FPGA Carry Chain Adder (1A)

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Please send corrections (or suggestions) to youngwlim@hotmail.com.

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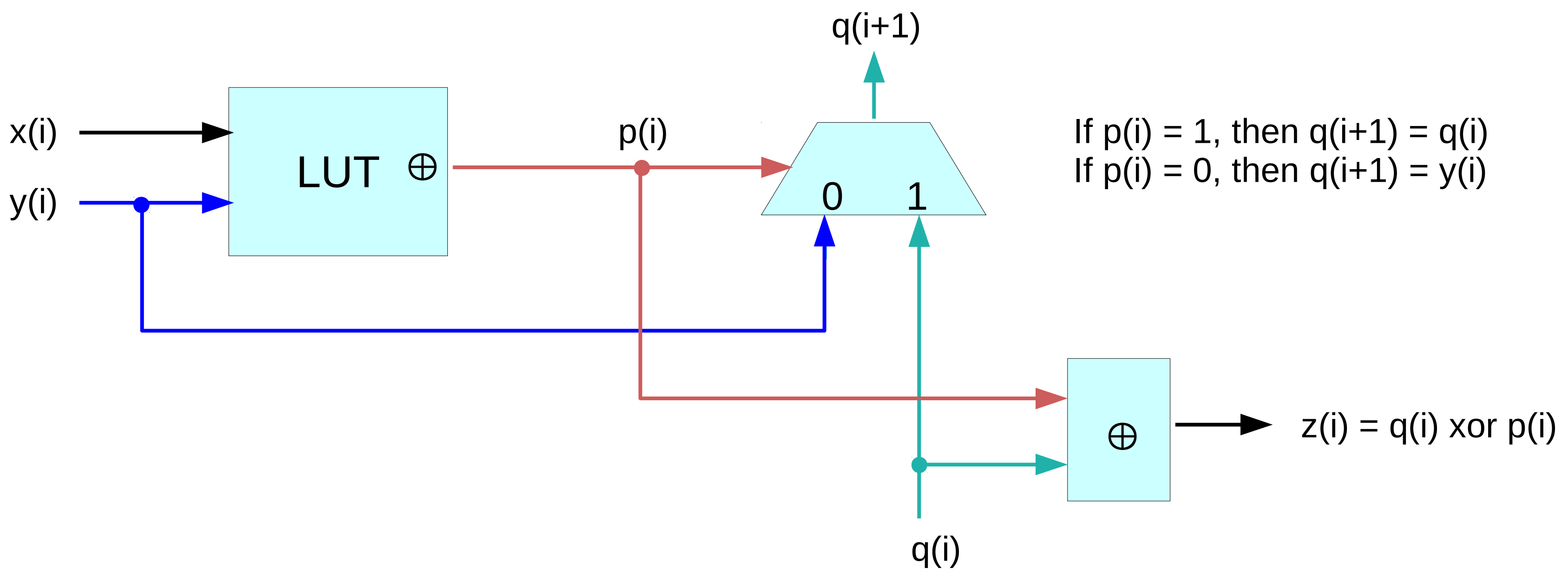
FPGA Carry Chain Cell



q(i+1)
x(i)
y(i)
LUT ⊕
p(i)
0
1
If p(i) = 1, then q(i+1) = q(i)
If p(i) = 0, then q(i+1) = y(i)
⊕
z(i) = q(i) xor p(i)
q(i)

$$s_i = (a_i \oplus b_i) \oplus c_i = p_i \oplus c_i$$

$$c_{i+1} = (a_i \cdot b_i) + (a_i \oplus b_i) c_i = \overline{p_i} \cdot g_i + p_i \cdot c_i = \overline{p_i} \cdot a_i + p_i \cdot c_i = \overline{p_i} \cdot b_i + p_i \cdot c_i$$

when $\overline{p}_i = 1$, *then* $a_i = b_i$

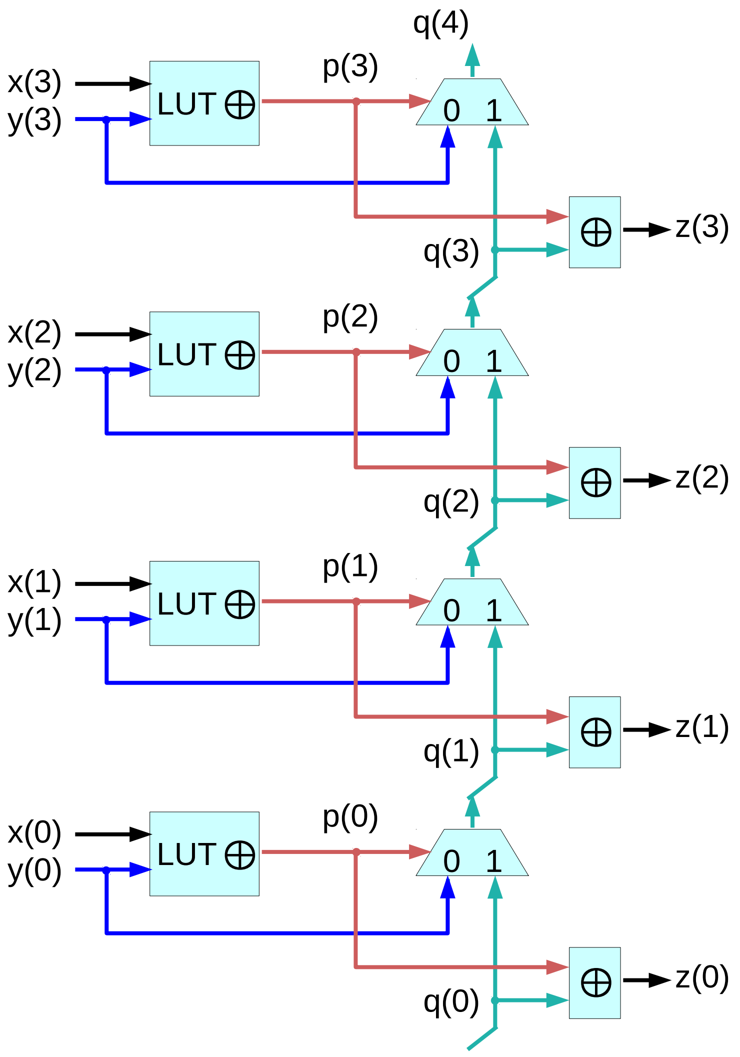
when $g_i = 1$, *then* $a_i = b_i = 1$

p(i)	0	1
0	0	1
1	1	0

g(i)	0	1
0	0	0
1	0	1

Synthesis of Arithmetic Circuits: FPGA, ASIC and Ebedded Systems, J-P Deschamps et al

FPGA Carry Chain Cell



Synthesis of Arithmetic Circuits: FPGA, ASIC and Ebedded Systems, J-P Deschamps et al

FPGA Carry Chain

FPGAs generally contain dedicated computation resources
for generating fast adders

The Virtex family programmable arrays include
logic gates (**XOR**) and **multiplexers** that along with the
general purpose **lookup tables** allow one to build effective carry-chain adders

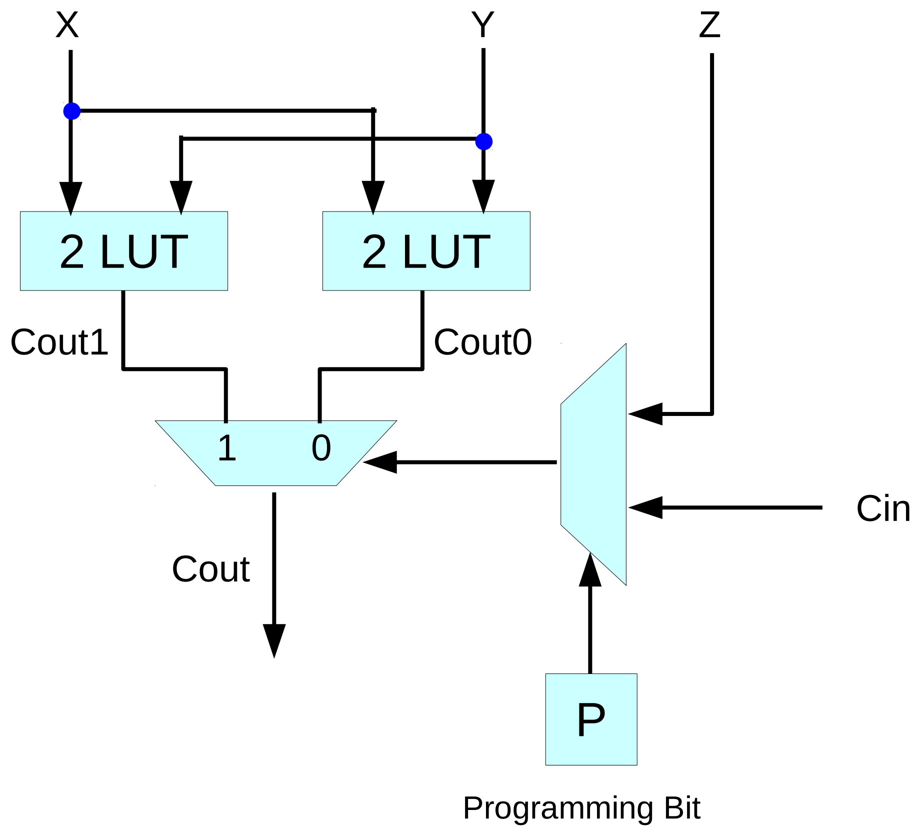
The carry chain is made up of multiplexers
belonging to adjacent configurable blocks

the lookup table is used for implementing the exclusive or function

p(i) = x(i) xor y(i)

https://en.wikipedia.org/wiki/Carry-lookahead_adder

FPGA Carry Chain Cell



Cout1, Cout2 : functions of X, Y, Cin

Cout1 = X+Y when Cin=1
Cout0 = X Y when Cin=0

Cout = (X + Y) Cin + X Y $\overline{\text{Cin}}$

Cout = P' Cin + G $\overline{\text{Cin}}$ … P' = relaxed P

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

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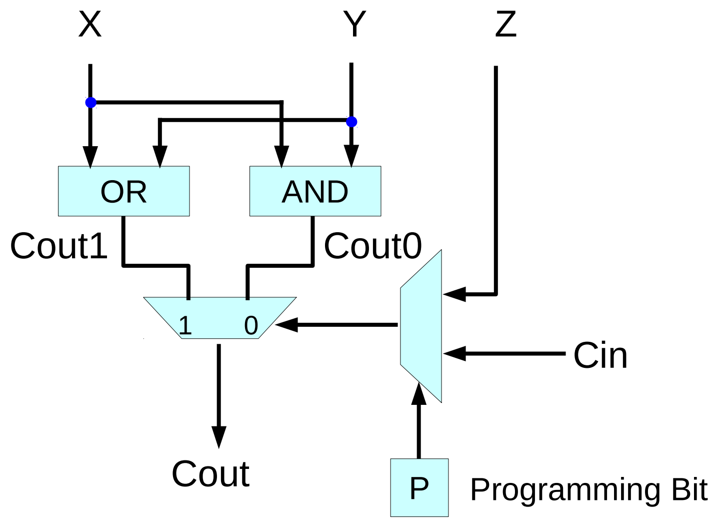
FPGA Carry Chain Cell



		Cin	$\overline{Cin}$	
X	Y	Cout1	Cout0	
0	0	0	0	$\overline{X}\,\overline{Y}$
0	1	1	0	$\overline{X}\,Y$
1	0	1	0	$X\,\overline{Y}$
1	1	1	1	$X\,Y$

Cout : functions of X, Y, Cin

Cout(X, Y, 1) = Cout1 = X + Y
Cout(X, Y, 0) = Cout0 = X Y

Cout1 = X + Y when Cin=1
Cout0 = XY when Cin=0

Cout1 = P' Cin … P' = relaxed P
Cout0 = G $\overline{Cin}$

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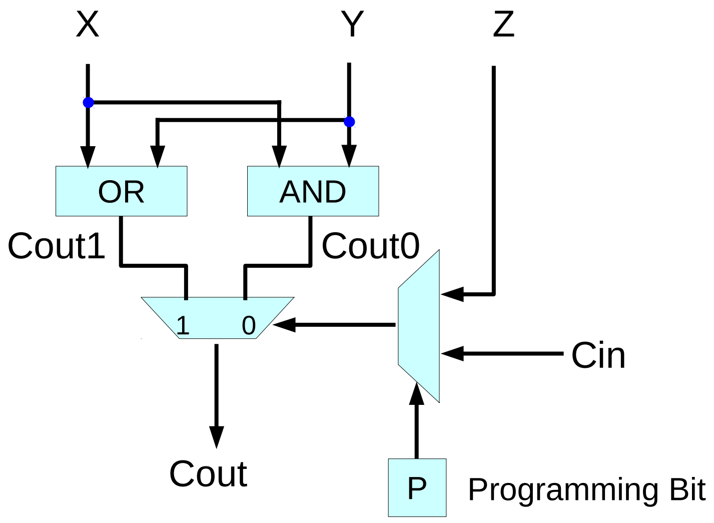
If Cin, then Cout = ($\overline{X}$ Y + X $\overline{Y}$ + X Y)
If $\overline{Cin}$, then Cout = X Y

Cin (X + Y) + $\overline{Cin}$ X Y
Cin ($\overline{X}$ Y + X $\overline{Y}$ + X Y) + $\overline{Cin}$ X Y
Cin ($\overline{X}$ Y + X $\overline{Y}$) + (Cin + $\overline{Cin}$) X Y
P Cin + G

Cin (X + Y) + $\overline{Cin}$ X Y
Cin P' + $\overline{Cin}$ G … P' : relaxed P

FPGA Carry Chain Cell



		Cin	$\overline{\text{Cin}}$	
X	Y	Cout1	Cout0	
0	0	0	0	$\overline{X}\,\overline{Y}$
0	1	1	0	$\overline{X}\,Y$
1	0	1	0	$X\,\overline{Y}$
1	1	1	1	$X\,Y$

X	Y	Cin	Cout	
0	0	0	0	Cout0
0	1	0	0	Cout0
1	0	0	0	Cout0
1	1	0	1	Cout0
0	0	1	0	Cout1
0	1	1	1	Cout1
1	0	1	1	Cout1
1	1	1	1	Cout1

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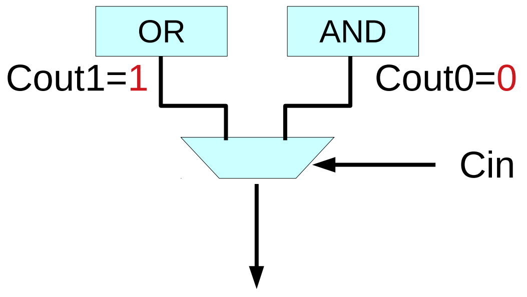
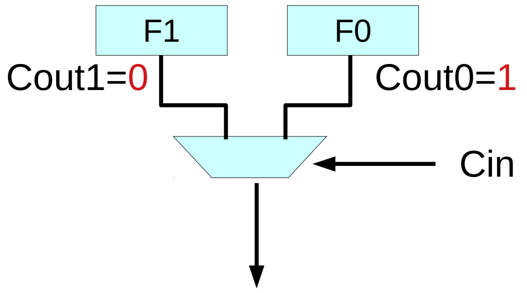
FPGA Carry Chain Cell



Cout1=1 when Cin=1
Cout0=0 when Cin=0
Cout = Cin



Cout1=0 when Cin=1
Cout0=1 when Cin=0
Cout = $\overline{\text{Cin}}$

Cout0	Cout1	Cout	Name
0	0	0	Kill
0	1	Cin	Propagate
1	0	$\overline{\text{Cin}}$	Inverse Propagate
1	1	1	Generate

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

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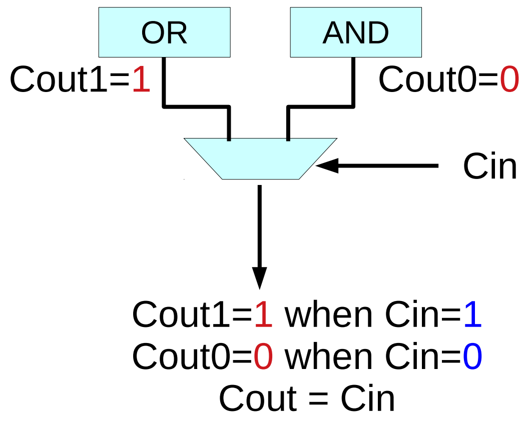
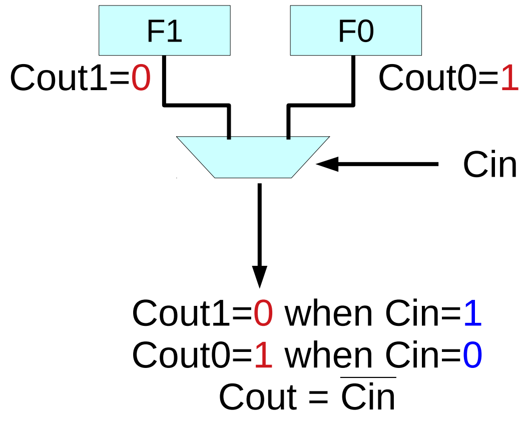
FPGA Carry Chain Cell





Cout0	Cout1	Cout	Name
0	0	0	Kill
0	1	Cin	Propagate
1	0	$\overline{\text{Cin}}$	Inverse Propagate
1	1	1	Generate

X	Y	Cin	Cout		Cout1	Cout0
0	0	0	0	Cout0	0	0
0	1	0	0	Cout0	1	0
1	0	0	0	Cout0	1	0
1	1	0	1	Cout0	1	1
0	0	1	0	Cout1	0	0
0	1	1	1	Cout1	1	0
1	0	1	1	Cout1	1	0
1	1	1	1	Cout1	1	1

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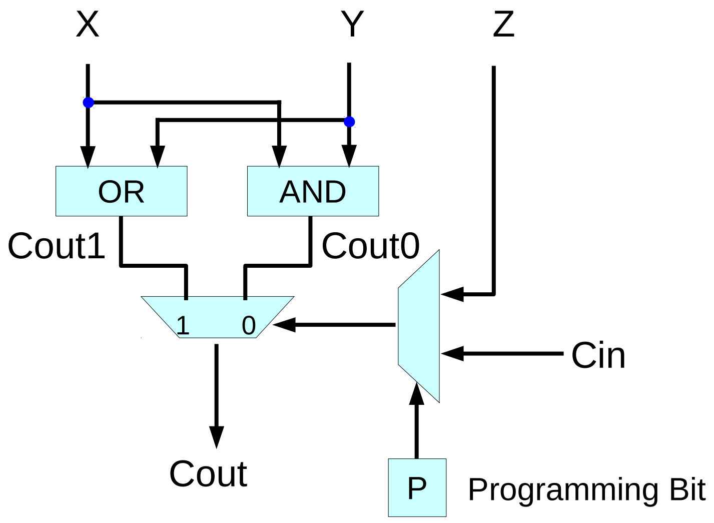
Carry Chain



		Cin	$\overline{Cin}$	
X	Y	Cout1	Cout0	
0	0	0	0	$\overline{X}\ \overline{Y}$
0	1	1	0	$\overline{X}\ Y$
1	0	1	0	$X\ \overline{Y}$
1	1	1	1	$X\ Y$

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	$\overline{Cin}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

Carry Out

X	Y	Cin	
0	0	Cin	Cin
0	1	Cin	$\overline{Cin}$
1	0	Cin	$\overline{Cin}$
1	1	Cin	Cin

