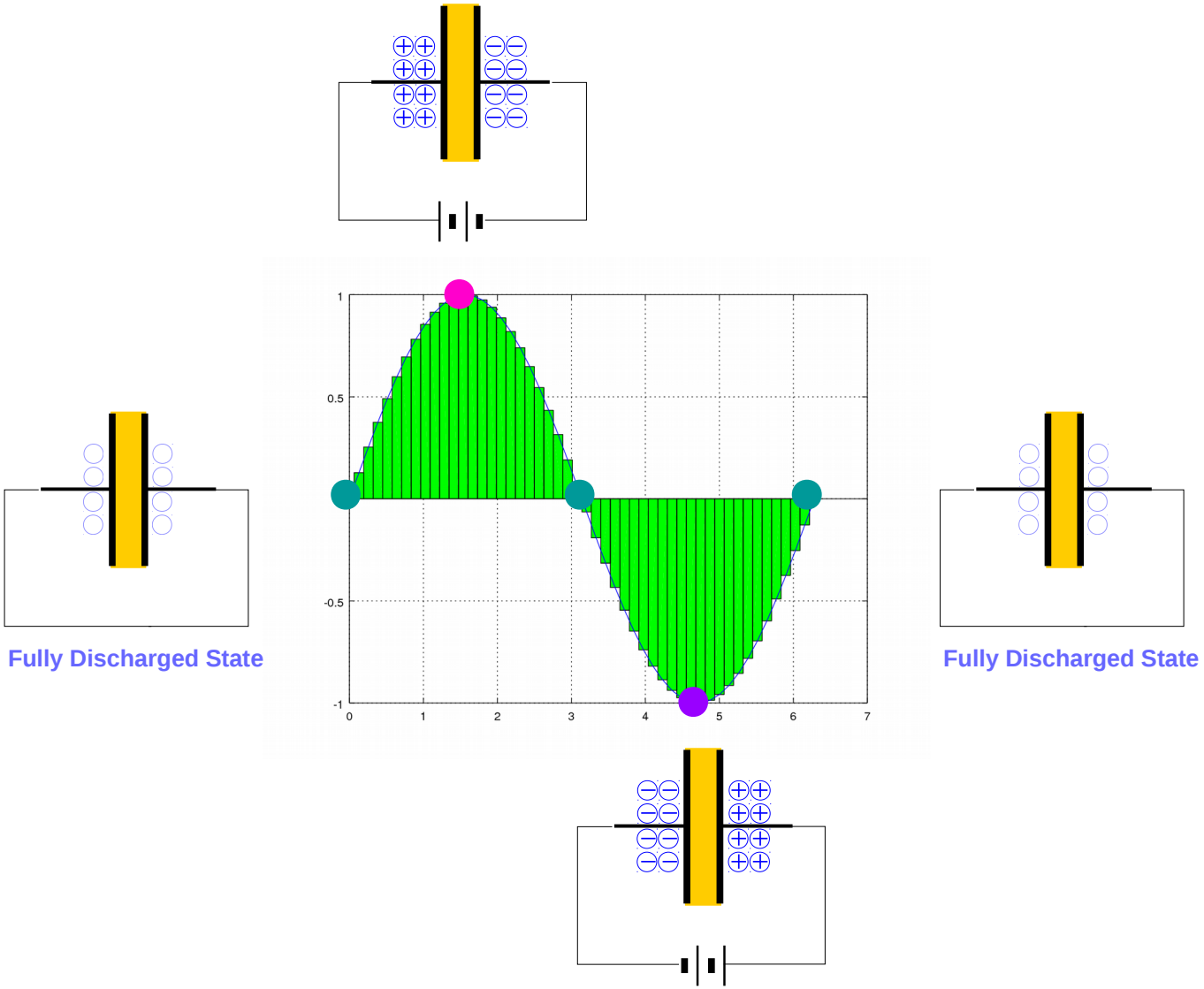


Fully Charged and Fully Discharged

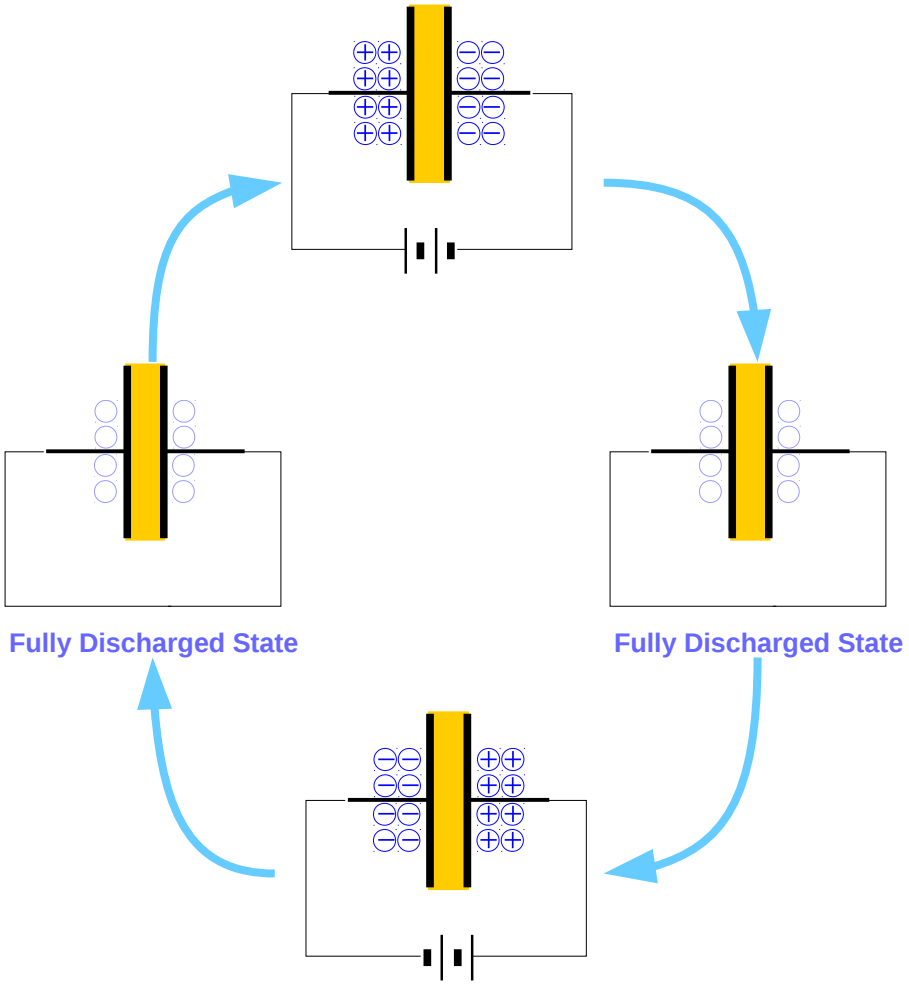


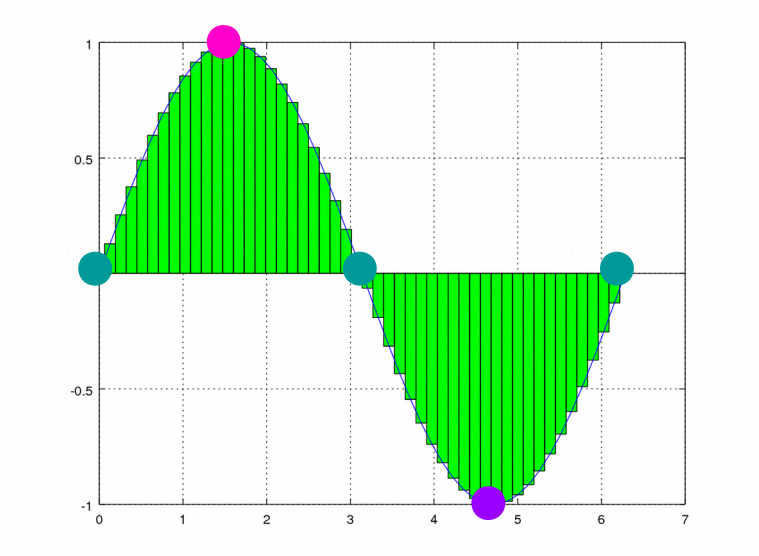
(+) Charging	(–) Charging	(–) Charging	(+) Charging
(+) current	(–) current	(–) current	(+) current
(+) Charging	(+) **Dis**charging	(–) Charging	(–) **Dis**charging

