Direct Form Filters

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Based on

Introduction to Signal Processing

S. J. Ofranidis

The necessities in DSP C Programming

FIR Filter (A.pdf) 20191114

Direct Form

Considering the widely used
Edge triggered
D-type Flip Flops

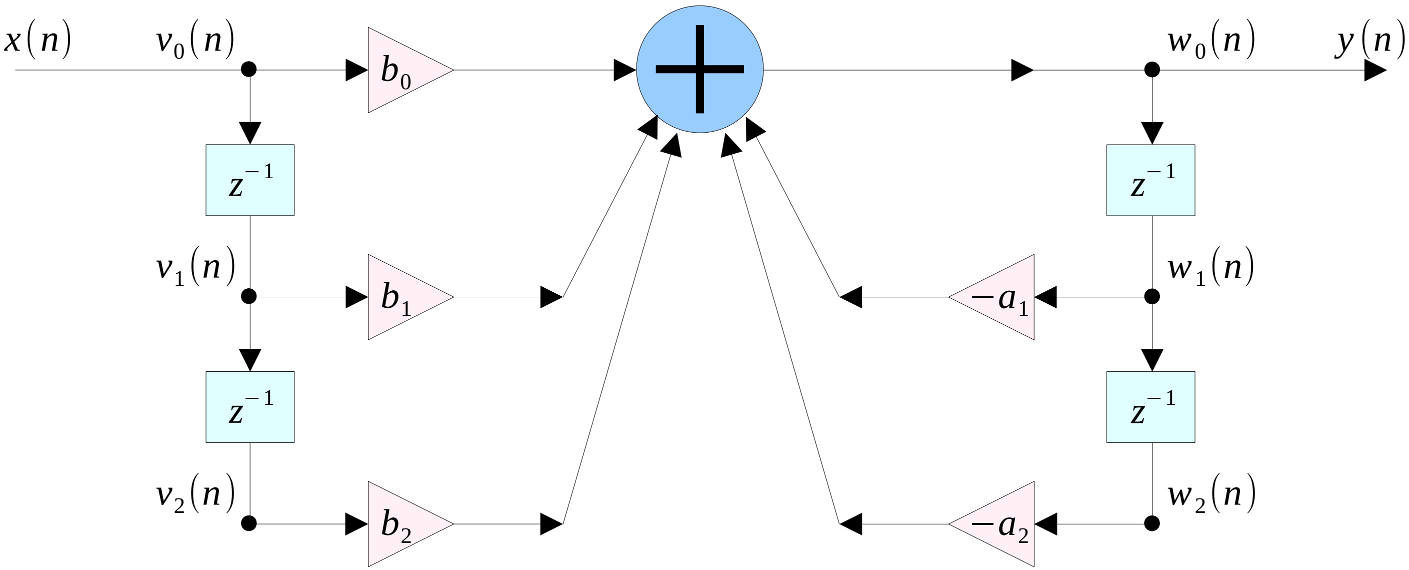
$$H(z) = \frac{N(Z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$y_n = -a_1 y_{n-1} - a_2 y_{n-2} + b_0 x_n + b_1 x_{n-1} + b_2 x_{n-2}$$

