Laurent Series and z-Transform
- Geometric Series
Double Pole Properties A

20190105 Sat

2 formulas of z

① $$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$$\Big\updownarrow z^{-1}$$

② $$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$$f(z) = \begin{cases} f_1(z) \\ f_2(z^{-1}) \end{cases} \qquad g(z) = \begin{cases} g_1(z) \\ g_2(z^{-1}) \end{cases}$$

$$X(z) = \begin{cases} X_1(z) \\ X_2(z^{-1}) \end{cases} \qquad Y(z) = \begin{cases} Y_1(z) \\ Y_2(z^{-1}) \end{cases}$$

$$\boxed{① \ \frac{3}{2} \frac{-1}{(z-0.5)(z-2)} \qquad ② \ \frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}}$$

$$\boxed{+\frac{1}{z-0.5} - \frac{1}{z-2}} \qquad \boxed{-\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}}$$

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \boxed{|z| < 0.5} \qquad \boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \boxed{|z| > 2}$$

causal $f_1(z)$

anti-causal $X_1(z)$

anti-causal $g_1(z)$

causal $Y_1(z)$

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \boxed{|z| > 2} \qquad \boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \boxed{|z| < 0.5}$$

anti-causal $f_2(z)$

causal $X_2(z)$

causal $g_2(z)$

anti-causal $Y_2(z)$

$$① \quad \frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} \qquad\qquad ② \quad \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$$

$$+\frac{1}{z-0.5} - \frac{1}{z-2} \qquad\qquad -\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}$$

$$\downarrow \cdot 2 \qquad \downarrow \cdot \frac{1}{2} \qquad\qquad \downarrow \cdot \frac{1}{z} \qquad \downarrow \cdot \frac{1}{z}$$

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad \boxed{|z|<0.5} \qquad\qquad -\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad \boxed{|z|>2}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad \boxed{|z|>2} \qquad\qquad +\frac{z}{1-0.5z} - \frac{z}{1-2z} \quad \boxed{|z|<0.5}$$

$$\uparrow \cdot \frac{1}{z} \qquad \uparrow \cdot \frac{1}{z} \qquad\qquad \uparrow \cdot \frac{1}{2} \qquad \uparrow \cdot 2$$

$$+\frac{1}{z-0.5} - \frac{1}{z-2} \qquad\qquad -\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}$$

$$\text{①}\ \frac{3}{2}\frac{-1}{(z-0.5)(z-2)} \qquad \text{②}\ \frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$$

$|2z|<1$ $\quad$ $|0.5z|<1$ $\qquad$ $|2z^{-1}|<1$ $\quad$ $|0.5z^{-1}|<1$

$|z|<0.5$ $\quad$ $|z|<2$ $\qquad$ $2<|z|$ $\quad$ $0.5<|z|$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \quad |z|<0.5 \qquad -\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \quad |z|>2$$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \quad |z|>2 \qquad +\frac{z}{1-0.5z}-\frac{z}{1-2z} \quad |z|<0.5$$

$|0.5z^{-1}|<1$ $\quad$ $|2z^{-1}|<1$ $\qquad$ $|0.5z|<1$ $\quad$ $|2z|<1$

$0.5<|z|$ $\quad$ $2<|z|$ $\qquad$ $|z|<2$ $\quad$ $|z|<0.5$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \quad |z|<0.5 \qquad\qquad -\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \quad |z|>2$$

$$\Big\downarrow \cdot\frac{1}{2z} \quad \Big\downarrow \cdot\frac{2}{z} \qquad\qquad \Big\downarrow \cdot\frac{z}{2} \quad \Big\downarrow \cdot 2z$$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \quad |z|>2 \qquad\qquad +\frac{z}{1-0.5z}-\frac{z}{1-2z} \quad |z|<0.5$$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \quad |z|<0.5 \qquad\qquad -\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \quad |z|>2$$

$$\Big\uparrow \cdot 2z \quad \Big\uparrow \cdot\frac{z}{2} \qquad\qquad \Big\uparrow \cdot\frac{2}{z} \quad \Big\uparrow \cdot\frac{1}{2z}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \quad |z|>2 \qquad\qquad +\frac{z}{1-0.5z}-\frac{z}{1-2z} \quad |z|<0.5$$

