Multiple Random Variables

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Based on
Probability, Random Variables and Random Signal Principles,
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Outline

Central Limit Theorem

Definition

the central limit theorem says that the probability distribution function of the sum of large number of random variables approaches a Gauassian distribution.
This theorem is known to apply some cases of statistically independent random variables.

Central Limit Theorem
Uneqaul Distribution Case

Definition

the sum Y of N independent random variables $X_1, X_2, ..., X_N$
Let $Y = X_1 + X_2 + \cdots + X_N$, then

$$\overline{Y}_N = \overline{X}_1 + \overline{X}_2 + \cdots + \overline{X}_N$$

$$\sigma^2_{Y_N} = \sigma^2_{X_1} + \sigma^2_{X_2} + \cdots + \sigma^2_{X_N}$$

the probability distribution of Y asymptotically approaches

to Gaussian distribution function as $N \to \infty$

Sufficient Conditions
Uneqaul Distribution Case

Definition

$$\sigma_{X_i}^2 > B_1 > 0 \qquad i = 1, 2, ..., N$$

$$E[|X_i - \overline{X}_i|^3] < B_2 \qquad i = 1, 2, ..., N$$

whre B_1 and B_2 are positive numbers

these conditions guarantee that no one random variable in the sum dominates

Distribution vs density functions
Uneqaul Distribution Case

the central limit theorem guarantees

- only that the distribution of the sum of random variables become Gaussian
- the density of the sum of random variables is not always Gaussian

- the sum of continuous random variables :
  - under certain conditions on individual random variables the density of the sum is always Gaussian
- the sum of discrete random variables :
  - the density function may contain impulses and thus is not Gaussian.

Discrete Random Variable Examples
distribution may contain impulses

the sum Y of N independent discrete random variables $X_1, X_2, ..., X_N$

- discrete random variable
- density function may contain impulses
- therefore the density function is not Gaussian
- although the distribution approaches Gaussian

- when the possible discrete values of each random variable are $kb, k = 0, \pm 1, \pm 2, ...,$ where b is a constant
  - the envelope of the impulses in the density of the sum will be Gaussian
  - with the mean Y_N and variance $\sigma^2_{Y_N}$