Laurent Series and z-Transform - Geometric Series Double Pole Examples B

20180216

2 formulas of z

① $$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\updownarrow\; z^{-1}$$

② $$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right).$$

$$= \left(\frac{1}{z-0.5}-\frac{1}{z-2}\right)$$

$$\Big\downarrow z^{-1}$$

$$\frac{3}{2}\frac{-1}{(z^{-1}-0.5)(z^{-1}-2)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{1}{z^{-1}-0.5}-\frac{1}{z^{-1}-2}\right)$$

$$= \left(\frac{2}{2z^{-1}-1}-\frac{0.5}{0.5z^{-1}-1}\right)$$

$$= \left(\frac{2z}{2-z}-\frac{0.5z}{0.5-z}\right)$$

$$= \left(\frac{-2z}{z-2}+\frac{0.5z}{z-0.5}\right)$$

$$= z\left(\frac{-2}{z-2}+\frac{0.5}{z-0.5}\right)$$

$$= z\left(\frac{-\frac{3}{2}z}{(z-2)(z-0.5)}\right)$$

$$= \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \frac{3}{2}\,\frac{2}{3}\left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

Ⓐ $f(z)$ Ⓑ $X(z)$

① $$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

①-Ⓐ $|z| < 0.5$ $$f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$f(z)$ $|z| > 2$ $$f(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

①-Ⓑ $|z| < 0.5$ $$X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

$X(z)$ $|z| > 2$ $$X(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

② $$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

②-Ⓐ $|z| < 0.5$ $$f(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z} \quad \boxed{-2^{n-1} + \left(\tfrac{1}{2}\right)^{n-1}} \quad (n \geq 1)$$

$f(z)$ $|z| > 2$ $$f(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}} \quad \boxed{+2^{n-1} - \left(\tfrac{1}{2}\right)^{n-1}} \quad (n < 1)$$

②-Ⓑ $|z| < 0.5$ $$X(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z} \quad \boxed{-\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1}} \quad (n < 0)$$

$X(z)$ $|z| > 2$ $$X(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}} \quad \boxed{+\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}} \quad (n \geq 0)$$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
Ⓐ $f(z)$	$\lvert z\rvert < \frac{1}{2}$	$-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n\geq 0)$	$-2^{n-1}+\left(\frac{1}{2}\right)^{n-1}$ $(n\geq 1)$
	$\lvert z\rvert > 2$	$+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$+2^{n-1}-\left(\frac{1}{2}\right)^{n-1}$ $(n<1)$
Ⓑ $X(z)$	$\lvert z\rvert < \frac{1}{2}$	$-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}$ $(n<1)$	$-\left(\frac{1}{2}\right)^{n+1}+2^{n+1}$ $(n<0)$
	$\lvert z\rvert > 2$	$+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}$ $(n\geq 1)$	$+\left(\frac{1}{2}\right)^{n+1}-2^{n+1}$ $(n\geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n\geq 0)$	$-2^{n-1}+\left(\frac{1}{2}\right)^{n-1}$ $(n\geq 1)$
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}$ $(n<1)$	$-\left(\frac{1}{2}\right)^{n+1}+2^{n+1}$ $(n<0)$
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$+2^{n-1}-\left(\frac{1}{2}\right)^{n-1}$ $(n<1)$
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}$ $(n\geq 1)$	$+\left(\frac{1}{2}\right)^{n+1}-2^{n+1}$ $(n\geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	causal $(n \geq 0)$	causal $(n \geq 1)$
$\lvert z\rvert > 2$	$f(z)$	anticausal $(n < 0)$	anticausal $(n < 1)$
$\lvert z\rvert < \frac{1}{2}$	$X(z)$	anticausal $(n < 1)$	anticausal $(n < 0)$
$\lvert z\rvert > 2$	$X(z)$	causal $(n \geq 1)$	causal $(n \geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	causal $(n \geq 0)$	causal $(n \geq 1)$
$\lvert z\rvert < \frac{1}{2}$	$X(z)$	anticausal $(n < 1)$	anticausal $(n < 0)$
$\lvert z\rvert > 2$	$f(z)$	anticausal $(n < 0)$	anticausal $(n < 1)$
$\lvert z\rvert > 2$	$X(z)$	causal $(n \geq 1)$	causal $(n \geq 0)$