Cout1=1 when Cin=1
Cout0=0 when Cin=0
Cout = Cin propagate

Cout1=0 when Cin=1
Cout0=1 when Cin=0
Cout = $\overline{Cin}$ inverse propagate

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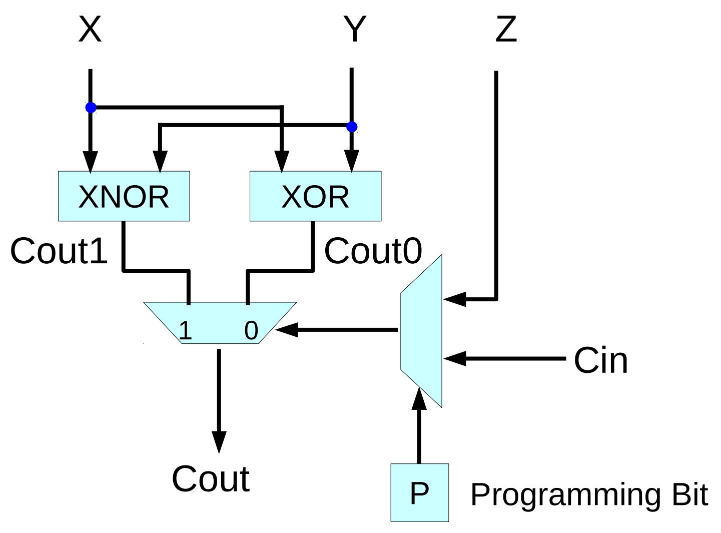
Parity Checker



X	Y	Cin Cout1	$\overline{\text{Cin}}$ Cout0	
0	0	1	0	$\overline{X}\,\overline{Y}$
0	1	0	1	$\overline{X}\,Y$
1	0	0	1	$X\,\overline{Y}$
1	1	1	0	$X\,Y$

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

Computing Parity

X ⊕ Y ⊕ Cin	
0 ⊕ 0 ⊕ Cin	Cin
0 ⊕ 1 ⊕ Cin	$\overline{\text{Cin}}$
1 ⊕ 0 ⊕ Cin	$\overline{\text{Cin}}$
1 ⊕ 1 ⊕ Cin	Cin

Cout1=1 when Cin=1
Cout0=0 when Cin=0
Cout = Cin propagate

Cout1=0 when Cin=1
Cout0=1 when Cin=0
Cout = $\overline{\text{Cin}}$ inverse propagate

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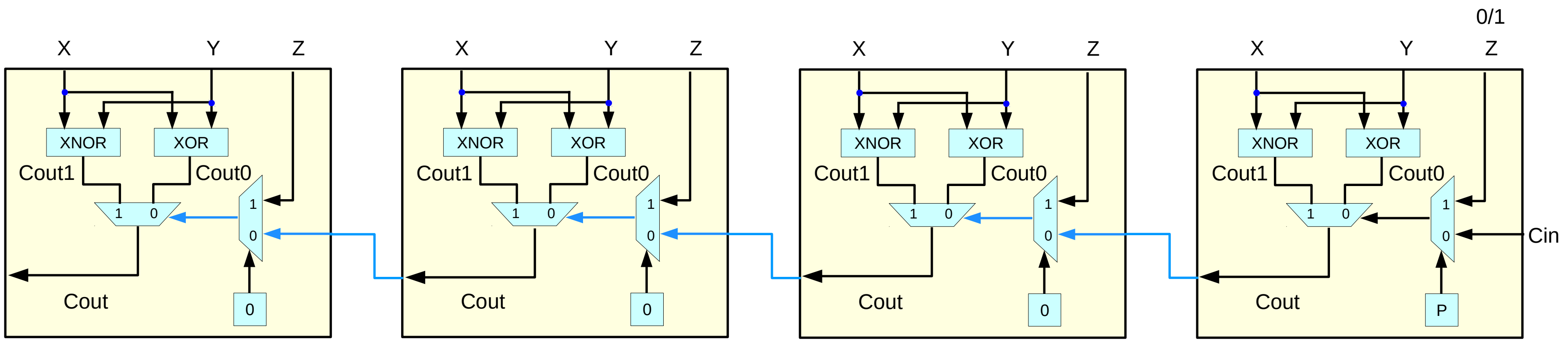
Ripple Carry Chain



the **first cell** in the chain

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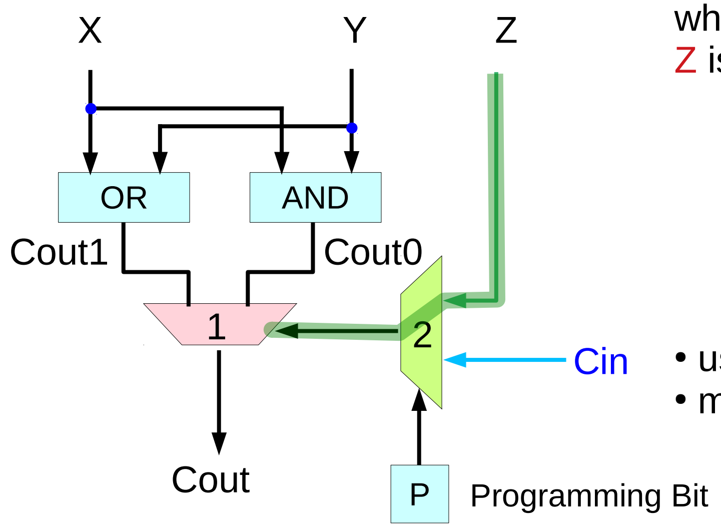
FPGA Carry Chain Cell



when Cin is ignored,
Z is routed to mux1

- used in combined adder/subtractors
- must be ignored, otherwise

the logic cells - resources to compute a function
the exact location of logic cells depends on the user.
a user can start or end a carry computation
at any place in an fpga.

But in many carry computations,
the first cell has only 2 inputs,
and forcing the carry chain
to wait for the arrival of an additional,
unnecessary input Z will only needlessly
slow down the circuit's computation.

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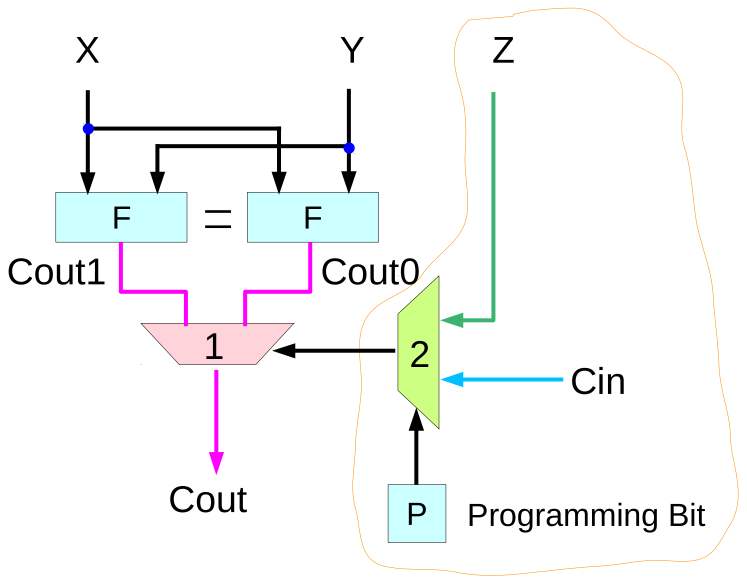
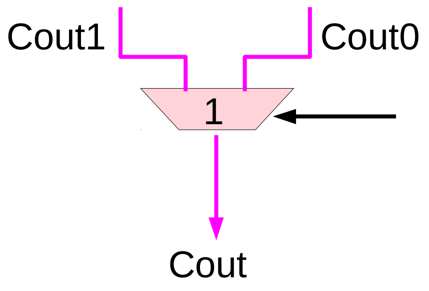
FPGA Carry Chain Cell



when Cin is ignored,
Z can also be ignored
by having the same LUTs



the **first cell** in the chain

the same LUTs

the same output
regardless of Z and Cin

Cout1 = Cout0 = Cout
regardless of the select

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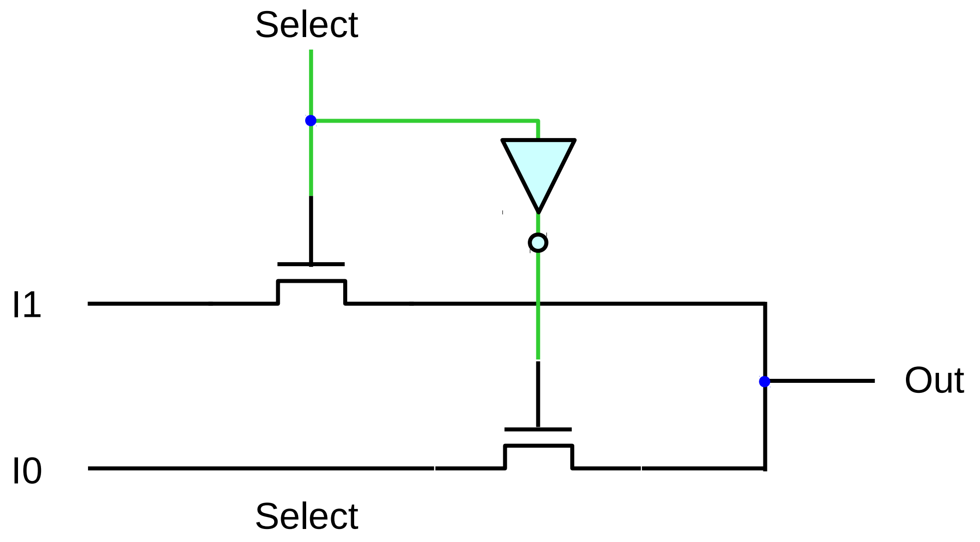
Ripple Carry Chain



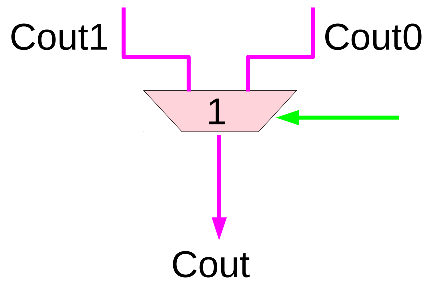


fig1b shows an implementation of a mux that does not obey this requirement

since the carry chain is part of an fpga, the input to this mux could be connected to some unused logic in another row which is generating unknown values.

if that unused logic had multiple transitions which caused the signal to change quicker than the gate could react, then it is possible that **the select signal** to this mux could be stuck midway between true and false (2.5V for 5V CMOS)

in this case, it will not be able to pass a true value from the input to the output and thus will not function properly for this application.

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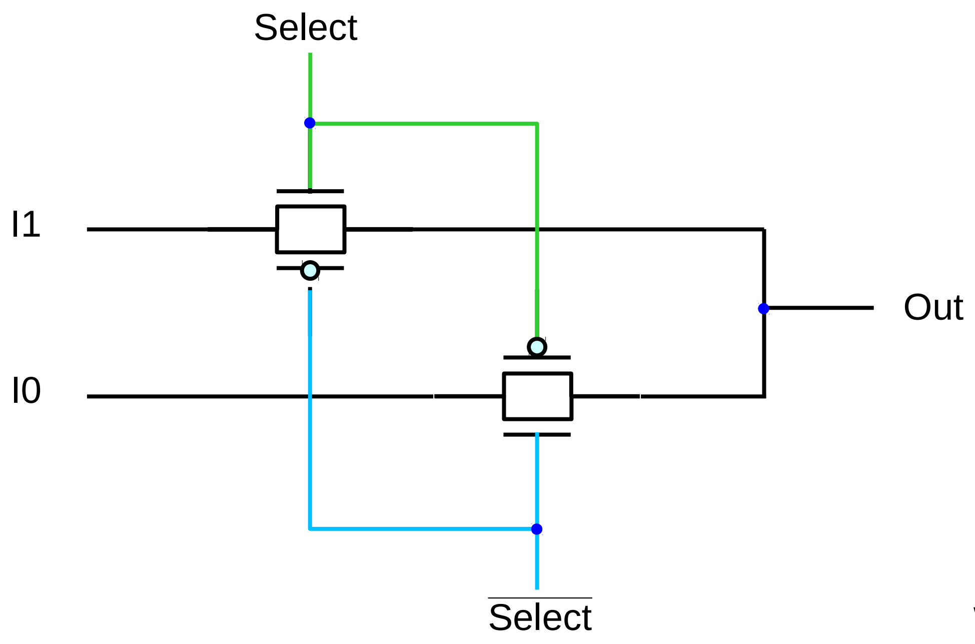
Ripple Carry Chain



however a mux built with
both n-transistor and p-transistor pass gates
will operate properly for this case

assume this mux implementation will be used

tristate driver based muxes could be used,
which restore signal drive and cut series RC chains

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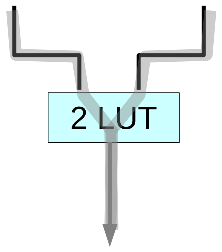
Unit Gate Delay Model

All simple gate of two or three inputs
that are directly implementable
in one logic level in CMOS
are considered to have a delay of one.

All other gate must be implemented by such gates,
and have the delay of the underlying circuit.

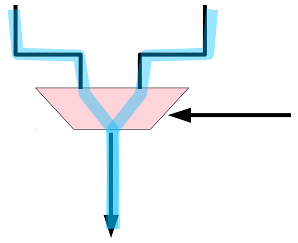
Delay of one
- inverters and
- 2 to 3 input NAND
- 2 to 3 input NOR gates

Delay of zero (constant delay)
- the delay of the 2-LUTs,
- any routing leading to them,

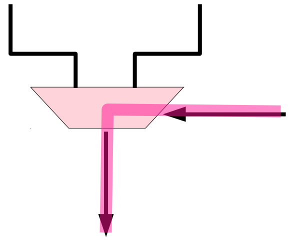


Delay of **0**

A 2:1 mux has a delay of one
from the I0 or I1 inputs to the output,
But has a delay of two
from the select input to the output
due to the Inverter delay



Delay of **1**



Delay of **2**

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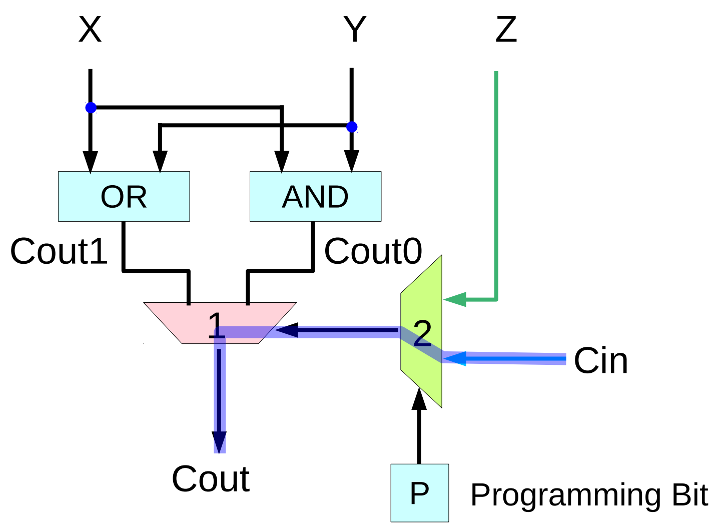
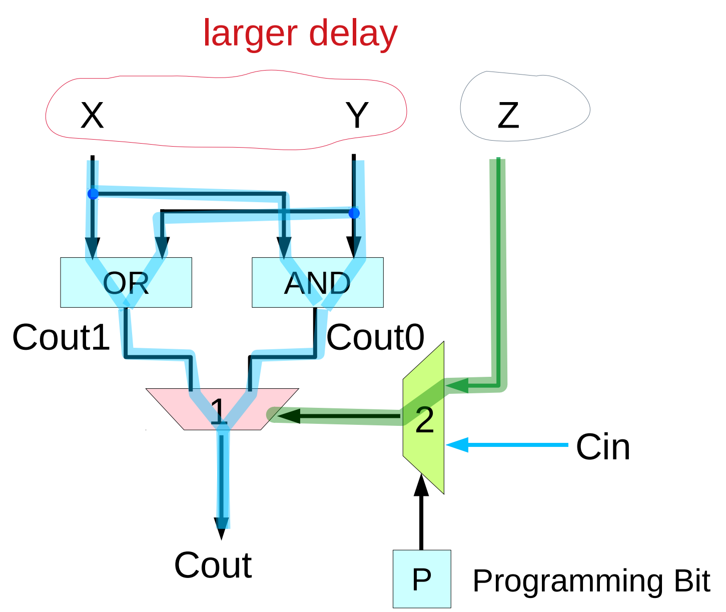
FPGA Carry Chain Cell



Delay 3



Delay 1

Significantly slower
two muxes on the carry chain in each cell

Delay 1 for first cell
Delay 3 for each additional cell in the carry chain
delay 1 for mux2
delays 2 for mux1

Overall 3n-2 for an n-cell carry chain

The critical path comes from the 2-LUTs
and not from the input Z
since the delay through the 2-LUTs
will be larger than through mux 2 in the first cell

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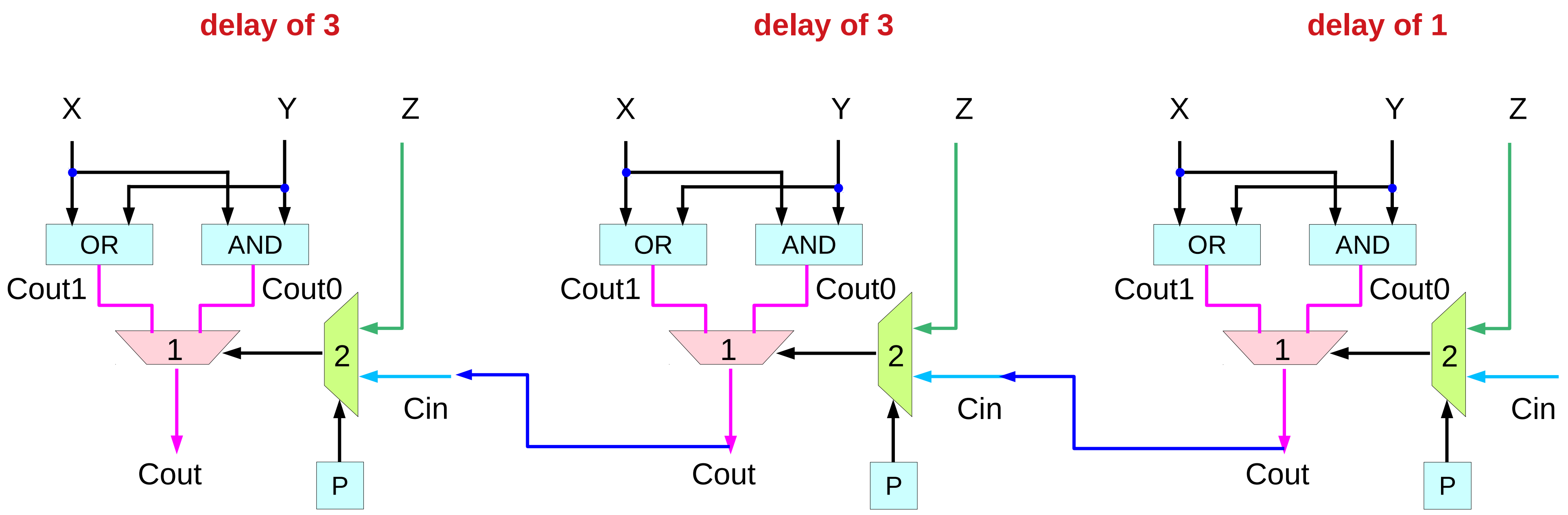
FPGA Carry Chain Cell



delay of 3n-2 for an n-bit ripple carry chain

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FPGA Carry Chain Cell

the linear delay growth of ripple carry adders

optimize a ripple carry chain structure for use in FPGAs

while this provides some performance gain
over the basis ripple carry scheme
found in many current FPGAs,

still much slower than what is done in custom logic

advanced adder techniques in custom logic
can be integrated into reconfigurable logic

