

Digital Signal Octave Codes (0A)

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Based on

M.J. Roberts, Fundamentals of Signals and Systems

S.K. Mitra, Digital Signal Processing : a computer-based approach 2nd ed

S.D. Stearns, Digital Signal Processing with Examples in MATLAB

A Cosine Waveform

```
n= [0:29];  
x= cos(2*pi*(n/10));
```

$$nT_s = n \cdot \frac{1}{10}$$

```
x= cos((2/10)*pi*n);
```

$$nT_s = n \cdot 1$$

$$\omega_0 nT_s = 2\pi f_0 nT_s = \frac{2\pi}{T_0} nT_s = 2\pi n \frac{T_s}{T_0}$$

$$\omega_0 t = 2\pi f t$$

$$\omega_0 nT_s = 2\pi f_0 nT_s = 2\pi \cdot 1 \cdot n \cdot \frac{1}{10}$$

$$\omega_0 nT_s = 2\pi f_0 nT_s = 2\pi \cdot \frac{1}{10} \cdot n \cdot 1$$

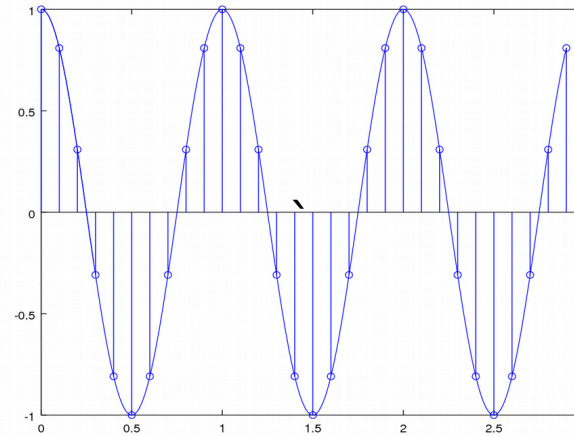
$$f_0 = 1 \quad T_0 = 1 \quad T_s = 0.1$$

$$f_0 = 0.1 \quad T_0 = 10 \quad T_s = 1$$

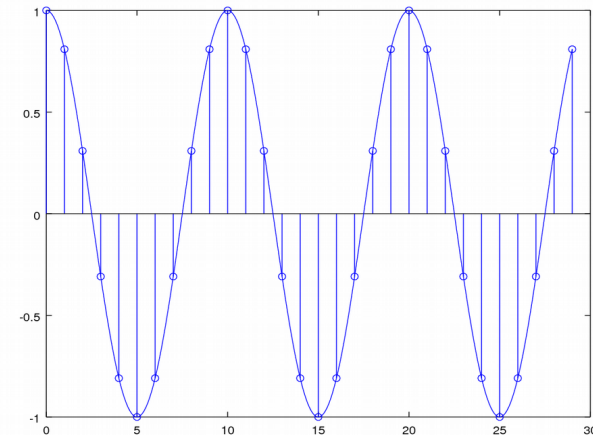
Many waveforms share the same sampled data

x

1.00000
0.80902
0.30902
-0.30902
-0.80902
-1.00000
-0.80902
-0.30902
0.30902
0.80902
1.00000
0.80902
0.30902
-0.30902
-0.80902
-1.00000
-0.80902
-0.30902
0.30902
0.80902
1.00000
0.80902
0.30902
-0.30902
-0.80902
-1.00000
-0.80902
-0.30902
0.30902
0.80902



3



30

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Cosine Wave 1

```
n = [0:29];  
x = cos(2*pi*n/10);
```

```
t = [0:29]/10;  
y = cos(2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:290]/100;  
y2 = cos(2*pi*t2);  
plot(t2, y2)
```

```
t = [0:29];  
y = cos(0.2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:290]/10;  
y2 = cos(0.2*pi*t2);  
plot(t2, y2)
```

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \quad f_0 = 1 \quad T_0 = 1 \quad T_s = 0.1$$

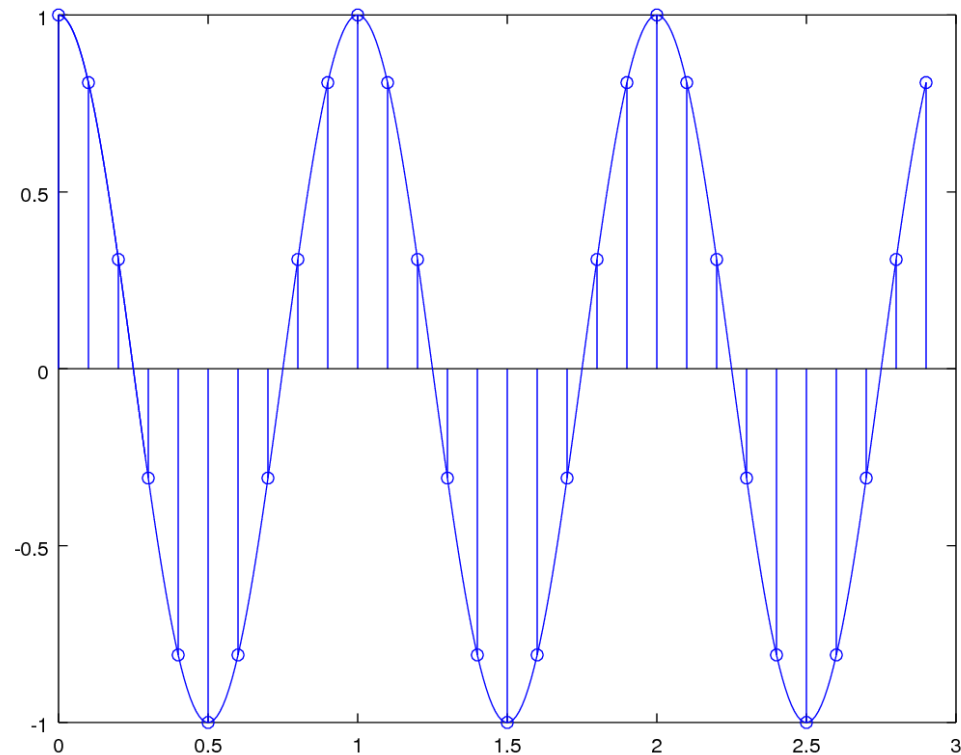
$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \quad f_0 = 0.1 \quad T_0 = 10 \quad T_s = 1$$

Cosine Wave 1

```
n = [0:29];  
x = cos(2*pi*n/10);
```

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \quad f_0 = 1 \quad T_0 = 1 \quad T_s = 0.1$$

```
t = [0:29]/10;  
y = cos(2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:290]/100;  
y2 = cos(2*pi*t2);  
plot(t2, y2)
```



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Cosine Wave 2

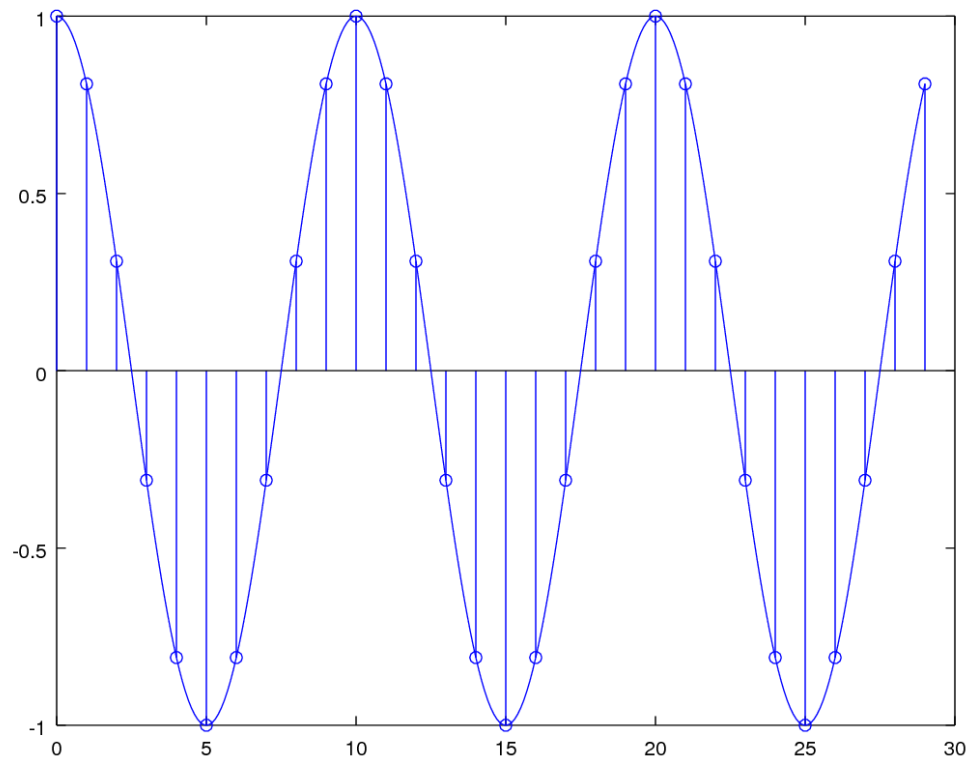
```
n = [0:29];  
x = cos(2*pi*n/10);
```