Causal sequence a_n & x_n

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5}$$

causal $f_1(z) =$

$$-\left[2+2^2z^1+2^3z^2+\cdots\right] \qquad -2^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2z^1+\left(\tfrac{1}{2}\right)^3z^2+\cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n+1}$$

0 1 2

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$$

causal $Y_1(z) =$

$$-\left[2^1z^0+2^2z^{-1}+2^3z^{-2}+\cdots\right] \qquad -2^{n+1}$$
$$+\left[\left(\tfrac{1}{2}\right)^1z^0+\left(\tfrac{1}{2}\right)^2z^{-1}+\left(\tfrac{1}{2}\right)^3z^{-2}+\cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n+1}$$

0 1 2

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$$

causal $X_2(z)$

$$+\left[\left(\tfrac{1}{2}\right)^0z^{-1}+\left(\tfrac{1}{2}\right)^1z^{-2}+\left(\tfrac{1}{2}\right)^2z^{-3}+\cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n-1}$$
$$-\left[2^0z^{-1}+2^1z^{-2}+2^2z^{-3}+\cdots\right] \qquad -2^{n-1}$$

1 2 3

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

causal $g_2(z)$

$$+\left[\left(\tfrac{1}{2}\right)^0z^1+\left(\tfrac{1}{2}\right)^1z^2+\left(\tfrac{1}{2}\right)^2z^3+\cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n-1}$$
$$-\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right] \qquad -2^{n-1}$$

1 2 3

Anti-causal sequence a_n & x_n

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5}$$

anti-causal $X_1(z)$

$$\boxed{\begin{array}{l} 2=\left(\frac{1}{2}\right)^{-1} \\ \left(\frac{1}{2}\right)=2^{-1} \end{array}}$$

$$-\left[\left(\tfrac{1}{2}\right)^{-1}+\left(\tfrac{1}{2}\right)^{-2}z^{1}+\left(\tfrac{1}{2}\right)^{-3}z^{2}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$+\left[2^{-1}+2^{-2}z^{1}+2^{-3}z^{2}+\cdots\right] \quad +2^{n-1}$$

0   -1   -2

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$$

anti-causal $g_1(z)$

$$-\left[\left(\tfrac{1}{2}\right)^{-1}z^{0}+\left(\tfrac{1}{2}\right)^{-2}z^{-1}+\left(\tfrac{1}{2}\right)^{-3}z^{-2}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$+\left[2^{-1}z^{0}+2^{-2}z^{-1}+2^{-3}z^{-2}+\cdots\right] \quad +2^{n-1}$$

0   -1   -2

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$$

anti-causal $f_2(z)$

$$\boxed{\begin{array}{l} 2=\left(\frac{1}{2}\right)^{-1} \\ \left(\frac{1}{2}\right)=2^{-1} \end{array}}$$

$$+\left[2^{0}z^{-1}+2^{-1}z^{-2}+2^{-2}z^{-3}+\cdots\right] \quad +2^{n+1}$$

$$-\left[\left(\tfrac{1}{2}\right)^{0}z^{-1}+\left(\tfrac{1}{2}\right)^{-1}z^{-2}+\left(\tfrac{1}{2}\right)^{-2}z^{-3}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

-1   -2   -3

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

anti-causal $Y_2(z)$

$$+\left[2^{0}z^{1}+2^{-1}z^{2}+2^{-2}z^{3}+\cdots\right] \quad +2^{n+1}$$

$$-\left[\left(\tfrac{1}{2}\right)^{0}z^{1}+\left(\tfrac{1}{2}\right)^{-1}z^{2}+\left(\tfrac{1}{2}\right)^{-2}z^{3}+\cdots\right] \quad -\left(\tfrac{1}{2}\right)^{n+1}$$

-1   -2   -3

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}} \quad \boxed{|z|<0.5}$$

causal $f_1(z) =$

$$-\left[2+2^2z^1+2^3z^2+\cdots\right] \qquad -2^{n+1}$$

$$+\left[\left(\tfrac{1}{2}\right)+\left(\tfrac{1}{2}\right)^2z^1+\left(\tfrac{1}{2}\right)^3z^2+\cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n+1}$$