A Cycle

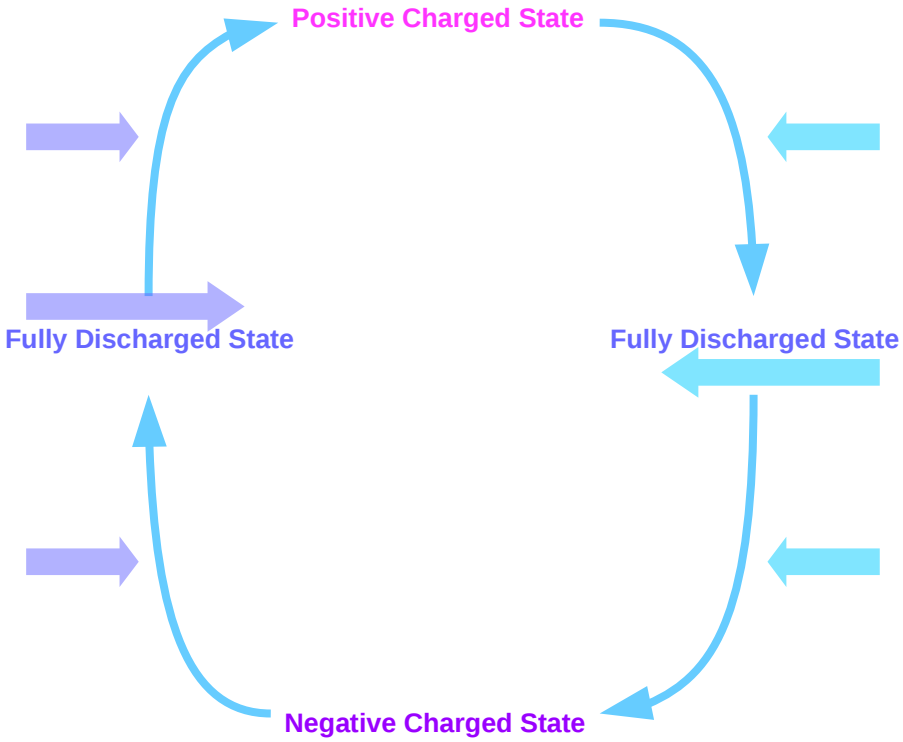

Fully Discharged State
Fully Discharged State
1
0.5
-0.5
-1
0
1
2
3
4
5
6
7

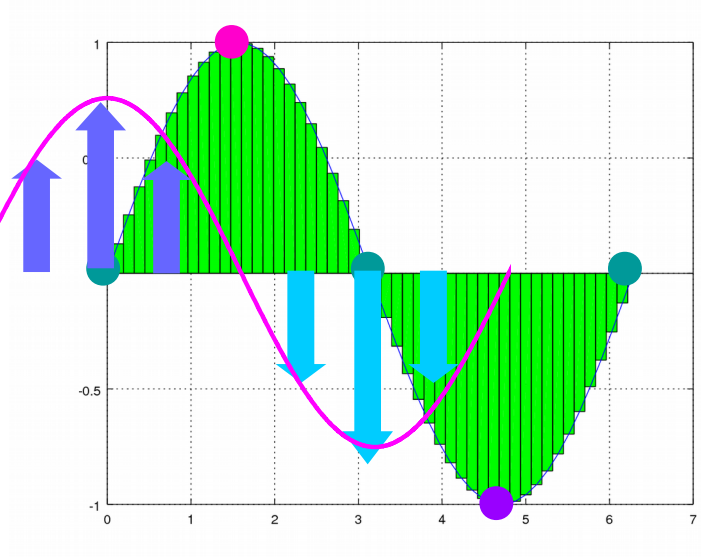
State Transition Diagram





Current Flow

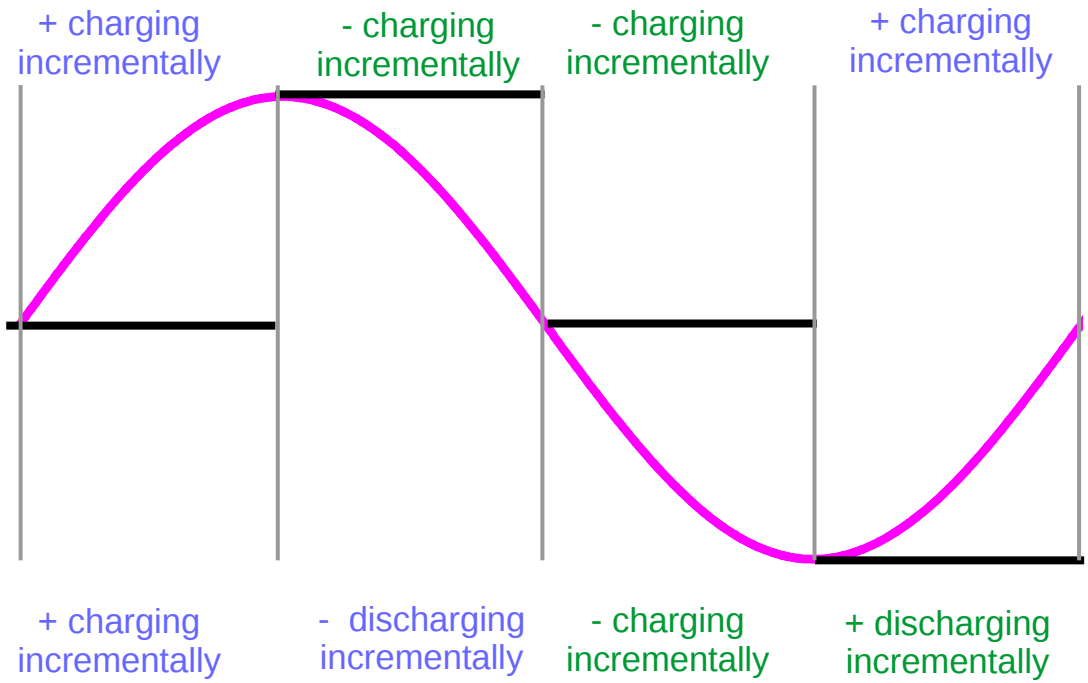




Continuous Charing and Discharging Operations

Incremental Voltage Increment ➡ + Charging incrementally

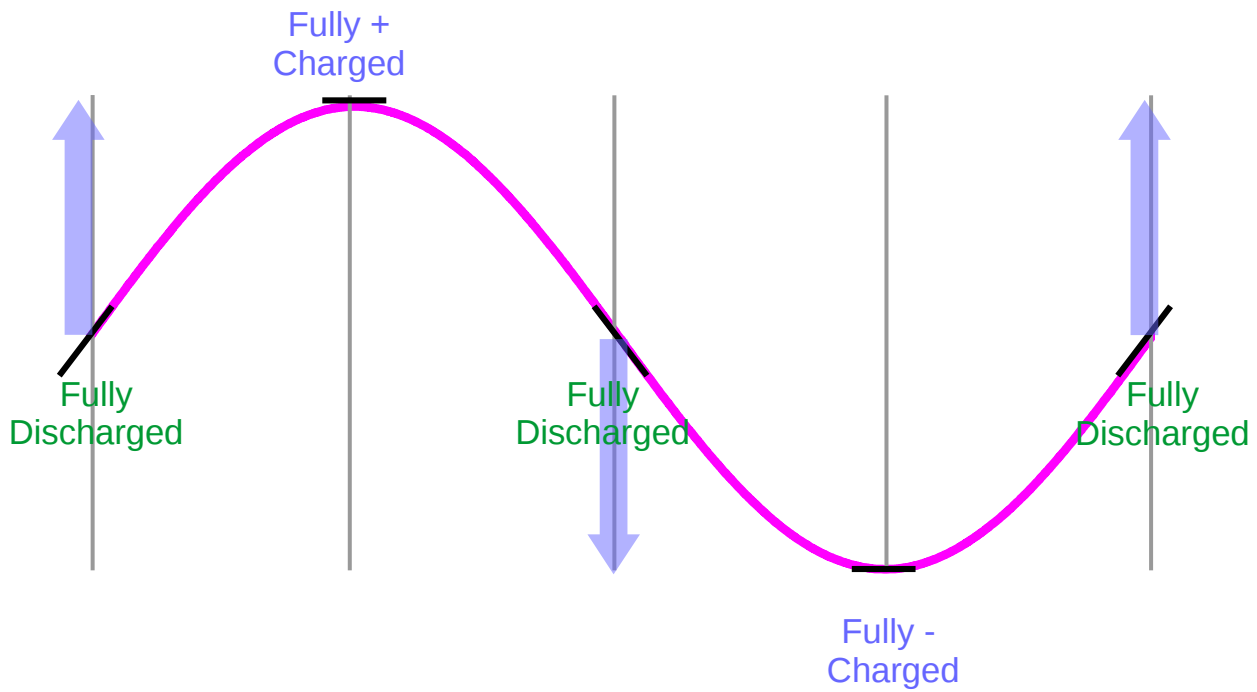
Incremental Voltage Decrement ➡ – Charging incrementally



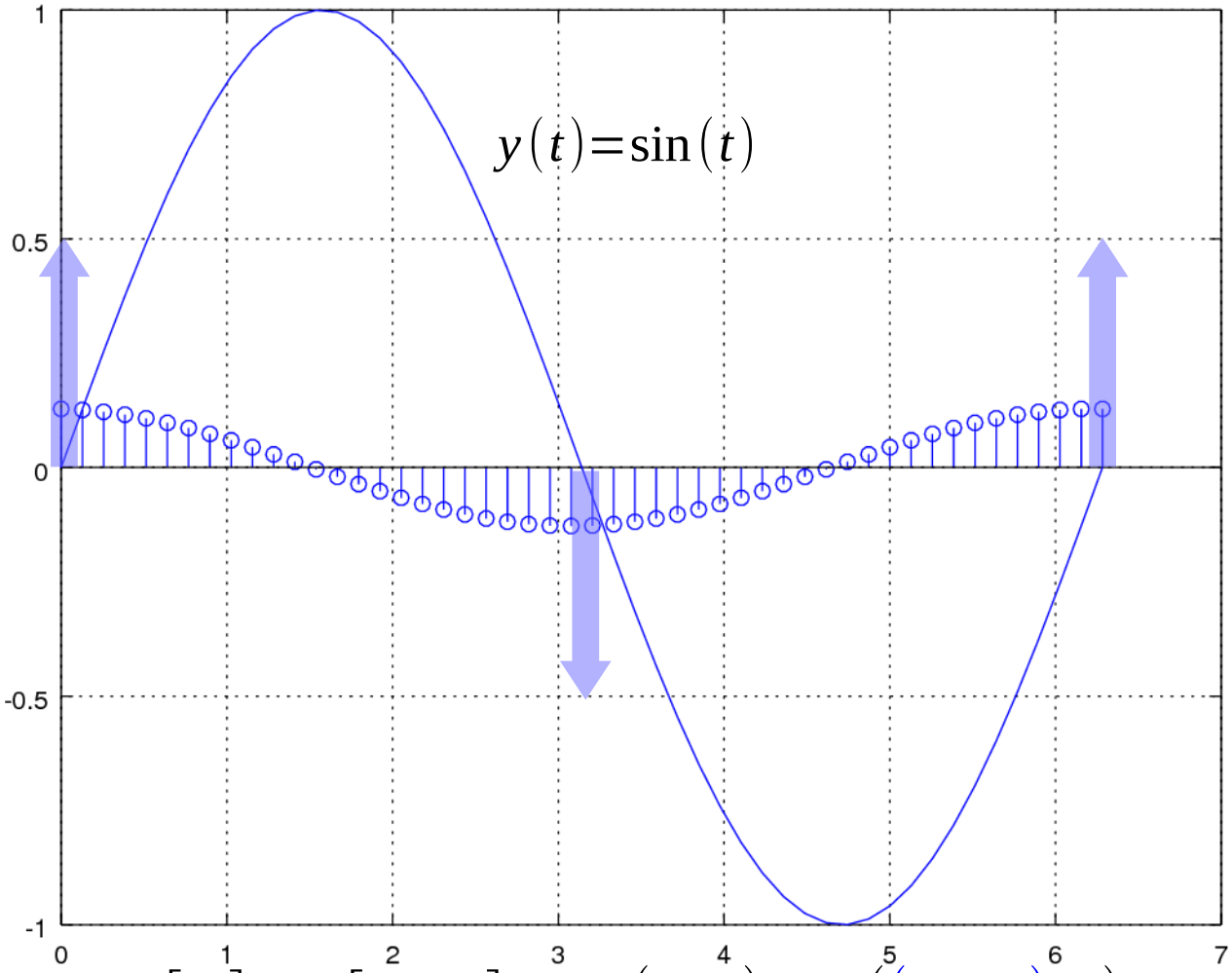
Fully Discharged : Large Current

Incremental Voltage Increment ➡ Continuous Charging

Incremental Voltage Decrement ➡ Continuous Discharging



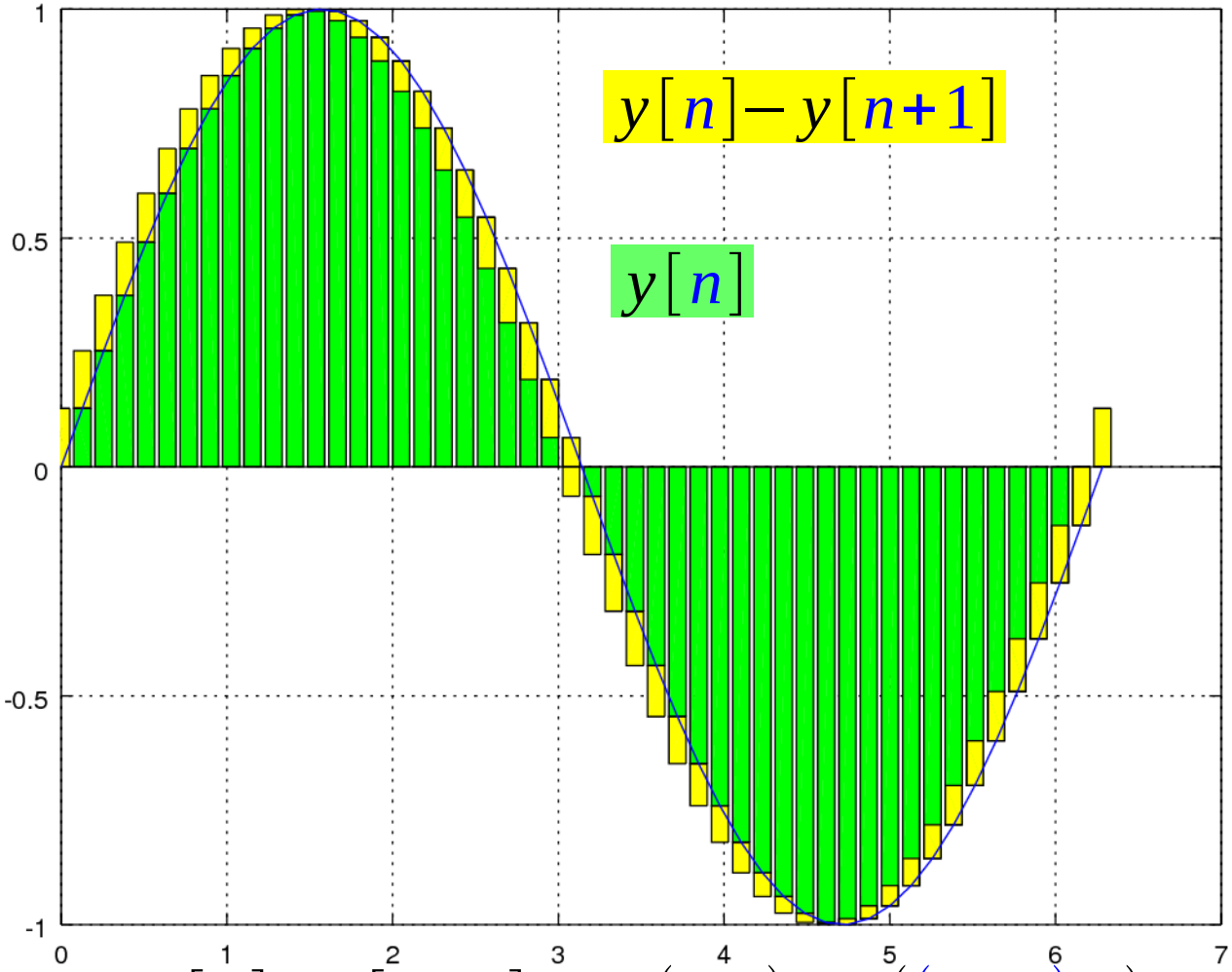
y[n+1] – y[n]




```
t = linspace(0, pi*2, 50);
t1 = t;
t2 = t + t(2);
y1 = sin(t1);
y2 = sin(t2) - sin(t1);
stem(t1, y2)
hold on
plot(t1, y1)
```

$$y[n]-y[n+1]=y(nT)-y((n+1)T)=\sin(nT)-\sin((n+1)T)$$

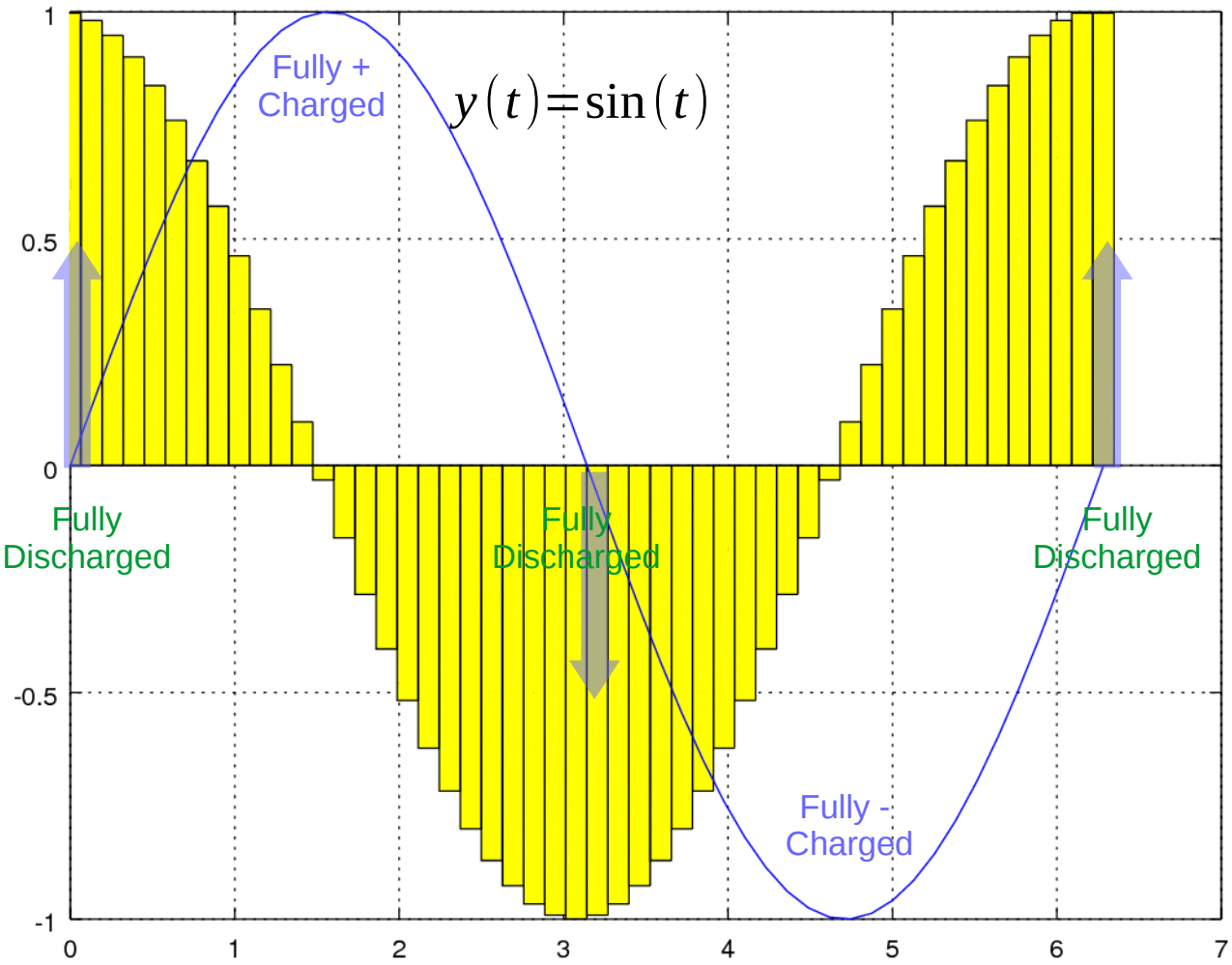
Fully Charged and Fully Discharged




```
 h = bar(t1, [y1' y2'], "stacked")
set(h(1), "facecolor", "g");
set(h(2), "facecolor", "y");
hold on
plot(t1, y1)
axis([0 7 -1 1]);
```