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \quad f_0 = 0.1 \quad T_0 = 10 \quad T_s = 1$$

```
t = [0:29];  
y = cos(0.2*pi*t);  
stem(t, y)  
hold on  
t2 = [0:290]/10;  
y2 = cos(0.2*pi*t2);  
plot(t2, y2)
```

$$\omega_0 n T_s = 2\pi f_0 n T_s = 2\pi \cdot \frac{1}{10} \cdot n \cdot 1$$

$$f_0 = 0.1 \quad T_0 = 10 \quad T_s = 1$$



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Sampled Sinusoids

$$g[n] = A e^{\beta n}$$

$$g[n] = A z^n \quad z = e^{\beta}$$

$$g[n] = A \cos(2\pi n/N_0 + \theta)$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$g[n] = A \cos(\Omega_0 n + \theta)$$

$$1/N_0$$

$$= F_0$$

$$= \Omega_0/2\pi$$

$$2\pi/N_0$$

$$= 2\pi F_0$$

$$= \Omega_0$$

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Sampling Period and Frequency

$$g(t) = A \cos(2\pi f_0 t + \theta)$$

$$g[n] = g(nT_s)$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$g(nT_s) = A \cos(2\pi f_0 T_s n + \theta)$$

$$F_0 = f_0 T_s = f_0 / f_s$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$T_s = \frac{1}{f_s}$$

sampling period

$$\frac{1}{T_s} = f_s$$

sampling frequency
sampling rate

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Periodic Condition of a Sampled Signal

$$2\pi F_0 n = 2\pi m$$

$$F_0 n = m \quad \text{Integers } n, m$$

$$F_0 = \frac{m}{n}$$

$$F_0 = \frac{m}{n} = \frac{f_0}{f_s} \quad \begin{array}{l} \text{Fundamental Frequency} \\ \text{Sampling Frequency} \end{array}$$

$$\text{Rational Number } F_0 = \frac{m}{n}$$

$$g(nT_s) = A \cos(2\pi f_0 T_s n + \theta)$$

$$F_0 = f_0 T_s = f_0 / f_s$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

Periodic Condition Examples

$$g(t) = A \cos(2\pi f_0 t + \theta)$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$g[t] = 4 \cos\left(\frac{72\pi t}{19}\right) = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot t\right)$$

$$g[n] = 4 \cos\left(\frac{72\pi n}{19}\right) = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot n\right) \quad T_s = 1$$

$$g(t) = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (t + T_0)\right)$$

$$T_0 = \frac{19}{36} \quad \text{Fundamental Period of } g(t)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (n + N_0)\right)$$

$$N_0 = 19 \quad \text{Fundamental Period of } g[n]$$

$$N_0 \neq \frac{1}{F_0} \quad \frac{N_0}{q} = \frac{1}{F_0} \quad \frac{q}{N_0} = F_0$$

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Periodic Condition Examples

$$g(t) = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (t + T_0)\right) \quad T_0 = \frac{19}{36} \quad \text{Fundamental Period of } g(t)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (n + N_0)\right) \quad N_0 = 19 \quad \text{Fundamental Period of } g[n]$$

$$F_0 = \frac{36}{19} = \frac{q}{N_0}$$

← the number of cycles in N_0 samples
← the smallest integer : fundamental period

$$N_0 \neq \frac{1}{F_0} \quad \frac{N_0}{q} = \frac{1}{F_0} \quad \frac{q}{N_0} = F_0$$

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Periodic Condition Examples

$$F_0 = \frac{36}{19} = \frac{q}{N_0}$$

← the number of cycles in N_0 samples
← the smallest integer : fundamental period

“When F_0 is not the reciprocal of an integer ($q=1$), a discrete-time sinusoid may not be immediately recognizable from its graph as a sinusoid.”

Periodic Condition Examples

$$g[n] = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (n + N_0)\right)$$

$$\frac{36}{19} \cdot (n + N_0)$$

integer

$$\frac{1}{19} \cdot N_0 = k$$

integer

$$N_0$$

integer

$N_0 = 19$ Fundamental period of $g[n]$

$$g(t) = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot (t + T_0)\right)$$

$$\frac{36}{19} \cdot (t + T_0)$$

integer

$$\frac{36}{19} \cdot T_0 = k$$

integer

$$T_0$$

~~integer~~

$T_0 = \frac{19}{36}$ Fundamental period of $g(t)$

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Periodic Condition Examples

$$g[n] = 4 \cos\left(2\pi \cdot \frac{1}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{2}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{3}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot n\right)$$

```
clf
n = [0:36]; t = [0:3600]/100;
y1 = 4*cos(2*pi*(1/19)*n);
y2 = 4*cos(2*pi*(2/19)*n);
y3 = 4*cos(2*pi*(3/19)*n);
y4 = 4*cos(2*pi*(36/19)*n);
yt1 = 4*cos(2*pi*(1/19)*t);
yt2 = 4*cos(2*pi*(2/19)*t);
yt3 = 4*cos(2*pi*(3/19)*t);
yt4 = 4*cos(2*pi*(36/19)*t);
```

```
subplot(4,1,1);
stem(n, y1); hold on;
plot(t, yt1);
subplot(4,1,2);
stem(n, y2); hold on;
plot(t, yt2);
subplot(4,1,3);
stem(n, y3); hold on;
plot(t, yt3);
subplot(4,1,4);
stem(n, y4); hold on;
plot(t, yt4);
```

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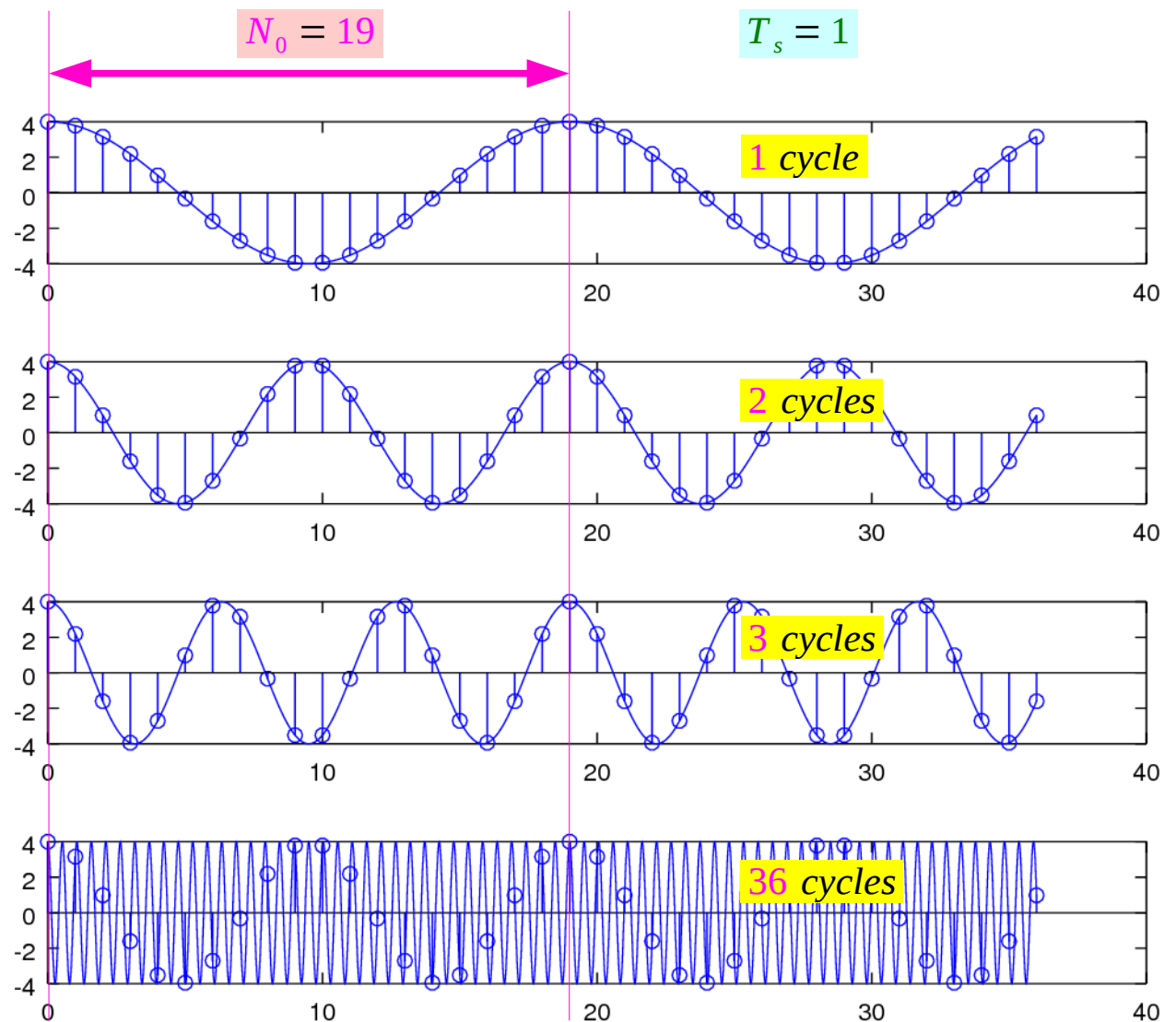
Periodic Condition Examples