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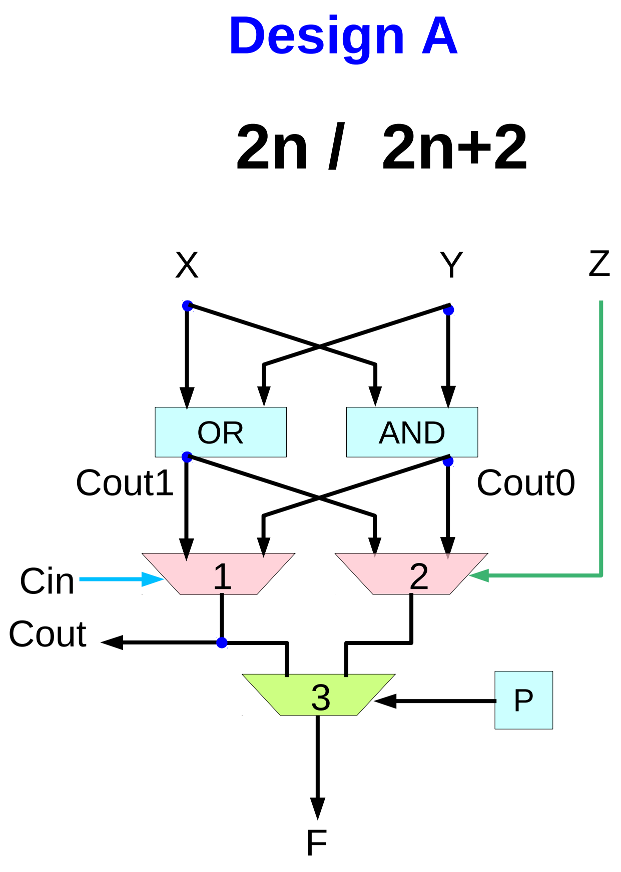
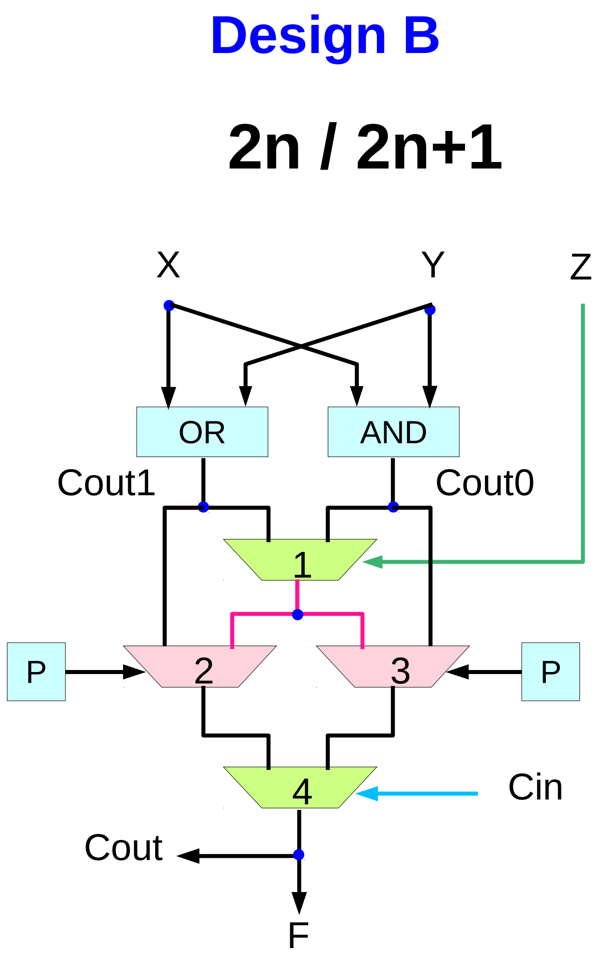
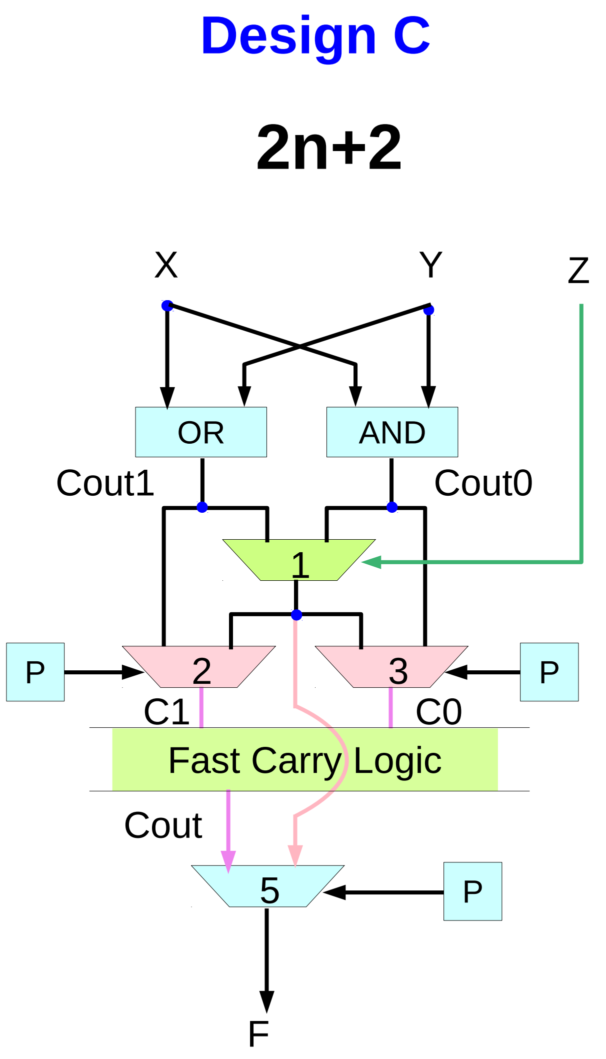
FPGA Carry Chain Cell







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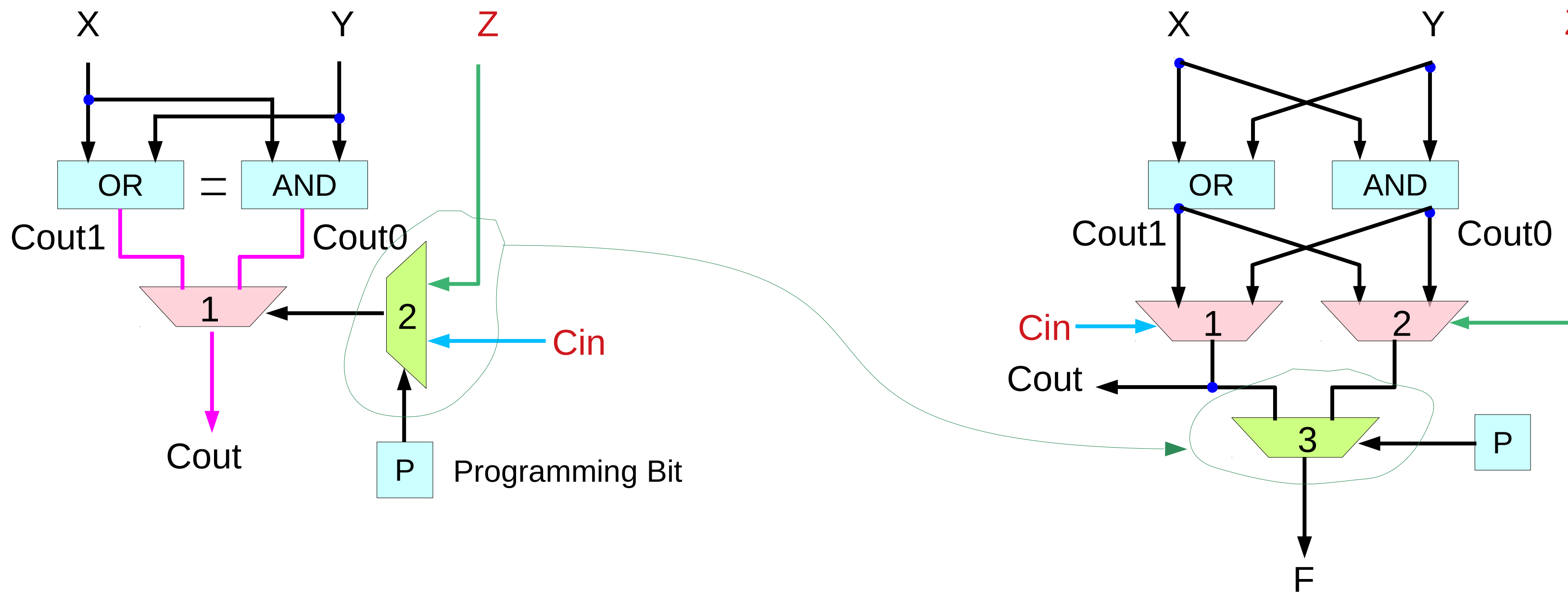
Design A

to reduce the delay of the ripple carry chain

- remove mux2 from the carry path.
- no need to choose between Cin and Z for the select line to the output mux1

- two separate muxes, mux1 and mux2, controlled by Cin and Z, respectively.
- the circuit chooses between these outputs with mux3.



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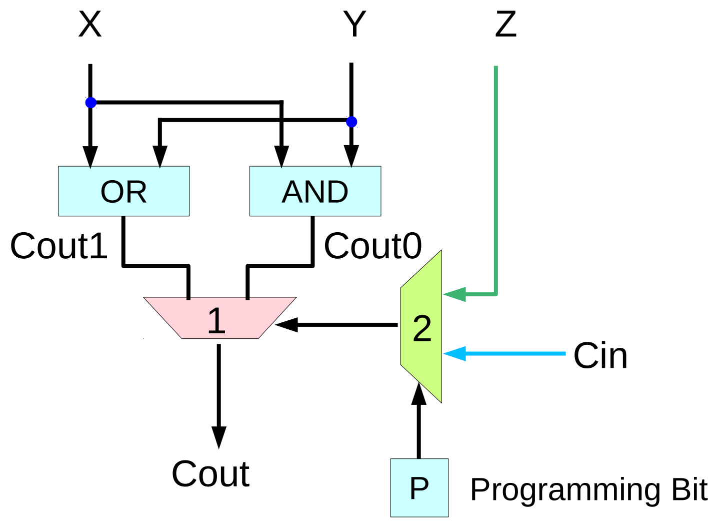
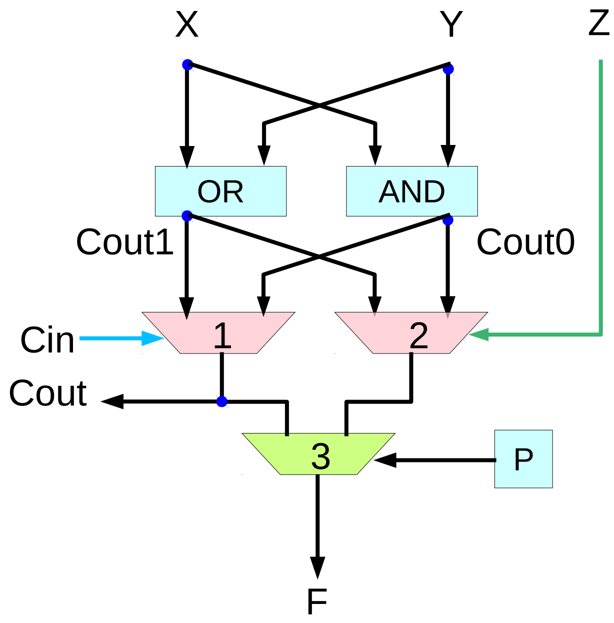
Design A





- not logically equivalent
- the Z input in the <u>first</u> cell cannot be used
  - Z is only attached to mux2
  - mux2 does not lead to the carry cells
  - not connected to Cout

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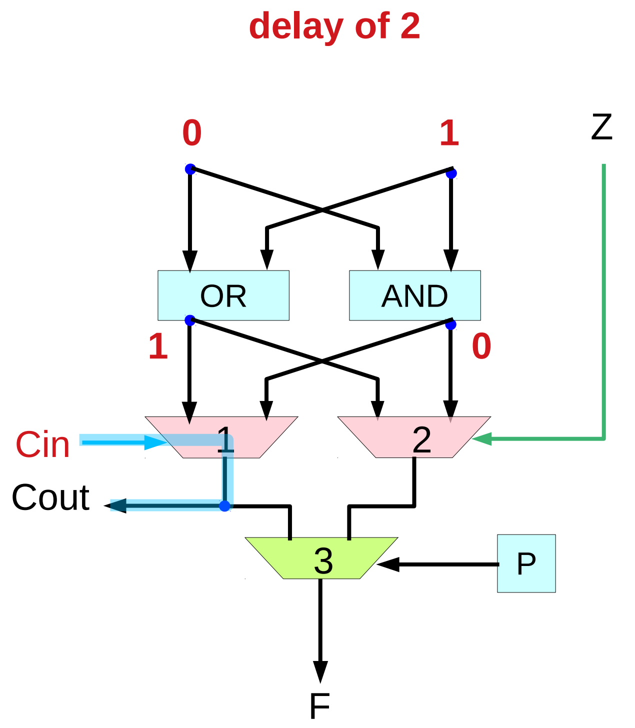
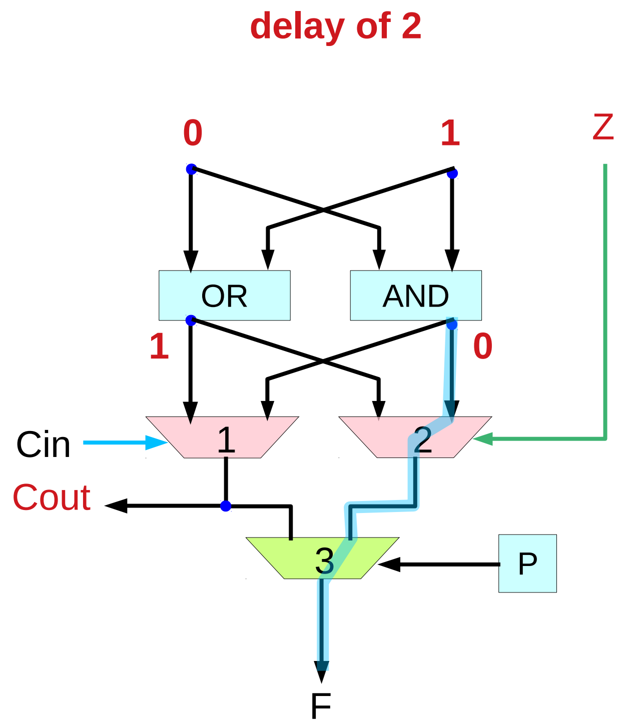
Design A



an additional cell
for generating Cin



- need an additional cell to use Z as a carry input

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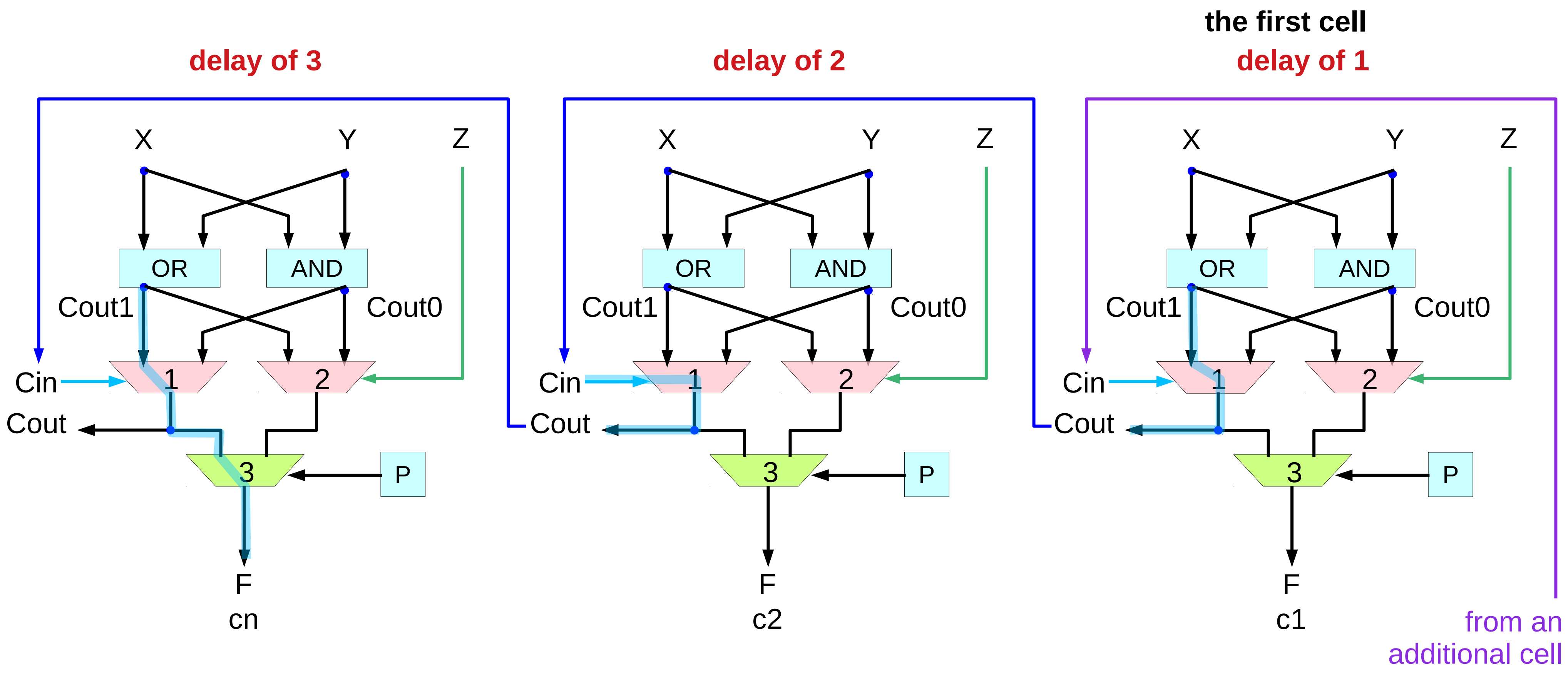
Design A



(2 for mux1, 1 for mux3)

50% faster circuit that the original design

delay of 2n for an n-bit ripple carry chain

without a carry input Z

delay of 2(n+1)

with a carry input Z

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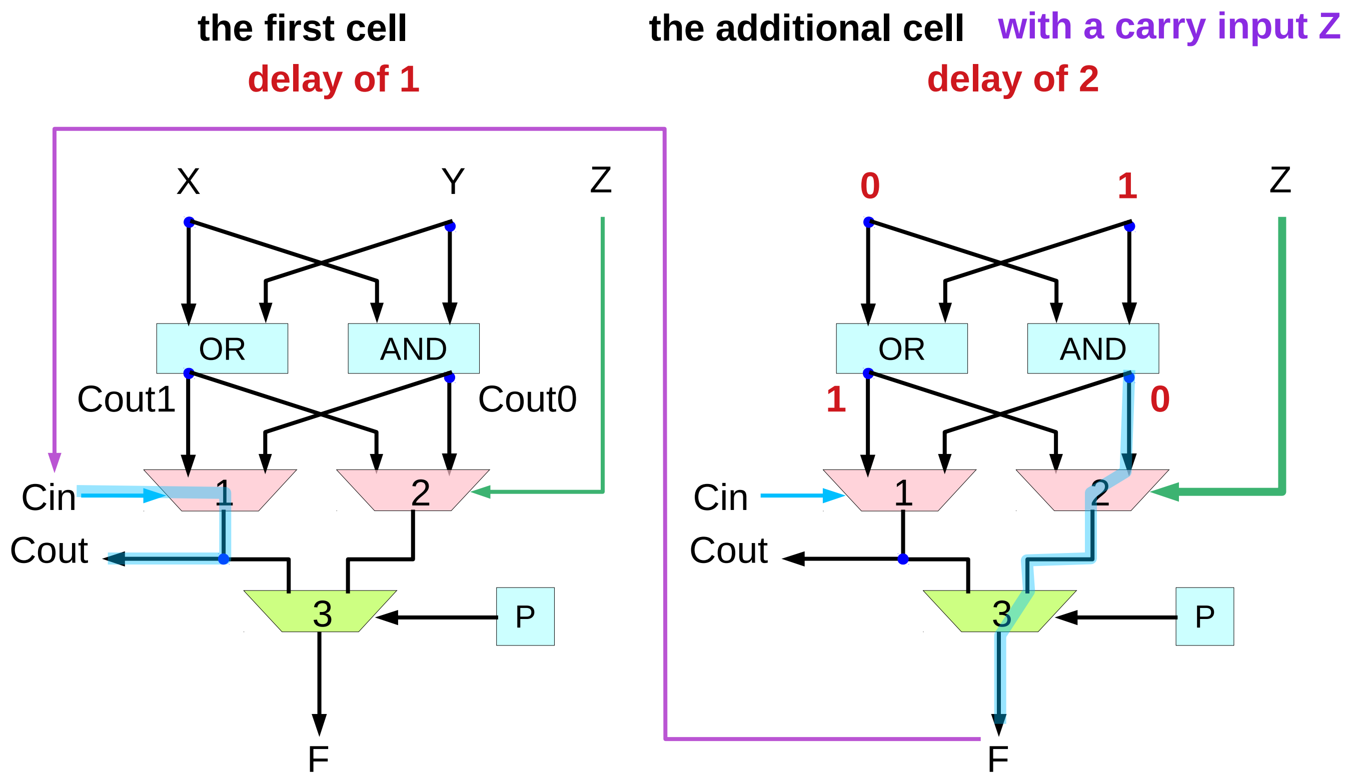
Design A



delay of 2(n+1) for an n-bit ripple carry chain with a carry input

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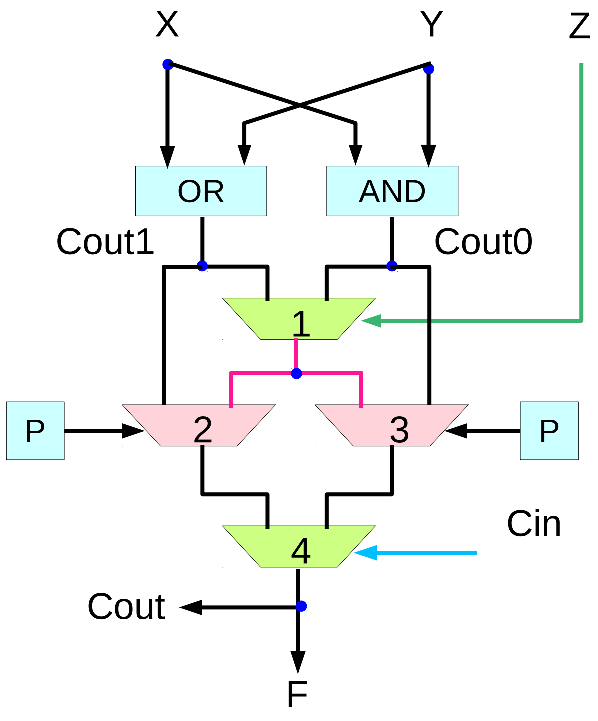
Design B

although this design is 1 gate delay slower than that of fig 2a,
it provides the ability to have a carry input
to the first cell in a carry chain,
something that is important in many computations.