$$y[n]-y[n+1] = y(nT)-y((n+1)T)=\sin(nT)-\sin((n+1)T)$$

Fully Charged and Fully Discharged

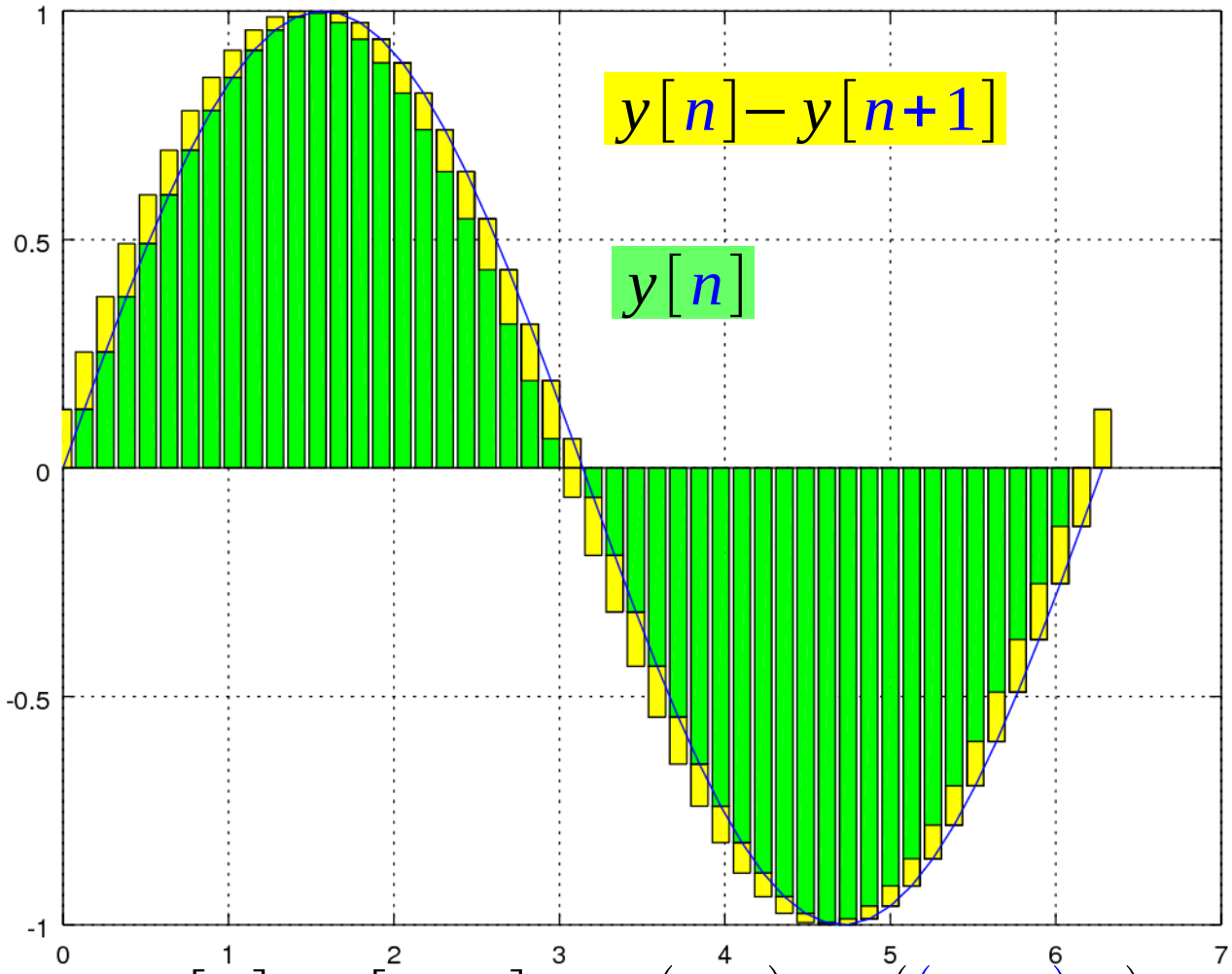



```
h = bar(t1, y2/t(2), "hist")
set(h(1), "facecolor", "y");
hold on
plot(t1, y1)
axis([0 7 -1 1]);
```

$$\frac{y[n]-y[n+1]}{T}$$

$$\propto \frac{dy}{dt}$$

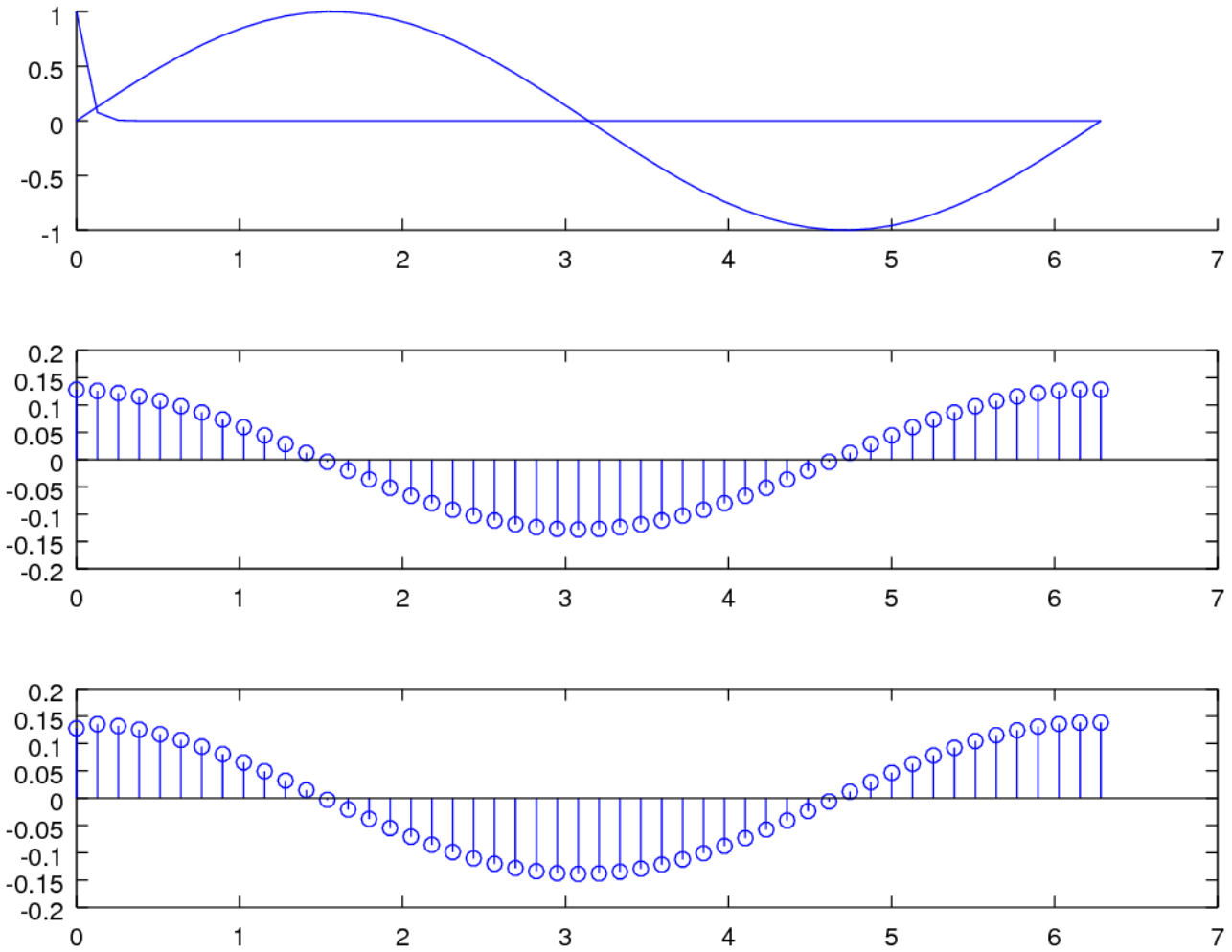
Fully Charged and Fully Discharged




```
 h = bar(t1, [y1' y2'], "stacked")
set(h(1), "facecolor", "g");
set(h(2), "facecolor", "y");
hold on
plot(t1, y1)
axis([0 pi]);
```

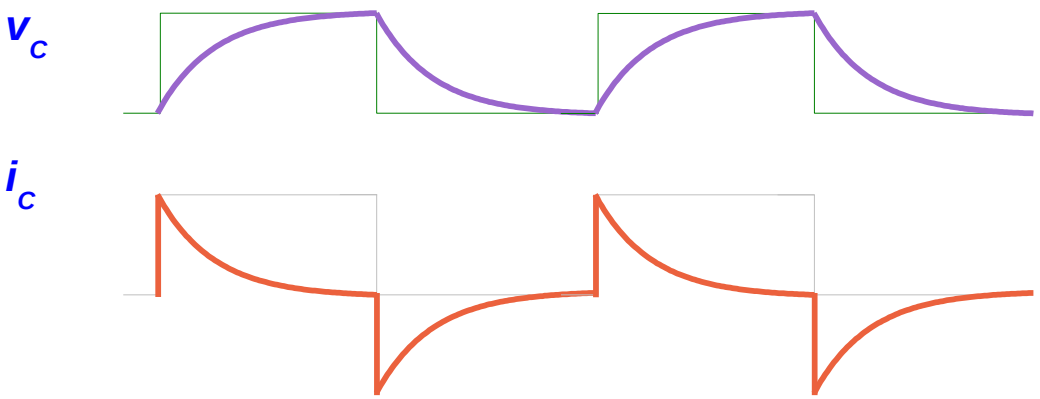
$$y[n]-y[n+1] = y(nT)-y((n+1)T)=\sin(nT)-\sin((n+1)T)$$

Fully Charged and Fully Discharged



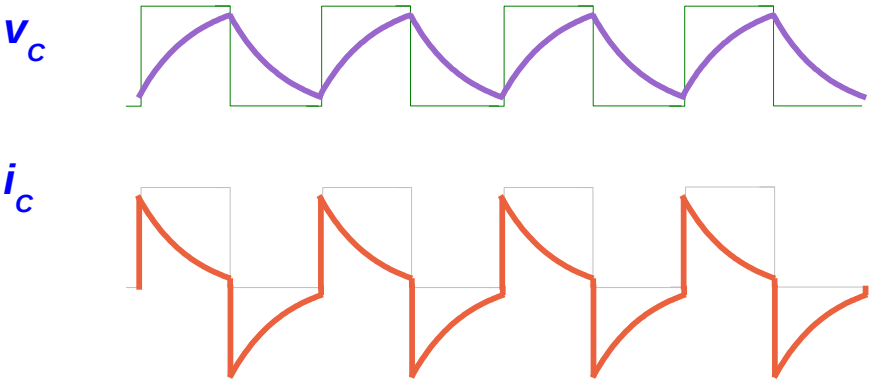

```
clf
t = linspace(0, pi*2, 50);
t1 = t;
t2 = t + t(2);
y1 = sin(t1);
y2 = sin(t2) - sin(t1);
y3 = e.^(-20*t);
y4 = conv(y2, y3);
y5 = y4([1:length(t1)]);

subplot(3, 1, 2);
stem(t1, y2)
subplot(3, 1, 1);
hold on
plot(t1, y1);
plot(t1, y3);
subplot(3, 1, 3);
stem(t1, y5);
```

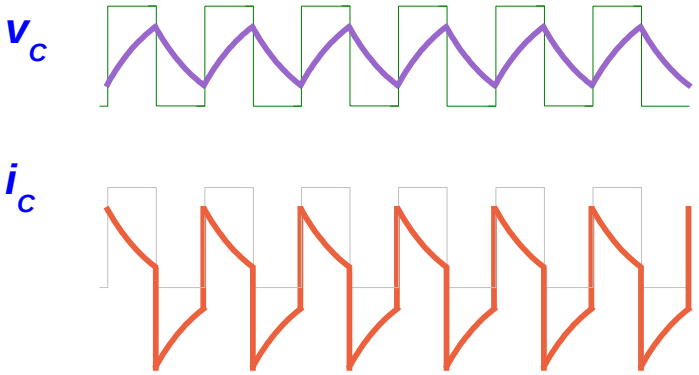
Pulse



$$i_C = C \frac{d v_C}{d t}$$

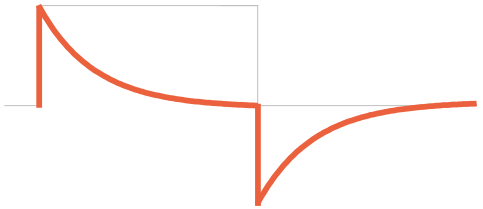
ω ↑ i_c ↑ x_c ↓



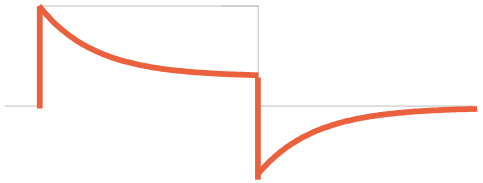


Time Constants

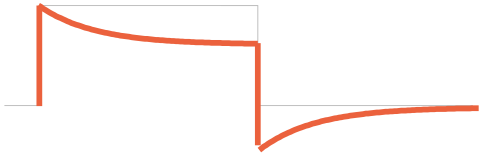
i_c



$\tau = RC$ *small time constant*



$\tau = RC$ *medium time constant*



$\tau = RC$ *large time constant*

Time Constants

i_c

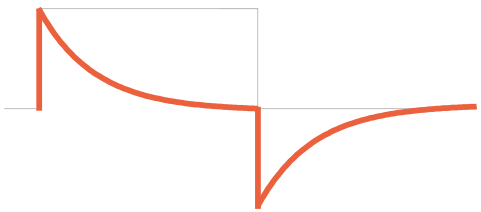
$$\tau_1 < \tau_2 < \tau_3$$
$$a_1 > a_2 > a_3$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}} = e^{-at}$$

