Angle Recoding 2. Wu
2. AR (Angle Recode)

20180914 Fri

① AR [Hu]

skip certain micro rotations

the rotation sequence $\mu(i) = \{-1, 0, +1\}$

$\mu(i) = 0 \rightarrow$ skip

desire to minimize

$$\sum_{i=0}^{N} |\mu(i)|$$

← greedy algorithm can minimize

so that the total number of CORDIC iterations can be minimized

Angle Recoding ← Multiplier Recoding

angle recoding method for efficient
implementation of the CORDIC algorithm
Hu & Naganathan, ISCAS 89

Greedy algorithm

the angle quantization error

$$\xi_{m,AR} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i)$$

Skipping rotations

$\mu(i) = 0$

null operations

$$\begin{cases} x(i+1) = x(i) \\ y(i+1) = y(i) \end{cases}$$

keep the iteration number N the same

in the AQ perspective, remove these steps
and use different index variable

"index variable conversion"

the iteration number N becomes N'

the number of subangles N_A of EAS

$N_A \Leftarrow N'$ not fixed but varying

Optimization Problem

given a rotation angle θ
find rotation sequence $\mu(i) \in \{-1, 0, +1\}$
for $0 \leq i \leq N-1$

such that angle quantization error

$$|\xi_{m,AR}| < a(N-1)$$

and the effective iteration number

$$N' = \sum_{i=0}^{N} |\mu(i)|$$ is minimized

Hu's greedy algorithm

$\theta(0) = \theta$, $\{\mu(i) = 0,\ 0 \leq i \leq N-1\}$, $k=0$.

repeat until $|\theta(k)| < a(N-1)\ D_0$

Choose i_k, $0 \leq i_k \leq N-1$

$$\big|\,|\theta(k)| - a(i_k)\big| = \min_{0 \leq i \leq N-1} \big|\,|\theta(k)| - a(i_k)\big|$$

$$\theta(k+1) = \theta(k) - \mu(i_k)\, a(i_k)$$

$$\mu(i_k) = \text{Sign}(\theta(k))$$

AQ vs. AR

$$\xi_{m,AR} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i) \qquad N \qquad AR$$

$$= \theta - \left[\sum_{j=0}^{N'-1} \tan^{-1}\left(\alpha(j) \cdot 2^{-s(j)}\right) \right] \qquad N' \qquad AQ$$

$$= \theta - \left[\sum_{j=0}^{N'-1} \tilde{\theta}(j) \right] \qquad N'$$

$$N' \triangleq \sum_{i=0}^{N-1} |\mu(i)|$$

the effective transition number

count non-zero rotation

$s(j) \in \{0, 1, \cdots, N-1\}$ the rotational sequence

determines the micro-rotation angle in the j-th iteration

$\alpha(j) \in \{-1, 0, +1\}$ the directional sequence

$\{-1, +1\}$ (after sampling) controls the direction of the j-th micro-rotation of $a(s(j))$

$\tilde{\theta}(j)$ the jth micro-rotation angle

$$\tilde{\theta}(j) = \tan^{-1}\left(\alpha(j) \cdot 2^{-s(j)}\right)$$

AR essentially tries to approximate θ with the combination of selected angle elements from a pre-defined **elementary angle set** (EAS).

the EAS consists of **all** possible values of $\tilde{\theta}(j)$'s

the EAS S_1 in AR

$$S_1 = \{ \tan^{-1}(\alpha^* \cdot 2^{-s^*}) : \alpha^* \in \{-1, 0, +1\}, \ s^* \in \{0, 1, \cdots, N-1\} \}$$

$$\xi_{m,AR} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i)$$

$$= \theta - \left[\sum_{j=0}^{N'} \tan^{-1}(\alpha(j) \cdot 2^{-s(j)}) \right]$$

$$= \theta - \left[\sum_{j=0}^{N'} \tilde{\theta}(j) \right]$$

$$\xi_m \triangleq \theta - \sum_{i=0}^{N_A - 1} \theta_i$$

AR performs AQ of the target angle θ

the sub-angle θ_i becomes $\tilde{\theta}(i) = \tan^{-1}(\alpha(i) \cdot 2^{-s(i)})$

$$N_A \longrightarrow \boxed{N' \triangleq \sum_{i=0}^{N-1} |\mu(i)|}$$

the effective transition number

given a rotation angle θ

AR

find rotation sequence $\mu(i) \in \{-1, 0, +1\}$
for $0 \le i \le N-1$

AQ

find the combination of elementary angles
from S_1

Such that angle quantization error

$$|\xi_{m,AR}| < \alpha(N-1)$$

and the effective iteration number

$$N' = \sum_{i=0}^{N} |\mu(i)| \quad \text{is minimized}$$

EAS S_1 AR

elementary angle	value
r(1) = atan(-2^{-0})	
r(2) = atan(-2^{-1})	
r(3) = atan(-2^{-2})	
r(4) = atan(-2^{-3})	
r(5) = atan(-2^{-4})	
r(6) = atan(-2^{-5})	
r(7) = atan(-2^{-6})	
r(8) = atan(-2^{-7})	
r(9) = atan(0)	
r(10) = atan(2^{-7})	
r(11) = atan(2^{-6})	
r(12) = atan(2^{-5})	
r(13) = atan(2^{-4})	
r(14) = atan(2^{-3})	
r(15) = atan(2^{-2})	
r(16) = atan(2^{-1})	
r(17) = atan(2^{-0})	