$$g[n] = 4 \cos\left(2\pi \cdot \frac{1}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{2}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{3}{19} \cdot n\right)$$

$$g[n] = 4 \cos\left(2\pi \cdot \frac{36}{19} \cdot n\right)$$



Periodic Condition Examples

$$g(t) = A \cos(2\pi f_0 t + \theta)$$

$$g_1(t) = 4 \cos(2\pi \cdot 1 \cdot t)$$

$$g_2(t) = 4 \cos(2\pi \cdot 2 \cdot t)$$

$$g_3(t) = 4 \cos(2\pi \cdot 3 \cdot t)$$

$$t \leftarrow nT_1$$

$$t \leftarrow nT_2$$

$$t \leftarrow nT_3$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$g_1[n] = 4 \cos(2\pi n T_{s1})$$

$$g_2[n] = 4 \cos(2\pi n T_{s2})$$

$$g_3[n] = 4 \cos(2\pi n T_{s3})$$

$$t \leftarrow nT_1$$

$$T_1 = \frac{1}{10}$$

$$n = 0, 1, 2, 3, \dots \rightarrow 1 \cdot t = 0, 0.1, 0.2, 0.3, \dots$$

$$t \leftarrow nT_2$$

$$T_2 = \frac{1}{20}$$

$$n = 0, 1, 2, 3, \dots \rightarrow 2 \cdot t = 0, 0.1, 0.2, 0.3, \dots$$

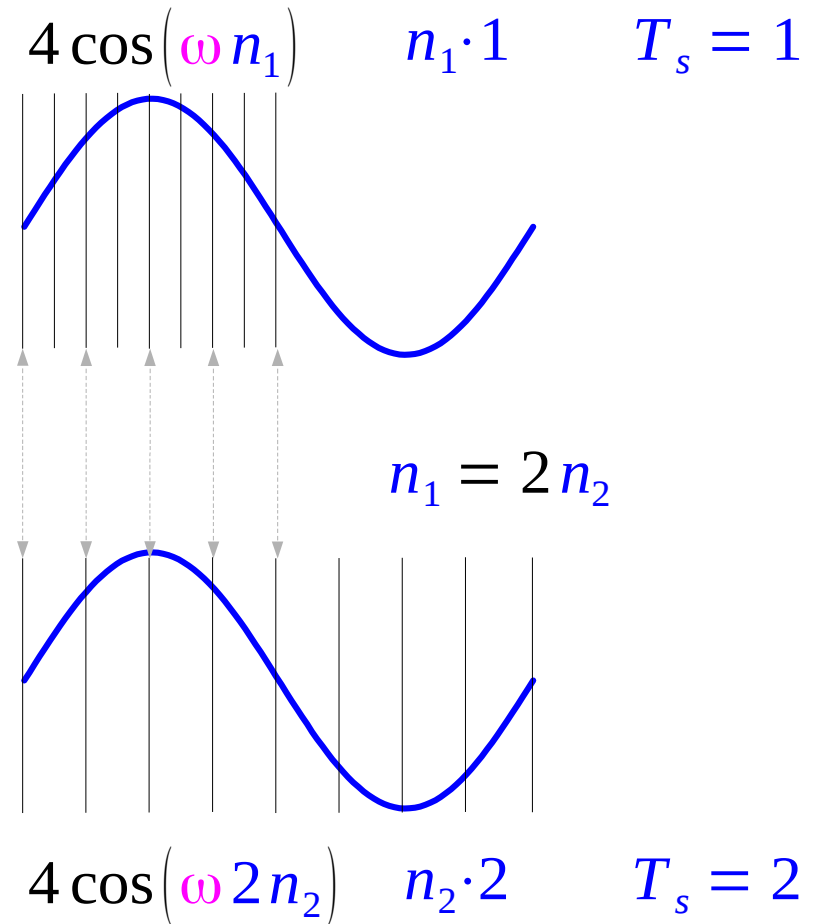
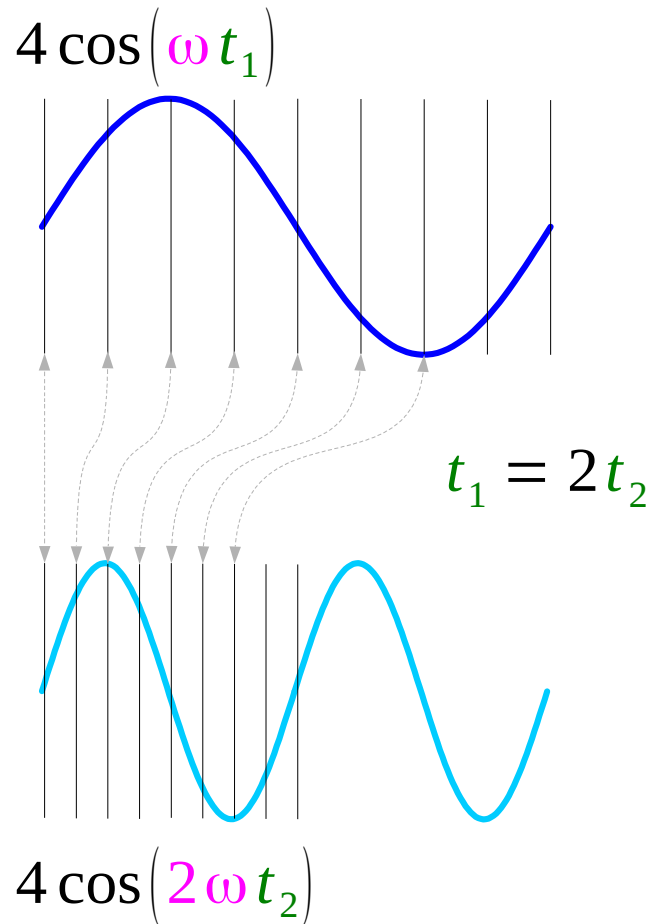
$$t \leftarrow nT_3$$