0 1 2

anti-causal $X_1(z)$

$$-\left[\left(\tfrac{1}{2}\right)^{-1}+\left(\tfrac{1}{2}\right)^{-2}z^1+\left(\tfrac{1}{2}\right)^{-3}z^2+\cdots\right] \qquad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$+\left[2^{-1}+2^{-2}z^1+2^{-3}z^2+\cdots\right] \qquad +2^{n-1}$$

0 -1 -2

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z|>2}$$

anti-causal $g_1(z)$

$$-\left[\left(\tfrac{1}{2}\right)^{-1}z^0+\left(\tfrac{1}{2}\right)^{-2}z^{-1}+\left(\tfrac{1}{2}\right)^{-3}z^{-2}+\cdots\right] \qquad -\left(\tfrac{1}{2}\right)^{n-1}$$

$$+\left[2^{-1}z^0+2^{-2}z^{-1}+2^{-3}z^{-2}+\cdots\right] \qquad +2^{n-1}$$

0 -1 -2

causal $Y_1(z) =$

$$-\left[2^1z^0+2^2z^{-1}+2^3z^{-2}+\cdots\right] \qquad -2^{n+1}$$

$$+\left[\left(\tfrac{1}{2}\right)^1z^0+\left(\tfrac{1}{2}\right)^2z^{-1}+\left(\tfrac{1}{2}\right)^3z^{-2}+\cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n+1}$$

0 1 2

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z|>2}$$

anti-causal $f_2(z)$

$$+\left[2^0z^{-1}+2^{-1}z^{-2}+2^{-2}z^{-3}+\cdots\right] \qquad +2^{n+1}$$

$$-\left[\left(\tfrac{1}{2}\right)^0z^{-1}+\left(\tfrac{1}{2}\right)^{-1}z^{-2}+\left(\tfrac{1}{2}\right)^{-2}z^{-3}+\cdots\right] \qquad -\left(\tfrac{1}{2}\right)^{n+1}$$

-1 -2 -3

causal $X_2(z)$

$$+\left[\left(\tfrac{1}{2}\right)^0z^{-1}+\left(\tfrac{1}{2}\right)^1z^{-2}+\left(\tfrac{1}{2}\right)^2z^{-3}+\cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n-1}$$

$$-\left[2^0z^{-1}+2^1z^{-2}+2^2z^{-3}+\cdots\right] \qquad -2^{n-1}$$

1 2 3

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}} \quad \boxed{|z|<0.5}$$

causal $g_2(z)$

$$+\left[\left(\tfrac{1}{2}\right)^0z^1+\left(\tfrac{1}{2}\right)^1z^2+\left(\tfrac{1}{2}\right)^2z^3+\cdots\right] \qquad +\left(\tfrac{1}{2}\right)^{n-1}$$

$$-\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right] \qquad -2^{n-1}$$

1 2 3

anti-causal $Y_2(z)$

$$+\left[2^0z^1+2^{-1}z^2+2^{-2}z^3+\cdots\right] \qquad +2^{n+1}$$

$$-\left[\left(\tfrac{1}{2}\right)^0z^1+\left(\tfrac{1}{2}\right)^{-1}z^2+\left(\tfrac{1}{2}\right)^{-2}z^3+\cdots\right] \qquad -\left(\tfrac{1}{2}\right)^{n+1}$$

-1 -2 -3

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$$f(z) = -\left[2 + 2^2 z + 2^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \qquad (n \geq 0)}$$

$$X(z) = -\left[\left(\tfrac{1}{2}\right)^{-1} + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right] + \left[2^{-1} + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right]$$

$$\boxed{x_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1} \qquad (n < 1)}$$

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$$f(z) = -\left[\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^{-1} + \left(\tfrac{1}{2}\right)^{-3} z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1} \qquad (n < 1)}$$

$$X(z) = -\left[2^1 z^0 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right] + \left[\left(\tfrac{1}{2}\right)^1 z^0 + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right]$$

$$\boxed{x_n = -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \qquad (n \geq 0)}$$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$f(z) = +\left[2^0 z^1 + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right]$$

$$\boxed{a_n = +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \qquad (n < 0)}$$

$$X(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right] - \left[2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right]$$

$$\boxed{x_n = +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \qquad (n \geq 1)}$$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

$$f(z) = +\left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^1 z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right] - \left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right]$$

$$\boxed{a_n = +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \qquad (n \geq 1)}$$

$$X(z) = +\left[2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right] - \left[\left(\tfrac{1}{2}\right)^0 z^1 + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right]$$

$$\boxed{x_n = +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \qquad (n < 0)}$$