$$\tau = RC = \frac{1}{a}$$

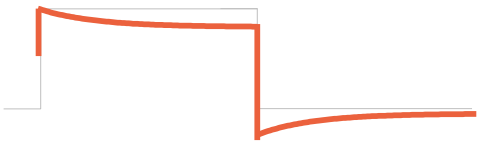
Time Constants

i_c



$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$



$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

small τ

small C

large $\frac{1}{\omega C} \gg R$

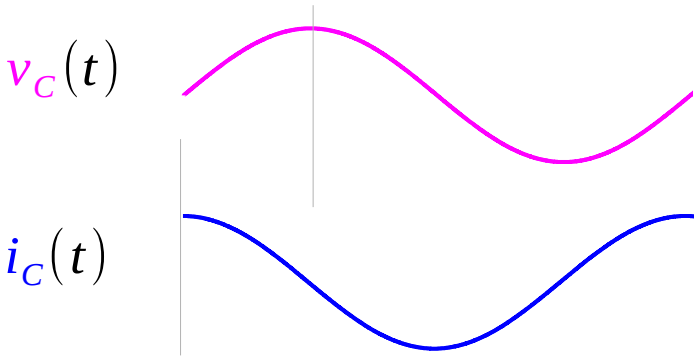
Fully Capacitative

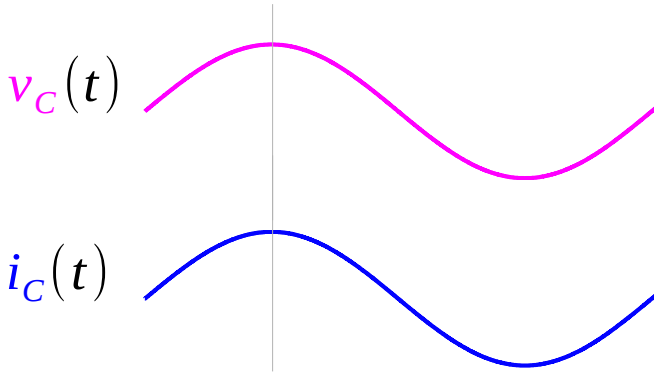
large τ

large C

small $\frac{1}{\omega C} \ll R$

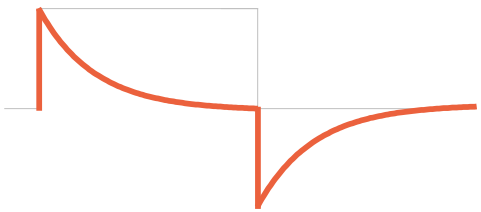
Fully Resistive





Time Constants

i_c



$$\tau = RC$$

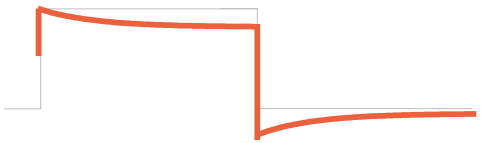
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