$$T_3 = \frac{1}{30}$$

$$n = 0, 1, 2, 3, \dots \rightarrow 3 \cdot t = 0, 0.1, 0.2, 0.3, \dots$$

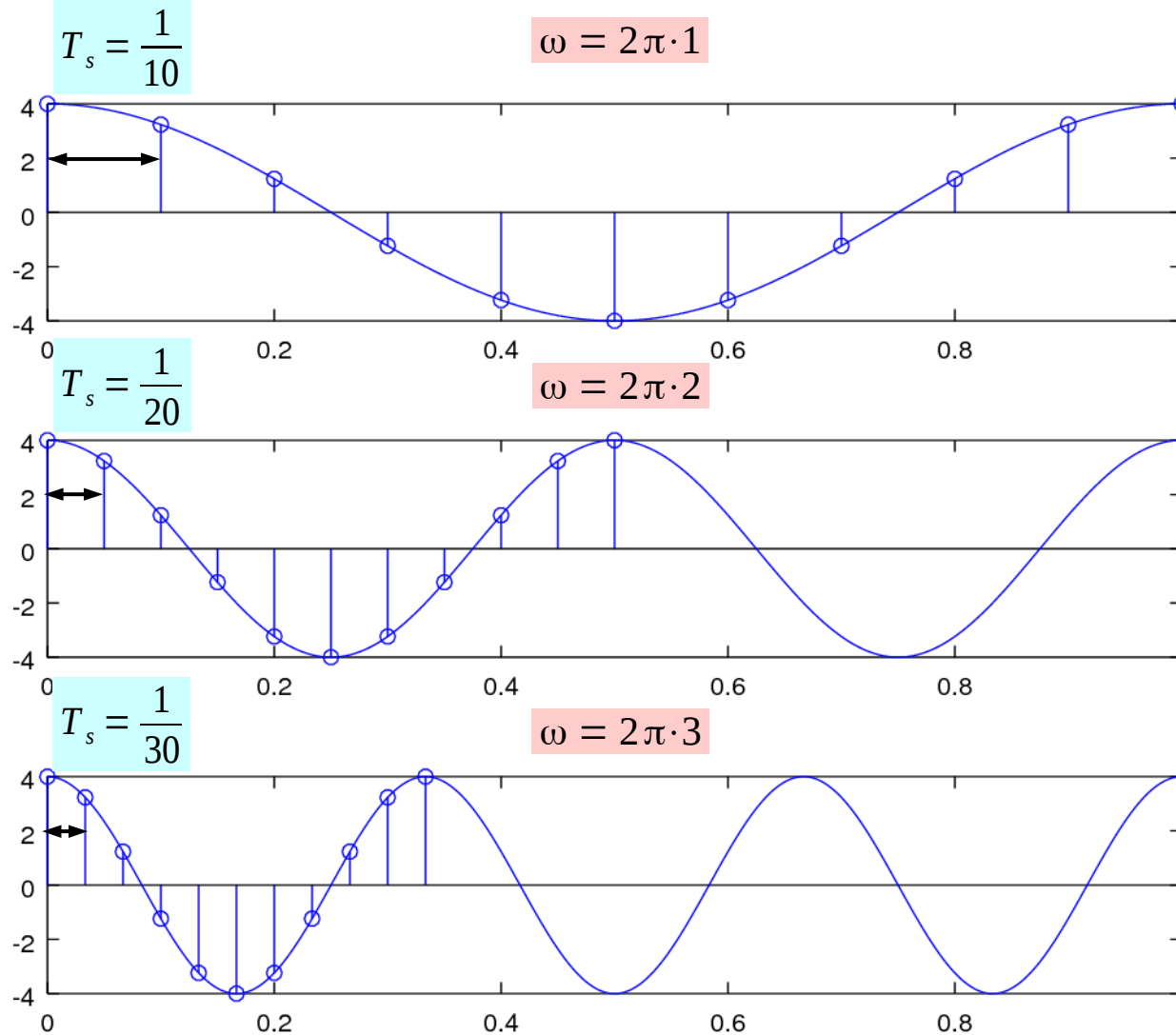
$$\{g_1[n]\} \equiv \{g_2[n]\} \equiv \{g_3[n]\}$$

Periodic Condition Examples



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Periodic Condition Examples

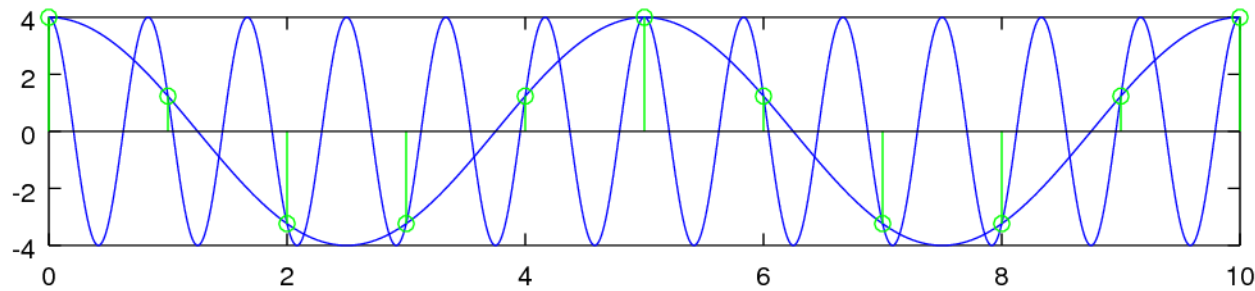
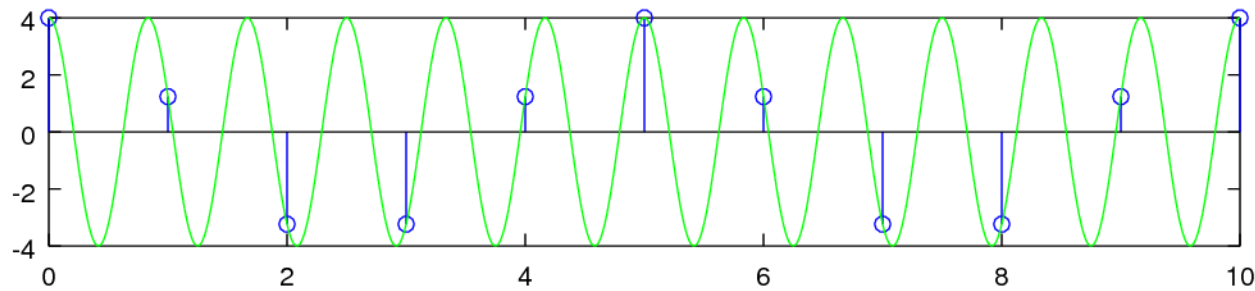
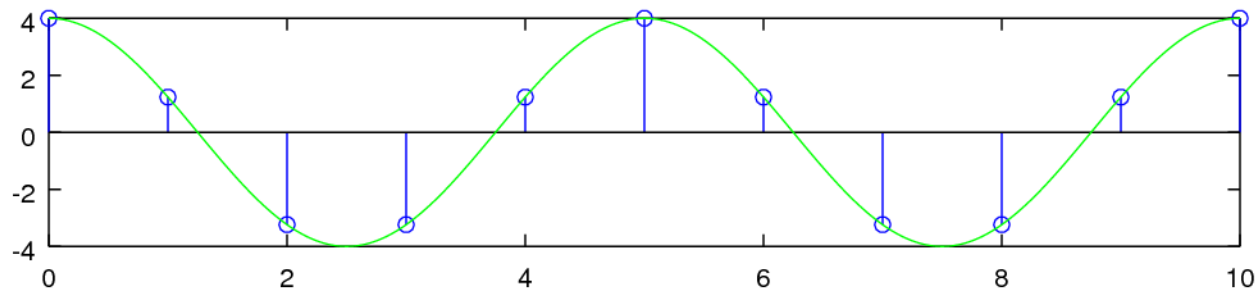


```
clf  
n = [0:10]; t = [0:1000]/1000;  
y1 = 4*cos(2*pi*1*n/10);  
y2 = 4*cos(2*pi*2*n/20);  
y3 = 4*cos(2*pi*3*n/30);  
yt1 = 4*cos(2*pi*t);  
yt2 = 4*cos(2*pi*2*t);  
yt3 = 4*cos(2*pi*3*t);
```

```
subplot(3,1,1);  
stem(n, y1); hold on;  
plot(t, yt1);  
subplot(3,1,2);  
stem(n/20, y2); hold on;  
plot(t, yt2);  
subplot(3,1,3);  
stem(n/30, y3); hold on;  
plot(t, yt3);
```