1-A

2-A

1-B

2-B

$f(z)$ $|z| < 0.5$ $|z| > 2$

causal anticausal

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

①-Ⓐ $\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}}$ $\boxed{|z| < 0.5}$ $\boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $\boxed{(n \geq 0)}$

$$-\left(2 + 2^2 z + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0$ $n=1$ $n=2$ $n=0$ $n=1$ $n=2$

①-Ⓑ $\boxed{\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}$ $\boxed{|z| > 2}$ $\boxed{+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}}$ $\boxed{(n < 0)}$

$$\left(z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(z^{-1} + 2z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$$\left(2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots\right) - \left(\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^{-1} z^{-2} + \left(\tfrac{1}{2}\right)^{-2} z^{-3} + \cdots\right)$$

$n=-1$ $n=-2$ $n=-3$ $n=-1$ $n=-2$ $n=-3$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

②-Ⓐ $\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}$ $\boxed{|z| > 2}$ $\boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\boxed{(n < 1)}$

$$-\left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} + \left(\tfrac{1}{2}\right)^{-2} z^{-1} + \left(\tfrac{1}{2}\right)^{-3} z^{-2} + \cdots\right) + \left(2^{-1} + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right)$$

$n=0$ $n=-1$ $n=-2$ $n=0$ $n=-1$ $n=-2$

②-Ⓑ $\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}}$ $\boxed{|z| < 0.5}$ $\boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}}$ $\boxed{(n \geq 1)}$

$$+\left(z + \left(\tfrac{1}{2}\right) z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right) - \left(z + 2z^2 + 2^2 z^3 + \cdots\right)$$

$n=1$ $n=2$ $n=3$ $n=1$ $n=2$ $n=3$

$X(z)$ $\quad |z| < 0.5 \quad |z| > 2$

anticausal $\quad$ causal

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

1-A $\quad \boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \quad \boxed{|z| < 0.5} \quad \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad \boxed{(n < 1)}$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad n=0 \quad n=-1 \quad n=-2$

1-B $\quad \boxed{\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \quad \boxed{|z| > 2} \quad \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad \boxed{(n \geq 1)}$

$$\left(\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$n=1 \quad n=2 \quad n=3 \qquad n=1 \quad n=2 \quad n=3$

$$\frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

2-A $\quad \boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \quad \boxed{|z| > 2} \quad \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad \boxed{(n \geq 0)}$

$$-\left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad n=0 \quad n=1 \quad n=2$

2-B $\quad \boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \quad \boxed{|z| < 0.5} \quad \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad \boxed{(n < 0)}$

$$+\left(z + \left(\tfrac{1}{2}\right) z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right) - \left(z + 2z^2 + 2^2 z^3 + \cdots\right)$$

$$+\left(2^0 z + 2^{-1} z^2 + 2^{-2} z^3 + \cdots\right) - \left(\left(\tfrac{1}{2}\right)^0 z + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right)$$

$n=-1 \quad n=-2 \quad n=-3 \qquad n=-1 \quad n=-2 \quad n=-3$

$f(z) \longleftrightarrow a_n$

$X(z) \longleftrightarrow x_n$

①-Ⓐ

$$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}} \quad (n<1)$$

②-Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}} \quad (n<1)$$

$$x_n = \boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}} \quad (n \geq 1)$$

②-Ⓑ

$$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}} \quad (n \geq 1)$$

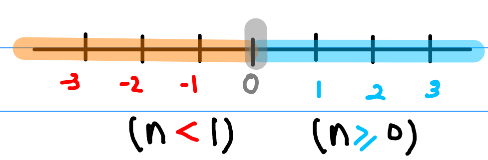
$$x_n = \boxed{+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