$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right) \qquad \left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$$\boxed{z} \ -\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad \boxed{z} \ -\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

$$\boxed{|z|<0.5} \quad |2z|<1 \quad |0.5z|<1 \qquad \boxed{|z|<0.5} \quad |2z|<1 \quad |0.5z|<1$$

$$\boxed{z^{-1}} \ \frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{z^{-1}} \ \frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$$\boxed{|z|>2} \quad |0.5z^{-1}|<1 \quad |2z^{-1}|<1 \qquad \boxed{|z|>2} \quad |0.5z^{-1}|<1 \quad |2z^{-1}|<1$$

$$\boxed{\begin{array}{cc} -\frac{2}{1-2z}+\frac{0.5}{1-0.5z} & -\frac{z}{1-2z}+\frac{z}{1-0.5z} \\ \cdot\frac{1}{2z}\downarrow\uparrow\cdot 2z \quad \cdot\frac{2}{z}\downarrow\uparrow\cdot\frac{z}{2} & \cdot\frac{1}{2z}\downarrow\uparrow\cdot 2z \quad \cdot\frac{2}{z}\downarrow\uparrow\cdot\frac{z}{2} \\ \frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} & \frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}} \end{array}}$$

$$\boxed{① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}}$$

$$\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right) \qquad \left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$\boxed{z}$ $-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$

$\boxed{|z|<0.5}$ $f(z)$ causal $(n \geq 0)$

$\boxed{z}$ $-\frac{z}{1-2z}+\frac{z}{1-0.5z}$

$\boxed{|z|<0.5}$ $f(z)$ causal $(n \geq 1)$

$\boxed{z^{-1}}$ $\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$

$\boxed{|z|>2}$

$X(z)$ causal $(n \geq 1)$

$\boxed{z^{-1}}$ $\frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$

$\boxed{|z|>2}$

$X(z)$ causal $(n \geq 0)$

$$\text{①}\ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \xleftrightarrow{z^{-1}} \text{②}\ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\left(\frac{1}{z-0.5}-\frac{1}{z-2}\right) \qquad \left(\frac{0.5z}{(z-0.5)}-\frac{2z}{(z-2)}\right)$$

$\boxed{z}$ $-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$ | $\boxed{z}$ $-\frac{z}{1-2z}+\frac{z}{1-0.5z}$

$\boxed{|z|<0.5}$ $f(z)$ causal $(n \geq 0)$ | $\boxed{|z|<0.5}$ $f(z)$ causal $(n \geq 1)$

$X(z)$ anticausal $(n<1)$ | $X(z)$ anticausal $(n<0)$

$\boxed{z^{-1}}$ $\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$ | $\boxed{z^{-1}}$ $\frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$

$\boxed{|z|>2}$ $f(z)$ anticausal $(n<0)$ | $\boxed{|z|>2}$ $f(z)$ anticausal $(n<1)$

$X(z)$ causal $(n \geq 1)$ | $X(z)$ causal $(n \geq 0)$

$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$|z| < 0.5$

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -\left[2 + 2^2 z^1 + 2^3 z^2 + \cdots\right] \qquad -2^{n+1}$$

$$+\left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n+1}$$

$|z| > 2$

$$\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n-1}$$

$$-\left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right] \qquad -2^{n-1}$$

$\ast$ $n =$ 1 2 3

$|z| < 0.5$

$$-\frac{z}{1-2z} + \frac{z}{1-0.5z}$$

$$f(z) = -\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right] \qquad -2^{n-1}$$

$$+\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^1 z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n-1}$$

$|z| > 2$

$$\frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n+1}$$

$$-\left[2^1 z^0 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right] \qquad -2^{n+1}$$

$\ast$ $n =$ 0 1 2

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$$① \ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \quad \overset{z^{-1}}{\longleftrightarrow} \quad ② \ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$$\boxed{\begin{array}{l} 2=\left(\tfrac{1}{2}\right)^{-1} \\ \left(\tfrac{1}{2}\right)=2^{-1} \end{array}}$$

$|z|<0.5$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$$

$$f(z)=-\left[2+2^2z^1+2^3z^2+\cdots\right] \quad -2^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2z^1+\left(\tfrac{1}{2}\right)^3z^2+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$

$$X(z)=-\left[\left(\tfrac{1}{2}\right)^{-1}+\left(\tfrac{1}{2}\right)^{-2}z^1+\left(\tfrac{1}{2}\right)^{-3}z^2+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$
$$+\left[2^{-1}+2^{-2}z^1+2^{-3}z^2+\cdots\right] \quad +2^{n-1}$$