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Aliasing Condition Examples

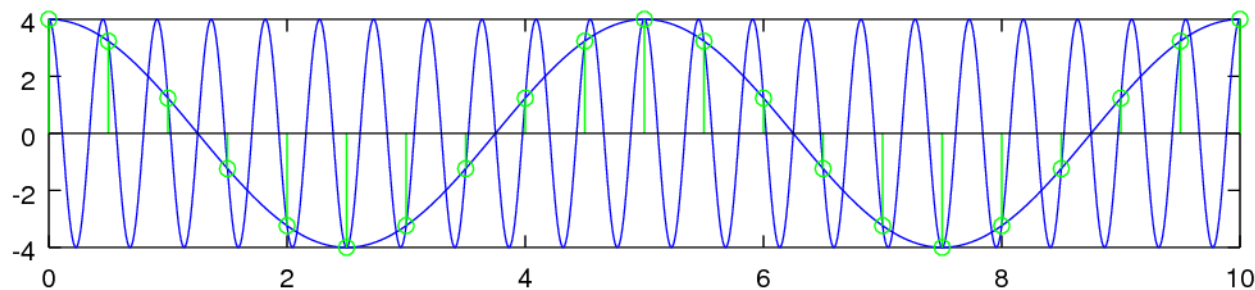
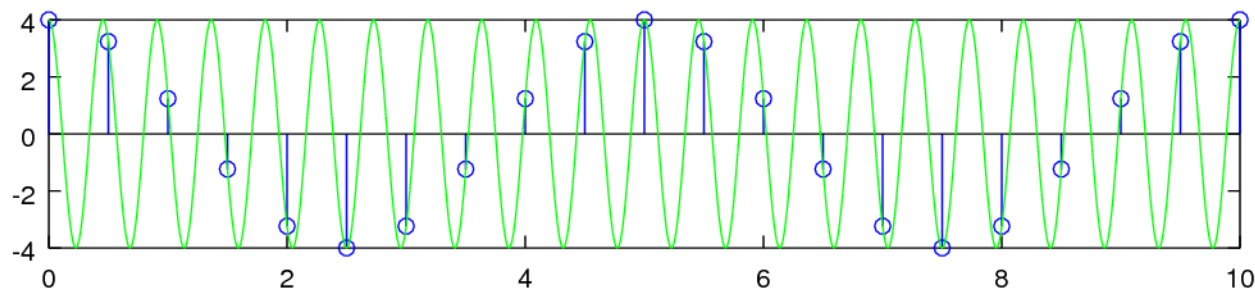
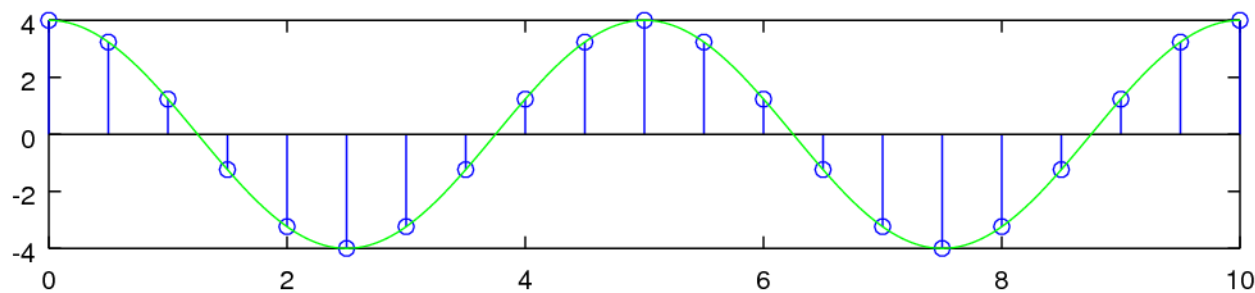


```
clf
n = [0:1:10];
t = [0:1000]/100;
y1 = 4*cos(2*pi*(1/5)*n);
y2 = 4*cos(2*pi*(6/5)*n);
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(6/5)*t);
```

```
subplot(3,1,1);
stem(n, y1); hold on;
plot(t, yt1, 'g');
subplot(3,1,2);
stem(n, y2); hold on;
plot(t, yt2, 'g');
subplot(3,1,3);
plot(t, yt1); hold on;
plot(t, yt2);
stem(n, y1, 'g');
```

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Aliasing Condition Examples



```
clf
n = [0:0.5:10];
t = [0:1000]/100;
y1 = 4*cos(2*pi*(1/5)*n);
y2 = 4*cos(2*pi*(11/5)*n);
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(11/5)*t);
```

```
subplot(3,1,1);
stem(n, y1); hold on;
plot(t, yt1, 'g');
subplot(3,1,2);
stem(n, y2); hold on;
plot(t, yt2, 'g');
subplot(3,1,3);
plot(t, yt1); hold on;
plot(t, yt2);
stem(n, y1, 'g');
```

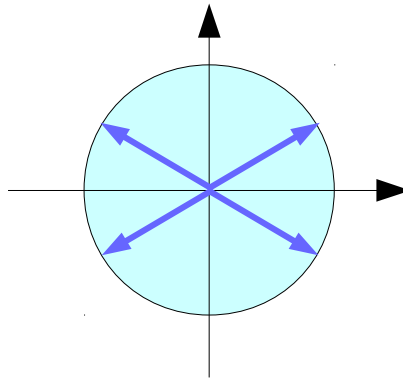
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Aliasing Condition Examples

$$\cos(\omega_1 t) = \cos(\omega_2 t)$$

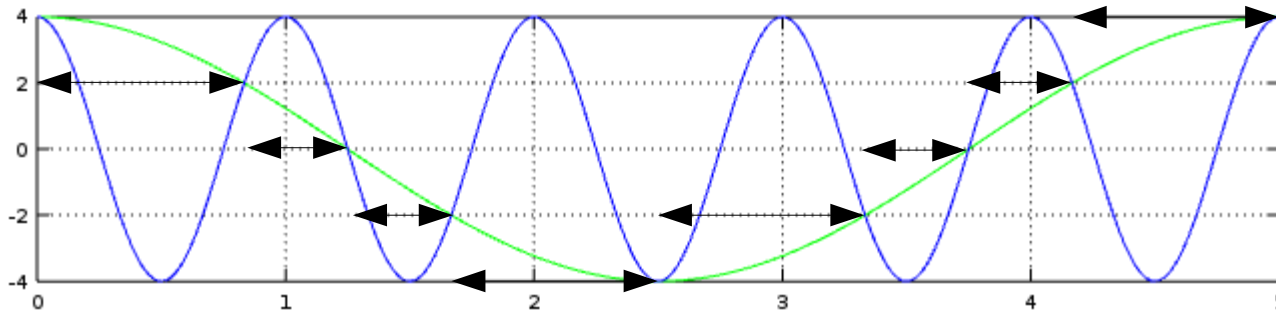
$$\omega_1 t - \omega_2 t = 2n\pi$$

$$\omega_1 t + \omega_2 t = 2n\pi$$



$$\begin{cases} \frac{5}{5}t + \frac{1}{5}t = n \\ \frac{5}{5}t - \frac{1}{5}t = n \end{cases}$$

$$\begin{cases} \frac{6}{5}t = n \\ \frac{4}{5}t = n \end{cases}$$



$$\frac{5}{6}, \frac{10}{6}, \frac{15}{6}, \dots$$



$$\frac{5}{4}, \frac{10}{4}, \frac{15}{4}, \dots$$

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Aliasing Condition Examples

$\frac{2}{5}t + \frac{1}{5}t = n$	$\frac{2}{5}t - \frac{1}{5}t = n$	$\frac{3}{5}t = n$	$\frac{1}{5}t = n$	$T_s = \frac{5}{3}$	$T_s = \frac{5}{1}$
$\frac{3}{5}t + \frac{1}{5}t = n$	$\frac{3}{5}t - \frac{1}{5}t = n$	$\frac{4}{5}t = n$	$\frac{2}{5}t = n$	$T_s = \frac{5}{4}$	$T_s = \frac{5}{2}$
$\frac{4}{5}t + \frac{1}{5}t = n$	$\frac{4}{5}t - \frac{1}{5}t = n$	$\frac{5}{5}t = n$	$\frac{3}{5}t = n$	$T_s = \frac{5}{5}$	$T_s = \frac{5}{3}$
$\frac{5}{5}t + \frac{1}{5}t = n$	$\frac{5}{5}t - \frac{1}{5}t = n$	$\frac{6}{5}t = n$	$\frac{4}{5}t = n$	$T_s = \frac{5}{6}$	$T_s = \frac{5}{4}$
$\frac{6}{5}t + \frac{1}{5}t = n$	$\frac{6}{5}t - \frac{1}{5}t = n$	$\frac{7}{5}t = n$	$\frac{5}{5}t = n$	$T_s = \frac{5}{7}$	$T_s = \frac{5}{5}$
$\frac{7}{5}t + \frac{1}{5}t = n$	$\frac{7}{5}t - \frac{1}{5}t = n$	$\frac{8}{5}t = n$	$\frac{6}{5}t = n$	$T_s = \frac{5}{8}$	$T_s = \frac{5}{6}$
$\frac{8}{5}t + \frac{1}{5}t = n$	$\frac{8}{5}t - \frac{1}{5}t = n$	$\frac{9}{5}t = n$	$\frac{7}{5}t = n$	$T_s = \frac{5}{9}$	$T_s = \frac{5}{7}$

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Aliasing Condition Examples

```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(2/5)*t);
yt3 = 4*cos(2*pi*(3/5)*t);
yt4 = 4*cos(2*pi*(4/5)*t);
yt5 = 4*cos(2*pi*(5/5)*t);
yt6 = 4*cos(2*pi*(6/5)*t);
yt7 = 4*cos(2*pi*(7/5)*t);
yt8 = 4*cos(2*pi*(8/5)*t);
```

```
n1 = 0: 5/2 : 5;
n2 = 0: 5/3 : 5;
n3 = 0: 5/4 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/6 : 5;
n6 = 0: 5/7 : 5;
n7 = 0: 5/8 : 5;
n8 = 0: 5/9 : 5;
```

```
y2 = 4*cos(2*pi*(2/5)*n2);
y3 = 4*cos(2*pi*(3/5)*n3);
y4 = 4*cos(2*pi*(4/5)*n4);
y5 = 4*cos(2*pi*(5/5)*n5);
y6 = 4*cos(2*pi*(6/5)*n6);
y7 = 4*cos(2*pi*(7/5)*n7);
y8 = 4*cos(2*pi*(8/5)*n8);
```

```
subplot(4,2,1);
plot(t, yt1, 'g'); hold on
```

```
subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');
```

```
subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n3, y3, 'r');
```

```
subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n4, y4, 'r');
```

