Measurement of Correlation Functions

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Based on
Probability, Random Variables and Random Signal Principles,
P.Z. Peebles,Jr. and B. Shi

Outline

in the real world, we can never measure the true correlation functions
of two random processes $X(t)$ and $Y(t)$

- each sample realization of a process is a time function
- collecting many independent sample functions costs a lot
- cannot have all sample functions of the ensemble
- can have only a portion of one sample function from each process

- can determine time averages based on finite time portion of single sample functions
- because we are able to work only with time functions, we are forced to presume that the given processe satisfy appropriate **ergodic theorems**

a possible system for measuring
the approximate time cross-correlation function
of two cross-correlation-ergodic and
stationary random processes $X(t)$ and $Y(t)$

- sample functions $x(t)$ and $y(t)$ are delayed by amounts T and $T - \tau$, respectively
- the product of the delayed waveforms
- integrate to form the output which equals the integral at time $t_1 + 2T_s$,
  where t_1 is arbitrary and $2T$ is the integration period

Definition

assume $x(t)$ and $y(t)$ exist at least during the interval $t \geq -T$ and that $t_1 \geq 0$ is an arbitrary time instant and that $\tau \leq T$ then the output is as follows

$$R_o(t_1 + 2T) = \frac{1}{2T} \int_{t_1 - T}^{t_{1+}T} x(t) y(t+\tau) dt$$

Definition

if we choose $t_1 = 0$ and assume T is large

$$R_o(t_1 + 2T) = \frac{1}{2T} \int_{t_1 - T}^{t_{1+} T} x(t)y(t+\tau)dt$$

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$$R_o(2T) = \frac{1}{2T} \int_{-T}^{+T} x(t)y(t+\tau)dt = R_{XY}(\tau)$$

Definition

for cross-correlation ergodic processes,
approximately measuring cross-correlation
τ is varied to obtain the complete function

$$R_o(2T) = \frac{1}{2T} \int_{-T}^{+T} x(t) y(t+\tau) dt = R_{XY}(\tau)$$

Definition

measuring auto-correlation functions

$$R_o(2T) = \frac{1}{2T} \int_{-T}^{+T} x(t)y(t+\tau)dt = R_{XY}(\tau)$$

$$R_o(2T) = \frac{1}{2T} \int_{-T}^{+T} x(t)x(t+\tau)dt = R_{XX}(\tau)$$

$$R_o(2T) = \frac{1}{2T} \int_{-T}^{+T} y(t)y(t+\tau)dt = R_{YY}(\tau)$$