$n=\quad 0 \quad -1 \quad -2$

$|z|<0.5$

$$-\frac{z}{1-2z}+\frac{z}{1-0.5z}$$

$$f(z)=-\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right] \quad -2^{n-1}$$
$$+\left[\left(\tfrac{1}{2}\right)^0z^1+\left(\tfrac{1}{2}\right)^1z^2+\left(\tfrac{1}{2}\right)^2z^3+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n-1}$$

$$X(z)=-\left[\left(\tfrac{1}{2}\right)^0z^1+\left(\tfrac{1}{2}\right)^{-1}z^2+\left(\tfrac{1}{2}\right)^{-2}z^3+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$
$$+\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right] \quad +2^{n+1}$$

$n=\quad -1 \quad -2 \quad -3$

$$\boxed{\begin{array}{l} 2=\left(\tfrac{1}{2}\right)^{-1} \\ \left(\tfrac{1}{2}\right)=2^{-1} \end{array}}$$

$|z|>2$

$$\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$$

$$f(z)=+\left[2^0z^{-1}+2^{-1}z^{-2}+2^{-2}z^{-3}+\cdots\right] \quad +2^{n+1}$$
$$-\left[\left(\tfrac{1}{2}\right)^0z^{-1}+\left(\tfrac{1}{2}\right)^{-1}z^{-2}+\left(\tfrac{1}{2}\right)^{-2}z^{-3}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

$$X(z)=+\left[\left(\tfrac{1}{2}\right)^0z^{-1}+\left(\tfrac{1}{2}\right)^1z^{-2}+\left(\tfrac{1}{2}\right)^2z^{-3}+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n-1}$$
$$-\left[2^0z^{-1}+2^1z^{-2}+2^2z^{-3}+\cdots\right] \quad -2^{n-1}$$

$n=\quad 1 \quad 2 \quad 3$

$|z|>2$

$$\frac{0.5}{1-0.5z^{-1}}-\frac{2}{1-2z^{-1}}$$

$$f(z)=+\left[2^{-1}z^0+2^{-2}z^{-1}+2^{-3}z^{-2}+\cdots\right] \quad +2^{n-1}$$
$$-\left[\left(\tfrac{1}{2}\right)^{-1}z^0+\left(\tfrac{1}{2}\right)^{-2}z^{-1}+\left(\tfrac{1}{2}\right)^{-3}z^{-2}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$X(z)=+\left[\left(\tfrac{1}{2}\right)^1z^0+\left(\tfrac{1}{2}\right)^2z^{-1}+\left(\tfrac{1}{2}\right)^3z^{-2}+\cdots\right] \quad +\left(\tfrac{1}{2}\right)^{n+1}$$
$$-\left[2^1z^0+2^2z^{-1}+2^3z^{-2}+\cdots\right] \quad -2^{n+1}$$

$n=\quad 0 \quad 1 \quad 2$

$$① \ \frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} \quad \xleftrightarrow{z^{-1}} \quad ② \ \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$$

$|z| < 0.5$

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -\left[2 + 2^2 z + 2^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^{-1} + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right] + \left[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1} \quad (n < 1)}$$

$|z| > 2$

$$\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$

$$f(z) = +\left[2^0 z^1 + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right]$$

$$\boxed{a_n = +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right] - \left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)}$$

$|z| < 0.5$

$$-\frac{z}{1-2z} + \frac{z}{1-0.5z}$$

$$f(z) = -\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right] + \left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^1 z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right]$$

$$\boxed{a_n = -2^{n-1} + \left(\tfrac{1}{2}\right)^{n-1} \quad (n \geq 1)}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right] + \left[2^0 z^1 + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1} \quad (n < 0)}$$