small τ

small C

large $\frac{1}{\omega C} \gg R$

Fully Capacitative



$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

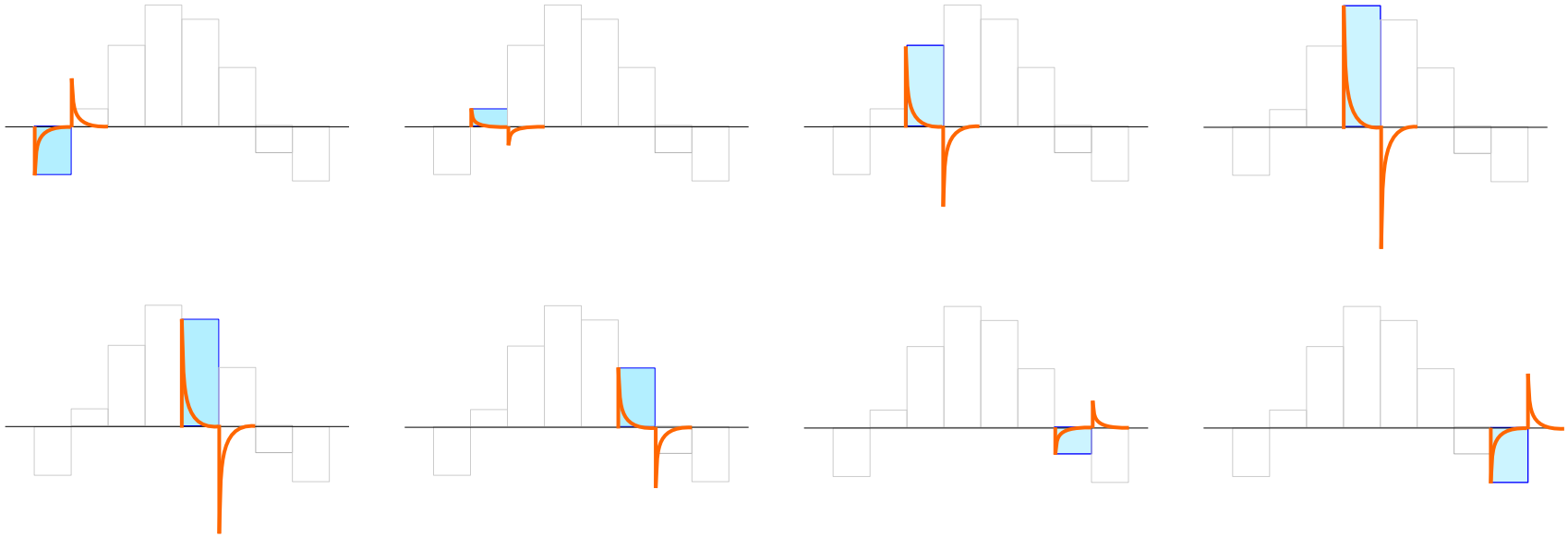
large τ

large C

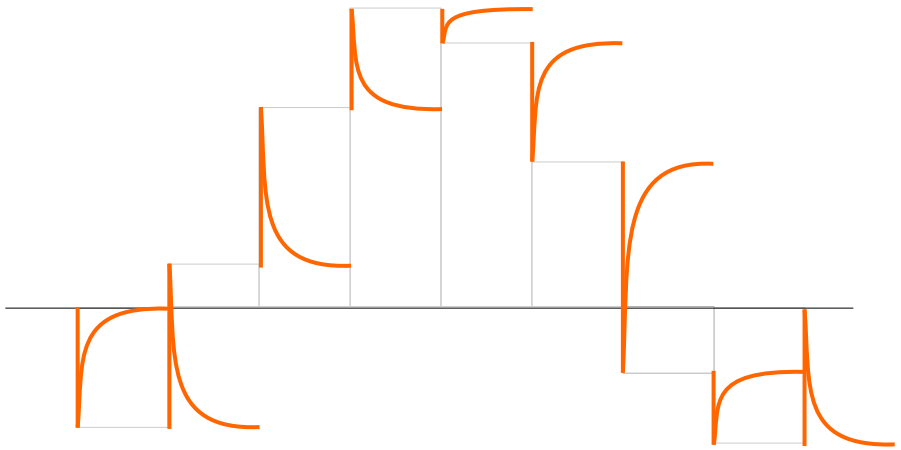
small $\frac{1}{\omega C} \ll R$

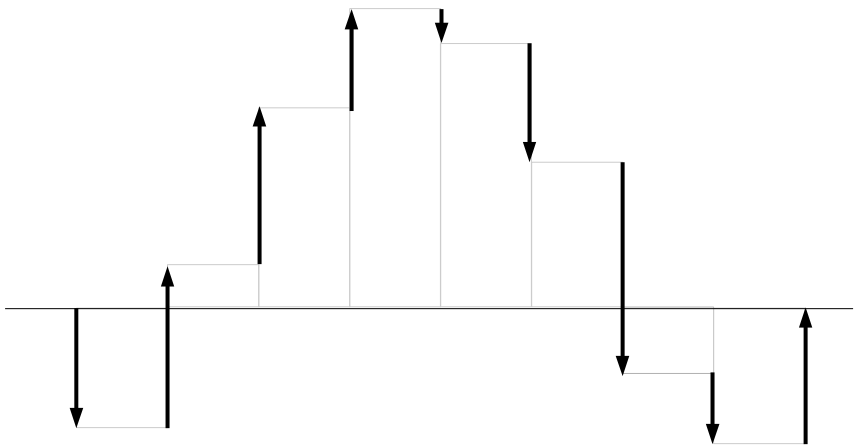
Fully Resistive

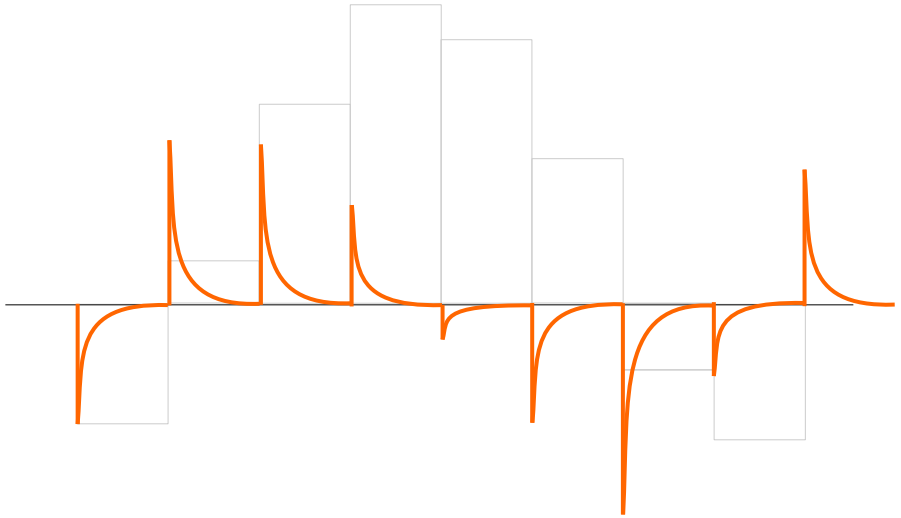
Superposition - Small Time Constant

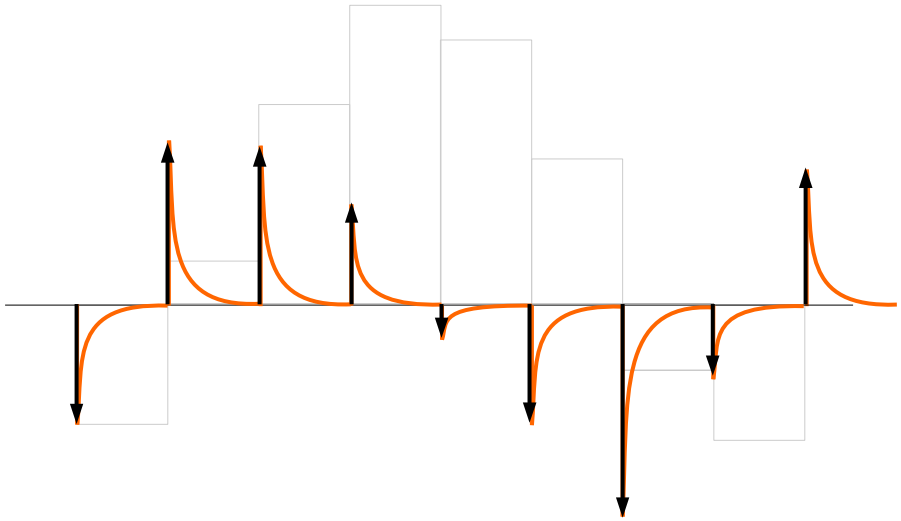


Small Time Constants

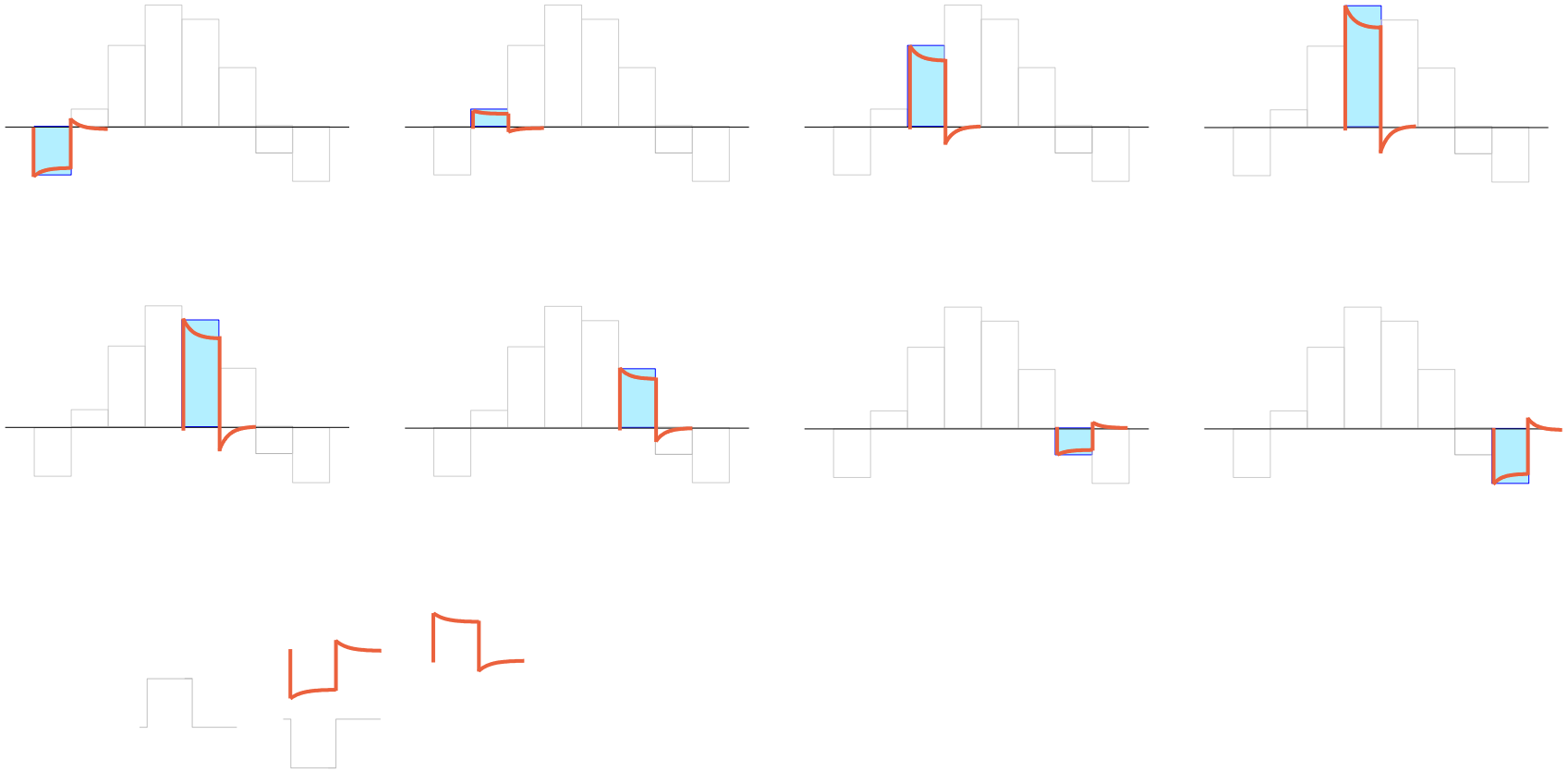




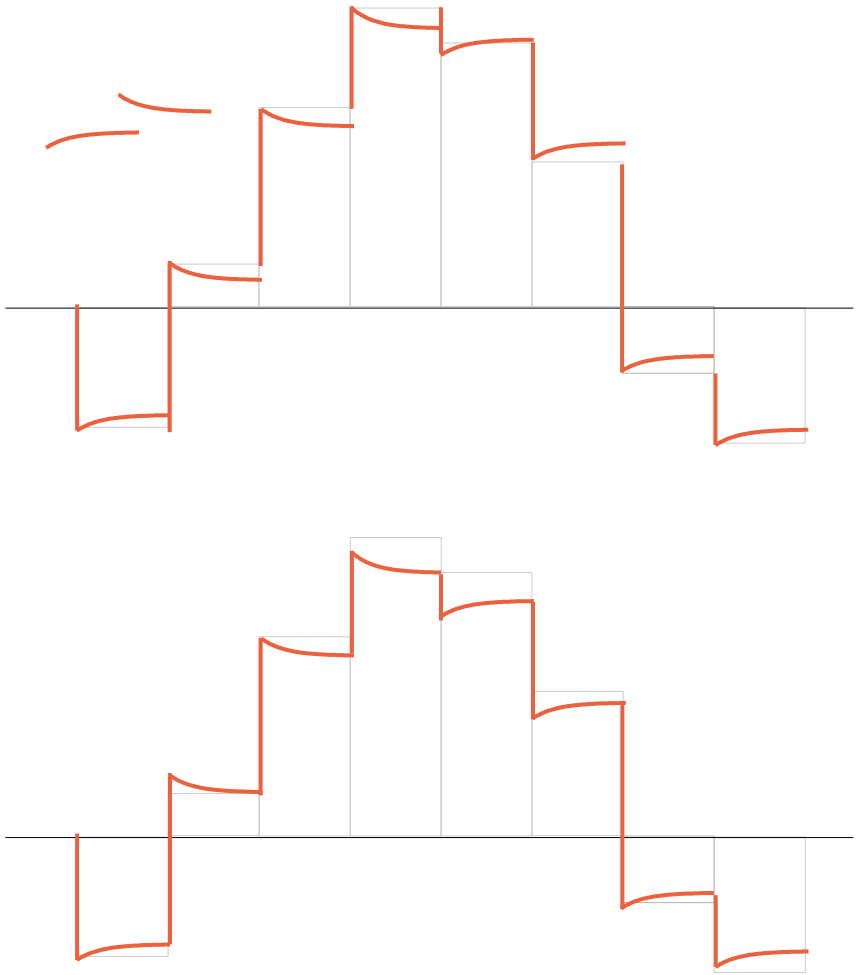




Superposition – Large Time Constant

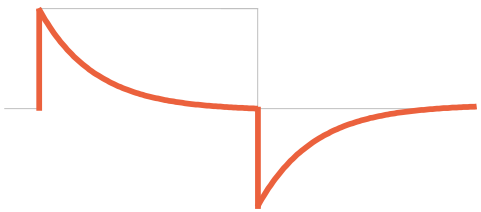


Large Time Constants



Time Constants

i_c



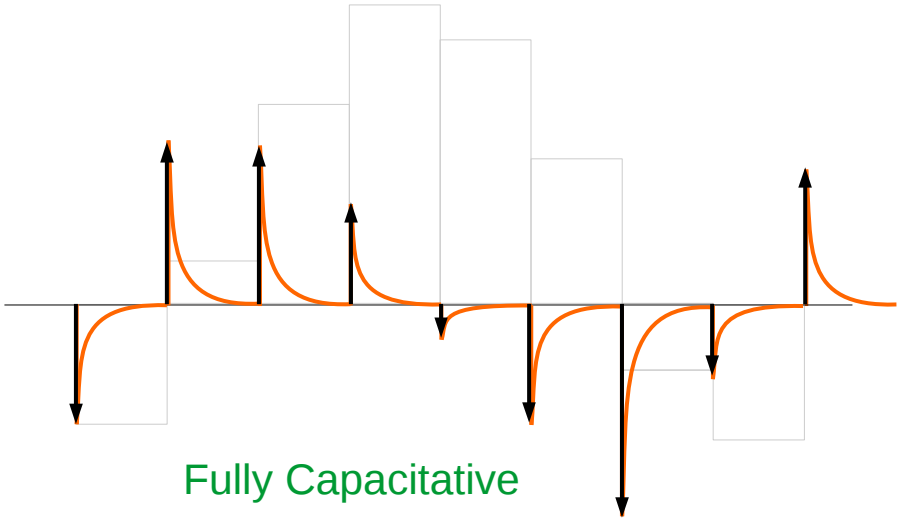
$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

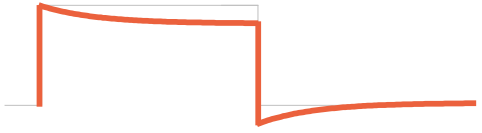
small τ

small C

large $\frac{1}{\omega C} \gg R$



Fully Capacitative



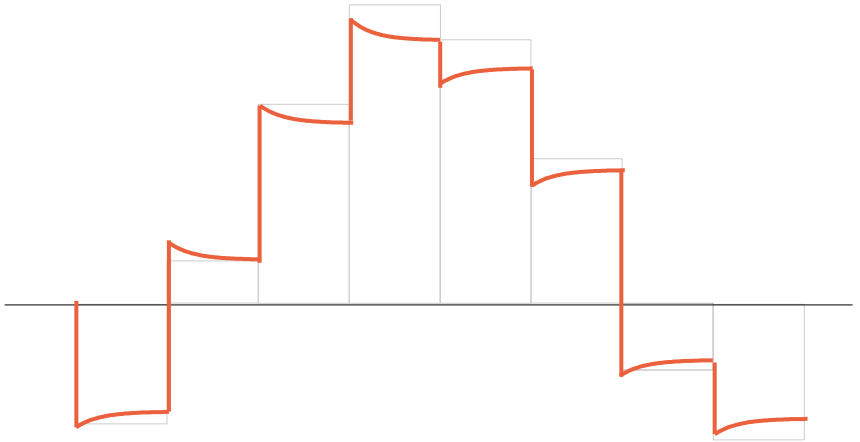
$$\tau = RC$$

$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

large τ

large C

small $\frac{1}{\omega C} \ll R$



Fully Resistive

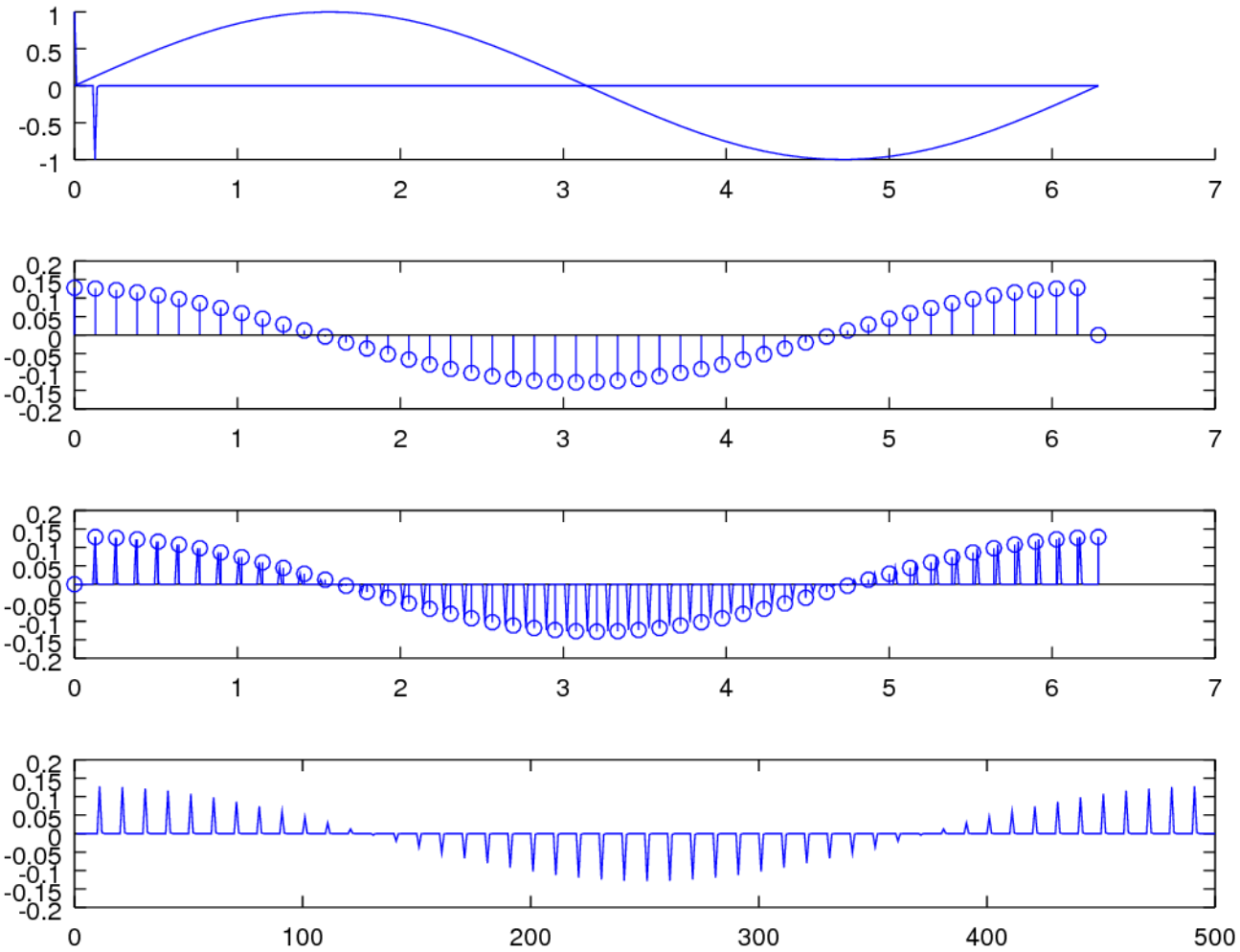
Plotting superposition results

```
clf
t = linspace(0, pi*2, 50);
tt= linspace(0, pi*2, 500);