Also, for carry computations that do not need this feature,
without a carry input
the first cell in a carry chain built from fig 2b
can be configured to bypass mux1,
reducing the overall delay to 2n,
which is identical to that of fig2a.



(2 for mux1, 1 for mux3)

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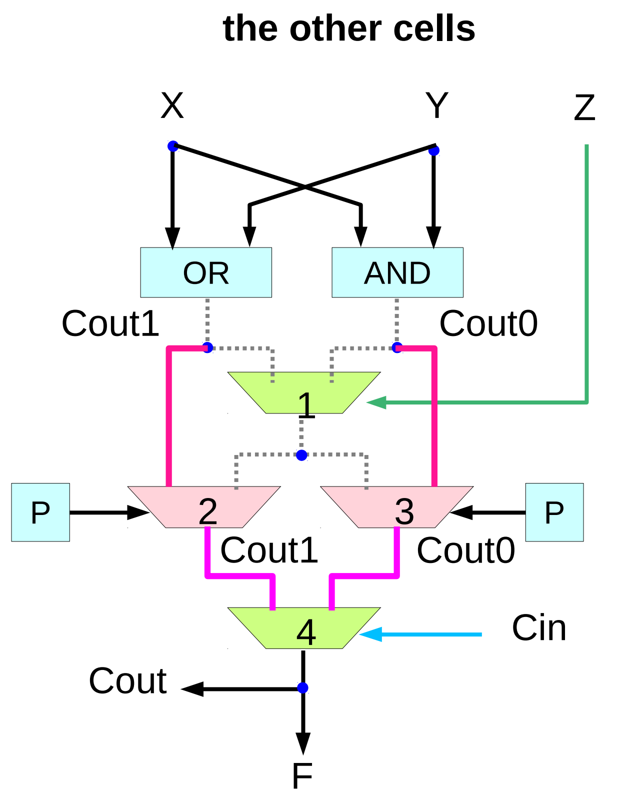
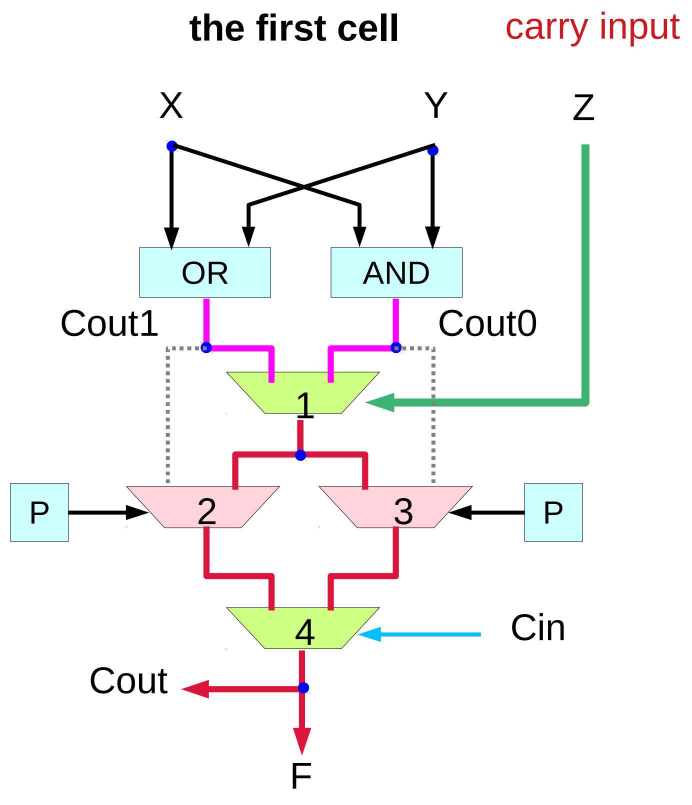
Design B





for cells in the middle of a carry chain
mux2 passes Cout1
mux3 passes Cout0
mux4 receives Cout1 and Cout0
provides a standard ripple carry path.

For the first cell in a carry chain
with a carry input (provided by input Z),
mux2 and mux3 both pass the value from mux1

the two main inputs to mux4 are identical
the output of mux4 (Cout) will be the same
as the output of mux1 (ignoring Cin)

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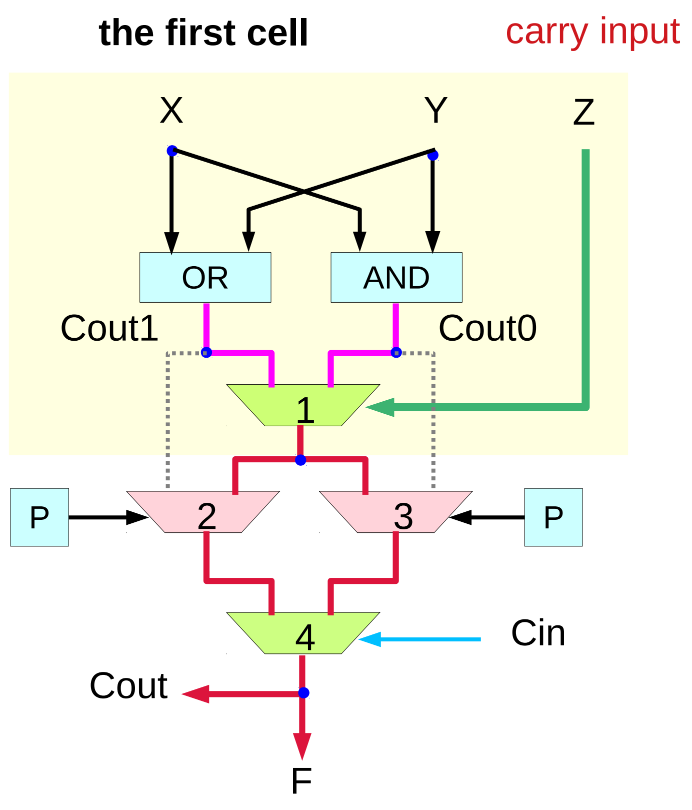
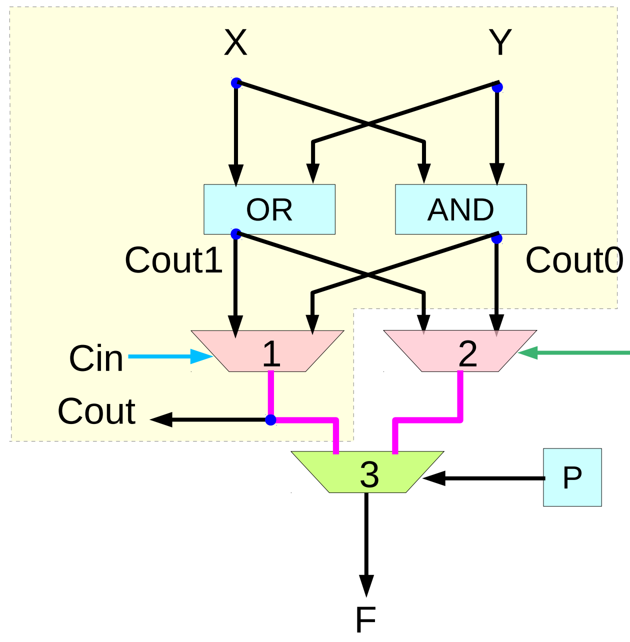
Design B



mux1's main inputs are driven
by two 2-LUTs (OR, AND) controlled by X and Y
mux1 forms a **3-LUT** with the other 2-LUTs

When mux2 and mux3 pass the value from mux1
(Cout1 and Cout2 respectively)
the circuit is configured to continue the carry chain

Functionally equivalent



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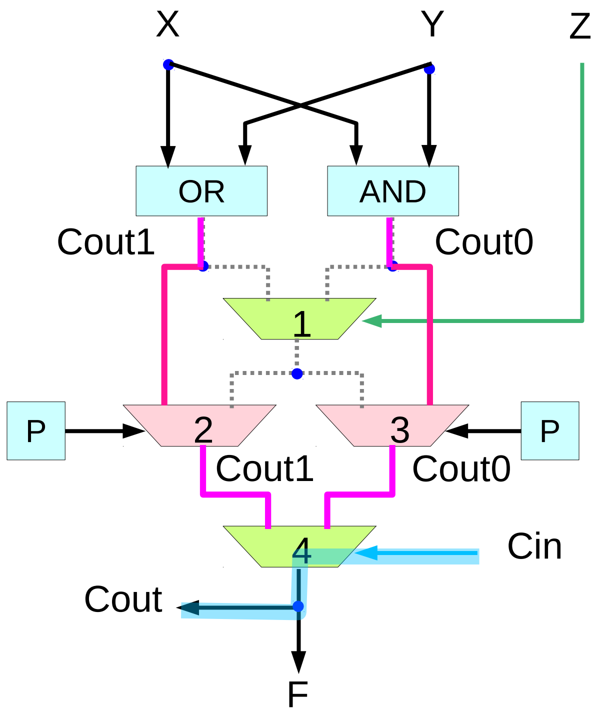
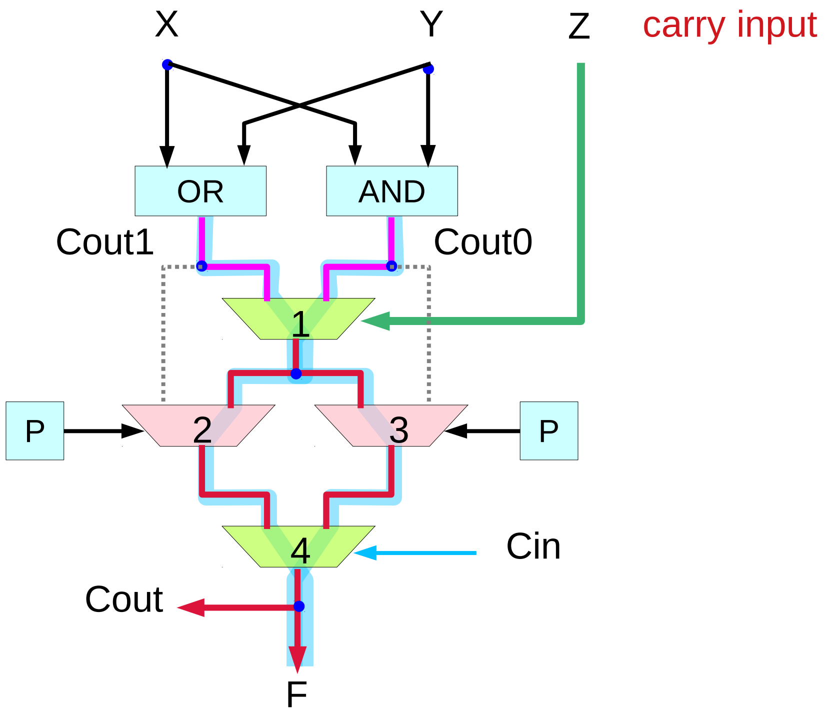
Design B

delay of 2 the other cells



delay of 3 the first cell with a carry input



A delay of 2 in all other cells
except the first cell in the carry chain

an total delay of **2n+1** for an n-bit carry chain
when a carry input to the first cell is enabled

1 gate delay slower than that of fig 2a,

a delay of 3 in the first cell
1 in mux1, 1 in mux2, 1 in mux4

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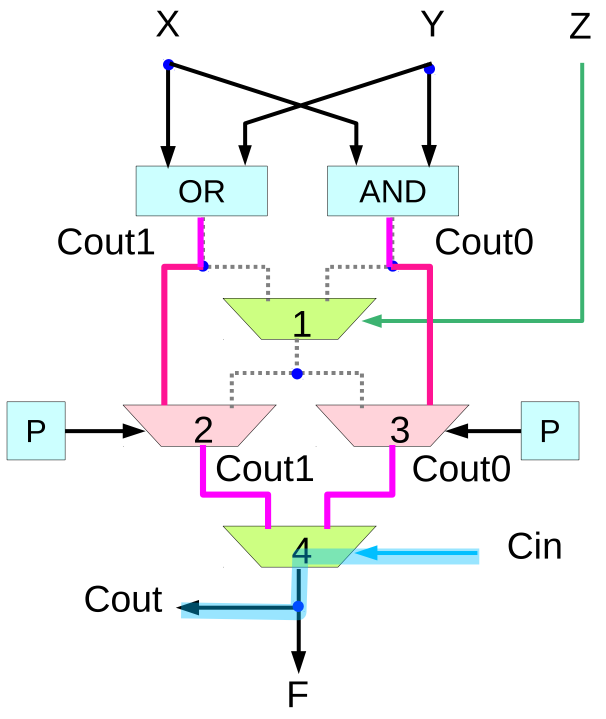
Design B

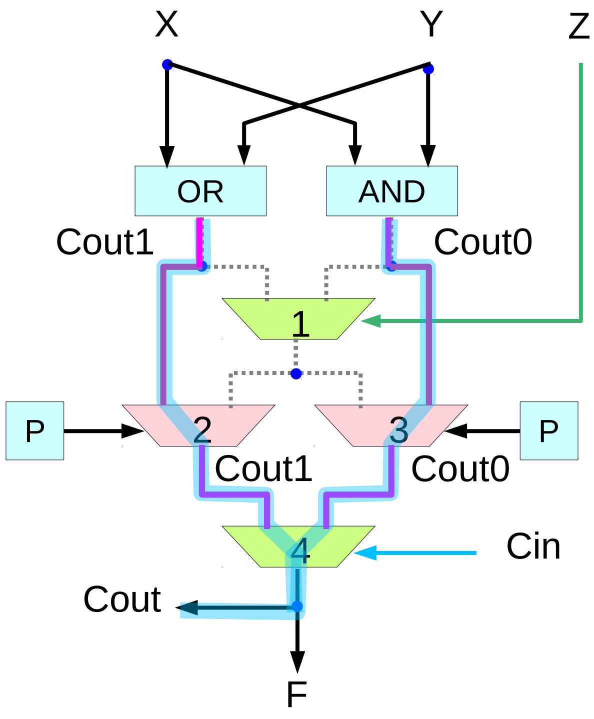
delay of 2 **the other cells**



A delay of 2 in all other cells
<u>except</u> the first cell in the carry chain

an total delay of **2n** for an n-bit carry chain
when a carry input to the first cell is disabled

delay of 2 **the first cell** **<u>without</u> a carry input**



a delay of 2 in the first cell
when a carry input is not used

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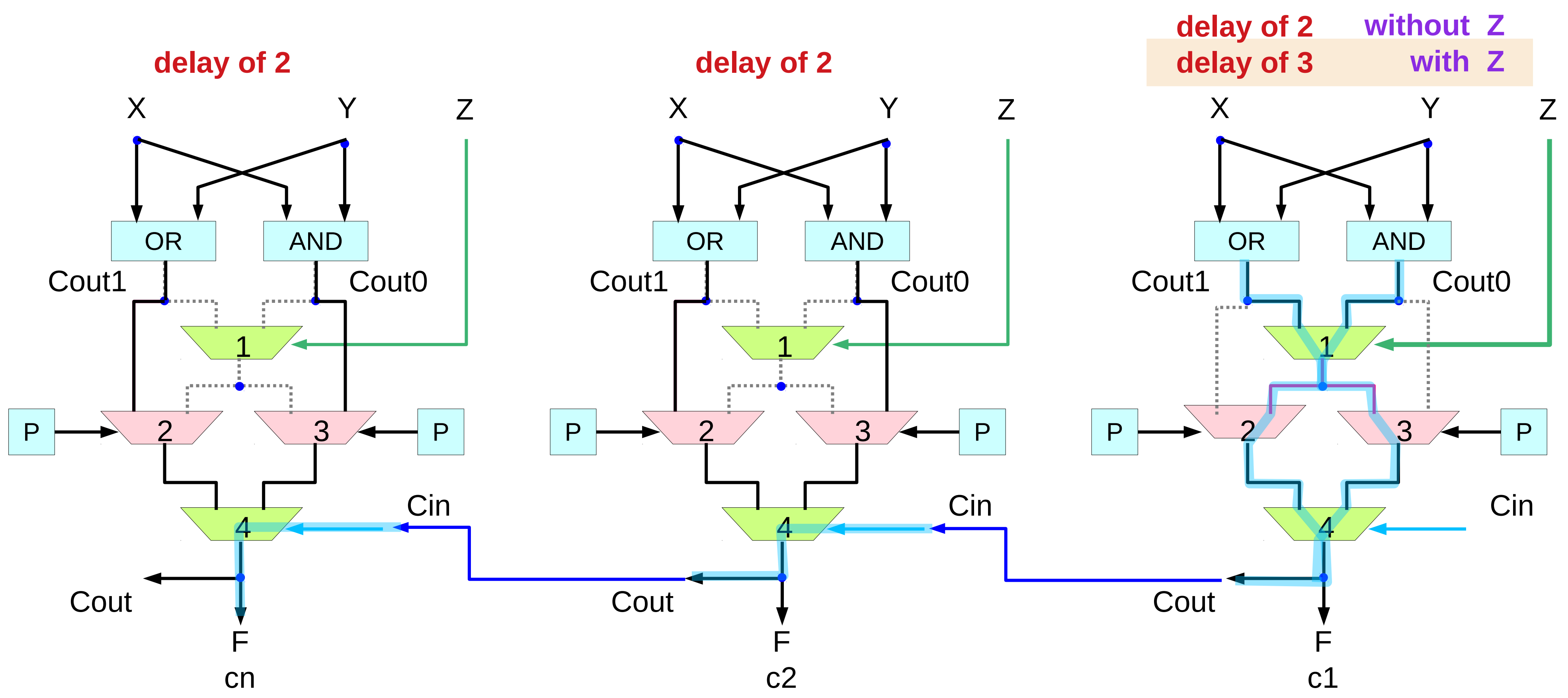
Design B