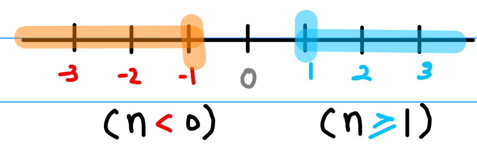
$$x_n = a_{-n}$$

$$a_n = x_{-n}$$

$(n \geq 0) \longleftrightarrow (n<1)$

$(n \geq 1) \longleftrightarrow (n<0)$





Ⅰ $(z^{-1},\ R^{-1}) \Leftrightarrow (a_{-n},\ -N)$

①-Ⓐ		②-Ⓐ
$\boxed{-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}}\ \boxed{\lvert z\rvert<0.5}$	$z^{-1},\ R^{-1}$ $\Longleftrightarrow$	$\boxed{-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}}\ \boxed{\lvert z\rvert>2}$
$a_n = \boxed{-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}}\quad (n\geq 0)$	$a_{-n},\ -N$ $\Longleftrightarrow$	$a_n = \boxed{-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}}\quad (n<1)$
$x_n = \boxed{-\left(\frac{1}{2}\right)^{n-1}+2^{n-1}}\quad (n<1)$	$a_{-n},\ -N$ $\Longleftrightarrow$	$x_n = \boxed{-2^{n+1}+\left(\frac{1}{2}\right)^{n+1}}\quad (n\geq 0)$

①-Ⓑ		②-Ⓑ
$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}}\ \boxed{\lvert z\rvert>2}$	$z^{-1},\ R^{-1}$ $\Longleftrightarrow$	$\boxed{+\frac{z}{1-0.5z}-\frac{z}{1-2z}}\ \boxed{\lvert z\rvert<0.5}$
$a_n = \boxed{+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}}\quad (n<0)$	$a_{-n},\ -N$ $\Longleftrightarrow$	$a_n = \boxed{+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}}\quad (n\geq 1)$
$x_n = \boxed{+\left(\frac{1}{2}\right)^{n-1}-2^{n-1}}\quad (n\geq 1)$	$a_{-n},\ -N$ $\Longleftrightarrow$	$x_n = \boxed{+2^{n+1}-\left(\frac{1}{2}\right)^{n+1}}\quad (n<0)$

II $(z,\ R^{-1}) \Leftrightarrow (-a_n,\ N^c)$

1 − A

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \quad \boxed{|z|<0.5}$$

$\Big\|\ z$ $\qquad \updownarrow R^{-1}$

1 − B

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \quad \boxed{|z|>2}$$

2 − A

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \quad \boxed{|z|>2}$$

$\Big\|\ z$ $\qquad \updownarrow R^{-1}$

2 − B

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \quad \boxed{|z|<0.5}$$

1 − A

$$a_n = \boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$\updownarrow -a_n \qquad \updownarrow N^c$

1 − B

$$a_n = \boxed{+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

2 − A

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}} \quad (n<1)$$

$\updownarrow -a_n \qquad \updownarrow N^c$

2 − B

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}} \quad (n \geq 1)$$

1 − A

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1}+2^{n-1}} \quad (n<1)$$

$\updownarrow -a_n \qquad \updownarrow N^c$

1 − B

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1}-2^{n-1}} \quad (n \geq 1)$$

2 − A

$$x_n = \boxed{-2^{n+1}+\left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

$\updownarrow -a_n \qquad \updownarrow N^c$

2 − B

$$x_n = \boxed{+2^{n+1}-\left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

Ⅲ

①

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

$$(z^{-1}, R^{-1}) \to (z, R^{-1}) \qquad (a_{-n}, -N) \to (-a_n, N^c)$$

①-Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \ \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

②-Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n<1)$$

①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

②-Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

①-Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}} \ \boxed{|z|<0.5}$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n<1)$$

②-Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}} \ \boxed{|z|>2}$$

$$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}} \ \boxed{|z|>2}$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

②-Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}} \ \boxed{|z|<0.5}$$

$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n<0)$$

III ②

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

$$(z^{-1}, R^{-1}) \rightarrow (z, R^{-1}) \qquad (a_{-n}, -N) \rightarrow (-a_n, N^c)$$