N = length(t);
NN= length(tt);

t1 = t;
t2 = [t(2:N), t(N)];
y1 = sin(t1);
y2 = sin(t2) - sin(t1);

yy = [y1; zeros(NN/N-1, N)];
yy2= yy(:)';
a = 1/300;
yy3= e.^(-a*tt);
yy3 =yy3 - [zeros(1, NN/N),
e.^(-a*tt)](1:NN);
```

```
svec = zeros(1, NN);
for i = 1:NN;
  tvec = zeros(1, NN);
  tvec = [zeros(1, i-1), yy3];
  tvec = yy2(i) * tvec(1:NN);
  svec = svec + tvec;
endfor
yy4 = svec;
% yy4= conv(yy2, yy3);
y5 = yy4([1:NN/N:NN]);
yy5= yy4([1:NN]);
```

```
subplot(4, 1, 2);
stem(t1, y2)
subplot(4, 1, 1);
hold on
plot(t1, y1);
plot(tt, yy3);
subplot(4, 1, 3);
stem(t1, y5); hold on
plot(tt, yy5)
subplot(4, 1, 4);
plot(yy4);
```

Small Time Constant




```
yy = [y1;
zeros(NN/N-1, N)];
yy2= yy(:)';
a = 300;
yy3= e.^(-a*tt);
yy3 =yy3 -
[zeros(1, NN/N),
e.^(-a*tt)](1:NN);
```

$$\tau = RC$$

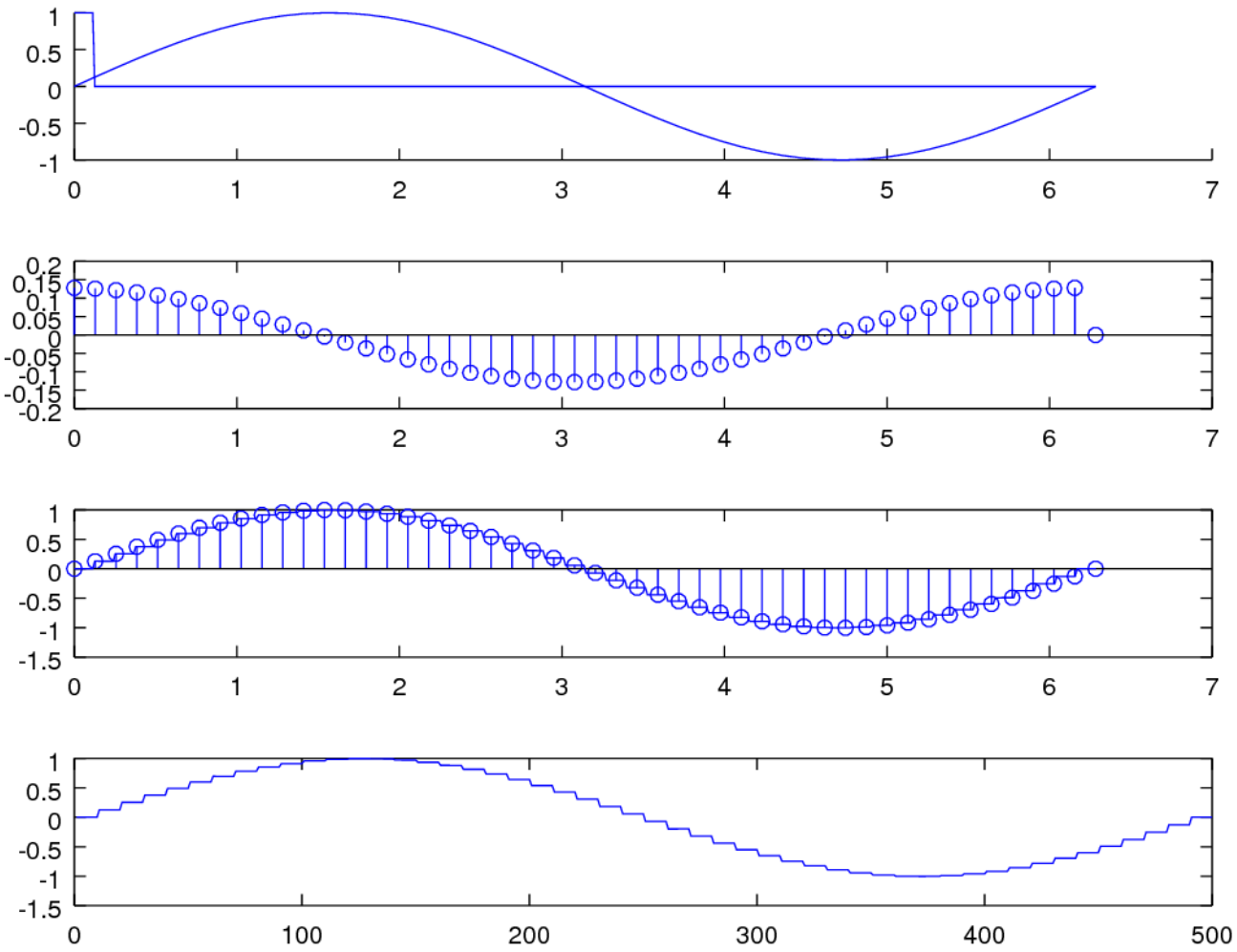
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

small τ

small C

large $\frac{1}{\omega C}$

Large Time Constant




```
yy = [y1;
zeros(NN/N-1, N)];
yy2= yy(:)';
a = 1/300;
yy3= e.^(-a*tt);
yy3 =yy3 -
[zeros(1, NN/N),
e.^(-a*tt)](1:NN);
```

$$\tau = RC$$

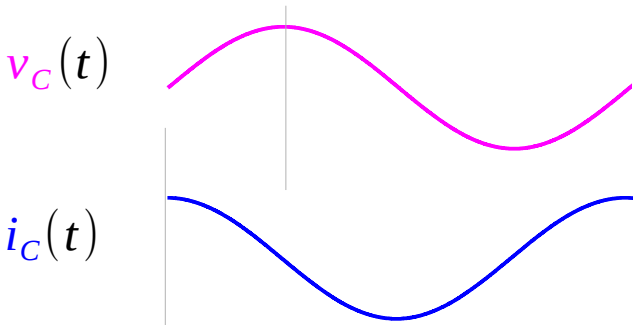
$$e^{-\frac{t}{\tau}} = e^{-\frac{t}{RC}}$$