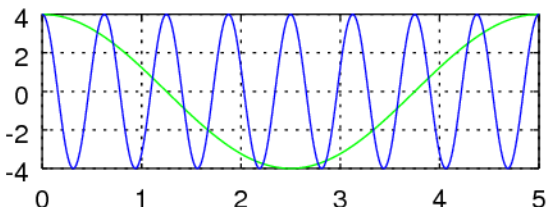
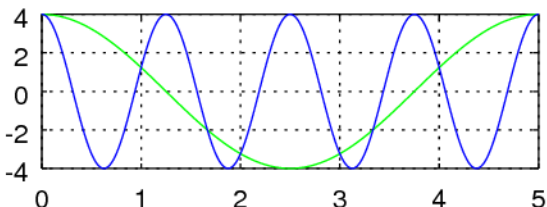
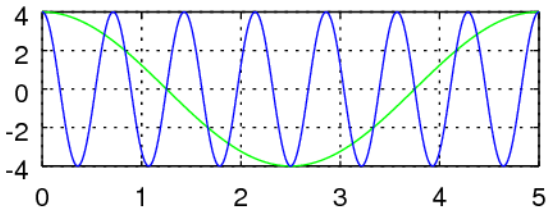
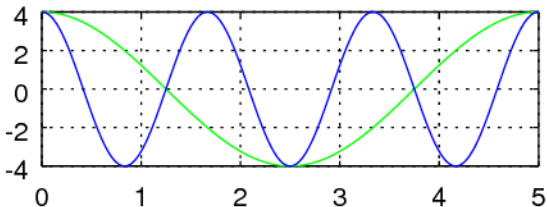
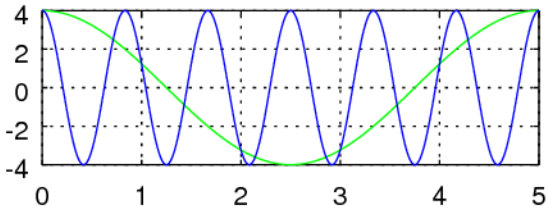
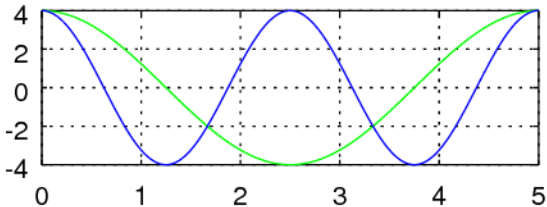
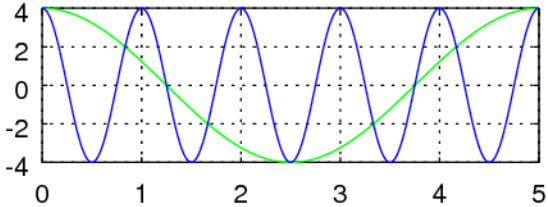
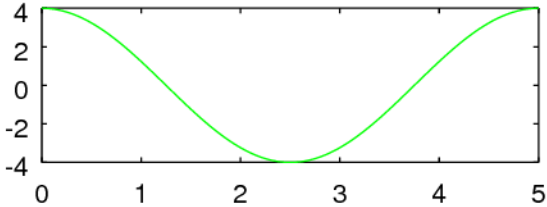
```
subplot(4,2,2);
plot(t, yt1, 'g'); hold on
plot(t, yt5, 'b'); grid on
stem(n5, y5, 'r');
```

```
subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n6, y6, 'r');
```

```
subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n7, y7, 'r');
```

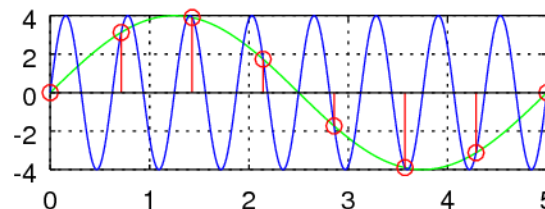
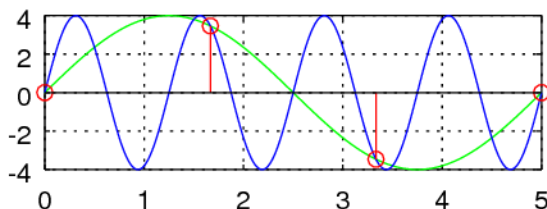
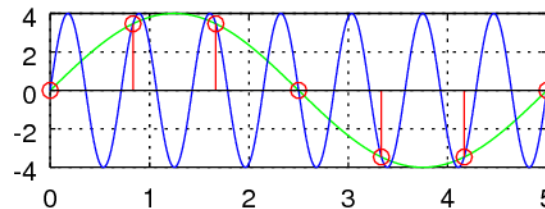
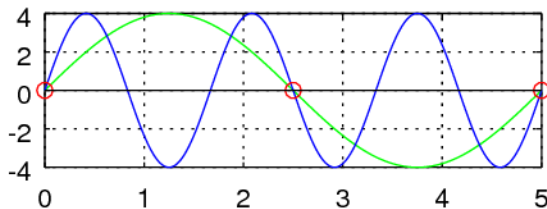
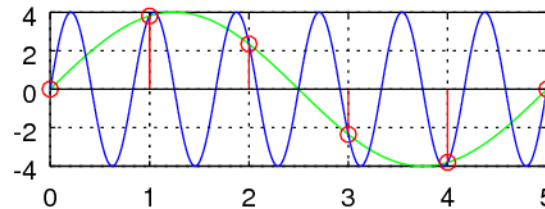
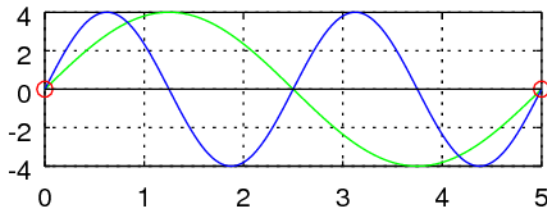
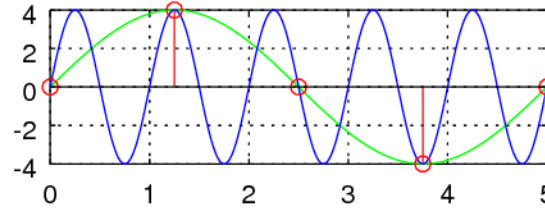
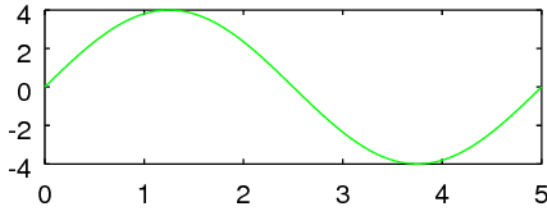
```
subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n8, y8, 'r');
```

Aliasing Condition Examples



```
[0:500]/100;
= 4*cos(2*pi*(1/5)*t);
= 4*cos(2*pi*(2/5)*t);
= 4*cos(2*pi*(3/5)*t);
= 4*cos(2*pi*(4/5)*t);
= 4*cos(2*pi*(5/5)*t);
= 4*cos(2*pi*(6/5)*t);
= 4*cos(2*pi*(7/5)*t);
= 4*cos(2*pi*(8/5)*t);
```