$|z| > 2$

$$\frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$

$$f(z) = +\left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^{-1} + \left(\tfrac{1}{2}\right)^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = +2^{n-1} - \left(\tfrac{1}{2}\right)^{n-1} \quad (n < 1)}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right] - \left[2^1 z^0 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1} \quad (n \geq 0)}$$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-2^{n-1}+\left(\frac{1}{2}\right)^{n-1}$ $(n \geq 1)$
		$\updownarrow$ -1	$\updownarrow$ -1
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$+2^{n-1}-\left(\frac{1}{2}\right)^{n-1}$ $(n<1)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$			
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}$ $(n<1)$	$-\left(\frac{1}{2}\right)^{n+1}+2^{n+1}$ $(n<0)$
$\lvert z\rvert > 2$		$\updownarrow$ -1	$\updownarrow$ -1
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}$ $(n \geq 1)$	$+\left(\frac{1}{2}\right)^{n+1}-2^{n+1}$ $(n \geq 0)$

$$2^{-n+1} = \left(\tfrac{1}{2}\right)^{n} \cdot 2 = \left(\tfrac{1}{2}\right)^{n-1} \qquad \left(\tfrac{1}{2}\right)^{-n-1} = 2^{n} \cdot 2 = 2^{n+1}$$

$$\left(\tfrac{1}{2}\right)^{-n+1} = 2^{n} \cdot \tfrac{1}{2} = 2^{n-1} \qquad 2^{-n-1} = \left(\tfrac{1}{2}\right)^{n} \cdot \tfrac{1}{2} = \left(\tfrac{1}{2}\right)^{n+1}$$

z^{-1}

z^{-1}

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-2^{n-1} + \left(\frac{1}{2}\right)^{n-1}$ $(n \geq 1)$
		$-n$	$-n$
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$	$+2^{n-1} - \left(\frac{1}{2}\right)^{n-1}$ $(n < 1)$

z^{-1}

z^{-1}

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$			
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $(n < 1)$	$-\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}$ $(n < 0)$
$\lvert z\rvert > 2$		$-n$	$-n$
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $(n \geq 1)$	$+\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}$ $(n \geq 0)$

z^{-1}

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-2^{n-1} + \left(\frac{1}{2}\right)^{n-1}$ $(n \geq 1)$
		$-n,$	-1
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$	$+2^{n-1} - \left(\frac{1}{2}\right)^{n-1}$ $(n < 1)$
		$-n,$	-1

z^{-1}

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$		$-n,$	-1
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $(n < 1)$	$-\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}$ $(n < 0)$
$\lvert z\rvert > 2$		$-n,$	-1
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $(n \geq 1)$	$+\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}$ $(n \geq 0)$

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$	$-n$
	$X(z)$	$-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}$ $(n<1)$	
$\lvert z\rvert > 2$	$f(z)$	$+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}$ $(n<0)$	$-n$
	$X(z)$	$+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}$ $(n \geq 1)$	

		① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$	② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$
$\lvert z\rvert < \frac{1}{2}$	$f(z)$	$-n$	$-2^{n-1}+\left(\frac{1}{2}\right)^{n-1}$ $(n \geq 1)$
	$X(z)$		$-\left(\frac{1}{2}\right)^{n+1}+2^{n+1}$ $(n<0)$
$\lvert z\rvert > 2$	$f(z)$	$-n$	$+2^{n-1}-\left(\frac{1}{2}\right)^{n-1}$ $(n<1)$
	$X(z)$		$+\left(\frac{1}{2}\right)^{n+1}-2^{n+1}$ $(n \geq 0)$

$f(z)$ $\quad |z| < 0.5 \quad |z| > 2$

causal $\quad$ anticausal

①-Ⓐ

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$

$$f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \qquad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$\frac{a}{1-az} \qquad \frac{z^{-1}}{a^{-1}z^{-1}-1}$$

$$-\left(\underset{n=0}{2} + \underset{n=1}{2^2 z} + \underset{n=2}{2^3 z^2} + \cdots\right) + \left(\underset{n=0}{\left(\tfrac{1}{2}\right)} + \underset{n=1}{\left(\tfrac{1}{2}\right)^2 z} + \underset{n=2}{\left(\tfrac{1}{2}\right)^3 z^2} + \cdots\right)$$

$|z| > 2$

$$f(z) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

$$\left(z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(z^{-1} + 2z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$$\left(\underset{n=-1}{2^0 z^{-1}} + \underset{n=-2}{2^{-1} z^{-2}} + \underset{n=-3}{2^{-2} z^{-3}} + \cdots\right) - \left(\underset{n=-1}{\left(\tfrac{1}{2}\right)^0 z^{-1}} + \underset{n=-2}{\left(\tfrac{1}{2}\right)^{-1} z^{-2}} + \underset{n=-3}{\left(\tfrac{1}{2}\right)^{-2} z^{-3}} + \cdots\right)$$