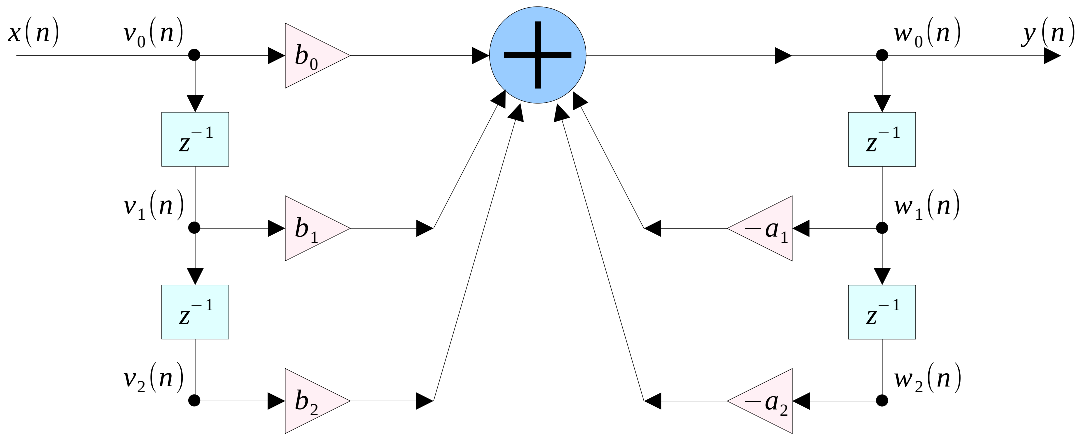
Direct Form

$$H(z) = \frac{N(Z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$y_n = -a_1 y_{n-1} - a_2 y_{n-2} + b_0 x_n + b_1 x_{n-1} + b_2 x_{n-2}$$



Direct Form



$$v_0(n) = x(n)$$
$$v_1(n) = x(n-1) = v_0(n-1)$$
$$v_2(n) = x(n-2) = v_1(n-1)$$

$$w_0(n) = y(n)$$
$$w_1(n) = y(n-1) = w_0(n-1)$$
$$w_2(n) = y(n-2) = w_1(n-1)$$

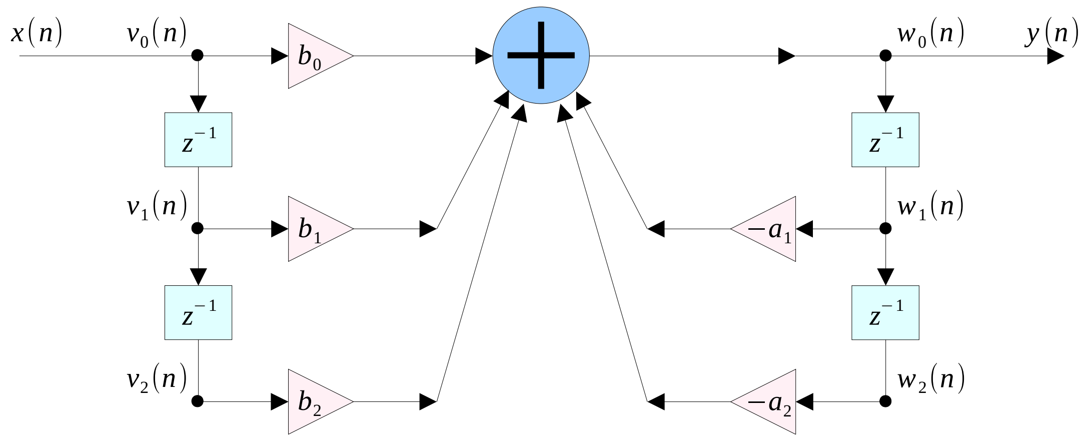
$$v_1(n+1) = v_0(n)$$
$$v_2(n+1) = v_1(n)$$