Periodic Condition Examples

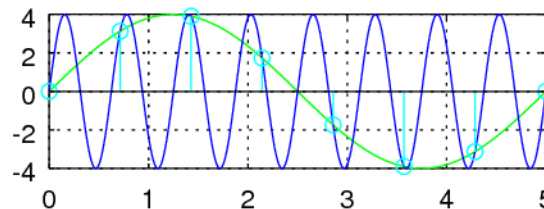
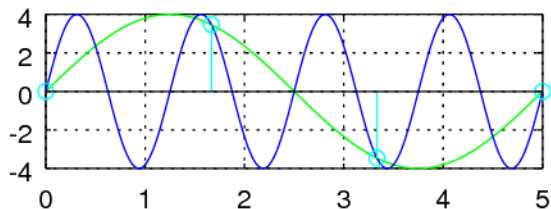
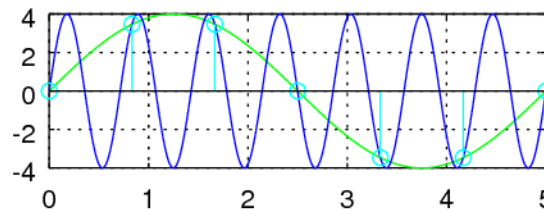
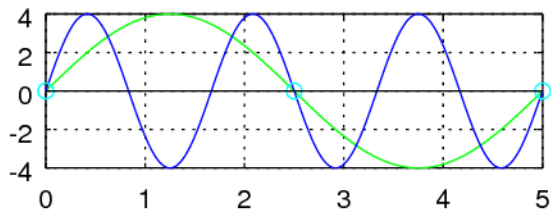
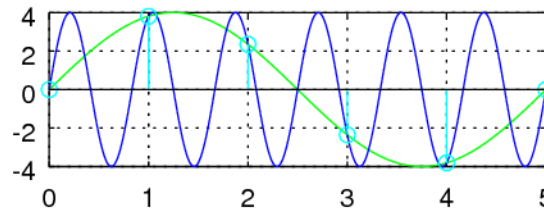
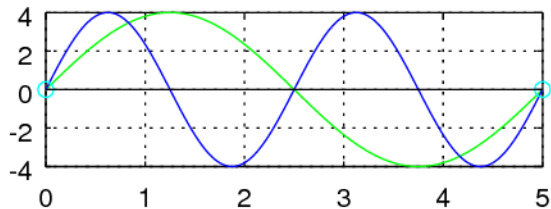
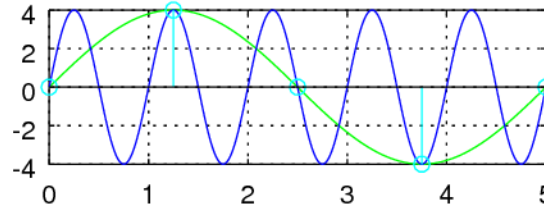
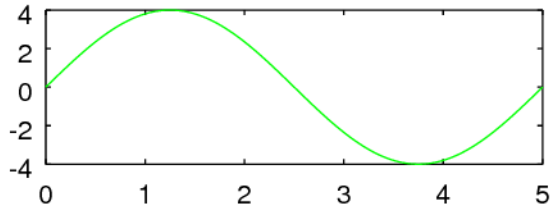


$n1 = 0: 5/2 : 5;$
 $n2 = 0: 5/3 : 5;$
 $n3 = 0: 5/4 : 5;$
 $n4 = 0: 5/5 : 5;$
 $n5 = 0: 5/6 : 5;$
 $n6 = 0: 5/7 : 5;$
 $n7 = 0: 5/8 : 5;$
 $n8 = 0: 5/9 : 5;$

$y2 = 4*\cos(2*\pi*(2/5)*n2);$
 $y3 = 4*\cos(2*\pi*(3/5)*n3);$
 $y4 = 4*\cos(2*\pi*(4/5)*n4);$
 $y5 = 4*\cos(2*\pi*(5/5)*n5);$
 $y6 = 4*\cos(2*\pi*(6/5)*n6);$
 $y7 = 4*\cos(2*\pi*(7/5)*n7);$
 $y8 = 4*\cos(2*\pi*(8/5)*n8);$

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Periodic Condition Examples



$n_1 = 0: 5/2 : 5;$
 $n_2 = 0: 5/1 : 5;$
 $n_3 = 0: 5/2 : 5;$
 $n_4 = 0: 5/3 : 5;$
 $n_5 = 0: 5/4 : 5;$
 $n_6 = 0: 5/5 : 5;$
 $n_7 = 0: 5/6 : 5;$
 $n_8 = 0: 5/7 : 5;$

$y_2 = 4*\cos(2*\pi*(2/5)*n_2);$
 $y_3 = 4*\cos(2*\pi*(3/5)*n_3);$
 $y_4 = 4*\cos(2*\pi*(4/5)*n_4);$
 $y_5 = 4*\cos(2*\pi*(5/5)*n_5);$
 $y_6 = 4*\cos(2*\pi*(6/5)*n_6);$
 $y_7 = 4*\cos(2*\pi*(7/5)*n_7);$
 $y_8 = 4*\cos(2*\pi*(8/5)*n_8);$

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Periodic Condition Examples

$$g(t) = A \cos(2\pi f_0 t + \theta)$$

$$g[n] = A \cos(2\pi F_0 n + \theta)$$

$$\begin{aligned} g[n] &= 4 \cos\left(\frac{72\pi n}{19}\right) \\ &= 4 \cos\left(2\pi \left(\frac{36}{19}\right) n\right) \\ &= 4 \cos\left(2\pi \left(\frac{36}{19} \cdot (n + N_0)\right)\right) \\ &\quad \text{smallest } N_0 = 19 \end{aligned}$$

$$2\pi F_0 n = 2\pi m$$

$$\frac{36}{19} n = m$$

smallest $n = 19$

$$\frac{36}{19} = \frac{m}{n}$$

$$\frac{36}{19} = \frac{m}{n} = \frac{f_0}{f_s}$$

$$F_0 = \frac{q}{N_0}$$

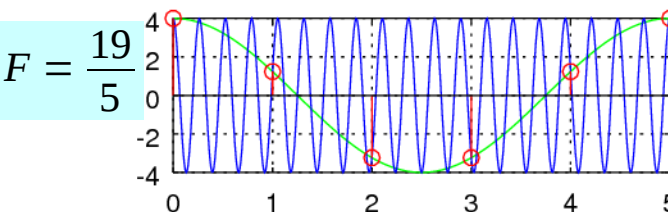
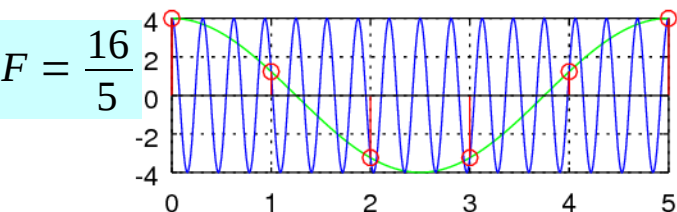
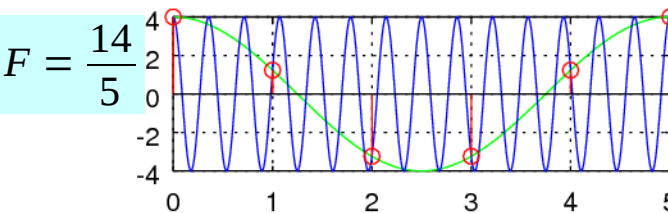
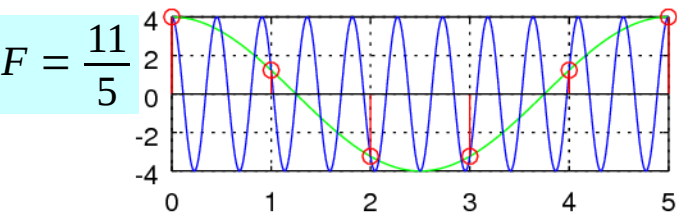
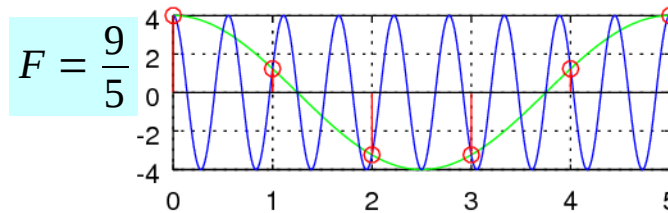
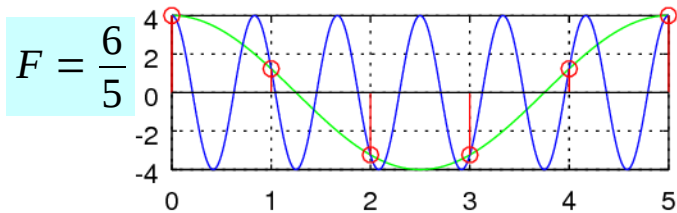
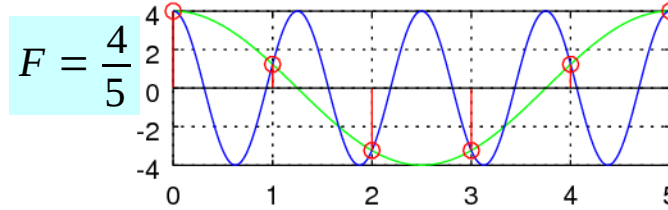
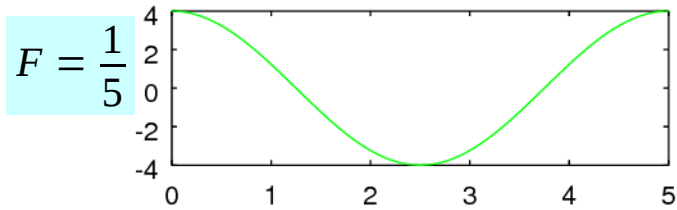
$$1/N_0$$

$$= F_0$$

$$= \Omega_0/2\pi$$

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Numerical Computation of DTFT