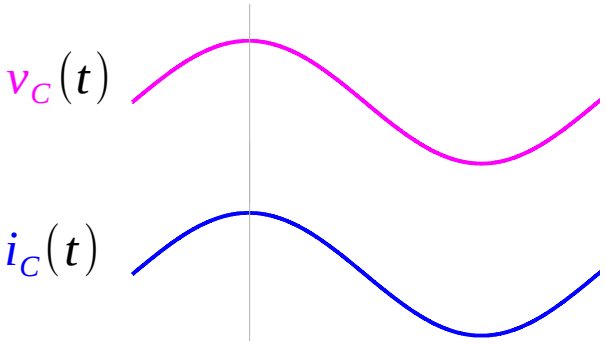
large τ

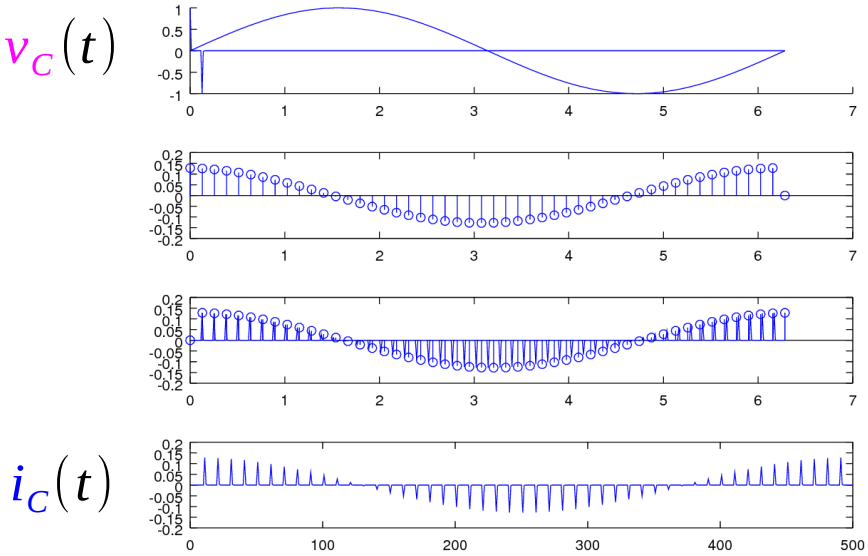
large C

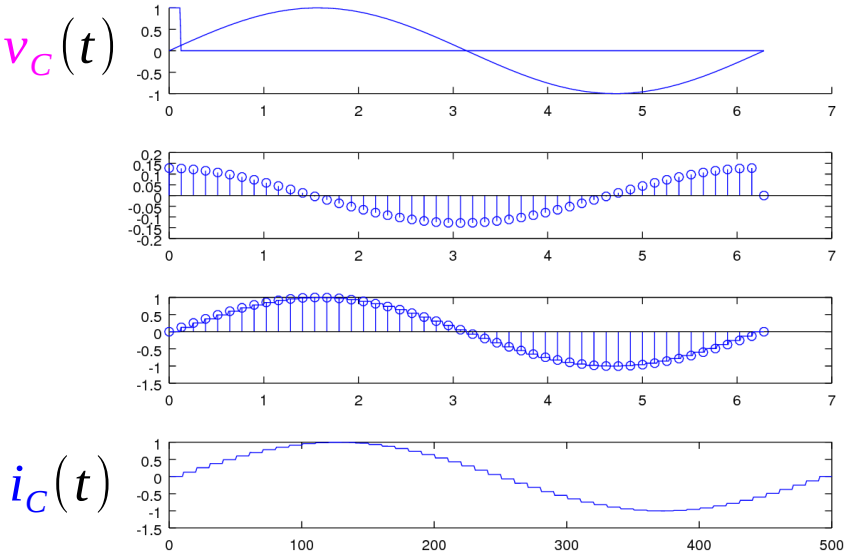
small $\frac{1}{\omega C}$

Time Constants

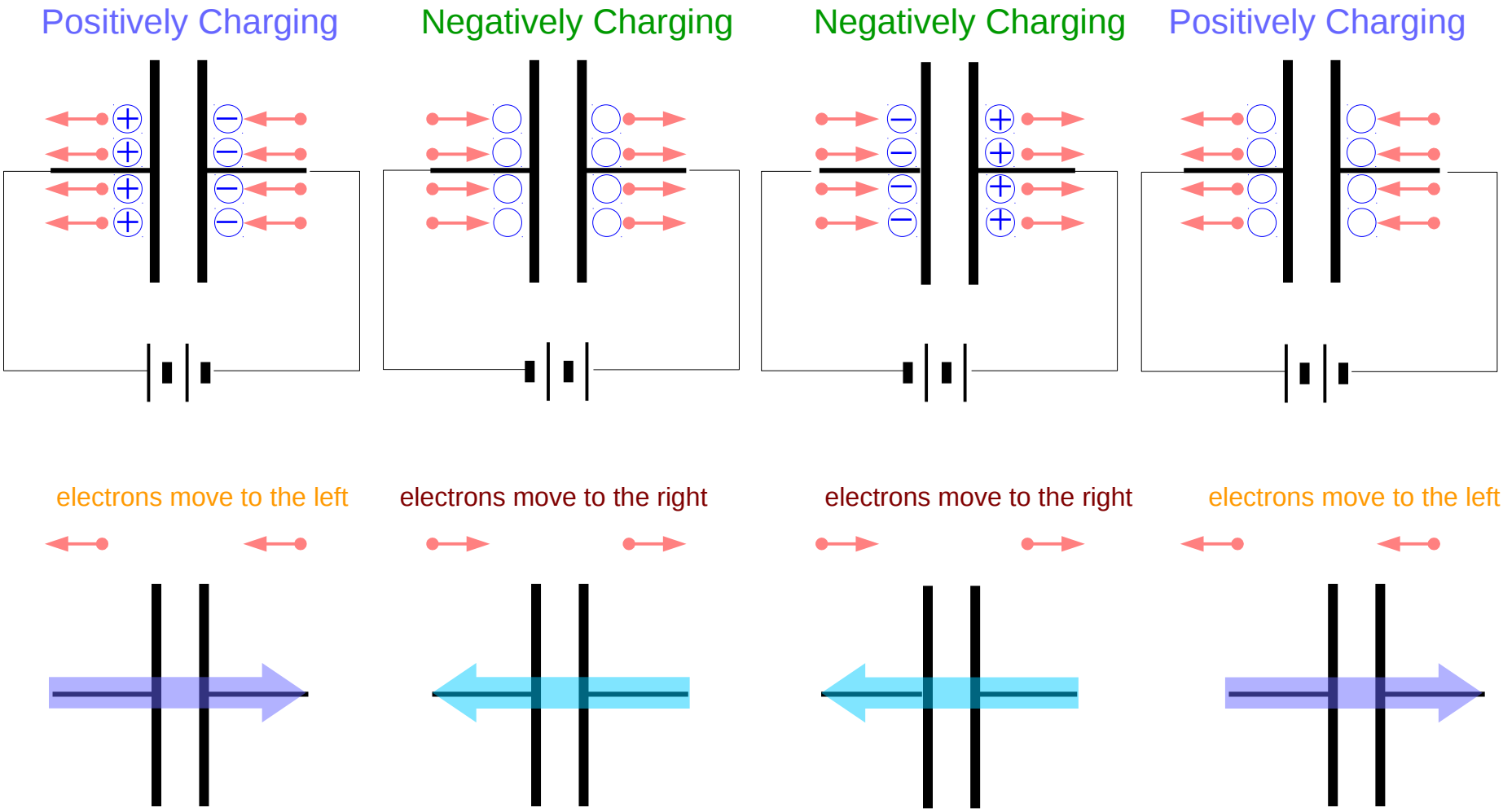




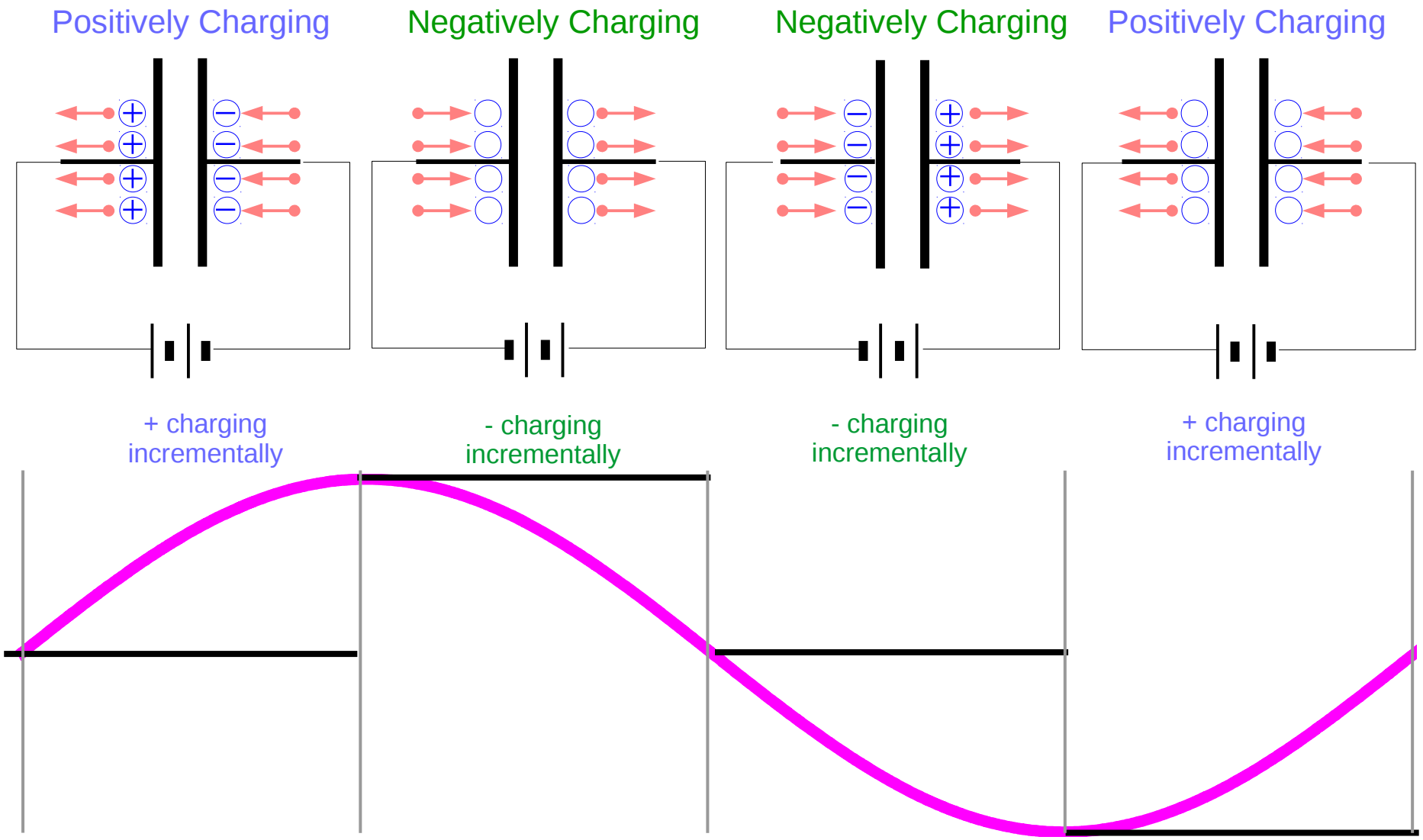




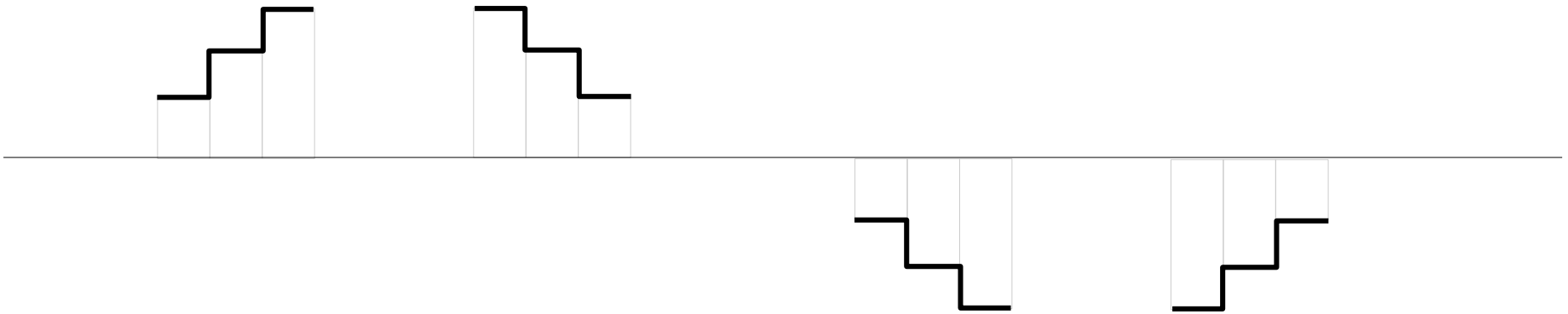
Evercharging signal pairs

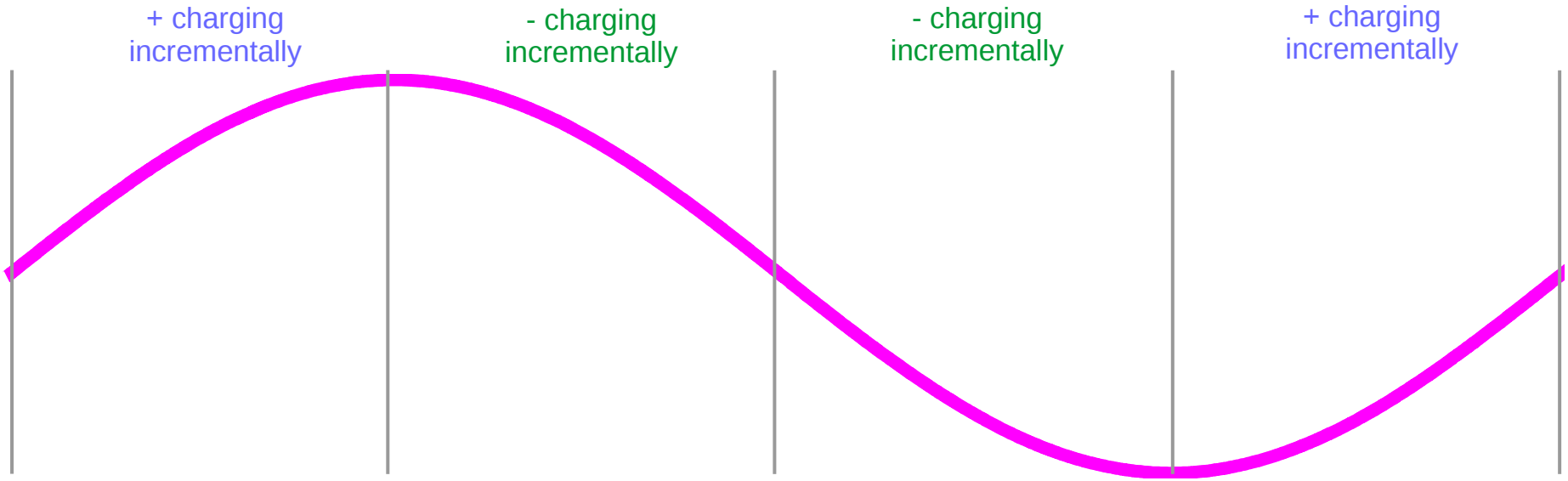


Continuous Charing and Discharging Operations

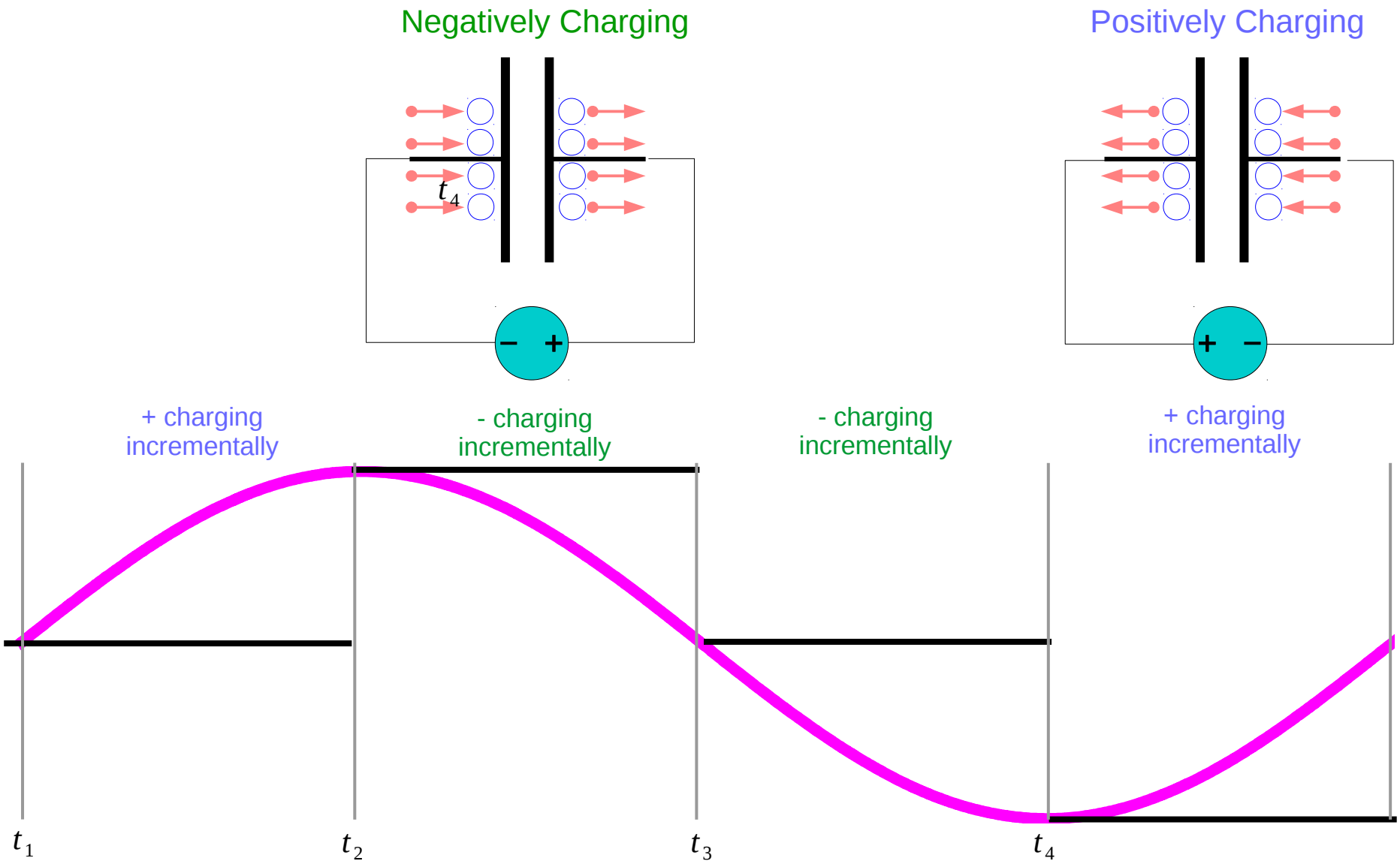


Continuous Charing and Discharging Operations

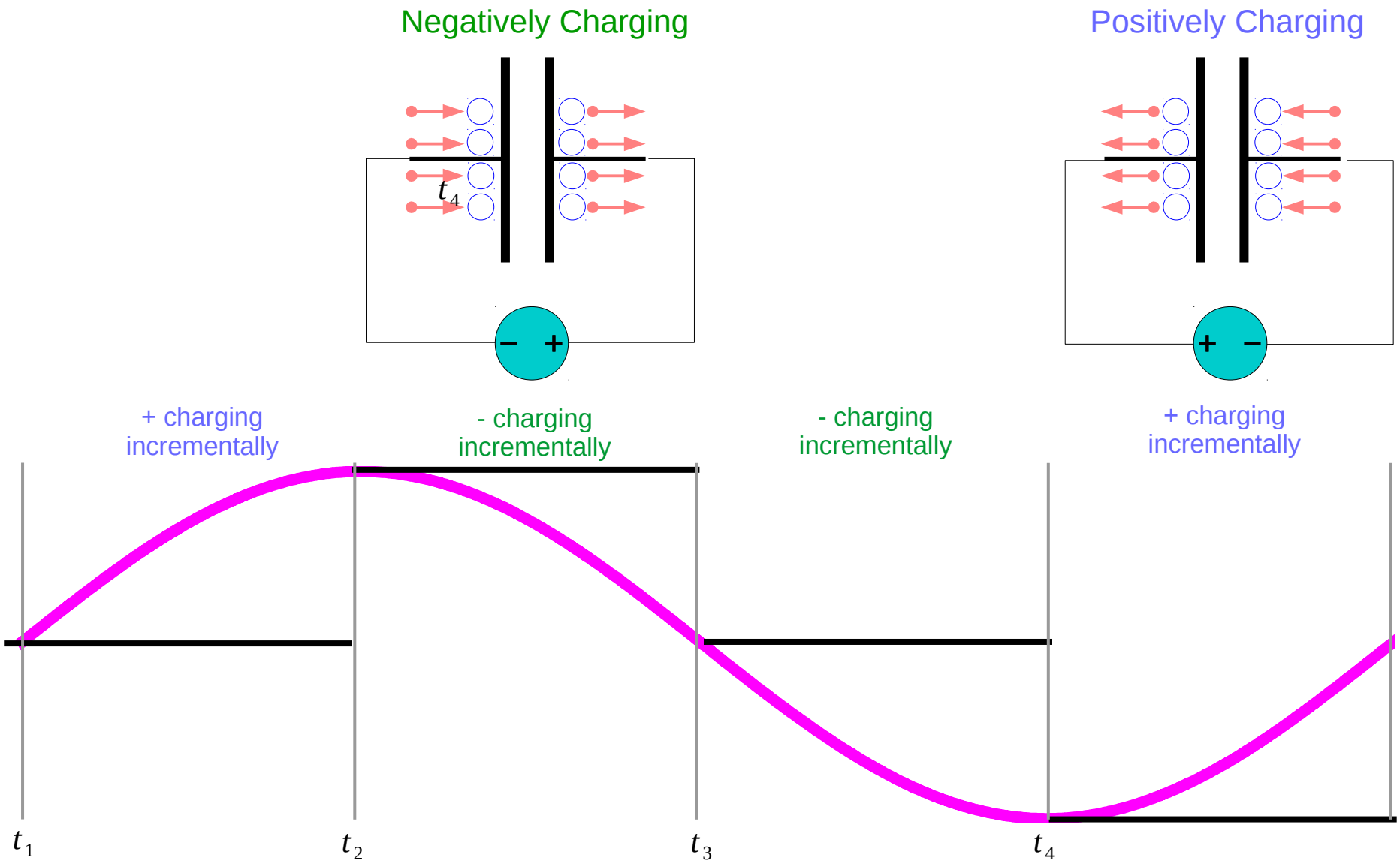




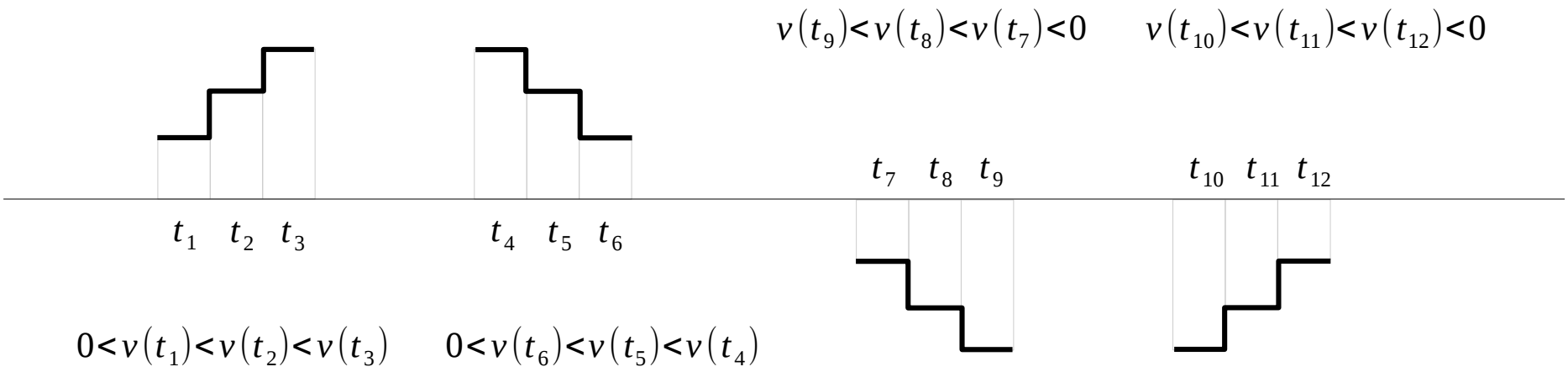
Continuous Charing and Discharging Operations

