Sampling Inspection

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Outline

Based on

"Probability with R: An Introduction with Computer Science Applications" Jane Horgan
https://en.wikipedia.org/wiki/Geometric_distribution

"Operations Management" Krajewski, Ritzman, Malhotra
http://wps.pearsoned.co.uk/ema_ge_krajewski_opsmgmt_9/140/35881/9185585.cw/index.html

TQM (Total Management Control)

1. a random sample
2. the sample passes the test, then entire quantity is accepted
3. the sample fails the test, (a) 100 percent inspection, all defective items repaired or replaced (b) or return the entire quantity

AQL (Acceptable Quality Level)

- quality level desired by the consumer
- for example, a quality level must not exceed one defective unit in 10,000, ie, AQL of 0.0001
- the producer's risk α
  - the risk that the sampling plan will fail to verify an acceptable lot's quality and thus reject it
  - Type I error : treat good as bad
  - commonly it is set at 0.05 (5 percent)

LTPD (Lot Tolerance Proportion Defective)

- the worst level of quality that the consumer can tolerate
- the definition of bad quality that the consumer would like to reject
- consumer's risk (β)
  - the probability of accepting a lot with LTPD quality
  - a type II error : treat bad as good
  - commonly it is set at 0.10 (10 percent)

Sampling Plan

- devised to provide a specified producer's and consumer's risk.
- must minimze ANI (average number of itmes inspected)
- types of sampling plans
    - Single Sampling Plan
        - easy to use
        - usually results in a larger ANI
    - Double Sampling Plan
    - Sequential Sampling Plan

Single Sampling Plan

- a decision rule to accept or reject a lot
  based on the results of one random sample from the lot
  - take random sample of size (n) and inspect each item
  - if the number of defects does not exceed a specified acceptance number (c), the consumer accepts the entire lot
    any defects found in the sample are either repaired or returned to the producter
  - if the number of defects in the sample is greater than c, the consumer
    - subjects the entire lot to 100 percent inspection
    - rejects the entire lot and returns it to the producer

Operating Characteristic (OC) Curves

- plotting the probability of accepting the lot for a range of proportions of defective units
- x: proportion defective
- y: proabability of acceptance
- how well a sampling plan discriminates between good and bad lots
- accepts lots with a quality level better than the AQL 100 percent
- rejects lots with a quality level worse than the AQL 0 percent
- such performance can be achieved only with 100 pecent inspection

OC Curve Parameters

- probability α of rejecting a good lot (producer's risk)
- proability β of accepting a bad lot (consumer's risk)
- managers are left with choosing a sample size n and an acceptance number c to achieve the level of performance specified by the AQL, α, LTPD, β

Example 1

- 100 items in a box.
- Each box must be tested by a sample of 10 items.
- Accept the box if it contains less than or equal to one defective item.
- Otherwise, the box is rejected
- $N = 100$
- $n = 10$
- $c = 1$

Example 2

- 1000 memory chips are packed in a batch
- Each batch must be tested by a sample of 20 memory chips.
- Reject the box if it contains more than two defective chips.
- Otherwise, the batch is accepted
- $N = 1000$
- $n = 20$
- $c = 2$

Example 3

- 10000 microchips in a batch.
- Each batch must be tested by a sample of 100 chips.
- Accept the box if it contains less than or equal to three defective chips.
- Otherwise, the batch is rejected
- $N = 10000$
- $n = 100$
- $c = 3$

Parameters

- N = the size of a batch
- n = the sample size taken from a batch
- c = the maximum number of defectives allowed for accepting a batch

- Ex1) $N = 100, n = 10, c = 1$
- Ex2) $N = 1000, n = 20, c = 2$
- Ex3) $N = 10000, n = 100, c = 3$

Proportion Defective

- $p =$ the proportion defective in the batch
- $q = 1 - p =$ the proportion nodefective
- Np defectives in a batch
- $N(1 - p)$ non-defectives in a batch
- p also called batch quality

Acceptance Probability

- X = the number of defectives found in a sample of size n
- accept a batch, if $X \leq c$
- the probability of acceptance of batch
- $P(acceptance, p) = \frac{\binom{Np}{0}\binom{N(1-p)}{n}}{\binom{N}{n}} + \frac{\binom{Np}{1}\binom{N(1-p)}{n-1}}{\binom{N}{n}} + \cdots \frac{\binom{Np}{c}\binom{N(1-p)}{n-c}}{\binom{N}{n}}$
- reject batches with low batch quality - high p
- accept batches with high batch quality - low p