① − Ⓐ

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad |z| < 0.5$$

② − Ⓑ

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z} \quad |z| < 0.5$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0) \quad \xleftrightarrow{-a_{-n},\,(-N)^c} \quad a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

① − Ⓑ

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$

② − Ⓐ

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 2$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0) \quad \xleftrightarrow{-a_{-n},\,(-N)^c} \quad a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

① − Ⓐ

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad |z| < 0.5$$

② − Ⓑ

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z} \quad |z| < 0.5$$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1) \quad \xleftrightarrow{-a_{-n},\,(-N)^c} \quad x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

① − Ⓑ

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z| > 2$$

② − Ⓐ

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z| > 2$$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1) \quad \xleftrightarrow{-a_{-n},\,(-N)^c} \quad x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

Ⅳ $(a_n, N) \Leftrightarrow (x_{-n}, -N)$

①-Ⓐ

$$\boxed{-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}}\ \boxed{|z|<0.5}$$

$$a_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

a_{-n} $\quad$ $-N$

$$x_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

②-Ⓐ

$$\boxed{-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}}\ \boxed{|z|>2}$$

$$a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}} \quad (n < 1)$$

a_{-n} $\quad$ $-N$

$$x_n = \boxed{-2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1}} \quad (n \geq 0)$$

①-Ⓑ

$$\boxed{+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}}\ \boxed{|z|>2}$$

$$a_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

a_{-n} $\quad$ $-N$

$$x_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

②-Ⓑ

$$\boxed{+\frac{z}{1-0.5z} - \frac{z}{1-2z}}\ \boxed{|z|<0.5}$$

$$a_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}} \quad (n \geq 1)$$

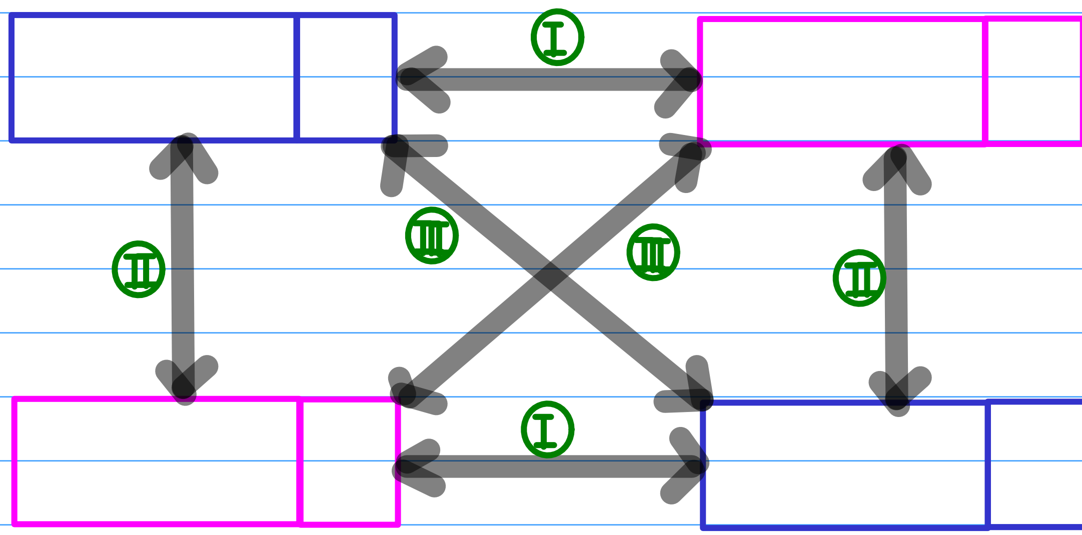
a_{-n} $\quad$ $-N$

$$x_n = \boxed{+2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1}} \quad (n < 0)$$

I $(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

II $(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$

III $(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$



$(a_n, N) \Leftrightarrow (x_{-n}, -N)$

ROC's of interests

$R1(z)$ $\quad$ $R1(z^{-1})$

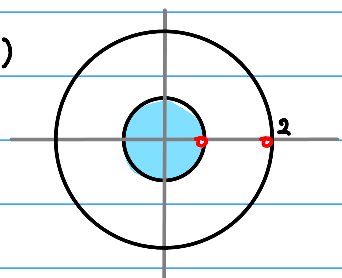
$R2(z)$ $\quad$ $R2(z^{-1})$

$$① \ \frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} \ \xleftrightarrow{z^{-1}} \ ② \ \frac{3}{2}\,\frac{-z^2}{(z-2)(z-0.5)}$$

$$\boxed{+\frac{1}{z-0.5}-\frac{1}{z-2}} \ \xleftrightarrow{z^{-1}} \ \boxed{-\frac{2z}{(z-2)}+\frac{0.5z}{(z-0.5)}}$$