delay of 2n for an n-bit ripple carry chain **without a carry input Z**

delay of 2n+1 **with a carry input Z**

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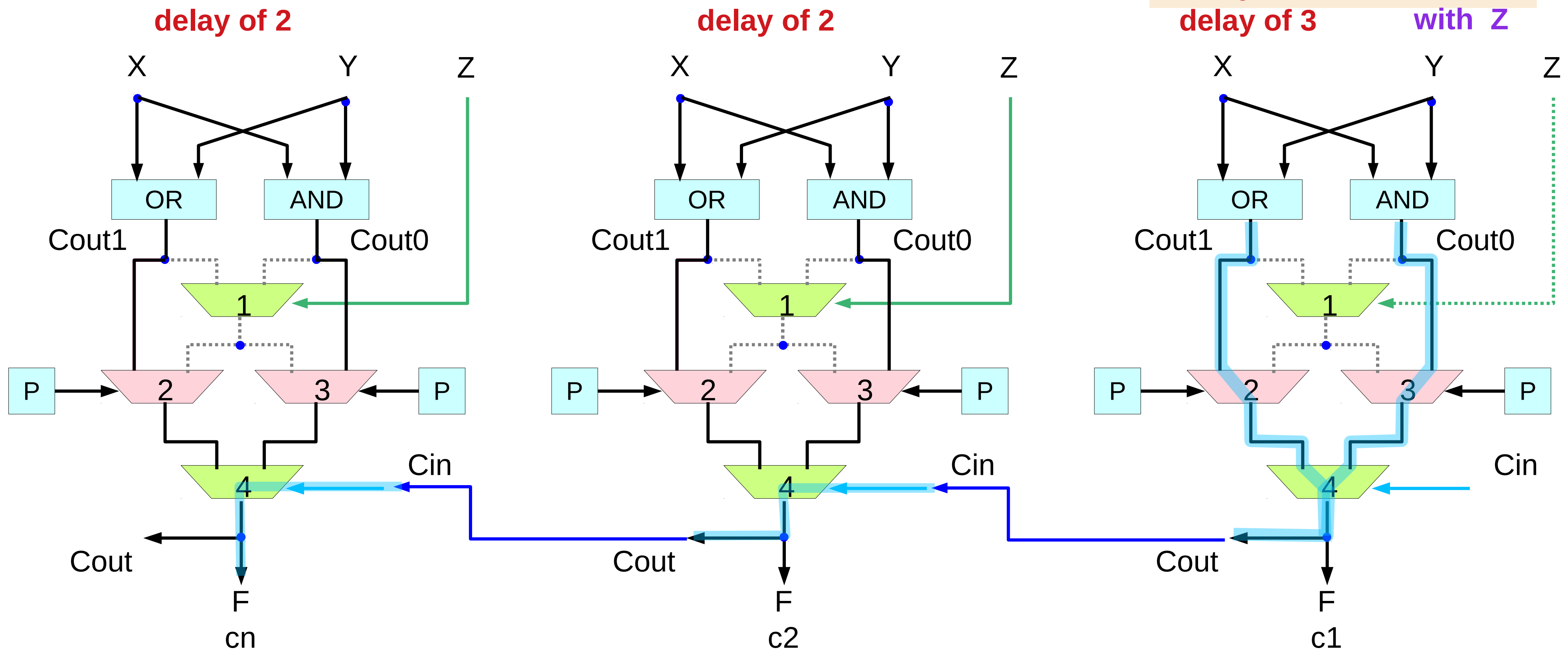
Design B

delay of 2 without Z
delay of 3 with Z



delay of 2n for an n-bit ripple carry chain without a carry input Z

delay of 2n+1 with a carry input Z

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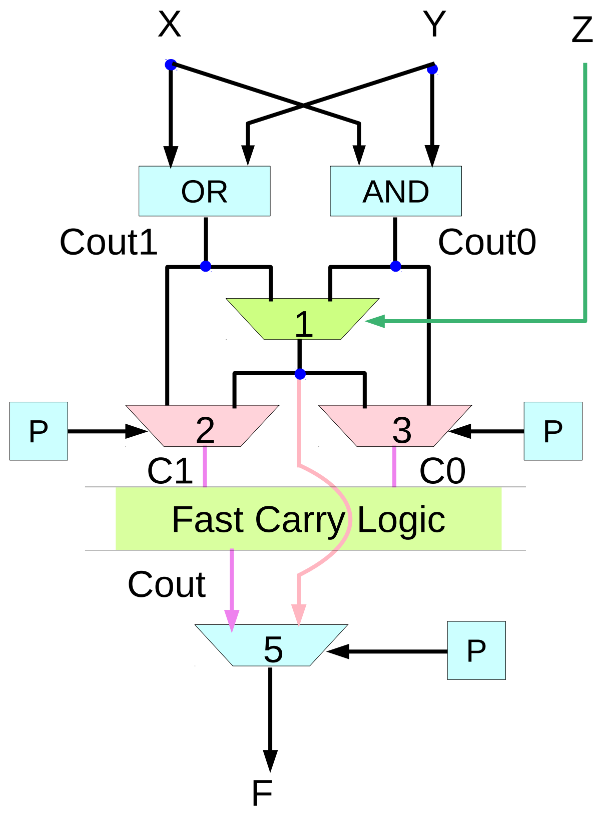
Design C (1)

various high performance carry chains
can be developed based on
the carry cell of Design C

very similar to Design B
except that the actual carry chain (mux4)
has been replaced by
an abstract fast carry logic unit
and mux5 has been added

this extra mux5 is present because
although some of our faster carry chains will
have much faster carry propagation
for long carry chains,
they incur significant delay
for non-carry computations

thus, when the cell is used as
a simple normal **3 LUT**,
using inputs X, Y, and Z
mux5 allows us to bypass the carry chain
by selecting the output of mux1



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Design C (1)

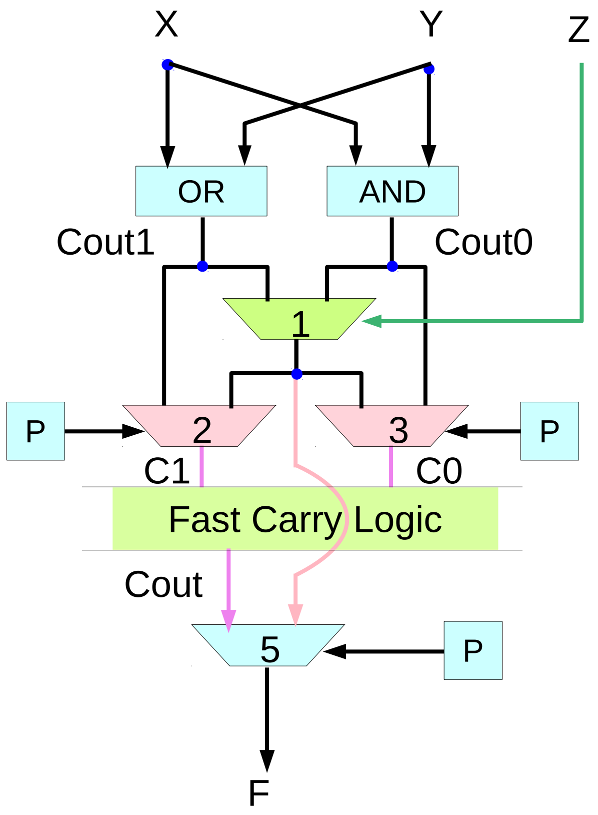
The important thing to realize about the logic of Design C is that any logic that can compute the value

$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

where i is the position of the cell within the carry chain, can provide the functionality necessary to support the needs of FPGA computations

thus, the fast carry logic unit can contain any logic structure implementing this (including Brent-Kung), Variable Bit, and Ripple Carry.

Note that because of the needs and requirements of carry chains for FPGAs, we will have to develop new circuits, inspired by the standard adder structures, but which are more appropriate for FPGAs



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Design C (2)

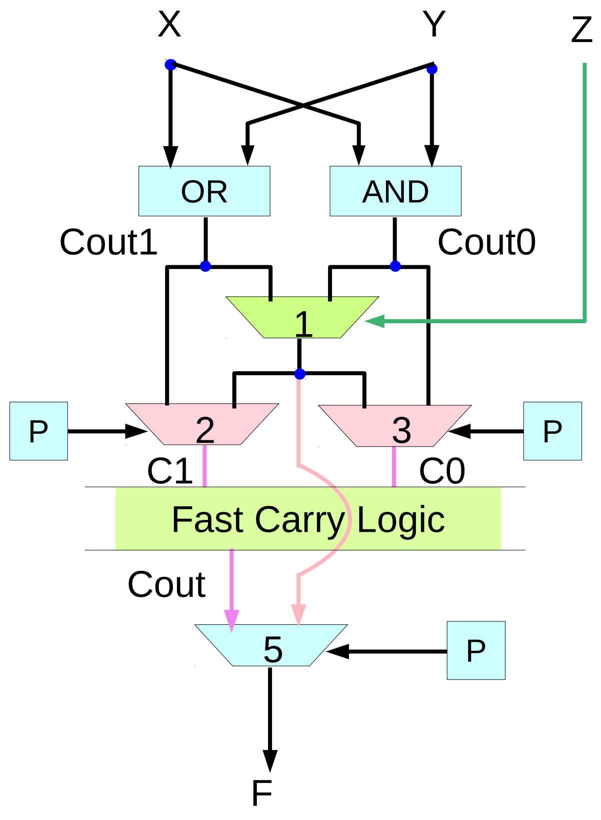
the main difference is to support all states

- Generate
- Propagate
- Kill
- Inverse Propagate

These 4 states are encoded
on signals C1 and C0

Also, while standard adders are concerened
only with the maximum delay
through an entire n-bit adder structure,
the delay concerns for FPGAs
are more complicated

Specifically, when an n-bit carry chain is built into
the architecture of an FPGA
it does <u>not</u> represent an <u>actual</u> computation,
but only the <u>potential</u> for a computation.



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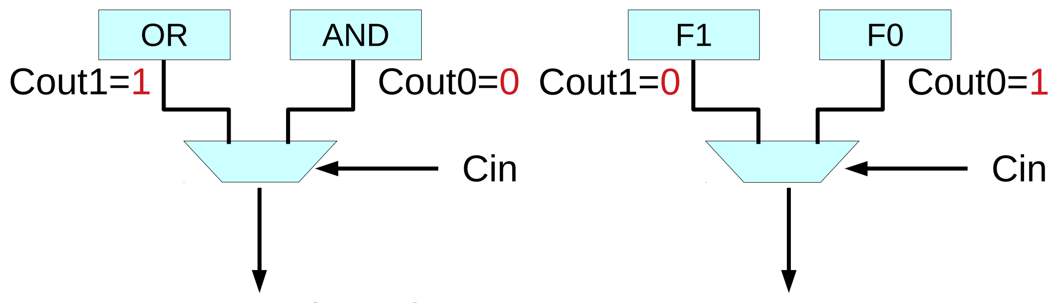
Design C (2)

		Cin	$\overline{\text{Cin}}$	
X	Y	Cout1	Cout0	
0	0	0	0	$\overline{X}\ \overline{Y}$
0	1	1	0	$\overline{X}\ Y$
1	0	1	0	$X\ \overline{Y}$
1	1	1	1	$X\ Y$

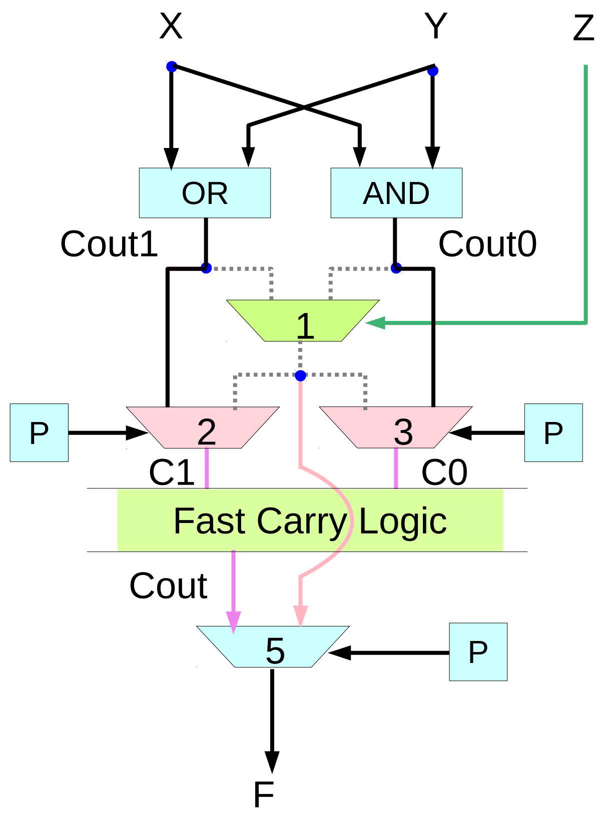
C1	C0	Name	
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

Cout1	Cout0	Cout	Name
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate



Cout1=1 when Cin=1
Cout0=0 when Cin=0
Cout = Cin

Cout1=0 when Cin=1
Cout0=1 when Cin=0
Cout = $\overline{\text{Cin}}$



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Design C (2)

X	Y	C1	C0	
0	0	0	0	$\overline{X}\,\overline{Y}$
0	1	1	0	$\overline{X}\,Y$
1	0	1	0	$X\,\overline{Y}$
1	1	1	1	$X\,Y$

$$C1_i = X_i + Y_i$$

$$C0_i = X_i \cdot Y_i$$

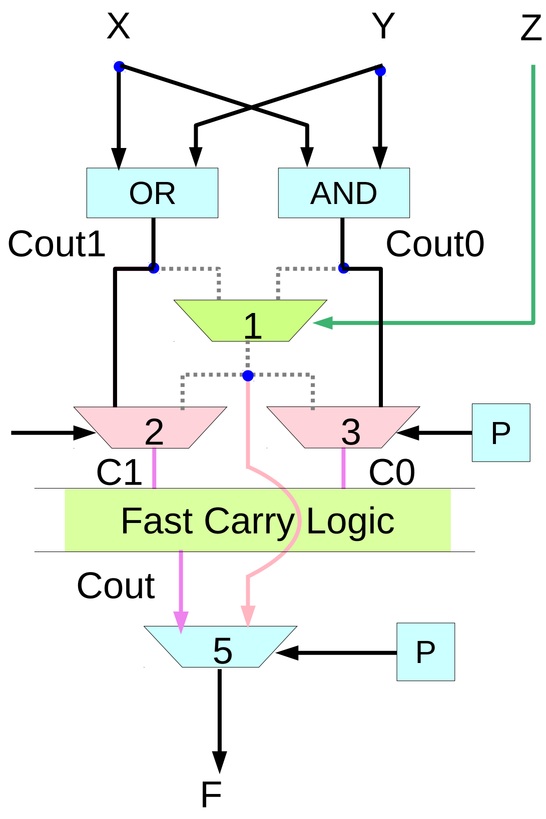
C1	C0	Name	
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

$$(Cout_{i-1} \cdot C1_i) = Cout_{i-1} \cdot (\overline{X}\,Y + X\,\overline{Y} + X\,Y)$$

$$(\overline{Cout_{i-1}} \cdot C0_i) = \overline{Cout_{i-1}} \cdot X\,Y$$

X	Y	$Cout_i$	$Cout_{i+1}$
0	0	0	0
0	1	0	0
1	0	0	0
1	1	0	1
0	0	1	0
0	1	1	1
1	0	1	1
1	1	1	1



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Generate and propagate

$C1 = \bar{X}Y + X\bar{Y} + XY$

X	Y	C1
0	0	0
0	1	1
1	0	1
1	1	1

$C0 = XY$

X	Y	C0
0	0	0
0	1	0
1	0	0
1	1	1

$C1 = \bar{X}Y + X\bar{Y} + XY$

$C0 = XY$

$\overline{C1} = \bar{X}\bar{Y}$

X	Y	$\overline{\text{C1}}$
0	0	0
0	1	1
1	0	1
1	1	1

$\overline{C0} = \bar{X}Y + X\bar{Y} + \bar{X}\bar{Y}$

X	Y	$\overline{\text{C0}}$
0	0	0
0	1	1
1	0	1
1	1	1

$\overline{C1} = \overline{(\bar{X}Y) + (X\bar{Y}) + (XY)} = \bar{X}\bar{Y}$

$\overline{C0} = \bar{X} + \bar{Y} = \bar{X}Y + X\bar{Y} + \bar{X}\bar{Y}$

$$\begin{aligned} Cout_3 &= \left(Cout_1 \cdot \left(C1_3 \cdot C1_2 + C0_3 \cdot \overline{C1_2}\right)\right) \\ &+ \left(\overline{Cout_1} \cdot \left(C1_3 \cdot C0_2 + C0_3 \cdot \overline{C0_2}\right)\right) \end{aligned}$$

$$\begin{aligned} &= \left(Cout_1 \cdot \left(C1_3 \cdot \left(\bar{X}_2 Y_2 + X_2 \bar{Y}_2 + X_2 Y_2\right) + C0_3 \cdot \bar{X}_2 \bar{Y}_2\right)\right) \\ &+ \left(\overline{Cout_1} \cdot \left(C1_3 \cdot X_2 Y_2 + C0_3 \cdot \left(\bar{X}_2 Y_2 + X_2 \bar{Y}_2 + \bar{X}_2 \bar{Y}_2\right)\right)\right) \end{aligned}$$

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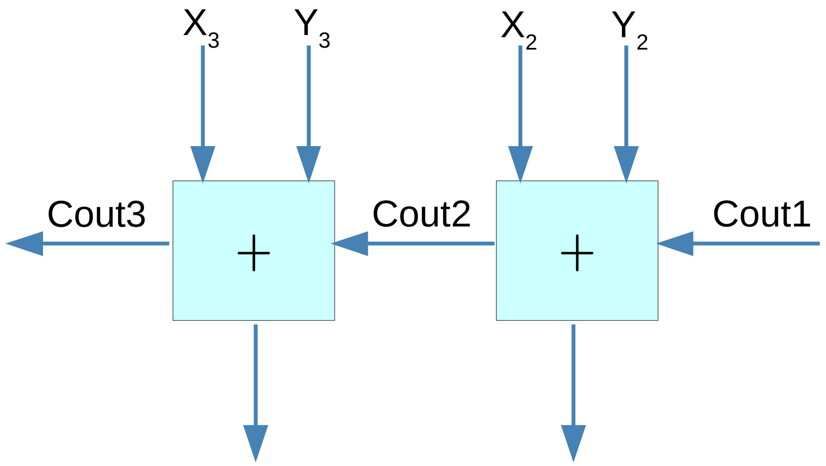
Cout3 in terms of Cout1

X_3 Y_3	X_2 Y_2	Cout2	Cout3	Cout3
0 0	0 0	0	0	0
0 0	0 1	Cout1	0	0
0 0	1 0	Cout1	0	0
0 0	1 1	1	0	0
0 1	0 0	0	Cou2	0
0 1	0 1	Cout1	Cou2	Cout1
0 1	1 0	Cout1	Cou2	Cout1
0 1	1 1	1	Cou2	1
1 0	0 0	0	Cou2	0
1 0	0 1	Cout1	Cou2	Cout1
1 0	1 0	Cout1	Cou2	Cout1
1 0	1 1	1	Cou2	1
1 1	0 0	0	1	1
1 1	0 1	Cout1	1	1
1 1	1 0	Cout1	1	1
1 1	1 1	1	1	1



$$
\begin{aligned}
Cout_3 &= \left(Cout_1 \cdot \left(C1_3 \cdot C1_2 + C0_3 \cdot \overline{C1_2}\right)\right) \\
&+ \left(\overline{Cout_1} \cdot \left(C1_3 \cdot C0_2 + C0_3 \cdot \overline{C0_2}\right)\right)
\end{aligned}
$$