②-Ⓐ

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$|z| < 0.5$

$$f(z) = -\frac{z}{1-2z} + \frac{z}{1-0.5z} \qquad \boxed{-2^{n-1} + \left(\tfrac{1}{2}\right)^{n-1}} \quad (n \geq 1)$$

$$-\left(\underset{n=1}{z} + \underset{n=2}{2z^2} + \underset{n=3}{2^2 z^3} + \cdots\right) + \left(\underset{n=1}{z} + \underset{n=2}{\left(\tfrac{1}{2}\right) z^2} + \underset{n=3}{\left(\tfrac{1}{2}\right)^2 z^3} + \cdots\right)$$

$|z| > 2$

$$f(z) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}} \qquad \boxed{+2^{n-1} - \left(\tfrac{1}{2}\right)^{n-1}} \quad (n < 1)$$

$$\left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right) + \left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right)$$

$$\left(\underset{n=0}{2^{-1}} + \underset{n=-1}{2^{-2} z^{-1}} + \underset{n=-2}{2^{-3} z^{-2}} + \cdots\right) + \left(\underset{n=0}{\left(\tfrac{1}{2}\right)^{-1}} + \underset{n=-1}{\left(\tfrac{1}{2}\right)^{-2} z^{-1}} + \underset{n=-2}{\left(\tfrac{1}{2}\right)^{-3} z^{-2}} + \cdots\right)$$

$X(z)$ $\quad |z| < 0.5 \quad |z| > 2$

anticausal $\quad$ causal

① − Ⓑ $\frac{3}{2} \dfrac{-1}{(z-0.5)(z-2)} = \left(\dfrac{1}{z-0.5} - \dfrac{1}{z-2} \right)$

$|z| < 0.5 \qquad X(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z} \qquad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \qquad (n<1)$

$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$

$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

$|z| > 2 \qquad X(z) = \dfrac{z^{-1}}{1-0.5z^{-1}} - \dfrac{z^{-1}}{1-2z^{-1}} \qquad \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \qquad (n \geq 1)$

$\left(\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

② − Ⓑ $\frac{3}{2} \dfrac{-z^2}{(z-2)(z-0.5)} = \left(\dfrac{0.5z}{(z-0.5)} - \dfrac{2z}{(z-2)} \right)$

$|z| < 0.5 \qquad X(z) = -\dfrac{z}{1-2z} + \dfrac{z}{1-0.5z} \qquad \boxed{-\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1}} \qquad (n<0)$

$-\left(z + 2z^2 + 2^2 z^3 + \cdots\right) + \left(z + \left(\tfrac{1}{2}\right) z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right)$

$-\left(\left(\tfrac{1}{2}\right)^0 z + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right) + \left(2^0 z + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right)$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$|z| > 2 \qquad X(z) = \dfrac{0.5}{1-0.5z^{-1}} - \dfrac{2}{1-2z^{-1}} \qquad \boxed{+\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}} \qquad (n \geq 0)$

$\left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right) + \left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right)$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

$$\frac{3}{2}\ \frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $X(z)$ $a_n = \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $(n < 1)$

$|z| > 2$ $X(z)$ $b_n = \boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}}$ $(n \geq 1)$

$\{\,|z| < 0.5\,\} \cap \{\,|z| > 2\,\} = \phi \Rightarrow a_n + b_n = 0$

$a_n = -b_n$

$|z| < a$ $X(z) = \sum_{n=0}^{\infty} a^{n+1} z^n$ a^{n+1} $n \geq 0$ $n \geq 1$ $n < 0$ $n < 1$

$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$ $\xrightarrow{-n}$

$|z| > a$ $X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$ a^{-n+1} $n < 0$ $n < 1$ $n \geq 0$ $n \geq$

$= \left(\frac{1}{a}\right)^{n-1}$

$a^{n+1} z^n$

$a\,(az)^n$

$a\left(\frac{1}{az}\right)^{-n}$

$\frac{a}{1-az}$ $\sum_{n=0}^{\infty} a^{n+1} z^n$ $\frac{z}{1-az}$ $\sum_{n=1}^{\infty} a^{n-1} z^n$

$-\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $-\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$ $-\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $-\sum_{n=1}^{\infty} a^{-n-1} z^{-n}$

$\sum_{n=-1}^{-\infty} a^{n+1} z^n$ $-\sum_{n=0}^{-\infty} a^{n-1} z^n$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad f(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\quad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}}$ $\quad (n \geq 0)$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0$ $\quad n=1$ $\quad n=2$ $\qquad n=0$ $\quad n=1$ $\quad n=2$