Acceptance Probability for Example 1

- allow up to one defective in samples of size 10 from batches of size 100
- batch is accepted if the number of defective in the sample $X \leq 1$
- $P(acceptance, p) = \frac{\binom{100p}{0}\binom{100(1-p)}{10}}{\binom{100}{10}} + \frac{\binom{100p}{1}\binom{100(1-p)}{9}}{\binom{100}{10}}$
- $P(acceptance, 0.1) = \frac{\binom{10}{0}\binom{90}{10}}{\binom{100}{10}} + \frac{\binom{10}{1}\binom{90}{9}}{\binom{100}{10}} = 0.330 + 0.408 = 0.738$
- $p = 0.05$ (5 defectives and 95 good), $P = 0.923$
- $p = 0.10$ (10 defectives and 90 good), $P = 0.738$
- $p = 0.15$ (15 defectives and 85 good), $P = 0.537$

Acceptance Probability for Example 2

- allow up to one defective in samples of size 20 from batches of size 1000
- batch is accepted if the number of defective in the sample $X \leq 2$
- when N is large, the binomial can approximate the hypergeometric
- $P(acceptance, p) = \frac{\binom{100p}{0}\binom{100(1-p)}{10}}{\binom{100}{10}} + \frac{\binom{100p}{1}\binom{100(1-p)}{9}}{\binom{100}{10}}$
  $\approx (1-p)^2 0 + \binom{20}{1} p^1 (1-p)^1 9 + \binom{20}{2} p^2 (1-p)^{18}$

O

- $P(acceptance, 0.1) = \approx (0.9)^2 0 + \binom{20}{1} 0.1^1 (0.9)^1 9 + \binom{20}{2} 0.1^2 (0.9)^{18}$
  $= 0.122 + 0.270 + 0.285 = 0.677$
- $p = 0.05$ (5 defectives and 95 good), $P = 0.924$
- $p = 0.10$ (10 defectives and 90 good), $P = 0.677$
- $p = 0.15$ (15 defectives and 85 good), $P = 0.405$

Acceptance Probability for Example 3

- allow up to one defective in samples of size 100 from batches of size 10000
- batch is accepted if the number of defective in the sample $X \leq 3$
- when N is large, the Poisson can approximate the hypergeometric by $\lambda = np$
- $P(acceptance, p) = \frac{\binom{100p}{0}\binom{100(1-p)}{10}}{\binom{100}{10}} + \frac{\binom{100p}{1}\binom{100(1-p)}{9}}{\binom{100}{10}}$
  $\approx e^{-\lambda} + e^{-\lambda}\lambda + e^{-\lambda}\frac{\lambda^2}{2!} + e^{-\lambda}\frac{\lambda^3}{3!} +$ where $\lambda = 100p$
- $P(acceptance, 0.1) = \approx e^{-10} + e^{-10}10 + e^{-10}\frac{10^2}{2!} + e^{-10}\frac{10^3}{3!} +$ where $\lambda = 100p = 10$
  $= 0.000005 + 0.00045 + 0.00227 + 0.00757 = 0.001034$
- $p = 0.05$ (5 defectives and 95 good), $P = 0.2650$
- $p = 0.10$ (10 defectives and 90 good), $P = 0.0103$
- $p = 0.15$ (15 defectives and 85 good), $P = 0.0002$

Acceptance Probability Computation using R

- Ex1) $N = 100, n = 10, c = 1$

```
p <- c(.05, .1, .15)
phyper(1, 100*p, 100*(1-p), 10)
[ 1] 0.9231433 0.7384715 0.5375491
```

- Ex2) $N = 1000, n = 20, c = 2$

```
p <- c(.05, .1, .15)
pbinom(2, 20, p)
[ 1] 0.9245163 0.6769268 0.4048963
```

- Ex3) $N = 10000, n = 100, c = 3$

```
p <- c(.05, .1, .15)
ppois(1, 100*p, 100*p)
round( ppois(1, 100*p, 100*p), 4)
[ 1] 0.2650 0.0103 0.0002
```

Hypergeometric distribution R function

```
phyper(q, m, n, k)

q : vector of quantiles representing the number of white balls drawn
    without replacement from an urn which contains both black and white balls.
m : the number of white balls in the urn.
n : the number of black balls in the urn.
k : the number of balls drawn from the urn.

The hypergeometric distribution is used for sampling without replacement.
The density of this distribution with parameters m, n and k
(named Np, N-Np, and n, respectively in the reference below)
is given by

p(x) = choose(m, x) choose(n, k-x) / choose(m+n, k)

for x = 0, ..., k.

Note that p(x) is non-zero only for max(0, k-n) <= x <= min(k, m).
```