N_A (r(1) – r(17))

N conventional CORDIC (r(9) – r(17))

```
>> mu = [1, 0, 0, -1, 0, 0, -1, -1, 0, 0, 0, 1, 0, 0, 0, 1]
>> length(mu)
ans =  16
>> s =[ 0: 15]
>> atan(1)
ans =  0.78540
>> pi/4
ans =  0.78540
>> sum(atan(2.^(-s)) .* mu)
ans =  0.63813
>> 13 * pi / 32
ans =  1.2763
```

in terms of s(j), α(j) of AQ

i	Conventional CORDIC	only $\{-1, +1\}$	Angle Recode	skip allowed $\{-1, 0, +1\}$
0	$S(0) = 0$	$\alpha(0) = +1$	$S(0) = 0$	$\alpha(0) = +1$
1	$S(1) = 1$	$\alpha(1) = -1$	$S(1) = 1$	$\alpha(1) = 0$
2	$S(2) = 2$	$\alpha(2) = +1$	$S(2) = 2$	$\alpha(2) = 0$
3	$S(3) = 3$	$\alpha(3) = +1$	$S(3) = 3$	$\alpha(3) = -1$
4	$S(4) = 4$	$\alpha(4) = -1$	$S(4) = 4$	$\alpha(4) = 0$
5	$S(5) = 5$	$\alpha(5) = +1$	$S(5) = 5$	$\alpha(5) = 0$
6	$S(6) = 6$	$\alpha(6) = -1$	$S(6) = 6$	$\alpha(6) = -1$
7	$S(7) = 7$	$\alpha(7) = -1$	$S(7) = 7$	$\alpha(7) = -1$
8	$S(8) = 8$	$\alpha(8) = +1$	$S(8) = 8$	$\alpha(8) = 0$
9	$S(9) = 9$	$\alpha(9) = -1$	$S(9) = 9$	$\alpha(9) = 0$
10	$S(10) = 10$	$\alpha(10) = -1$	$S(10) = 10$	$\alpha(10) = 0$
11	$S(11) = 11$	$\alpha(11) = +1$	$S(11) = 11$	$\alpha(11) = +1$
12	$S(12) = 12$	$\alpha(12) = +1$	$S(12) = 12$	$\alpha(12) = 0$
13	$S(13) = 13$	$\alpha(13) = -1$	$S(13) = 13$	$\alpha(13) = 0$
14	$S(14) = 14$	$\alpha(14) = +1$	$S(14) = 14$	$\alpha(14) = 0$
15	$S(15) = 15$	$\alpha(15) = +1$	$S(15) = 15$	$\alpha(15) = +1$

$$\xi_{m,AR} \equiv \theta - \sum_{i=0}^{N-1} \mu(i)\, a(i) \qquad N \qquad AR$$

$$= \theta - \left[\sum_{j=0}^{N'-1} \tan^{-1}\left(\alpha(j) \cdot 2^{-S(j)}\right) \right] \qquad N' \qquad AQ$$

$$= \theta - \left[\sum_{j=0}^{N'-1} \tilde{\theta}(j) \right] \qquad N'$$

$$N' \triangleq \sum_{i=0}^{N-1} |\mu(i)|$$

the effective transition number

i	Angle Recode		
0	$S(0) = 0$	$\alpha(0) = +1$	0
1	$S(1) = 1$	$\alpha(1) = 0$	1
2	$S(2) = 2$	$\alpha(2) = 0$	2
3	$S(3) = 3$	$\alpha(3) = -1$	3
4	$S(4) = 4$	$\alpha(4) = 0$	4
5	$S(5) = 5$	$\alpha(5) = 0$	5
6	$S(6) = 6$	$\alpha(6) = -1$	6
7	$S(7) = 7$	$\alpha(7) = -1$	7
8	$S(8) = 8$	$\alpha(8) = 0$	8
9	$S(9) = 9$	$\alpha(9) = 0$	9
10	$S(10) = 10$	$\alpha(10) = 0$	10
11	$S(11) = 11$	$\alpha(11) = +1$	11
12	$S(12) = 12$	$\alpha(12) = 0$	12
13	$S(13) = 13$	$\alpha(13) = 0$	13
14	$S(14) = 14$	$\alpha(14) = 0$	14
$W-1 = 15$	$S(15) = 15$	$\alpha(15) = +1$	15