$|z| < 0.5$ $\quad X(z) = -\dfrac{2}{1-2z} + \dfrac{0.5}{1-0.5z}$ $\quad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \leq 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0$ $\quad n=-1$ $\quad n=-2$ $\qquad n=0$ $\quad n=-1$ $\quad n=-2$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n$ $\qquad a^{n+1}$ $\qquad n \geq 0$ $\quad n \geq 1$ $\quad n < 0$ $\quad n < 1$

$\|$ $\quad \|$ $\quad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$ $\qquad -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$ $\qquad a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$ $\qquad n < 0$ $\quad n < 1$ $\quad n \geq 0$ $\quad n \geq 1$

$$\text{ROC} \qquad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^{n} \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$$

$$z^{-1} \updownarrow \qquad z^{-1} \updownarrow \qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n} \qquad \big\downarrow{-n} \qquad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$\text{ROC} \qquad X(z) = \sum_{k=0}^{-\infty} (a)^{k-1} z^{-k} \qquad \left(\frac{1}{a}\right)^{-n+1} \qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$$

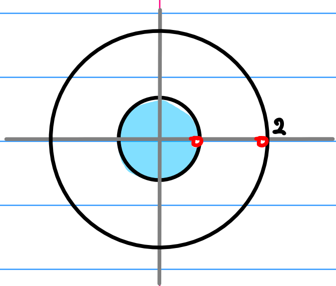
$$= a^{n-1}$$

Causal $\quad f(z) \quad X(z)$

$|z| < 0.5 \quad |z| > 2$

$$\frac{3}{2}\;\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 2 \qquad |z| < 0.5$



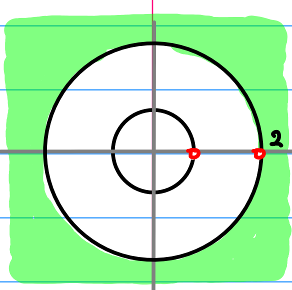
$$f(z) = (-2)\frac{0.5}{0.5-z} + (0.5)\frac{2}{2-z} \qquad (|z| < 0.5)$$

$$a_n = (-2)\;2^n + (0.5)\;\left(\tfrac{1}{2}\right)^n \qquad (n \geq 0)$$

$$-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}$$

$$\frac{3}{2}\;\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) \qquad |z| > 2$$

$|z| > 2 \qquad |z| > 0.5$



$$X(z) = 0.5\,\frac{z}{z-0.5} - 2\,\frac{z}{z-2} \qquad (|z| > 2)$$

$$a_n = (0.5)\;\left(\tfrac{1}{2}\right)^n - 2 \cdot 2^n \qquad (n \geq 0)$$

$$\left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}$$

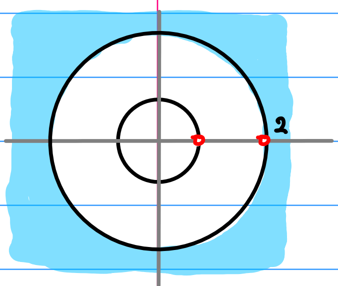
Anti-causal

$f(z)$ $X(z)$

$|z| > 2$ $|z| < 0.5$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| > 2$ $|z| > 0.5$



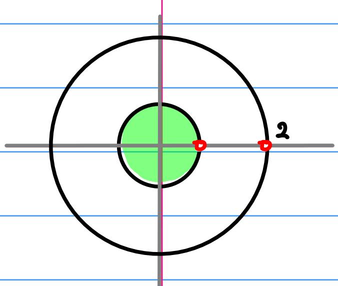
$$f(z) = (-2)\,\frac{-0.5}{0.5 - z} + (0.5)\,\frac{-2}{2 - z} \qquad (|z| > 0.5)$$

$$a_n = (+2)\,2^n - (0.5)\left(\frac{1}{2}\right)^n \qquad (n < 0)$$

$$+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) \qquad |z| < 2$$

$|z| < 2$ $|z| < 0.5$



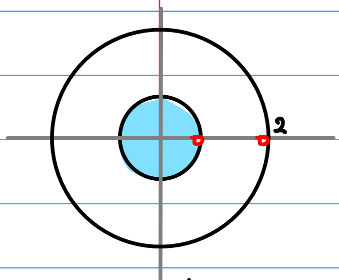
$$X(z) = 0.5\,\frac{-z}{z-0.5} - 2\,\frac{-z}{z-2} \qquad (|z| < 2)$$

$$a_n = -(0.5)\left(\frac{1}{2}\right)^n + 2 \cdot 2^n \qquad (n < 0)$$

$$-\left(\frac{1}{2}\right)^{n+1} + 2^{n+1}$$

① - Ⓐ $\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \boxed{f(z)}$ $|z| < 0.5$ causal $|z| > 2$ anticausal