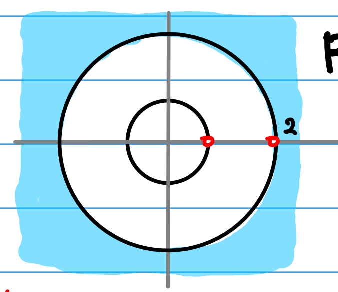
$$\begin{aligned} p_1 &= 0.5 \\ p_2 &= 2 \end{aligned} \ \xleftrightarrow{z^{-1}} \ \begin{aligned} p_1^{-1} &= 2 \\ p_2^{-1} &= 0.5 \end{aligned}$$

$R1(z)$



$|z| < 0.5$

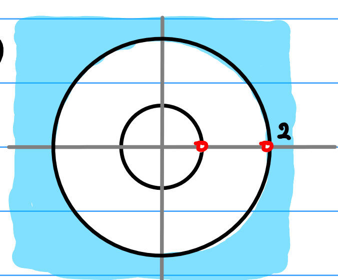
$\xleftrightarrow{z^{-1}}$



$R1(z^{-1})$

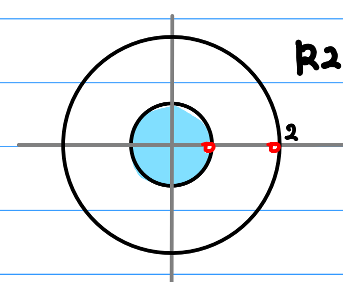
$|z^{-1}| < 0.5$ $\quad$ $|z| > 2$

$R2(z)$



$|z| > 2$

$\xleftrightarrow{z^{-1}}$



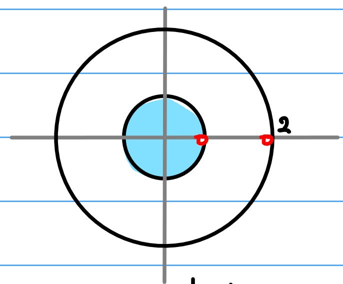
$R2(z^{-1})$

$|z^{-1}| > 2$ $\quad$ $|z| < 0.5$

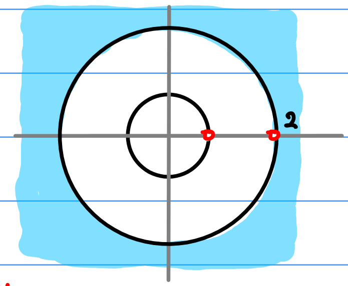
$$R1(z^{-1}) = R2(z)$$
$$R2(z^{-1}) = R1(z)$$

R^{-1}

$R1(z) \triangleq R1$ $\quad$ $R1(z^{-1}) \triangleq R1^{-1}$

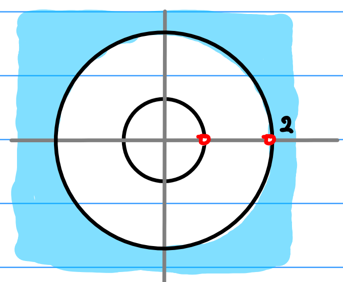


$\xleftrightarrow{z^{-1}}$

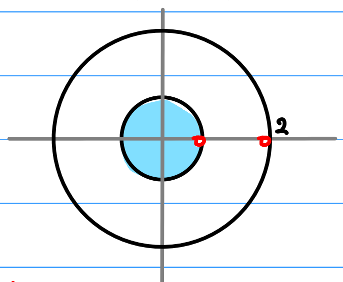


$|z| < 0.5$ $\quad$ $|z^{-1}| < 0.5$ $\quad$ $|z| > 2$

$R2(z)$ $\quad$ $R2(z^{-1}) \triangleq R2^{-1}$

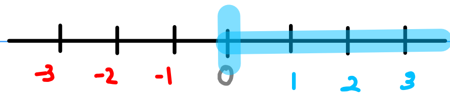


$\xleftrightarrow{z^{-1}}$



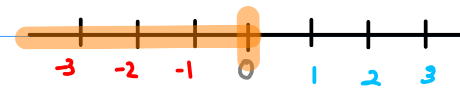
$|z| > 2$ $\quad$ $|z^{-1}| > 2$ $\quad$ $|z| < 0.5$