Binomial distribution R function

```
pbinom(q, size, prob)

q    : vector of quantiles
size : number of trials (zero or more).
prob : probability of success on each trial.

The binomial distribution with size = n and prob = p has density

p(x) = choose(n, x) p^x (1-p)^(n-x)

for x = 0, ..., n.

Note that binomial coefficients can be computed by choose in R.
```

Poisson distribution R function

```
ppois(q, lambda)

q      : vector of quantiles
lambda : vector of (non-negative) means.

The Poisson distribution has density

p(x) = ^x exp(-)/x!

for x = 0, 1, 2, ... . The mean and variance are E(X) = Var(X) = .

Note that  = 0 is really a limit case (setting 0^0 = 1)
resulting in a point mass at 0, see also the example.
```

Simulation for Example 1

```
rhyper(nn, m, n, k)
random generation for the hypergeometric distribution
nn : number of observations

rhyper(100,  5, 95, 10)
rhyper(100, 10, 90, 10)
rhyper(100, 15, 85, 10)

p <- c(.05, .10, .15)
plot(rhyper(100, 100*p, 100*(1-p), 10), ylib= c(0,5),
     xlab = "batch no",
     ylab = "no of defectives",
     main = p)
abline(h = 1.2)
```

Simulation for Example 2

```
rbinom(n, size, prob)
random generation for the binomial distribution
n : number of observations

rbinom(100, 20, .05)
rbinom(100, 20, .10)
rbinom(100, 20, .15)

p <- c(.05, .10, .15)
plot(rbinom(100, 20, p), ylim= c(0,8),
     xlab = "batch no",
     ylab = "no of defectives",
     main = p)
abline(h = 2.2)
```

Simulation for Example 3

```
rpois(n, lambda)
random generation for the Poisson distribution
n : number of random values to return
```

```
rpois(100,  5)
rpois(100, 10)
rpois(100, 15)

p <- c(.05, .10, .15)
plot(rpois(100, 100*p), ylim= c(0,20),
     xlab = "batch no",
     ylab = "no of defectives",
     main = p)
abline(h = 3.2)
```

OC Curve Plot

```
par(mfrwo = c(1,3))

p <- seq(0, 1, .01)
P <- phyper(1, 100*p, 100*(1-p), 10)
plot(p, P, type="l", xlim=c(0,.5),
     xlab="proportion defective",
     ylab="acceptance probability")

p <- seq(0, 1, .001)
P <- pbinom(2, 20, p)
plot(p, P, type="l", xlim=c(0,.5),
     xlab="proportion defective",
     ylab="acceptance probability")

p <- seq(0, 1, .0001)
P <- ppois(3, 100*p)
plot(p, P, type="l", xlim=c(0,.5),
     xlab="proportion defective",
     ylab="acceptance probability")
```

Ideal OC Curve Plot

```
x <- .04
y <- 1

plot(x, y, xlim=c(0,.1), ylim=c(0,1),
     xlab="proportion of defectives in a batch (p)",
     ylab="acceptance probability (P)")

lines(c(0,.04), c(1,1), lty=1)
lines(c(.04, .04), c(0,1), lty=1)

lines(c(0,0), c(0,1), lty=1)
lines(c(.04, .1), c(0,0), lty=1)
```

Producer's Risks

- the producer's risk: rejecting batches wih acceptable defective rates
- if the producer gives a guarantee that batches contain no more than 4 %% defectives, then batches with less than or equal to 4 %% defectives are considered good and should be accepted
- rejecting these involve unecessary extra costs to the producer
- this low proportion of defectives (4%%) is called the acceptable quality level (AQL)

Consumer's Risk

- the consumer's risk: accepting batches with unacceptiably high proportions of defectives
- batches containing 10%% defectives may considered as bad and should be rejected
- accepting a bad batch means that the consumer is given a batch with an unacceptably high level of defectives
- this proportion of defectives (10%) is called the limiting quality (LQ)

Computing Producer's Risk

```
p = .04  # good batches

1 - phyper(1, 100*p, 100*(1-p), 10)

1 - pbinom(2, 20, p)

1 - ppois(3, 100*p)
```

Computing Consumer's Risk

```
p = .10  # good batches

phyper(1, 100*p, 100*(1-p), 10)

pbinom(2, 20, p)

ppois(3, 100*p)
```