Statistical Inference Overview

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2024-04-22 Mon

Outline

Based on

"Understanding Statistics in the Behavioral Sciences" R. R. Pagano

Populations and Samples (1)

- population: everything in the group that we want to learn about.
- sample: a part of the population.

- Examples of populations and a sample from those populations:

Population	Sample
All of the people in Germany	500 Germans
All of the customers of Netflix	300 Netflix customers
Every car manufacturer	Tesla, Toyota, BMW, Ford

https://www.w3schools.com/statistics/statistics_statistical_inference.php

Populations and Samples (2)

- For good statistical analysis,
  the sample needs to be as *similar*
  as possible to the population.
- If they are *similar enough*, we say
  that the sample is representative of the population.
- The sample is used to make conclusions
  about the whole population.

https://www.w3schools.com/statistics/statistics_statistical_inference.php

Populations and Samples (3)

- If the sample is not *similar enough* to the whole population, the conclusions could be useless.

- Many words have specific meanings in statistics.
- The word population normally refers to a group of people.
- In statistics, it is any specific group that we are interested in learning about.

https://www.w3schools.com/statistics/statistics_statistical_inference.php

Statistical Inference

- Using data analysis and statistics to make conclusions about a population is called statistical inference.
- The main types of statistical inference are:
  - Estimation
  - Hypothesis testing

https://www.w3schools.com/statistics/statistics_statistical_inference.php

Estimation (1)

- Statistics from a sample are used to estimate population parameters.
- *The most likely* value is called a point estimate.
- There is always uncertainty when estimating.

https://www.w3schools.com/statistics/statistics_statistical_inference.php

Estimation (2)

- The uncertainty is often expressed as confidence intervals defined by a *likely* lowest and highest value for the parameter.
- An example could be a confidence interval for the number of bicycles a Dutch person owns:
  - The average number of bikes a Dutch person owns is between 3.5 and 6.

https://www.w3schools.com/statistics/statistics_statistical_inference.php

Hypothesis Testing (1)

- a method to check if a claim about a population is true.
- checks how likely it is that a hypothesis is true is based on the sample data.
- different types of hypothesis testing.
- the steps of the test depends on:
    - Type of data (categorical or numerical)
    - If you are looking at:
        - A single group
        - Comparing one group to another
        - Comparing the same group before and after a change

https://www.w3schools.com/statistics/statistics_statistical_inference.php

Hypothesis Testing (2)

- A hypothesis is a claim about a population parameter.
- A hypothesis test is a formal procedure to check if a hypothesis is true or not.
- Examples of claims that can be checked:
    - The average height of people in Denmark is more than 170 cm.
    - The share of left handed people in Australia is not 10%.
    - The average income of dentists is less the average income of lawyers.

https://www.w3schools.com/statistics/statistics_hypothesis_testing.php

The Null and Alternative Hypothesis

- Hypothesis testing is based on making two different claims about a population parameter.
- The null hypothesis (H_0) and the alternative hypothesis (H_1) are the claims.
- The two claims needs to be mutually exclusive, meaning only one of them can be true.
- The alternative hypothesis is typically what we are trying to prove.
- For example, we want to check the following claim:
  - "The average height of people in Denmark is more than 170 cm."

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Summary (1) comparing means

tests	
• one-sample test	comparing sample mean, population mean
• two-sample test	comparing two independent sample means
• paired test	comparing two related sample means

tests	test conditions
• t-test	1. when the population variance is *known*
	2. when the sample size is *large*
• z-test	1, when the population variance is *unknown*
	2. the sample size is *small*

https://www.qualitygurus.com/common-types-of-hypothesis-tests/

Summary (2) comparing means

one sample z-test	sample mean, population mean known population var / large sample size
one sample t-test	sample mean, population mean unknown population var / small sample size
two sample z-test	two *independent* sample means known population var / large sample size
two sample t-test	two *independent* sample means unknown population var / small sample size
paired t-test	two *related* sample means unknown population var / small sample size

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Summary (3) comparing proportions

one sample propotion test	sample proportion, population proportion when $np \geq 10$ and $n(1-p) \geq 10$
two sample proportion test	two *independent* sample proportions when $np \geq 10$ and $n(1-p) \geq 10$

test conditions
the normal approximation is used when both $np \geq 10$ and $n(1-p) \geq 10$ (data should have at least 10 "successes" and at least 10 "failures")

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Summary (4)

compare variances between	
sample variance, known population variance	Chi-square test
two independent sample variances	F-test
observed frequencies, expected frequencies	goodness of fit test
observed frequencies, expected frequencies	contingency tables
means of three or more independent samples	ANOVA (Analysis of Variance

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Tests for Comparing Means (1)

- One-sample z-test:
  - used to compare the mean of a sample to a known population mean
  - used when the population variance is known, or the sample size is *large* ($n > 30$).
- Two-sample z-test:
  - used to compare the means of two independent samples.
  - used when the population variances are known, or the sample sizes are *large* ($n > 30$).

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Tests for Comparing Means (2)

- One-sample t-test:
  - used to compare the mean of a sample to a known population mean.
  - used when the population variance is unknown, and the sample size is *small* ($n < 30$).
- Two-sample t-test:
  - used to compare the means of two independent samples.
  - used when the population variances are unknown, and the sample sizes are *small* ($n < 30$).