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$

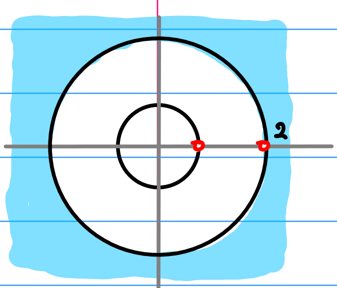


$|z| < 0.5$

$$f(z) = \boxed{\begin{aligned} &\frac{(-2)}{1-(2z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty} (2)^{n+1} (z)^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} (z)^n \\ &= -\sum_{n=0}^{\infty} (2)^{n+1} z^n + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^n \end{aligned}}$$

$(n \geq 0)$ $a_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



$|z| > 2$

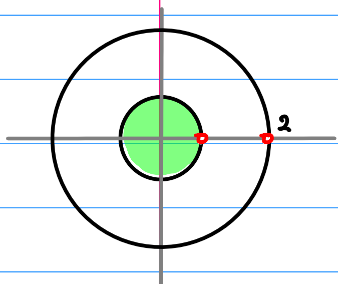
$$f(z) = \boxed{\begin{aligned} &\frac{(\frac{1}{z})}{1-(\frac{1}{2z})} - \frac{(\frac{1}{z})}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^n \left(\tfrac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty} (2)^n \left(\tfrac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty} \left(\tfrac{1}{2}\right)^{n-1} z^{-n} - \sum_{n=1}^{\infty} (2)^{n-1} z^{-n} \end{aligned}} \neq$$

$$= \sum_{n=-1}^{-\infty} (2)^{n+1} z^n - \sum_{n=-1}^{-\infty} \left(\tfrac{1}{2}\right)^{n+1} z^n$$

$(n < 0)$ $a_n = 2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$

① - Ⓑ $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \boxed{X(z)}$ $|z| < 0.5$ anticausal $|z| > 2$ causal

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{-2}{1-2z} + \frac{0.5}{1-0.5z}$$



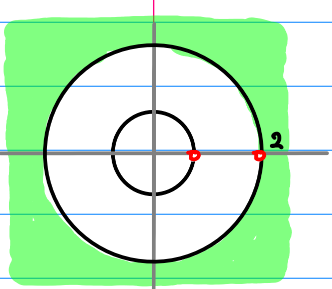
$|z| < 0.5$

$$\boxed{\begin{aligned} X(z) &= \frac{(-2)}{1-(2z)} + \frac{(\frac{1}{2})}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^{n+1}(z)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}(z)^n \\ &= -\sum_{n=0}^{\infty}(2)^{n+1} z^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^n \end{aligned}}$$

$$= -\sum_{n=0}^{-\infty}\left(\frac{1}{2}\right)^{n-1} z^{-n} + \sum_{n=0}^{-\infty}(2)^{n-1} z^{-n}$$

$(n \leq 0)$ $a_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right) = \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$$



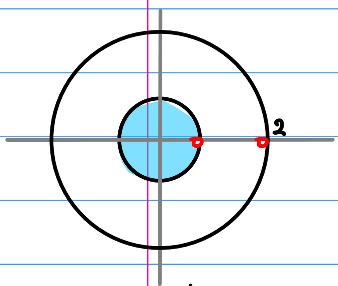
$|z| > 2$

$$\boxed{\begin{aligned} X(z) &= \frac{(\frac{1}{z})}{1-(\frac{1}{2z})} - \frac{(\frac{1}{z})}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n\left(\frac{1}{z}\right)^{n+1} - \sum_{n=0}^{\infty}(2)^n\left(\frac{1}{z}\right)^{n+1} \\ &= \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1} z^{-n} - \sum_{n=1}^{\infty}(2)^{n-1} z^{-n} \end{aligned}}$$ $\neq$