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Cout3 in terms of Generate and Propagate

						Cout1	Cout1	$\overline{\text{Cout1}}$	$\overline{\text{Cout1}}$	
$X_3\ Y_3$	$X_2\ Y_2$	$C1_3$	$C0_3$	$C1_2$	$C0_2$	$C1_3C1_2$	$C0_3\overline{C1_2}$	$C1_3C0_2$	$C0_3\overline{C0_2}$	Cout3
0 0	0 0	0	0	0	0	0	0	0	0	0
0 0	0 1	0	0	1	0	0	0	0	0	0
0 0	1 0	0	0	1	0	0	0	0	0	0
0 0	1 1	0	0	1	1	0	0	0	0	0
0 1	0 0	1	0	0	0	0	0	0	0	0
0 1	0 1	1	0	1	0	1	0	0	0	Cout1
0 1	1 0	1	0	1	0	1	0	0	0	Cout1
0 1	1 1	1	0	1	1	1	0	1	0	1
1 0	0 0	1	0	0	0	0	0	0	0	0
1 0	0 1	1	0	1	0	1	0	0	0	Cout1
1 0	1 0	1	0	1	0	1	0	0	0	Cout1
1 0	1 1	1	0	1	1	1	0	1	0	1
1 1	0 0	1	1	0	0	0	1	0	1	1
1 1	0 1	1	1	1	0	1	0	0	1	1
1 1	1 0	1	1	1	0	1	0	0	1	1
1 1	1 1	1	1	1	1	1	0	1	0	1

$$\begin{aligned} Cout_3 &= \left(Cout_1 \cdot \left(C1_3 \cdot C1_2 + C0_3 \cdot \overline{C1_2}\right)\right) \\ &+ \left(\overline{Cout_1} \cdot \left(C1_3 \cdot C0_2 + C0_3 \cdot \overline{C0_2}\right)\right) \end{aligned}$$

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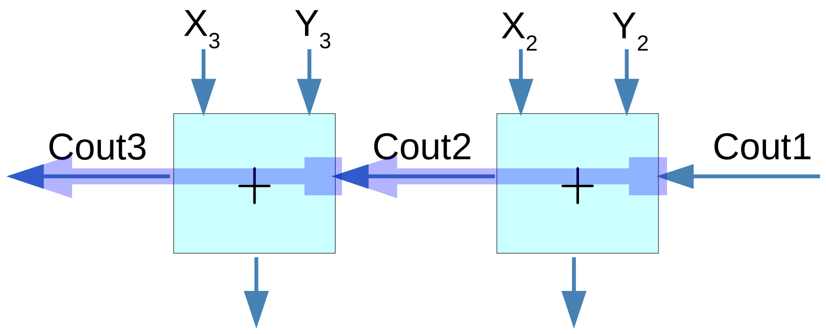
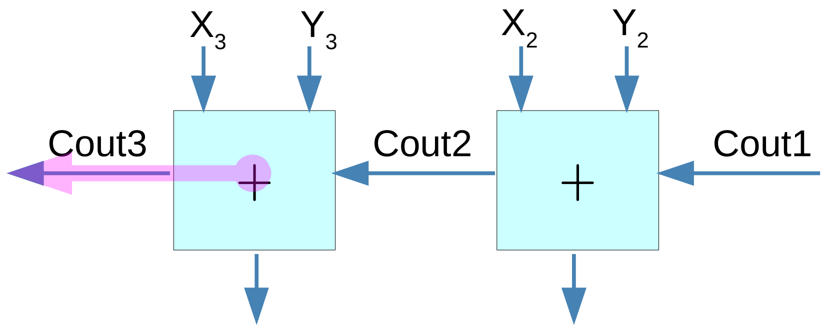
When Cout1 = 1





$$C1_3 \cdot C1_2 \cdot Cout_1$$

prop	prop
$\overline{X}_3 Y_3$	$\overline{X}_2 Y_2$
$X_3 \overline{Y}_3$	$X_2 \overline{Y}_2$
$X_3 Y_3$	$X_2 Y_2$

$$C0_3 \cdot \overline{C1_2} \cdot Cout_1$$

gen	prop
$X_3 Y_3$	$\overline{X}_2 \overline{Y}_2$

$$\begin{aligned} Cout_3 &= \left(Cout_1 \cdot \left(C1_3 \cdot C1_2 + C0_3 \cdot \overline{C1_2}\right)\right) \\ &+ \left(\overline{Cout_1} \cdot \left(C1_3 \cdot C0_2 + C0_3 \cdot \overline{C0_2}\right)\right) \end{aligned}$$

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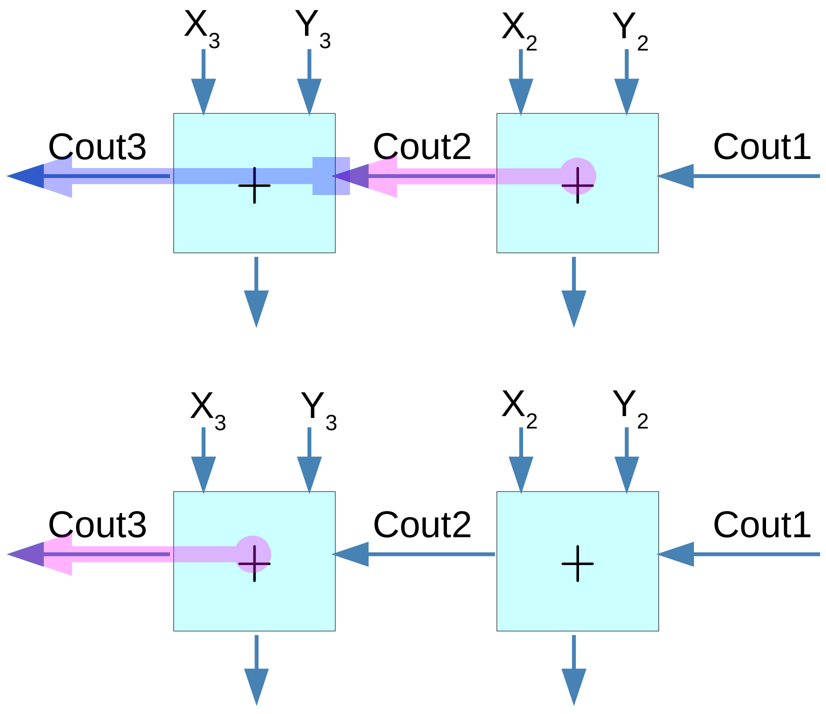
When Cout1 = 0



$$C1_3 \cdot C0_2 \cdot \overline{Cout}_1$$

prop	gen
$\overline{X}_3 Y_3$	$X_2 Y_2$
$X_3 \overline{Y}_3$	
$X_3 Y_3$	

$$C0_3 \cdot \overline{C0}_2 \cdot \overline{Cout}_1$$

gen	$\overline{\text{gen}}$
$X_3 Y_3$	$\overline{X}_2 Y_2$
	$X_2 \overline{Y}_2$
	$\overline{X}_2 \overline{Y}_2$

$$\begin{aligned} Cout_3 &= \left(Cout_1 \cdot \left(C1_3 \cdot C1_2 + C0_3 \cdot \overline{C1_2}\right)\right) \\ &+ \left(\overline{Cout}_1 \cdot \left(C1_3 \cdot C0_2 + C0_3 \cdot \overline{C0_2}\right)\right) \end{aligned}$$

$(C1_3 C1_2 + C0_3 \overline{C1}_2) Cout_1 + (C1_3 C0_2 + C0_3 \overline{C0}_2) \overline{Cout}_1$

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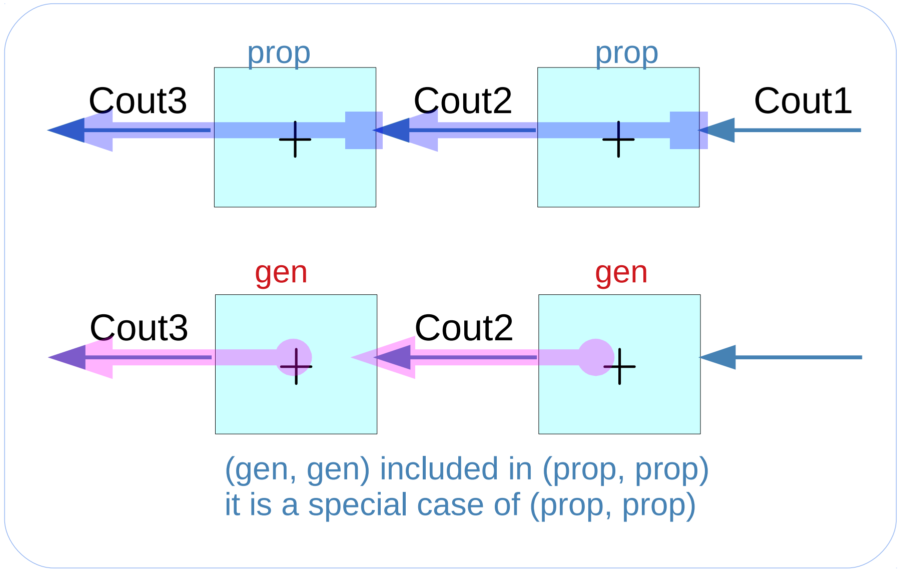
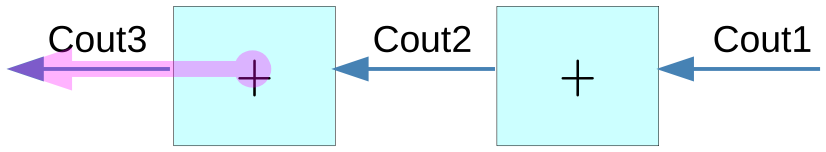
When Cout1 = 1



(gen, gen) included in (prop, prop)
it is a special case of (prop, prop)

$$C1_3 \cdot C1_2 \cdot Cout_1$$

prop	prop
$\overline{X_3}Y_3$	$\overline{X_2}Y_2$
$X_3\overline{Y_3}$	$X_2\overline{Y_2}$
X_3Y_3	X_2Y_2



$$\begin{aligned} Cout_3 &= \left(Cout_1 \cdot \left(C1_3 \cdot C1_2 + C0_3 \cdot \overline{C1_2}\right)\right) \\ &+ \left(\overline{Cout_1} \cdot \left(C1_3 \cdot C0_2 + C0_3 \cdot \overline{C0_2}\right)\right) \end{aligned}$$

$$C0_3 \cdot \overline{C1_2} \cdot Cout_1$$

gen	$\overline{\text{prop}}$
X_3Y_3	$\overline{X_2}\,\overline{Y_2}$

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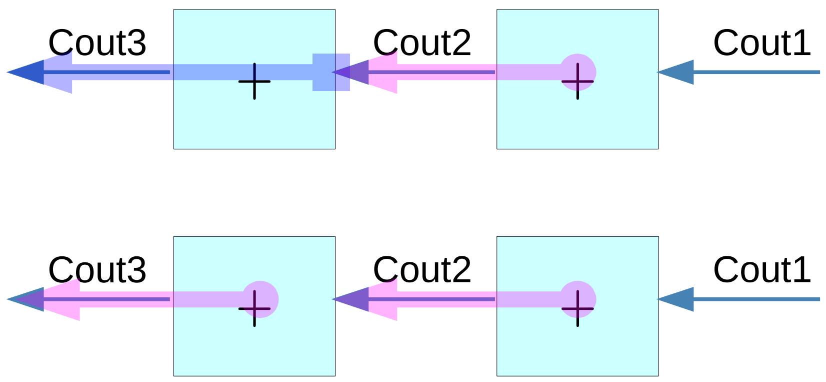
When Cout1 = 0

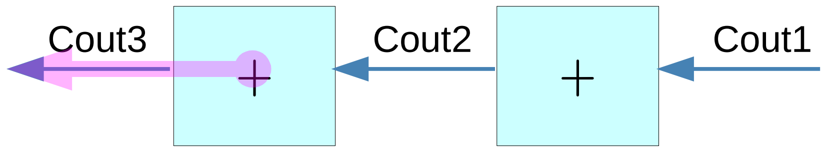


(gen, gen) included in (prop, gen)
it is a special case of (prop, gen)

$$C1_3 \cdot C0_2 \cdot \overline{Cout_1}$$

prop: $\overline{X_3}Y_3$, $X_3\overline{Y_3}$, X_3Y_3

gen: X_2Y_2



$$\begin{aligned} Cout_3 &= \left(Cout_1 \cdot \left(C1_3 \cdot C1_2 + C0_3 \cdot \overline{C1_2}\right)\right) \\ &+ \left(\overline{Cout_1} \cdot \left(C1_3 \cdot C0_2 + C0_3 \cdot \overline{C0_2}\right)\right) \end{aligned}$$

$$C0_3 \cdot \overline{C0_2} \cdot \overline{Cout_1}$$

gen: X_3Y_3

$\overline{\text{gen}}$: $\overline{X_2}Y_2$, $X_2\overline{Y_2}$, $\overline{X_2}\,\overline{Y_2}$

$(C1_3 C1_2 + C0_3\overline{C1_2})Cout_1 + (C1_3 C0_2 + C0_3\overline{C0_2})\overline{Cout_1}$

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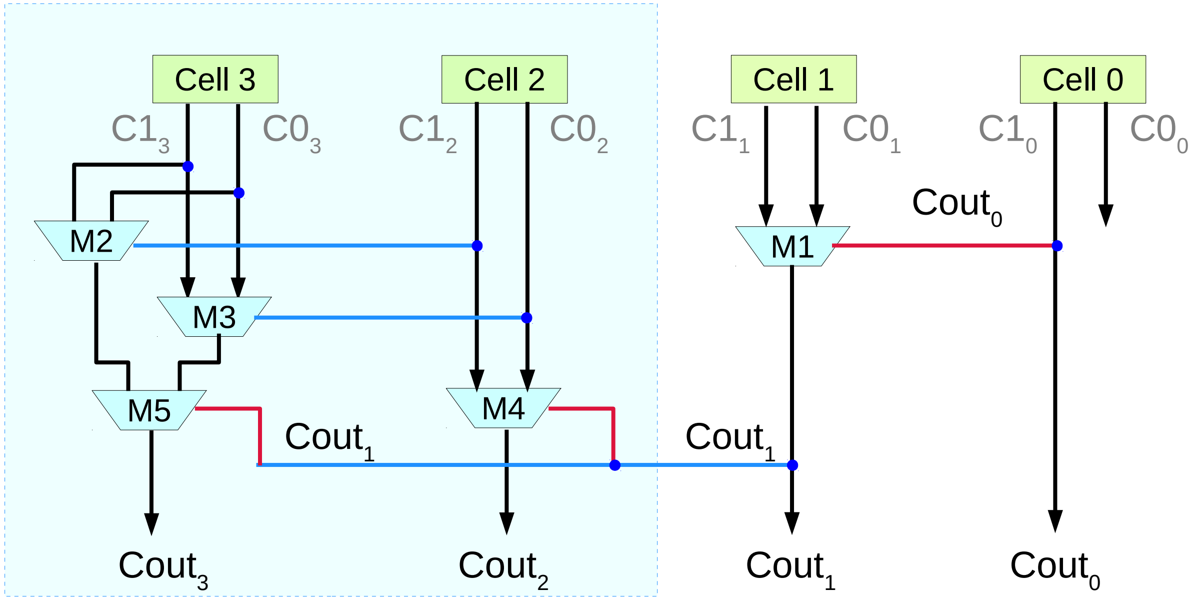
Design C - Carry Select (1)



$C1 = \bar{X}\,Y + X\,\bar{Y} + X\,Y$ $\quad C0 = X\,Y$

$\overline{C1} = \bar{X}\,\bar{Y}$ $\quad \overline{C0} = \bar{X}\,Y + X\,\bar{Y} + \bar{X}\,\bar{Y}$

$= (Cout_2 \cdot C1_3)$
$+ (\overline{Cout_2} \cdot C0_3)$

$= (Cout_1 \cdot C1_2)_2)$
$+ (\overline{Cout_1} \cdot C0_2)_2)$

$= (Cout_0 \cdot C1_1)$
$+ (\overline{Cout_0} \cdot C0_1)$

$$Cout_3 \begin{aligned} &= (Cout_1 \cdot (C1_3 \cdot C1_2 + C0_3 \cdot \overline{C1_2})) \\ &+ (\overline{Cout_1} \cdot (C1_3 \cdot C0_2 + C0_3 \cdot \overline{C0_2})) \end{aligned} \quad \begin{aligned} &= (Cout_1 \cdot (C1_3 \cdot (\bar{X}_2 Y_2 + X_2 \bar{Y}_2 + X_2 Y_2) + C0_3 \cdot \bar{X}_2 \bar{Y}_2)) \\ &+ (\overline{Cout_1} \cdot (C1_3 \cdot X_2 Y_2 + C0_3 \cdot (\bar{X}_2 Y_2 + X_2 \bar{Y}_2 + \bar{X}_2 \bar{Y}_2))) \end{aligned}$$

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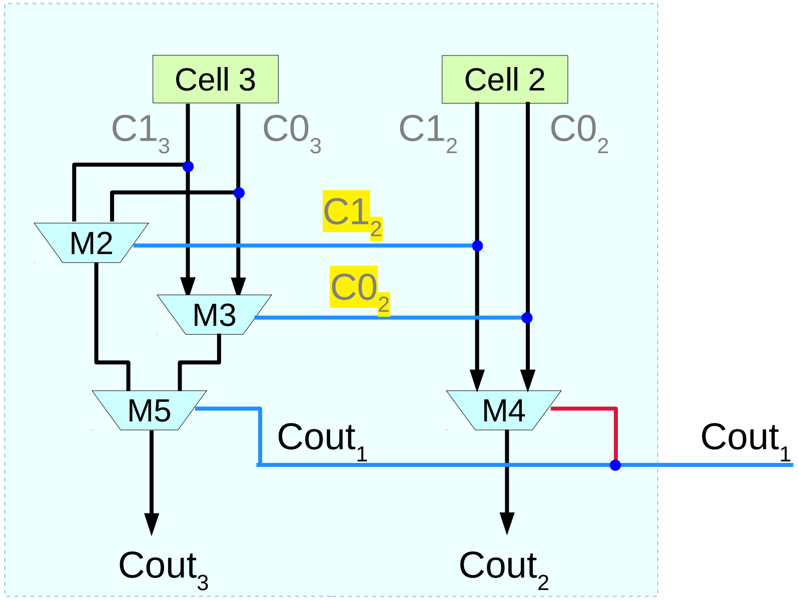
FPGA Carry Chain Cell



$$Cout_2 = (Cout_1 \cdot C1_2) + (\overline{Cout_1} \cdot C0_2)$$

$$Cout_3 = (Cout_2 \cdot C1_3) + (\overline{Cout_2} \cdot C0_3)$$

$$= (((Cout_1 \cdot C1_2) + (\overline{Cout_1} \cdot C0_2)) \cdot C1_3)$$

$$+ (\overline{((Cout_1 \cdot C1_2) + (\overline{Cout_1} \cdot C0_2))} \cdot C0_3)$$

$$(((Cout_1 \cdot C1_2) + (\overline{Cout_1} \cdot C0_2)) \cdot C1_3)$$

$$= (C1_3 C1_2 Cout_1 + C1_3 C0_2 \overline{Cout_1})$$

$$((\overline{(Cout_1 \cdot C1_2)} \cdot \overline{(\overline{Cout_1} \cdot C0_2)}) \cdot C0_3)$$

$$= (((\overline{Cout_1} + \overline{C1_2}) \cdot (Cout_1 + \overline{C0_2})) \cdot C0_3)$$

$$= (\overline{Cout_1} Cout_1 + \overline{C1_2} Cout_1 + \overline{Cout_1}\,\overline{C0_2} + \overline{C1_2}\,\overline{C0_2}) \cdot C0_3$$

$$= (\overline{C1_2} Cout_1 + \overline{C0_2}\,\overline{Cout_1}) \cdot C0_3$$

$$= (C0_3 \overline{C1_2} Cout_1 + C0_3 \overline{C0_2}\,\overline{Cout_1})$$

$(C1_3 C1_2 + C0_3 \overline{C1_2}) Cout_1 + (C1_3 C0_2 + C0_3 \overline{C0_2}) \overline{Cout_1}$