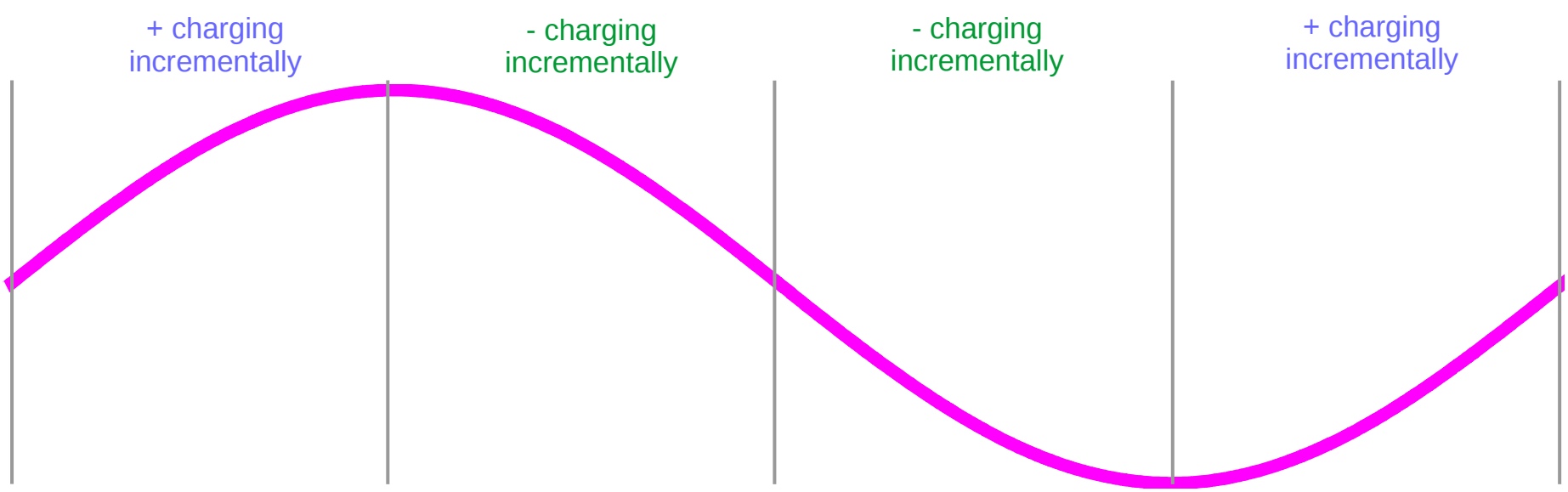


Continuous Charing and Discharging Operations

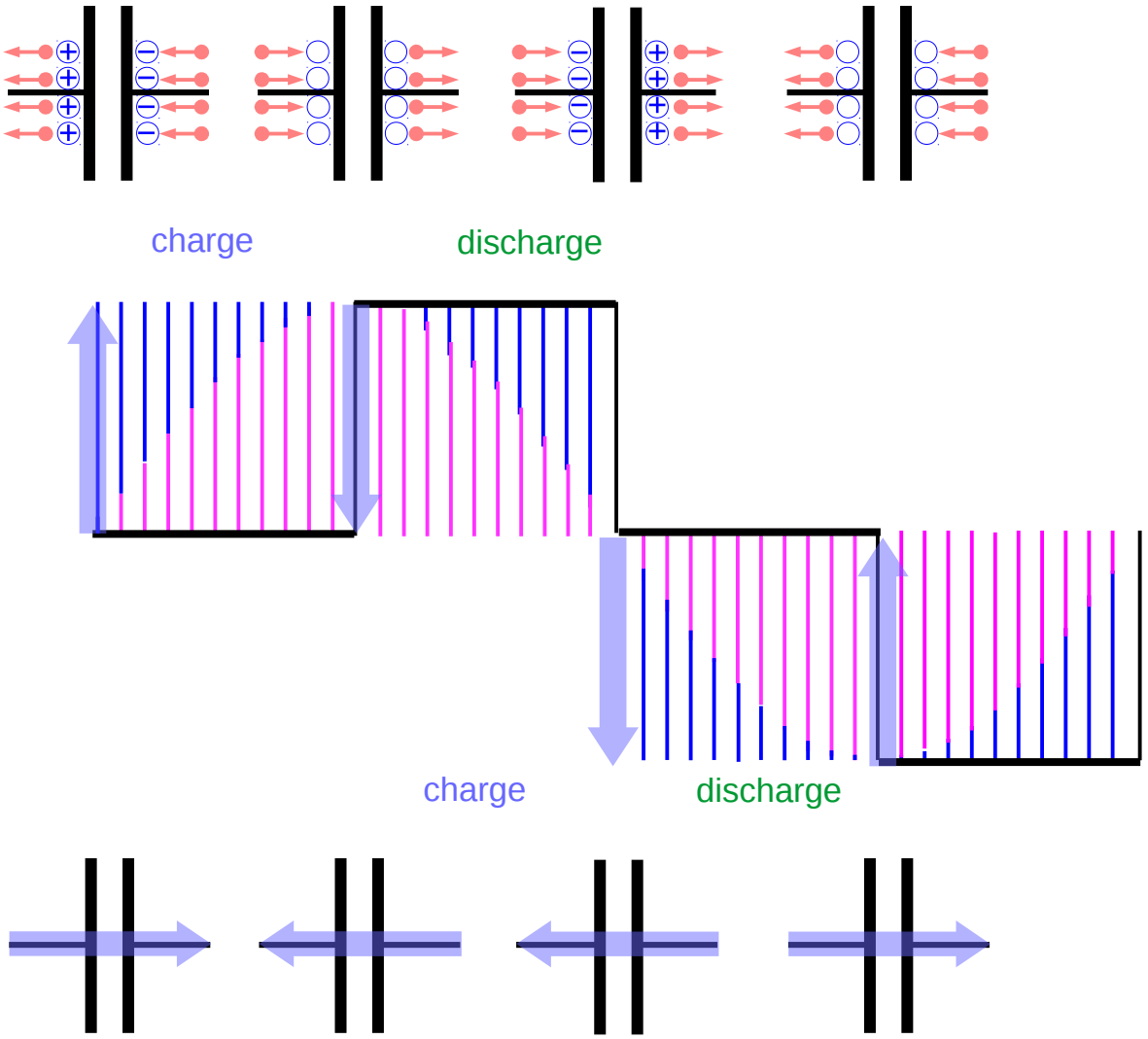


Incremental Charging

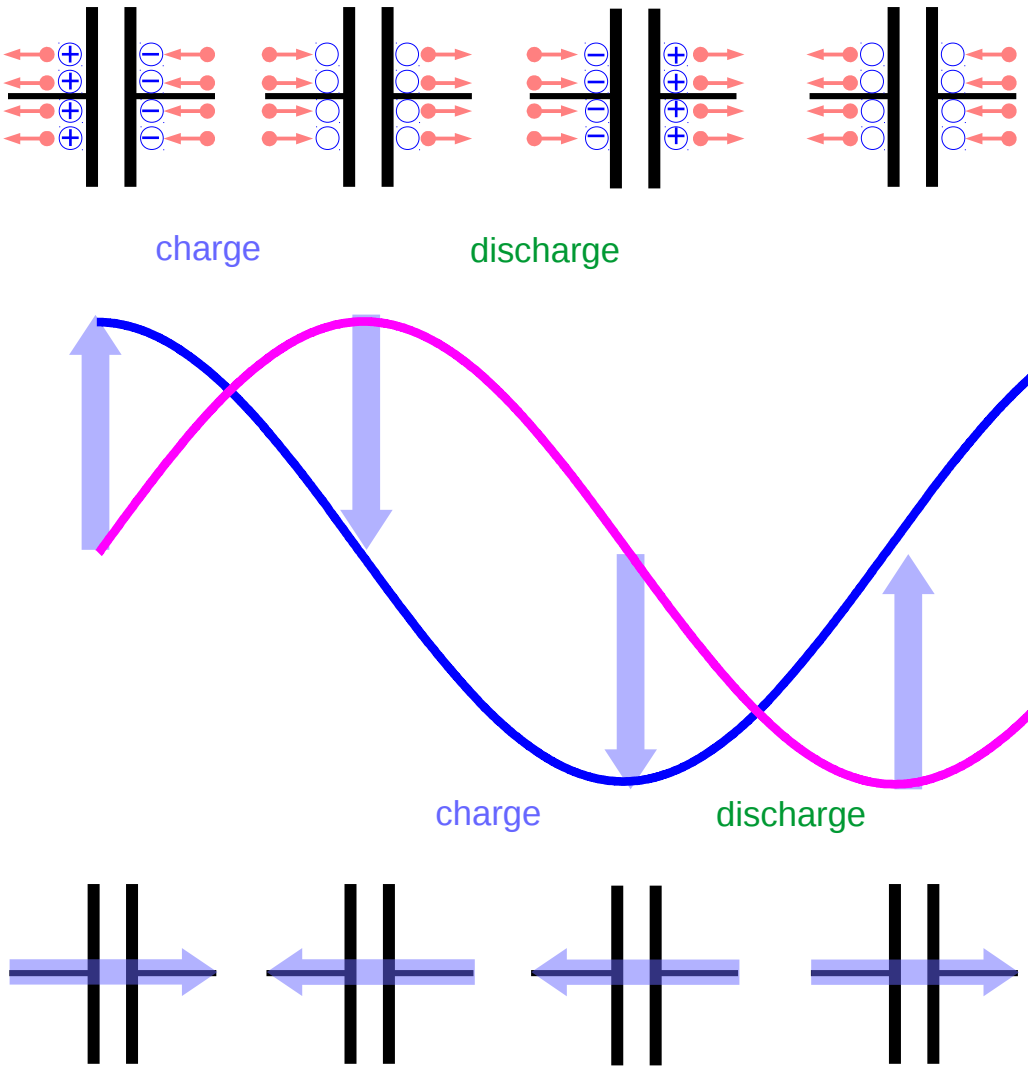




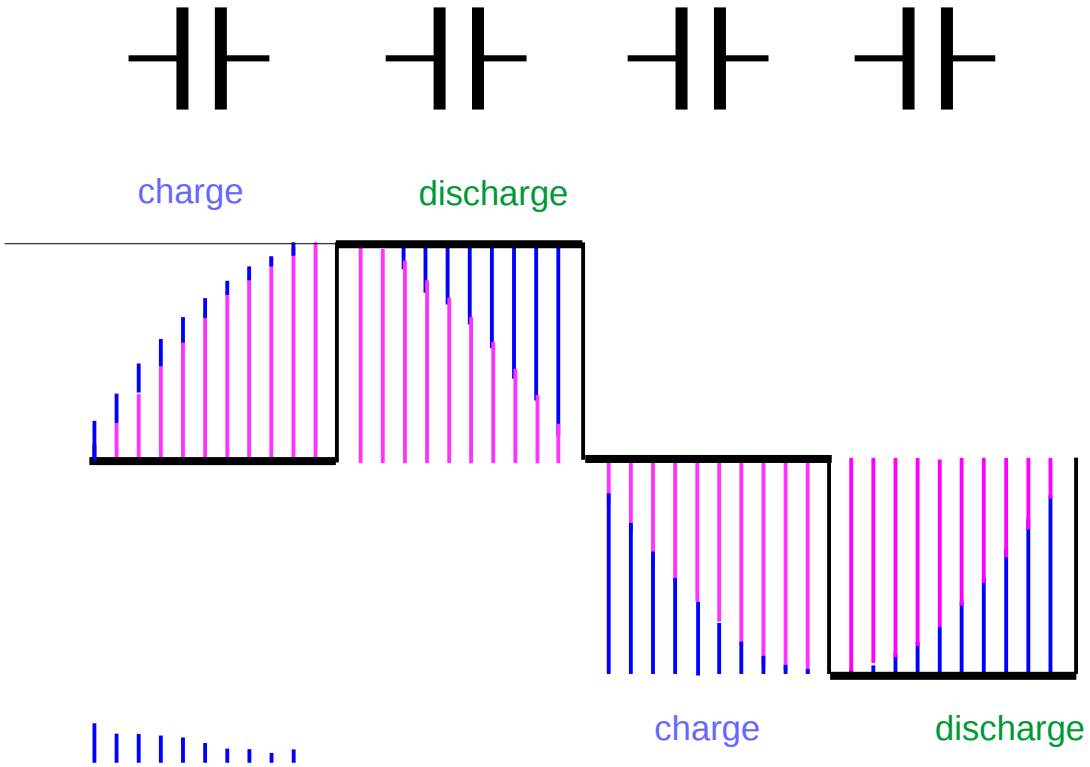
Everchanging signal pairs



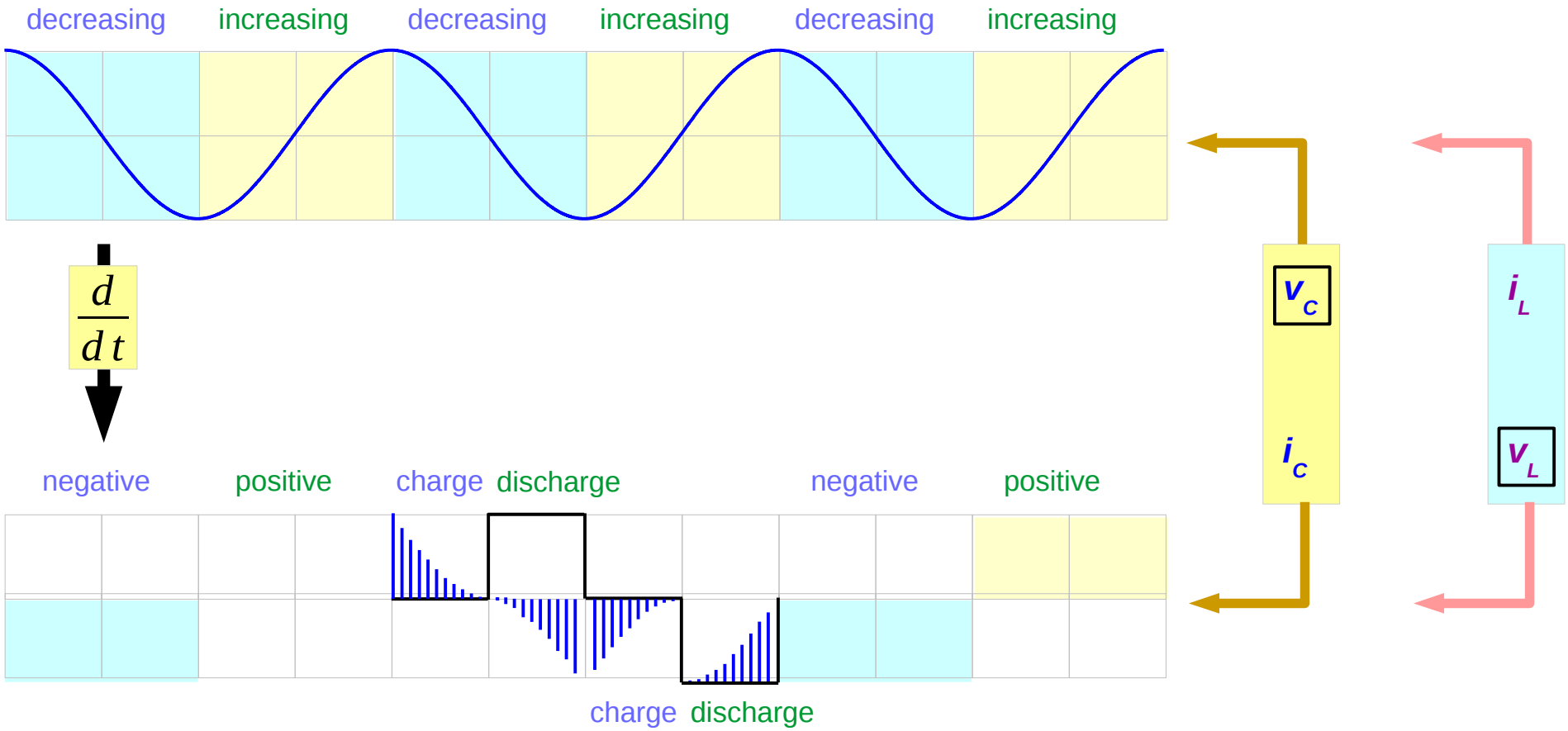
Everchanging signal pairs



Everchanging signal pairs

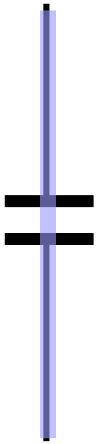


Everchanging signal pairs



I leads V by 90°

Initial charge

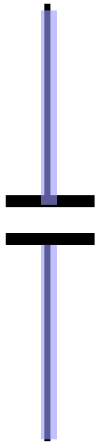


SHORT

V = 0

I : peak

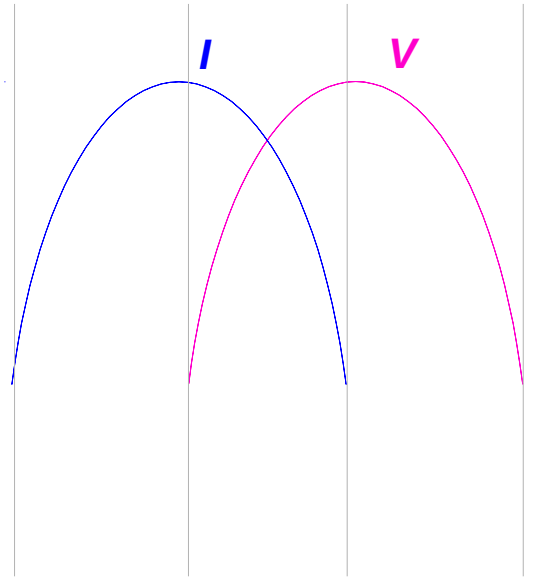
Full charge



OPEN

I = 0

V : peak



References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003