$$w_1(n+1) = w_0(n)$$
$$w_2(n+1) = w_1(n)$$

Direct Form

$$v_1(n+1) = v_0(n)$$

$$v_2(n+1) = v_1(n)$$



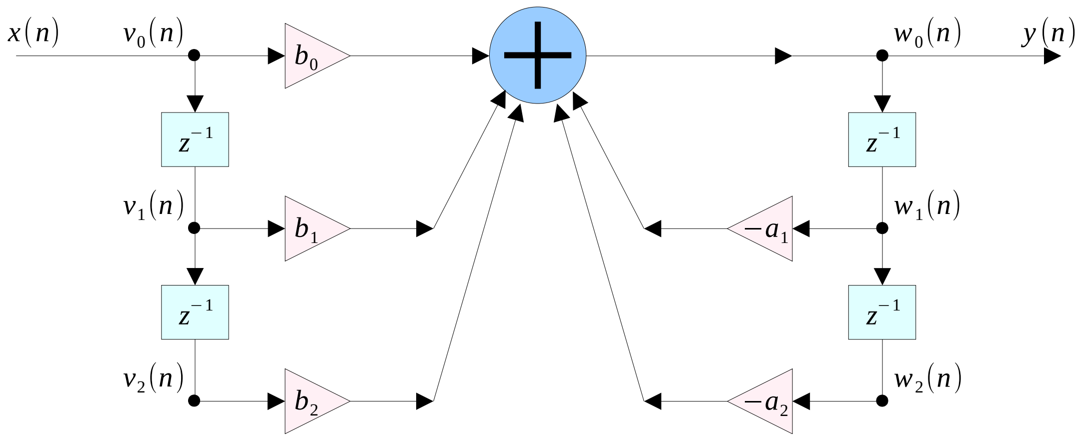
$$w_1(n+1) = w_0(n)$$

$$w_2(n+1) = w_1(n)$$

$$y_n = -a_1 y_{n-1} - a_2 y_{n-2} + b_0 x_n + b_1 x_{n-1} + b_2 x_{n-2}$$

$$y_n = -a_1 y_{n-1} - a_2 y_{n-2} + b_0 v_0(n) + b_1 v_1(n) + b_2 v_2(n)$$

Direct Form



$$v_0(n) = x(n)$$

$$w_0(n) = b_0 v_0(n) + b_1 v_1(n) + b_2 v_2(n) - a_1 w_1(n) - a_2 w_2(n)$$

$$y(n) = w_0(n)$$

$$v_2(n+1) = v_1(n) \qquad w_2(n+1) = w_1(n)$$

$$v_1(n+1) = v_0(n) \qquad w_1(n+1) = w_0(n)$$

for each input sample x do:

$$v_0 = x$$

$$w_0 = b_0 v_0 + b_1 v_1 + b_2 v_2 - a_1 w_1 - a_2 w_2$$

$$y = w_0$$

$$v_2 = v_1 \qquad w_2 = w_1$$

$$v_1 = v_0 \qquad w_1 = w_0$$

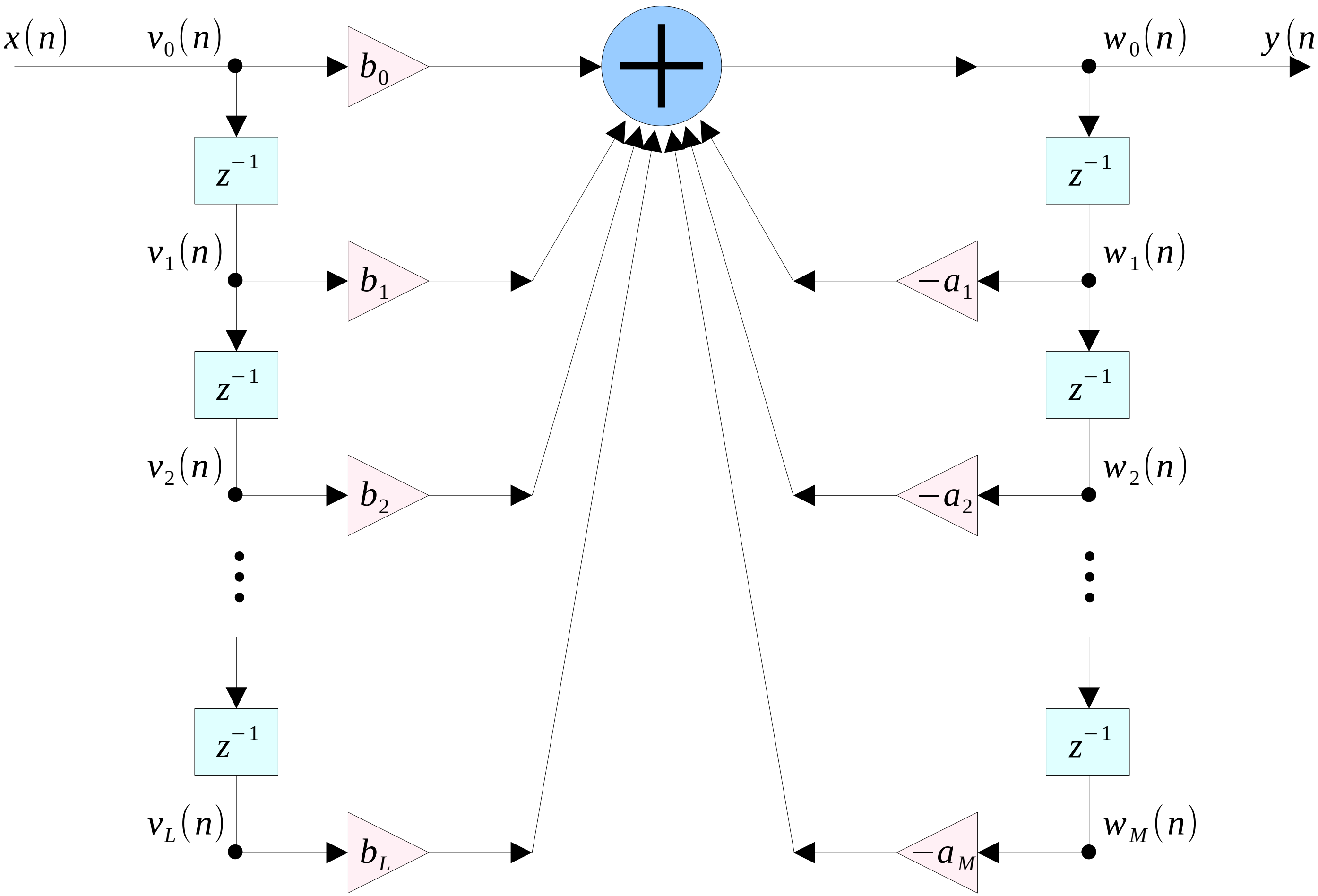
Direct Form

$$H(z) = \frac{N(Z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_L z^{-L}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_M z^{-M}}$$

$$y_n = -a_1 y_{n-1} - a_2 y_{n-2} - \cdots - a_M y_{n-M} + b_0 x_n + b_1 x_{n-1} + b_2 x_{n-2} + \cdots + b_L x_{n-L}$$

$$\begin{aligned} y_n = \; & b_0 x_n + b_1 x_{n-1} + b_2 x_{n-2} + \cdots + b_L x_{n-L} \\ & - a_1 y_{n-1} - a_2 y_{n-2} - \cdots - a_M y_{n-M} \end{aligned}$$

Direct Form

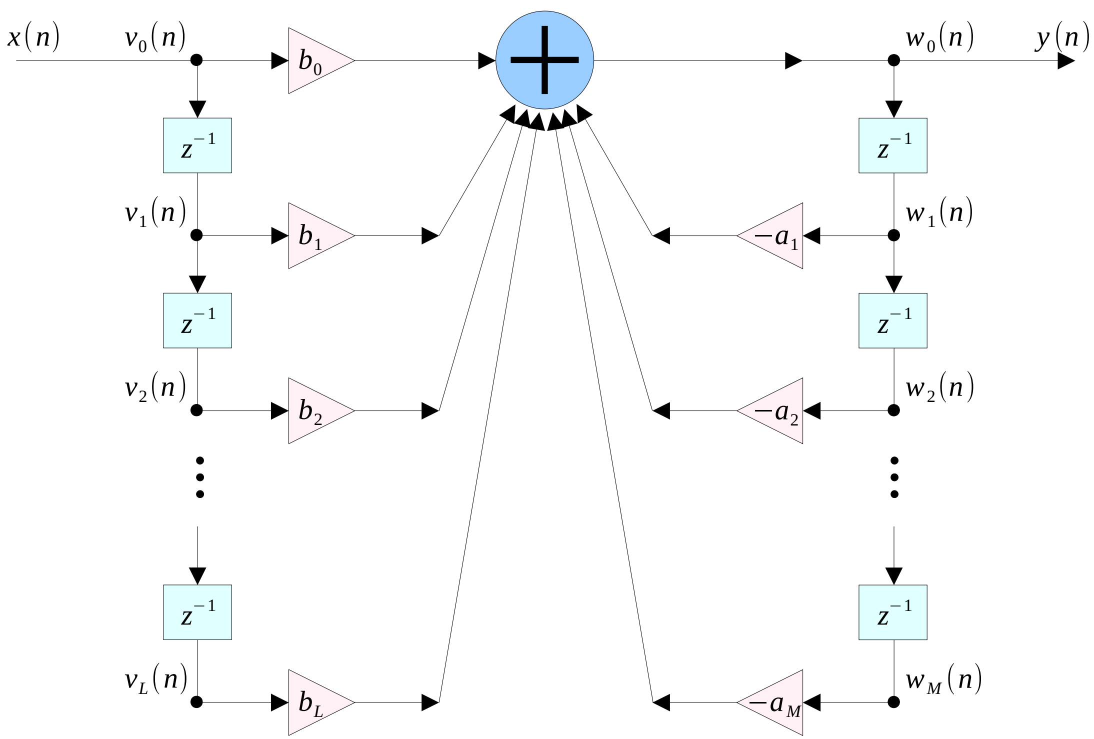


$x(n)$
$v_0(n)$
b_0
$w_0(n)$
$y(n)$
z^{-1}
$v_1(n)$
b_1
$-a_1$
$w_1(n)$
z^{-1}
z^{-1}
$v_2(n)$
b_2
$-a_2$
$w_2(n)$
z^{-1}
z^{-1}
$v_L(n)$
b_L
$-a_M$
$w_M(n)$