$C1 = \bar{X} Y + X \bar{Y} + X Y$ $\quad C0 = X Y$

$\overline{C1} = \bar{X} \bar{Y}$ $\quad \overline{C0} = \bar{X} Y + X \bar{Y} + \bar{X} \bar{Y}$

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Design C - Carry Select (1)

$C1 = \bar{X}Y + X\bar{Y} + XY$ $\qquad$ $C0 = XY$

$\overline{C1} = \bar{X}\bar{Y}$ $\qquad$ $\overline{C0} = \bar{X}Y + X\bar{Y} + \bar{X}\bar{Y}$

C1	C0	Name	
0	0	0	Kill
0	1	$\overline{\text{Cin}}$	Inverse Propagate
1	0	Cin	Propagate
1	1	1	Generate

$$Cout_1 = (Cout_0 \cdot C1_1) + (\overline{Cout_0} \cdot C0_1)$$

$$Cout_2 = (Cout_1 \cdot C1_2) + (\overline{Cout_1} \cdot C0_2)$$

$$Cout_3 = (Cout_2 \cdot C1_3) + (\overline{Cout_2} \cdot C0_3)$$

$$\begin{aligned} Cout_3 &= (Cout_1 \cdot (C1_3 \cdot C1_2 + C0_3 \cdot \overline{C1_2})) \\ &+ (\overline{Cout_1} \cdot (C1_3 \cdot C0_2 + C0_3 \cdot \overline{C0_2})) \end{aligned}$$

$$\begin{aligned} &= Cout_1 \cdot [(\bar{X}Y + X\bar{Y} + XY)_3 \cdot (\bar{X}Y + X\bar{Y} + XY)_2 \; + \; (XY)_3 \cdot (XY)_2] \\ &+ Cout_1 \cdot [(\bar{X}Y + X\bar{Y} + XY)_3 \cdot (XY)_2 \; + \; (XY)_3 \cdot (\bar{X}Y + X\bar{Y} + \bar{X}\bar{Y})_2] \end{aligned}$$

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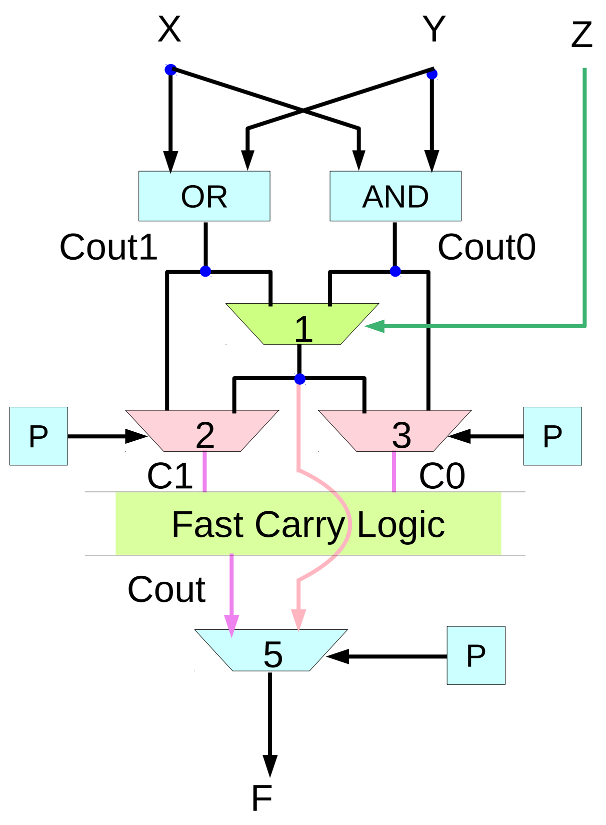
$$\begin{aligned} &= (Cout_1 \cdot (C1_3 \cdot (\bar{X}_2 Y_2 + X_2 \bar{Y}_2 + X_2 Y_2) + C0_3 \cdot \bar{X}_2 \bar{Y}_2)) \\ &+ (\overline{Cout_1} \cdot (C1_3 \cdot X_2 Y_2 + C0_3 \cdot (\bar{X}_2 Y_2 + X_2 \bar{Y}_2 + \bar{X}_2 \bar{Y}_2))) \end{aligned}$$

Design C (3)

A carry chain resource may span
the entire height of a column in the FPGA,
but a mapping to the logic may use
only a small portion of this chain,
with the carry logic in the mapping
starting and ending
at arbitrary points in the column

concerned with not just the **carry delay**
from the first to the last position
in a carry chain, but must consider
the delay for a **carry computation**
beginning at any point within this column.

For example,
even though the FPGA architecture may
provide support for **carry chains** of up to 32 bits,
it must also efficiently support 8 bit
carry computations placed at any point
within this carry chain resource



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Design C (4)

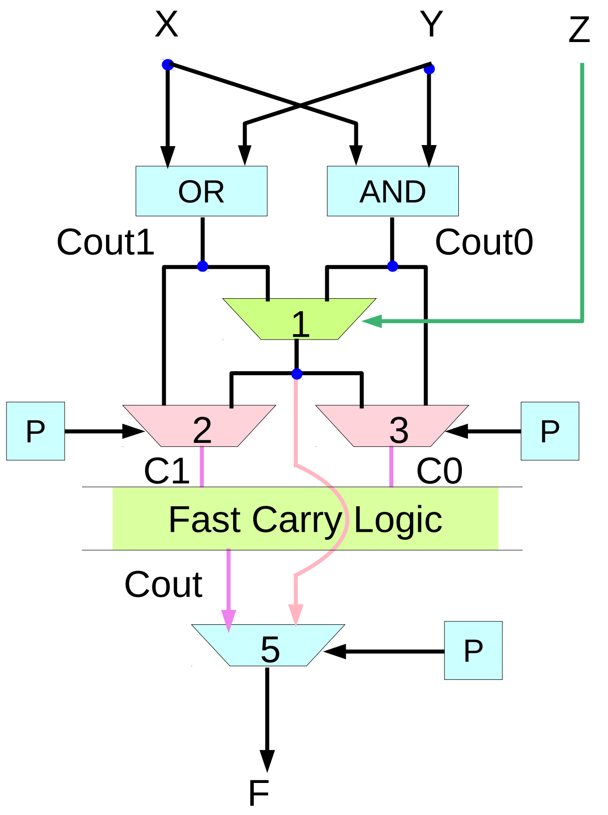
Carry Select

the problem with a ripple carry structure is that
the computation of the Cout for bit position i
cannot begin until after the computation has been
completed in bit positions 0 .. i-1

A carry select structure overcomes this limitation

the main observation is that for any bit position,
the only information it received
from the previous bit positions is its Cin signal,
which can be either **true** or **false**.

In a carry select adder
the **carry chain** is broken at a specific column,
and two separate additions occur



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Design C (5)

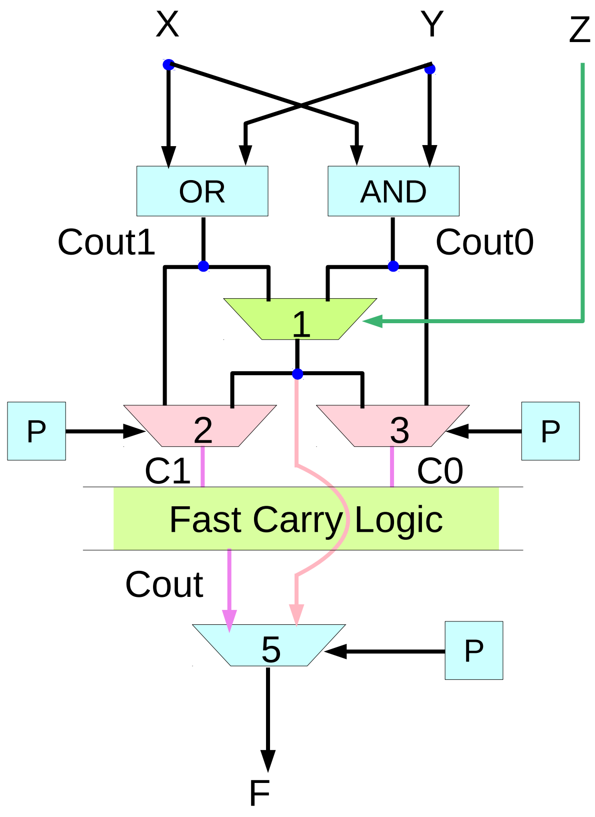
one assuming the Cin signal is **true**,
the other assuming it is **false**

These computations can take place before
the previous columns complete their operation
since they do not depend on the actual value of
the Cin signal

This Cin signal is instead used to determine
which adder's outputs should be used

if the Cin signal is **true**, the output of the following
stages comes from the adder that assumed
that the Cin would be **true**

likewise, a **false** Cin chooses the other adder's output



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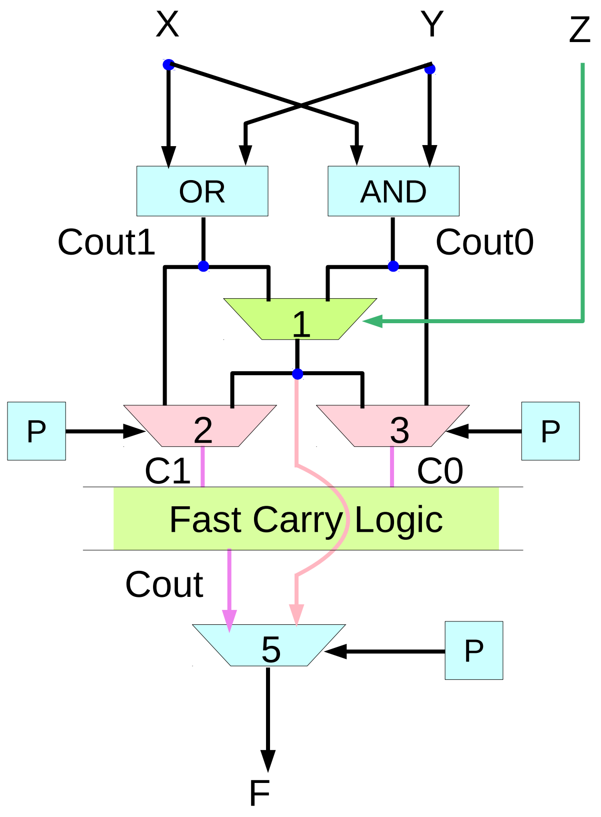
Design C (6)

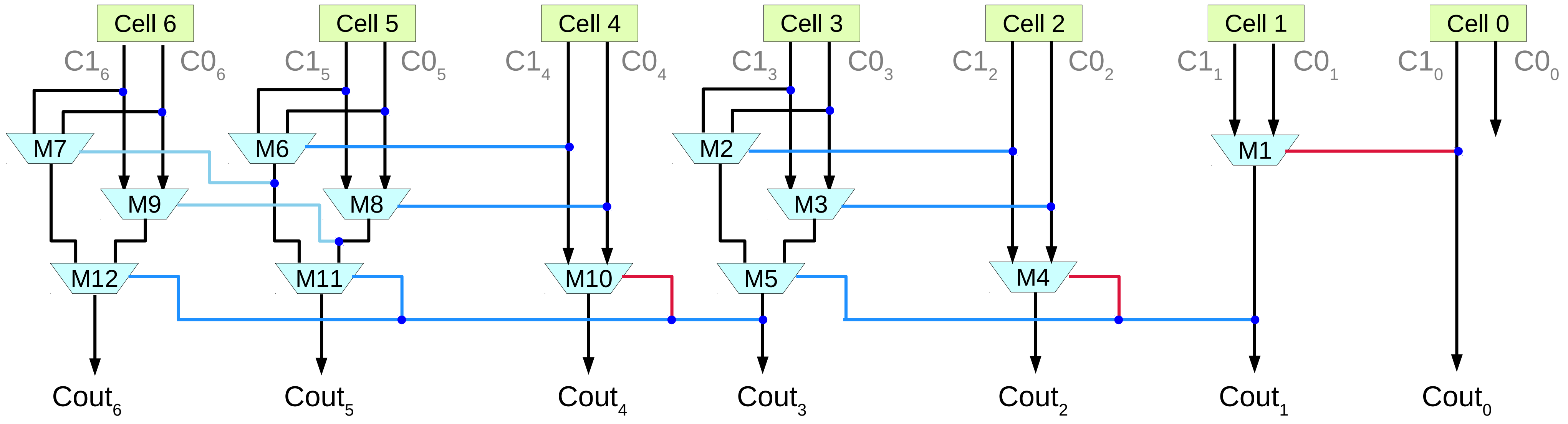
This splitting of the **carry chain**
can be done multiple times,
breaking the computation
into several pairs of short adders
with output muxes choosing
which adder's output to select

the length of the adders and
the breakpoint are carefully chosen
such that the small adders finish computation
just as their Cin signals become available

Short adders handle the low-order bits,
and the adder length is increased
further along the carry chain,
since later computations have more time
until their Cin signal is available



FPGA Carry Chain Cell



$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

$$Cout_1 = (Cout_0 \cdot C1_1) + (\overline{Cout_0} \cdot C0_1)$$

$$Cout_1 = (C1_0 \cdot C1_1) + (\overline{C1_0} \cdot C0_1)$$

$$Cout_{i+1} = (Cout_i \cdot C1_{i+1}) + (\overline{Cout_i} \cdot C0_{i+1})$$

$$Cout_{i+1} = ([(Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)] \cdot C1_{i+1}) + (\overline{[(Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)]} \cdot C0_{i+1})$$

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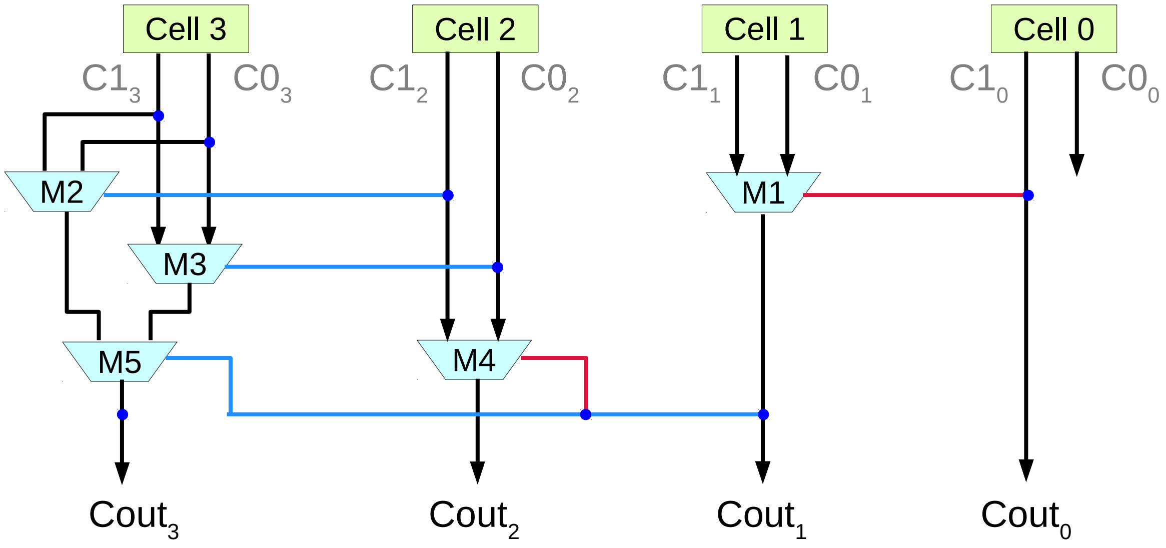
Design C - Carry Select (1)

A **Carry Select carry chain** structure
for use in FPGAs
the carry computation
for the first two cells is performed
with the simple **ripple-carry** structure
implemented by mux1

For cell2 and cell3 we use
two **ripple carry adders**,
with one adder (implemented by mux2)
assuming the Cin is **true**,
and the other (mux3)
assuming the Cin is **false**

Then mux4 and mux5 pick
between these two adders' outputs
based on the actual Cin coming from mux1.



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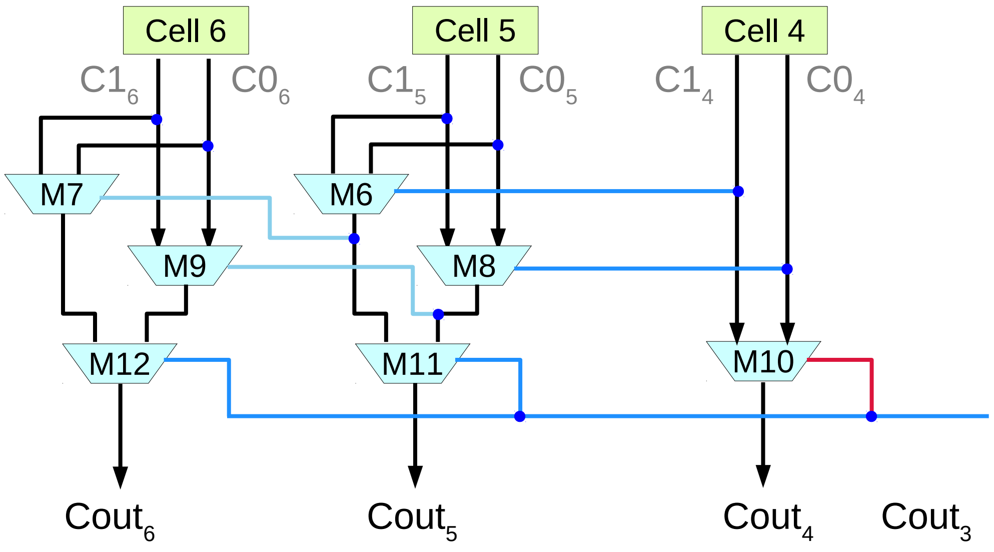
Design C - Carry Select (2)

Similarly, cell4, cell5, cell6
have two ripple carry adders
(mux6 & mux7 for a Cin of 1,
mux8 & mux9 for a Cin of 0),
with output muxes (mux10, mux11, mux12)
deciding between the two based
upon the actual Cin (from mux5).

