Angle Recoding 2. Wu
1. Conventional CORDIC

20180825 Sat

⓪ Conventional CORDIC

elementary angle $a(i) = \tan^{-1}(2^{-i})$

the number of elementary angles N

the rotation sequence $\mu(i) = \{-1, +1\}$

$+1, -1, -1, +1, +1, \ldots\ldots$

the i-the rotation angle $a(i)$

the W-bit word length

the iteration number $N \leq W$

the angle quantization error

$$\boxed{\xi_{m,\,\mathrm{CORDIC}} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i)}$$

AQ & conventional CORDIC

EAS (Elementary Angle Set)

comprises of all $a(i)$ for $0 \le i \le N-1$

$$S = \{a(i) : 0 \le i \le N-1\}$$

the CORDIC algorithm essentially performs AQ

tries to perform the rotation

by sequential applications of

micro-rotations of all elementary angles

given a target rotation angle θ

the first rotation sequence $\mu(0)$

for the most significant elementary angle $a(0)$

the second rotation sequence $\mu(1)$

for the next most significant elementary angle $a(1)$

repeated until the last elementary angle is applied.

the sub-angle θ_i in AQ

$\theta_i = \mu(i)\, a(i)$ in CORDIC

$\mu(i) = \{-1, +1\}$

$a(i) = \tan^{-1}(2^{-i})$

the number of sub-angles

N_A in AQ

N in CORDIC

CORDIC algorithm sequentially apply all θ_i's for $i = 0, 1, \cdots, N-1$ to approximate the target angle θ

iteration number	elementary angle	value in radian
i=0	a(0)=atan(2^{-0})	
i=1	a(1)=atan(2^{-1})	
i=2	a(1)=atan(2^{-2})	
i=3	a(1)=atan(2^{-3})	
i=4	a(1)=atan(2^{-4})	
i=5	a(1)=atan(2^{-5})	
i=6	a(1)=atan(2^{-6})	
i=7	a(1)=atan(2^{-7})	

$$\xi_{m,\,CORDIC} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i) \qquad \mu(i) = \{-1,\ 0,\ +1\}$$

$$= \theta - \sum_{j=0}^{N'} \tilde{\theta}(j)$$

$$N' \equiv \sum_{i=0}^{N-1} |\mu(i)| \qquad \{\,|+1|,\ |0|,\ |+1|\,\}$$

the effective iteration number N'

$S(j)$ the rotational sequence

determines the micro-rotation angle in the j-th iteration

$$i \leftarrow S(j)$$

index $0, 1, \cdots, N'$ (reduced number)

index $0, 1, \cdots, N$ (original number)

$$\mu(S(j)) \leftarrow \alpha(j)$$

$$\Downarrow \qquad\qquad \Uparrow$$

$$\{-1, +1\}$$

$$\mu(i) = \begin{cases} \mu(S(j)) & i = S(j) \\ 0 & i \neq S(j) \end{cases}$$ --- reduced index

er

$i =$	$0,$	$1,$	$2,$	$3,$	$\cdots,$	$N-1$		
$s(j) =$	$0,$	$1,$	$2,$	$3,$	$\cdots,$	$N-1$	rotational sequence	
$\alpha(j) =$	$-1,$	$0,$	$0,$	$+1,$	$\cdots,$	-1	directional sequence	
$j =$	$0,$	$-,$	$-,$	$1,$	$\cdots,$	$N'-1$	effective iteration number	

$N' = N-2$

the j-th micro-rotation of $a(s(j))$

elementary angle

$$a(i) = \tan^{-1}(2^{-i})$$
$$a(s(j)) = \tan^{-1}(2^{-s(j)})$$

$$\alpha(j)\, a(s(j)) = \alpha(j) \tan^{-1}(2^{-s(j)}) \qquad \alpha(j) \in \{-1, +1\}$$

$$\Leftrightarrow \mu(i)\, a(i) \qquad \mu(i) \in \{-1, 0, +1\}$$

$$\xi_{m,\,CORDIC} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i) \qquad \mu(i) \in \{-1, 0, +1\}$$

$$= \theta - \left[\sum_{j=0}^{N'} \tilde{\theta}(j) \right]$$

$$= \theta - \left[\sum_{j=0}^{N'} \tan^{-1}\left(\alpha(j) \cdot 2^{-s(j)} \right) \right] \qquad \alpha(j) \in \{-1, +1\}$$

$$\tilde{\theta}(j) = \alpha(j) \tan^{-1}\left(2^{-s(j)}\right)$$

$$= \tan^{-1}\left(\alpha(j) \cdot 2^{-s(j)} \right)$$

$$S_1 = \left\{ \tan^{-1}\left(\boxed{\alpha} \cdot 2^{-\boxed{s}} \right) \quad \boxed{\alpha} \in \{-1, 0, +1\},\ \boxed{s} \in \{0, 1, 2, \cdots N-1\} \right\}$$

```
>> mu = [1, -1, 1, 1, -1, 1, -1, -1, 1, -1, -1, 1, 1, -1, 1, 1]
>> length(mu)
ans =  16
>> s =[ 0: 15]
>> atan(1)
ans =  0.78540
>> pi/4
ans =  0.78540
>> sum(atan(2.^(-s)) .* mu)
ans =  0.63815
>> 13 * pi / 32
ans =  1.2763
```