Direct Form

$$H(z) = \frac{N(Z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2^{-2} + \cdots + b_L z^{-L}}{1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_M z^{-M}}$$

$$y_n = b_0 x_n + b_1 x_{n-1} + b_2 x_{n-2} + \cdots + b_L x_{n-L} - a_1 y_{n-1} - a_2 y_{n-2} - \cdots - a_M y_{n-M}$$



Direct Form

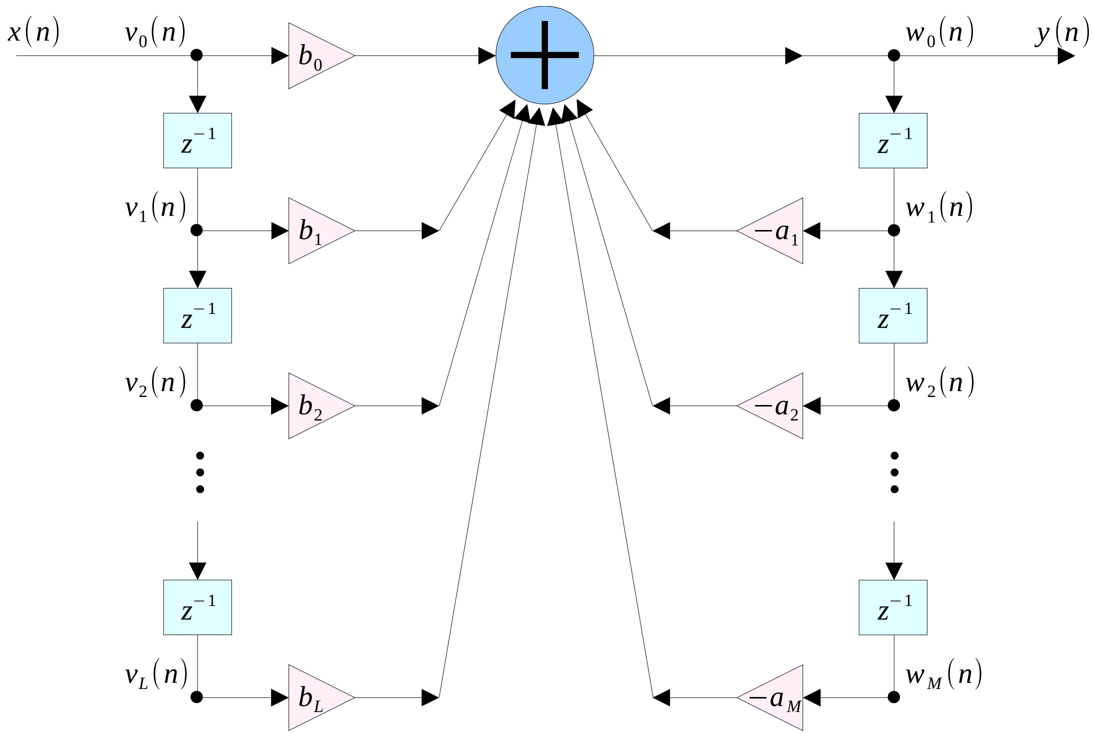
$v_i(n) = x(n-i), \;\; i=0,1,\ldots,L$

$w_i(n) = y(n-i), \;\; i=0,1,\ldots,M$

$v_i(n+1)=v_{i-1}(n), \;\; i=0,1,\ldots,L$

$w_i(n+1)=w_{i-1}(n), \;\; i=1,2,\ldots,M$

$$w_i(n+1) = y((n+1)-i) = y(n-(i-1)) = w_{i-1}(n)$$



References

[1] S. J. Ofranidis , Introduction to Signal Processing