Numerical Computation of DTFT

```
clf
t = [0:500]/100;
yt1 = 4*cos(2*pi*(1/5)*t);
yt2 = 4*cos(2*pi*(6/5)*t);
yt3 = 4*cos(2*pi*(11/5)*t);
yt4 = 4*cos(2*pi*(16/5)*t);
yt5 = 4*cos(2*pi*(4/5)*t);
yt6 = 4*cos(2*pi*(9/5)*t);
yt7 = 4*cos(2*pi*(14/5)*t);
yt8 = 4*cos(2*pi*(19/5)*t);

n1 = 0: 5/5 : 5;
n2 = 0: 5/5 : 5;
n3 = 0: 5/5 : 5;
n4 = 0: 5/5 : 5;
n5 = 0: 5/5 : 5;
n6 = 0: 5/5 : 5;
n7 = 0: 5/5 : 5;
n8 = 0: 5/5 : 5;

y2 = 4*cos(2*pi*(6/5)*n2);
y3 = 4*cos(2*pi*(11/5)*n2);
y4 = 4*cos(2*pi*(16/5)*n2);
y5 = 4*cos(2*pi*(4/5)*n5);
y6 = 4*cos(2*pi*(9/5)*n5);
y7 = 4*cos(2*pi*(14/5)*n5);
y8 = 4*cos(2*pi*(19/5)*n5);

subplot(4,2,1);
plot(t, yt1, 'g'); hold on

subplot(4,2,2);
plot(t, yt1, 'g'); hold on
stem(n5, y5, 'r');

subplot(4,2,3);
plot(t, yt1, 'g'); hold on
plot(t, yt2, 'b'); grid on
stem(n2, y2, 'r');

subplot(4,2,4);
plot(t, yt1, 'g'); hold on
plot(t, yt6, 'b'); grid on
stem(n5, y6, 'r');

subplot(4,2,5);
plot(t, yt1, 'g'); hold on
plot(t, yt3, 'b'); grid on
stem(n2, y3, 'r');

subplot(4,2,6);
plot(t, yt1, 'g'); hold on
plot(t, yt7, 'b'); grid on
stem(n5, y7, 'r');

subplot(4,2,7);
plot(t, yt1, 'g'); hold on
plot(t, yt4, 'b'); grid on
stem(n2, y4, 'r');

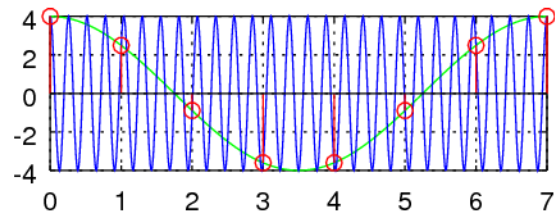
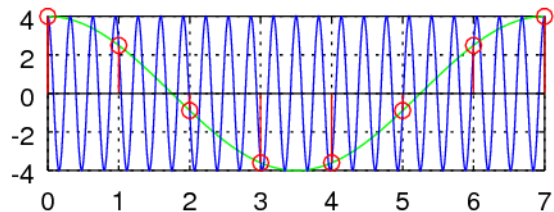
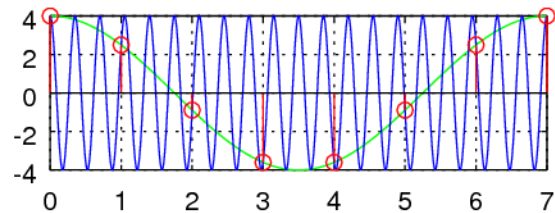
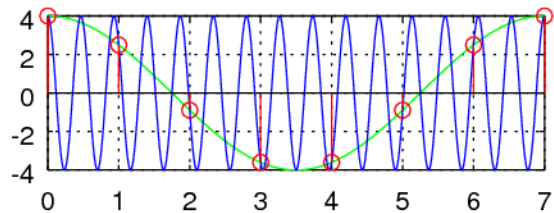
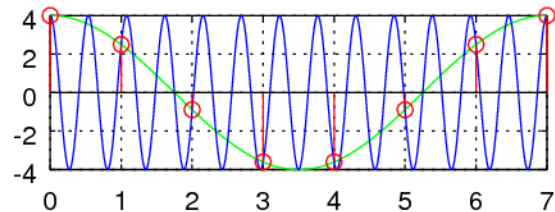
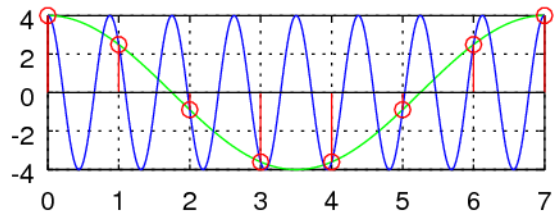
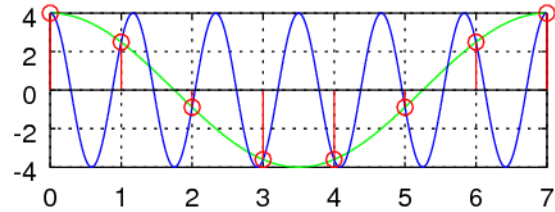
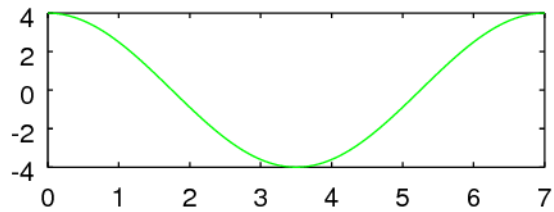
subplot(4,2,8);
plot(t, yt1, 'g'); hold on
plot(t, yt8, 'b'); grid on
stem(n5, y8, 'r');
```

Aliasing Condition Examples

$\frac{6}{5}t + \frac{1}{5}t = n$	$\frac{6}{5}t - \frac{1}{5}t = n$	$\frac{7}{5}t = n$	$\frac{5}{5}t = n$	$T_s = \frac{5}{7}$	$T_s = \frac{5}{1}$
$\frac{11}{5}t + \frac{1}{5}t = n$	$\frac{11}{5}t - \frac{1}{5}t = n$	$\frac{12}{5}t = n$	$\frac{10}{5}t = n$	$T_s = \frac{5}{12}$	$T_s = \frac{5}{2}$
$\frac{16}{5}t + \frac{1}{5}t = n$	$\frac{16}{5}t - \frac{1}{5}t = n$	$\frac{17}{5}t = n$	$\frac{15}{5}t = n$	$T_s = \frac{5}{17}$	$T_s = \frac{5}{3}$
$\frac{4}{5}t + \frac{1}{5}t = n$	$\frac{4}{5}t - \frac{1}{5}t = n$	$\frac{5}{5}t = n$	$\frac{3}{5}t = n$	$T_s = \frac{5}{5}$	$T_s = \frac{5}{4}$
$\frac{9}{5}t + \frac{1}{5}t = n$	$\frac{9}{5}t - \frac{1}{5}t = n$	$\frac{10}{5}t = n$	$\frac{8}{5}t = n$	$T_s = \frac{2}{10}$	$T_s = \frac{5}{5}$
$\frac{14}{5}t + \frac{1}{5}t = n$	$\frac{14}{5}t - \frac{1}{5}t = n$	$\frac{15}{5}t = n$	$\frac{13}{5}t = n$	$T_s = \frac{5}{15}$	$T_s = \frac{5}{6}$
$\frac{19}{5}t + \frac{1}{5}t = n$	$\frac{19}{5}t - \frac{1}{5}t = n$	$\frac{20}{5}t = n$	$\frac{18}{5}t = n$	$T_s = \frac{5}{20}$	$T_s = \frac{5}{7}$

M.J. Roberts, Fundamentals of Signals and Systems

Numerical Computation of DTFT



References

- [1] <http://en.wikipedia.org/>
- [2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
- [3] M.J. Roberts, Fundamentals of Signals and Systems
- [4] S.J. Orfanidis, Introduction to Signal Processing
- [5] K. Shin, et al., Fundamentals of Signal Processing for Sound and Vibration Engineerings

- [6] A “graphical interpretation” of the DFT and FFT, by Steve Mann