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Tests for Comparing Means (3)

- Paired t-test:
  - used to compare the means of two *related* samples, such as the before and after measurements of the same group of subjects.
  - used when the population variances are unknown, and the sample size is *small* ($n < 30$).

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Tests for Comparing Proportions (1)

- Let us consider the parameter p
  of the population proportion
- For instance, we might want to know
  the proportion of males
  within a total population of adults
  when we conduct a survey.
- A test of proportion will assess
  whether or not a sample from a population represents
  the true proportion of the entire population

https://online.stat.psu.edu/statprogram/reviews/statistical-concepts/proportions

Tests for Comparing Proportions (2-1)

- an example
  - newborn babies are more likely to be boys than girls.
  - a random sample found 13,173 boys were born among 25,468 newborn children
  - the sample proportion of boys was 0.5172.
  - is this sample evidence that the birth of boys is more common than the birth of girls in the entire population?

https://online.stat.psu.edu/statprogram/reviews/statistical-concepts/proportions

Tests for Comparing Proportions (2-2)

- so far, all of our examples involved testing whether a single population proportion p equals some value .
- Now, let's turn our attention for a bit towards testing whether one population proportion equals a second population proportion
- Additionally, most of our examples thus far have involved left-tailed tests in which the alternative hypothesis involved or right-tailed tests in which the alternative hypothesis involved
- Here, let's consider an example that tests the equality of two proportions against the alternative that they are not equal.

https://online.stat.psu.edu/stat415/lesson/9/9.4

Tests for Comparing Proportions (2-3)

- Time magazine reported the result of a telephone poll of 800 adult Americans.
- The question posed of the Americans who were surveyed was: "Should the federal tax on cigarettes be raised to pay for health care reform?"
- The results of the survey were:

Non-smokers	Somkers
$n_1 = 605$	$n_2 = 195$
$y_1 = 351$ said yes	$y_2 = 41$ said yes
$\hat{p}_1 = 351 over 605 = 0.58$	$\hat{p}_2 = 41 over 195 = 0.21$

https://online.stat.psu.edu/stat415/lesson/9/9.4

Tests for Comparing Proportions (3)

- One-sample proportion test :
  - used to compare the proportion of a sample
    to a known population proportion.
  - the normal approximation is used
    when both $np \geq 10$ and $n(1-p) \geq 10$
    (data should have at least 10 "successes" and at least 10 "failures")
    (in some books, it is 5)

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Tests for Comparing Proportions (4)

- Two-sample proportion test :
  - used to compare the proportions of two independent samples.
  - the normal approximation is used
    when both $np \geq 10$ and $n(1-p) \geq 10$
    (data should have at least 10 "successes" and at least 10 "failures")
    (in some books, it is 5)

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Tests for Comparing Variance

- Chi-square test for variance :
  - used to compare the variance of a sample
    to a known population variance
- F-test for variance :
  - used to compare the variances of two *independent* samples

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Other Common Tests (1)

- Goodness of fit test :
- used to determine whether a sample fits a *specific* distribution.
- used to compare the observed frequencies of a *categorical variable* to the expected frequencies under a *particular* distribution.

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Other Common Tests (2)

- Testing for independence of two attributes (Contingency Tables) :
- used to determine whether there is a relationship between two *categorical variables*.
- often used in the form of a chi-square test, which compares the observed frequencies in a contingency table to the expected frequencies under the assumption of independence.

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Other Common Tests (3)

- ANOVA (Analysis of Variance) :
- used to compare the means of three or more independent samples.
- used to determine whether there is a significant difference between the means of the groups.

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One-sample z-test

- used to test a hypothesis about the *population* mean
- based on the assumption that the sample is drawn from a normally distributed population.
  - the null hypothesis
    the *population* mean is equal to a specific value
  - the alternative hypothesis
    the *population* mean is not equal to that value

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Two-sample z-test

- based on the assumption that both samples are drawn from normally distributed populations with equal variances.
- the two-sample z-test requires
  that the population standard deviations be known or
  that the sample sizes be *large* (30 or more),
  - the null hypothesis
    the means of the two samples are equal
  - the alternative hypothesis
    the means are not equal

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One-sample t-test

- used to test a hypothesis about the *population* mean
- based on the assumption that the sample is drawn from a normally distributed population
    - the null hypothesis
      the *population* mean is equal to a specific value
    - the alternative hypothesis
      the *population* mean is not equal to that value

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Two-sample t-test

- based on the assumption that the samples are drawn from populations with normal distributions.
- the two-sample t-test
  that the population standard deviations need not be known or
  that the sample sizes need not be *large* (30 or more),
  - the null hypothesis
    the means of the two samples are equal
  - the alternative hypothesis
    the means are not equal

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Paired t-test

- used to test a hypothesis about the difference between the means of the two samples
- based on the assumption that the differences between the pairs are normally distributed
- In a dependent two-sample t-test (a paired t-test), the samples in the two groups being compared are *related* in some way.
  - the null hypothesis
    there is no difference between the means of the two samples
  - the alternative hypothesis
    there is a difference between the means

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Two proportions z-test

- used to test a hypothesis about the difference between the proportions of the two samples and
- based on the assumption that the samples are drawn from populations with a normal distribution
  - the null hypothesis :
    there is no difference between the proportions of the two samples
  - the alternative hypothesis :
    there is a difference between the proportion

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