$(n > 0)$ $a_n = \left(\frac{1}{2}\right)^{n-1} - (2)^{n-1}$

② - Ⓐ $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \boxed{f(z)}$ $|z| < 0.5$ causal $|z| > 2$ anticausal

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$$

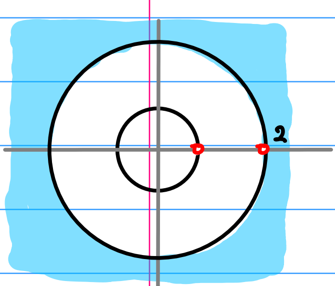


$|z|<0.5$

$$\boxed{\begin{aligned} f(z) &= -\frac{(z)}{1-(2z)} + \frac{(z)}{1-(\frac{z}{2})} \\ &= -\sum_{n=0}^{\infty}(2)^n(z)^{n+1} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n(z)^{n+1} \\ &= -\sum_{n=1}^{\infty}(2)^{n-1}z^n + \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}z^n \end{aligned}} \quad \neq$$

$(n > 0)$ $\quad a_n = -2^{n-1} + \left(\frac{1}{2}\right)^{n-1}$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$



$|z|>2$

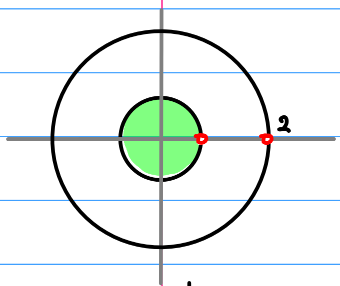
$$\boxed{\begin{aligned} f(z) &= \frac{(\frac{1}{2})}{1-(\frac{1}{2z})} - \frac{(2)}{1-(\frac{2}{z})} \\ &= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}\left(\frac{1}{z}\right)^n - \sum_{n=0}^{\infty}(2)^{n+1}\left(\frac{1}{z}\right)^n \\ &= \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1}z^{-n} - \sum_{n=0}^{\infty}(2)^{n+1}z^{-n} \end{aligned}}$$

$$= \sum_{n=0}^{-\infty}(2)^{n-1}z^n - \sum_{n=0}^{-\infty}\left(\frac{1}{2}\right)^{n-1}z^n$$

$(n \leq 0)$ $\quad a_n = 2^{n-1} - \left(\frac{1}{2}\right)^{n-1}$

② - Ⓑ $\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \boxed{X(z)}$ $|z| < 0.5$ anticausal $|z| > 2$ causal

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = -\frac{z}{1-2z} + \frac{z}{1-0.5z}$$



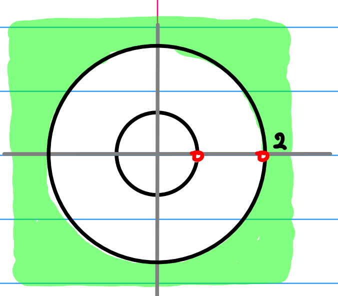
$|z| < 0.5$

$$\begin{aligned} X(z) &= -\frac{(z)}{1-(2z)} + \frac{(z)}{1-\left(\frac{z}{2}\right)} \\ &= -\sum_{n=0}^{\infty} (2)^n (z)^{n+1} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^n (z)^{n+1} \\ &= -\sum_{n=1}^{\infty} (2)^{n-1} z^n + \sum_{n=1}^{\infty} \left(\tfrac{1}{2}\right)^{n-1} z^n \\ &= -\sum_{n=-1}^{-\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{-n} + \sum_{n=-1}^{-\infty} (2)^{n+1} z^{-n} \end{aligned}$$

$\neq$

$(n < 0)$ $a_n = -\left(\tfrac{1}{2}\right)^{n+1} + 2^{n+1}$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right) = \frac{0.5}{1-0.5z^{-1}} - \frac{2}{1-2z^{-1}}$$



$|z| > 2$

$$\begin{aligned} X(z) &= \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} - \frac{(2)}{1-\left(\frac{2}{z}\right)} \\ &= \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} \left(\tfrac{1}{z}\right)^n - \sum_{n=0}^{\infty} (2)^{n+1} \left(\tfrac{1}{z}\right)^n \\ &= \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{-n} - \sum_{n=0}^{\infty} (2)^{n+1} z^{-n} \end{aligned}$$

$(n \geq 0)$ $a_n = \left(\tfrac{1}{2}\right)^{n+1} - 2^{n+1}$