Subsequent stages will continue
to grow in length by one,
with cells7, cell8, cell9, cell10 in one block,
cell11, cell12, cell13, cell14, cell15 in another, and so on.

timing values showing
the delay of the Carry Select carry chain
relative to other carry chain will be presented later



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Design C - Carry Select (3)

A Carry Select carry chain structure for use in FPGAs
The carry computation for the first two cells is performed with the simple ripple-carry structure implemented by mux1

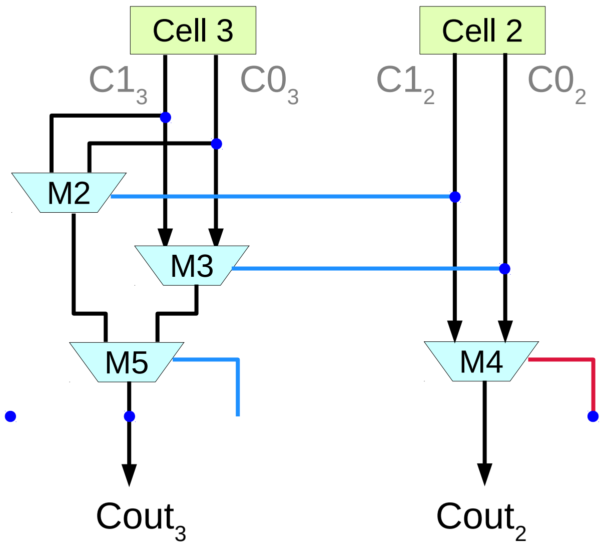
For cells 2 and 3 we use two ripple carry adders, with one adder (implemented by mux2) assuming the Cin is true, and the other (mux3) assuming the Cin is false

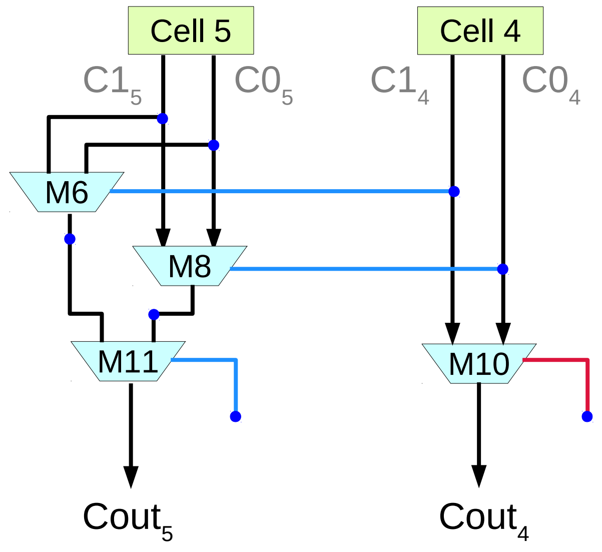
Then muxes 4 and 5 pick between these two adders' outputs based on the actual Cin coming from mux1.

Similarly, celss 4-6 have two ripple carry adders (mux6 & mux7 for a Cin of 1, mux8 & mux9 for a Cin of 0), with output muxes (muxes 10-12) deciding between the two based upon the actual Cin (from mux5).

Subsequent stages will continue to grow in length by one, with cells 7-10 in one block, cells 11-15 in another, and so on.

timing values showing the delay of the Carry Select carry chain relative to other carry chain will be presented later





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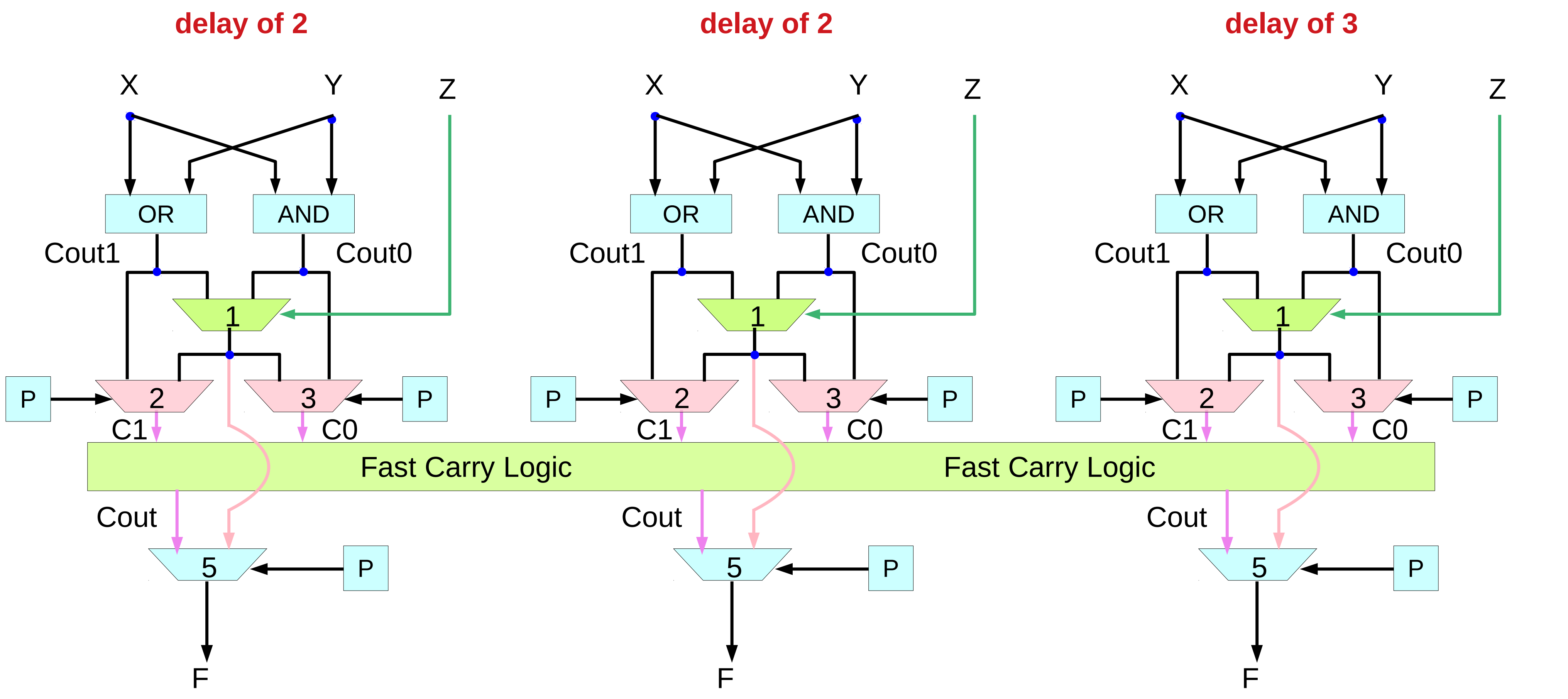
Design C



(1 for mux1, 1 for mux2, 1 in mux4)

delay of 2n+2 for an n-bit ripple carry chain

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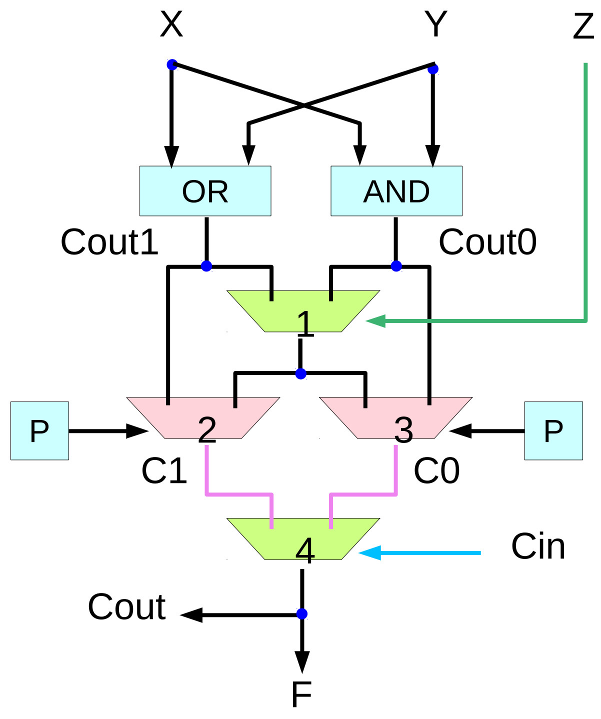
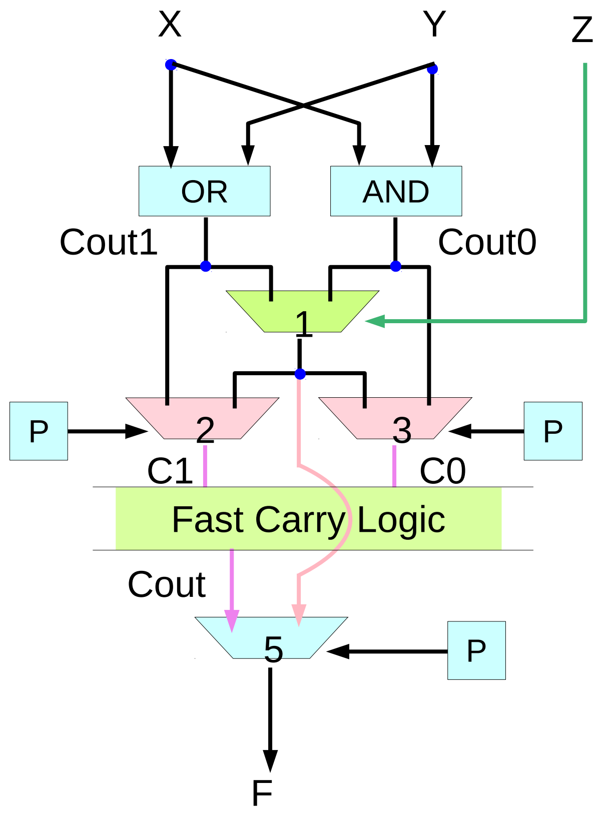
FPGA Carry Chain Cell





$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

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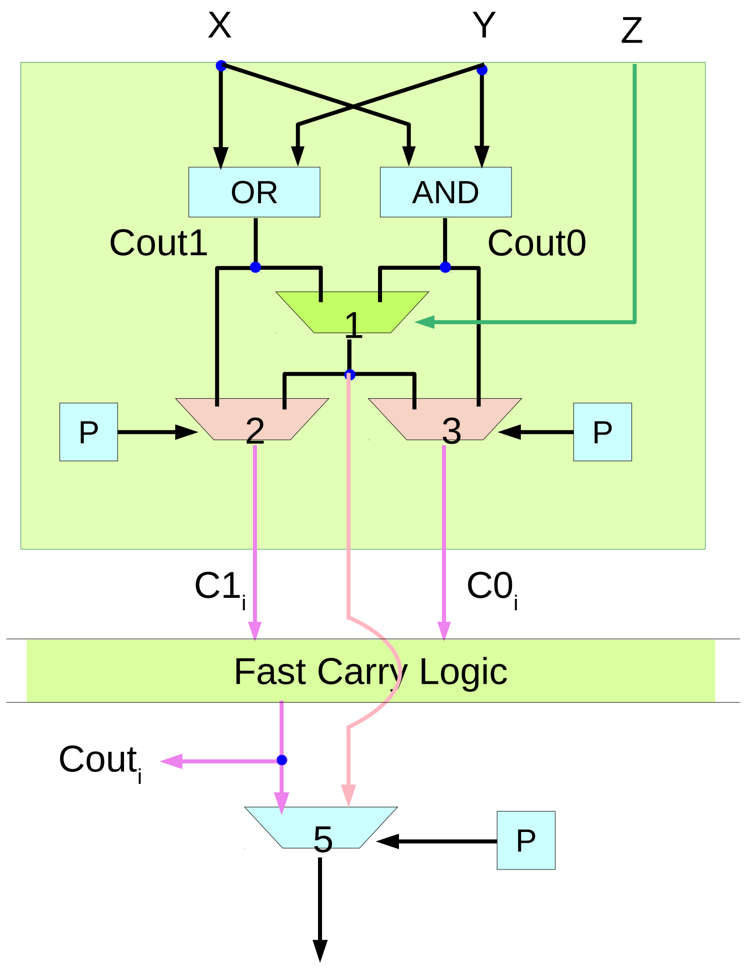
FPGA Carry Chain Cell



$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

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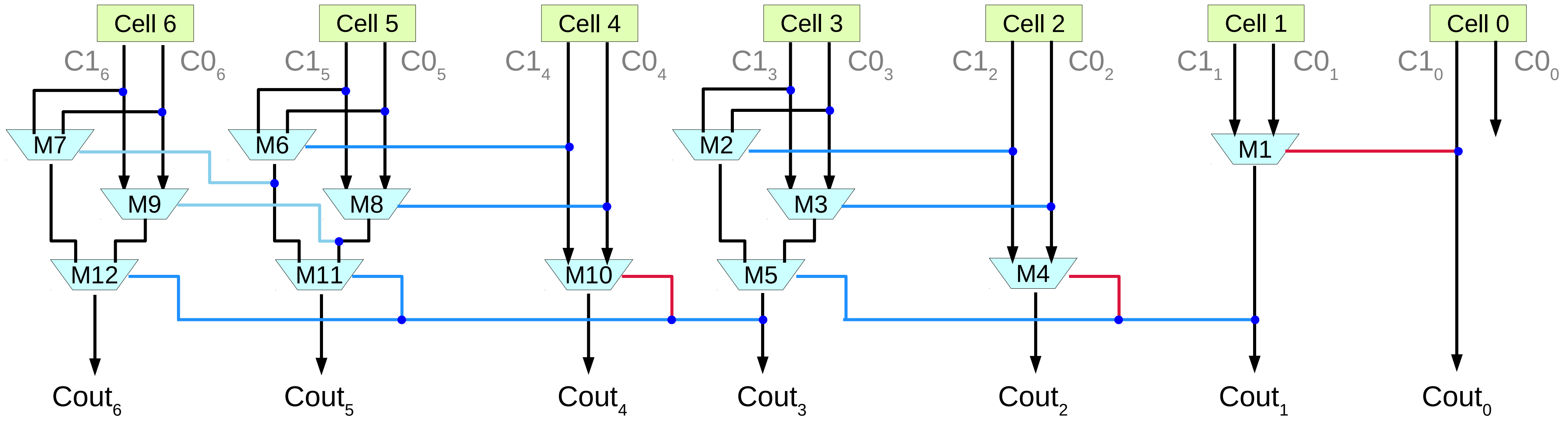
Fast Carry Logc

Carry Select Adder
Carry Lookahead Adder
        Brent-Kung
Variable Block
Ripple Carry Adder

https://en.wikipedia.org/wiki/Carry-lookahead_adder

FPGA Carry Chain Cell



$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

$$Cout_1 = (Cout_0 \cdot C1_1) + (\overline{Cout_0} \cdot C0_1)$$

$$Cout_1 = (C1_0 \cdot C1_1) + (\overline{C1_0} \cdot C0_1)$$

$$Cout_{i+1} = (Cout_i \cdot C1_{i+1}) + (\overline{Cout_i} \cdot C0_{i+1})$$

$$Cout_{i+1} = ([(Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)] \cdot C1_{i+1}) + (\overline{[(Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)]} \cdot C0_{i+1})$$

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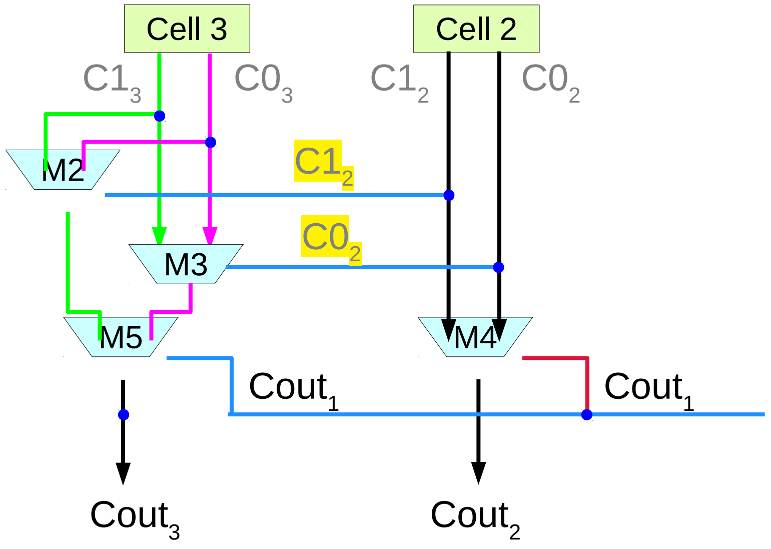
FPGA Carry Chain Cell



$$= \left(\cancel{\overline{Cout_1}Cout_1} + \overline{C1_2}Cout_1 + \overline{Cout_1}\,\overline{C0_2} + \overline{C1_2}\,\overline{C0_2}\right)\cdot C0_3$$

$$= \left(\overline{C1_2}Cout_1 + \overline{C0_2}\,\overline{Cout_1}\right)\cdot C0_3$$

$$= \left(C0_3\overline{C1_2}Cout_1 + C0_3\overline{C0_2}\,\overline{Cout_1}\right)$$

$(C1_3\,C1_2 + C0_3\overline{C1_2})Cout_1 + (C1_3\,C0_2 + C0_3\overline{C0_2})\overline{Cout_1}$

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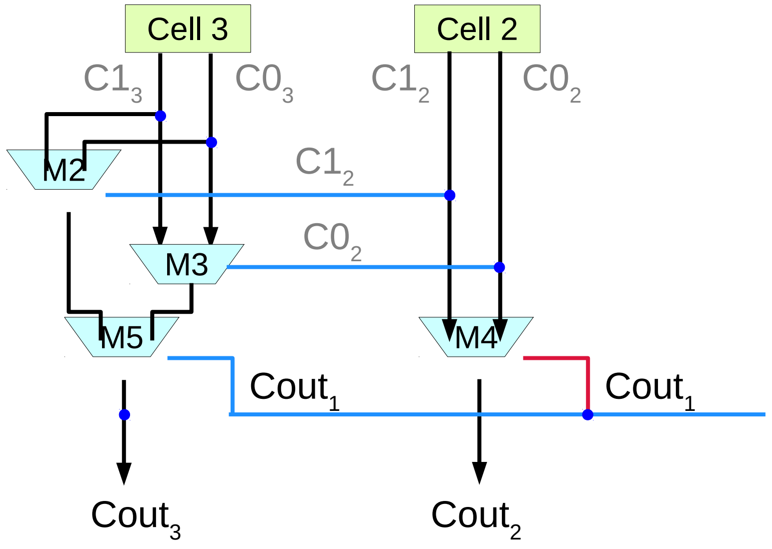
FPGA Carry Chain Cell

$C1_3\, C1_2 + C0_3 \overline{C1_2}$

$C1_3\, C0_2 + C0_3 \overline{C0_2}$



$(C1_3\, C1_2 + C0_3 \overline{C1_2}) Cout_1 + (C1_3\, C0_2 + C0_3 \overline{C0_2}) \overline{Cout_1}$

$$= C1_3 \cdot (C1_2 Cout_1 + C0_2 \overline{Cout_1})$$
$$+ C0_3 \cdot (\overline{C1_2} Cout_1 + \overline{C0_2}\, \overline{Cout_1})$$

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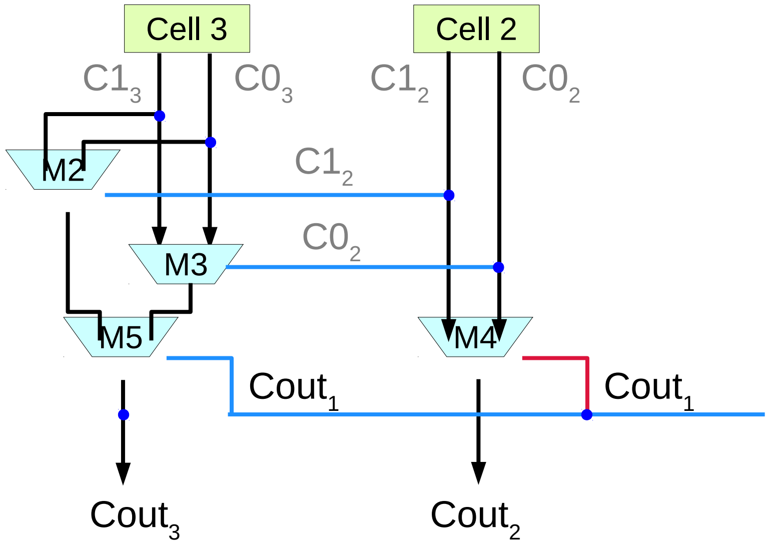
FPGA Carry Chain Cell



$$Cout_i = (Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)$$

$$Cout_{i+1} = (Cout_i \cdot C1_{i+1}) + (\overline{Cout_i} \cdot C0_{i+1})$$

$$Cout_{i+1} = ([(Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)] \cdot C1_{i+1})$$

$$+ (\overline{[(Cout_{i-1} \cdot C1_i) + (\overline{Cout_{i-1}} \cdot C0_i)]} \cdot C0_{i+1})$$

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References

[1] http://en.wikipedia.org/
[2] J-P Deschamps,et. al., “Synthesis of Arithmetic Circuits”, 2006