Angle Quantize	
$S(0) = 0$	$\alpha(0) = +1$
$S(1) = 3$	$\alpha(1) = -1$
$S(2) = 6$	$\alpha(2) = -1$
$S(3) = 7$	$\alpha(3) = -1$
$S(4) = 11$	$\alpha(4) = +1$
$S(5) = 15$	$\alpha(5) = +1$

removing skipped angle

$N'-1$

i	Conventional CORDIC	only $\{-1, +1\}$
0	$S(0) = 0$	$\alpha(0) = +1$
1	$S(1) = 1$	$\alpha(1) = -1$
2	$S(2) = 2$	$\alpha(2) = +1$
3	$S(3) = 3$	$\alpha(3) = +1$
4	$S(4) = 4$	$\alpha(4) = -1$
5	$S(5) = 5$	$\alpha(5) = +1$
6	$S(6) = 6$	$\alpha(6) = -1$
7	$S(7) = 7$	$\alpha(7) = -1$
8	$S(8) = 8$	$\alpha(8) = +1$
9	$S(9) = 9$	$\alpha(9) = -1$
10	$S(10) = 10$	$\alpha(10) = -1$
11	$S(11) = 11$	$\alpha(11) = +1$
12	$S(12) = 12$	$\alpha(12) = +1$
13	$S(13) = 13$	$\alpha(13) = -1$
14	$S(14) = 14$	$\alpha(14) = +1$
15	$S(15) = 15$	$\alpha(15) = +1$

W (range $i = 0$ to 15)

Angle Quant	skip allowed $\{-1, 0, +1\} = \{-1, +1\}$
$S(0) = 0$	$\alpha(0) = +1$
$S(1) = 3$	$\alpha(1) = -1$
$S(2) = 6$	$\alpha(2) = -1$
$S(3) = 7$	$\alpha(3) = -1$
$S(4) = 11$	$\alpha(4) = +1$
$S(5) = 15$	$\alpha(5) = +1$

N' (range 0 to 5)

```
#include <stdio.h>
#include <math.h>

#define N 16


int find_index(double a[], double atheta) {
  int i, ik = N;
  double minval = 1.0e+10;

  for (i=0; i<N; ++i) {
    if (minval > fabs(atheta - a[i])) {
      ik = i;
      minval = fabs(atheta - a[i]);
    }
  }

  return ik;
}

int main(void) {
  double a[N];
  double theta = 0.63761;
  int ik, uik;
  int k, i;
  int u[N];
  double angle;


  for (i=0; i<N; ++i)
    a[i] = atan(1./pow(2, i));

  for (i=0; i<N; ++i)
    u[i] = 0;
```

```
  k = 0;
  while ((fabs(theta) >= a[N-1]) && (k < N)) {
    ik = find_index(a, fabs(theta));

    uik = (theta >= 0) ? +1 : -1;

    printf("k=%2d theta=%10.7f ", k, theta);
    printf("ik=%2d uik=%+d ", ik, uik);

    theta = theta - uik * a[ik];

    printf("a[%2d]=%10.7f, new theta=%10.7f \n",
    ik, a[ik], theta);

    u[ik] = uik;

    k++;
  }


  angle = 0.0;
  for (i=0; i<N; ++i) {
    angle += u[i] * a[i];

    printf("i=%2d u[%2d]=%+d a[%2d]=%f angle=%f \n",
    i, i, u[i], i, a[i], angle);
  }





}
```

```
k= 0 theta= 0.6376100 ik= 0 uik=+1 a[ 0]= 0.7853982, new theta=-0.1477882
k= 1 theta=-0.1477882 ik= 3 uik=-1 a[ 3]= 0.1243550, new theta=-0.0234332
k= 2 theta=-0.0234332 ik= 5 uik=-1 a[ 5]= 0.0312398, new theta= 0.0078067
k= 3 theta= 0.0078067 ik= 7 uik=+1 a[ 7]= 0.0078123, new theta=-0.0000057
i= 0 u[ 0]=+1 a[ 0]=0.785398 angle=0.785398
i= 1 u[ 1]=+0 a[ 1]=0.463648 angle=0.785398
i= 2 u[ 2]=+0 a[ 2]=0.244979 angle=0.785398
i= 3 u[ 3]=-1 a[ 3]=0.124355 angle=0.661043
i= 4 u[ 4]=+0 a[ 4]=0.062419 angle=0.661043
i= 5 u[ 5]=-1 a[ 5]=0.031240 angle=0.629803
i= 6 u[ 6]=+0 a[ 6]=0.015624 angle=0.629803
i= 7 u[ 7]=+1 a[ 7]=0.007812 angle=0.637616
i= 8 u[ 8]=+0 a[ 8]=0.003906 angle=0.637616
i= 9 u[ 9]=+0 a[ 9]=0.001953 angle=0.637616
i=10 u[10]=+0 a[10]=0.000977 angle=0.637616
i=11 u[11]=+0 a[11]=0.000488 angle=0.637616
i=12 u[12]=+0 a[12]=0.000244 angle=0.637616
i=13 u[13]=+0 a[13]=0.000122 angle=0.637616
i=14 u[14]=+0 a[14]=0.000061 angle=0.637616
i=15 u[15]=+0 a[15]=0.000031 angle=0.637616
```