$-N$, N^c

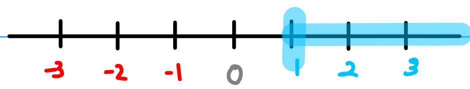


$(n \geq 0)$

$-N$

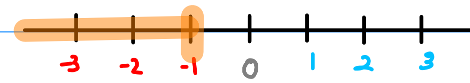


$(n < 1)$

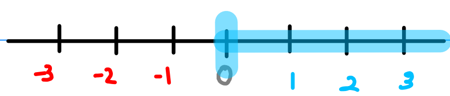


$(n \geq 1)$

$-N$

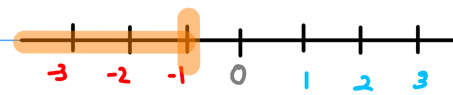


$(n < 0)$

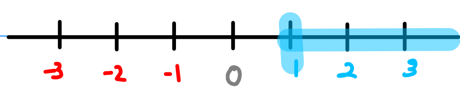


$(n \geq 0)$

$N^c = \overline{N}$

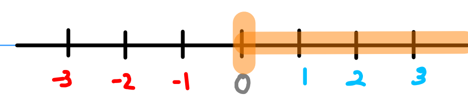


$(n < 0)$



$(n \geq 1)$

$N^c = \overline{N}$



$(n < 1)$

$-$	$\overline{\quad}$	« »
-1	complement	
$-N \triangleq RNG(-n)$	$N^c \triangleq \overline{RNG(n)}$	$(-N)^c = -(N^c)$

$$(z, R) \Leftrightarrow (a_n, N)$$

$f(z)$	$ROC(z)$	$\Leftrightarrow$	a_n	$RNG(n)$
	$\lvert z\rvert < p$			$n \geq 0$

Ⅰ $$(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

$f(z^{-1})$	$ROC(z^{-1})$	$\Leftrightarrow$	a_{-n}	$RNG(-n)$
	$\lvert z\rvert > \frac{1}{p}$			$n < 1$

Ⅱ $$(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$$

$f(z)$	$ROC(z^{-1})$	$\Leftrightarrow$	$-a_n$	$\overline{RNG}(n)$
	$\lvert z\rvert > \frac{1}{p}$			$n < 0$

Ⅲ $$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$

$f(z^{-1})$	$ROC(z)$	$\Leftrightarrow$	$-a_{-n}$	$\ll RNG(n) \gg$
	$\lvert z\rvert < p$			$n \geq 1$

Ⅰ + Ⅱ

Ⅳ $$(a_n, N) \Leftrightarrow (x_{-n}, -N)$$

$X(z)$	$ROC(z)$	$\Leftrightarrow$	a_{-n}	$RNG(-n)$
	$\lvert z\rvert < p$			$n < 1$

$$\text{III} = \text{I} + \text{II}$$

III $\quad \boxed{f(z^{-1}) \quad ROC(z)\ \ |z| < p \quad \Longleftrightarrow \quad -a_{-n} \quad \ll RNG(n) \gg\ \ n \geq 1}$ $\quad$ I + II

$$\boxed{f(z) \quad ROC(z)\ \ |z| < p \quad \Longleftrightarrow \quad a_n \quad RNG(n)\ \ n \geq 0}$$

I $\quad \boxed{f(z^{-1}) \quad ROC(z^{-1})\ \ |z| > \frac{1}{p} \quad \Longleftrightarrow \quad a_{-n} \quad RNG(-n)\ \ n < 1}$

II $\quad \boxed{f(z) \quad ROC(z^{-1})\ \ |z| > \frac{1}{p} \quad \Longleftrightarrow \quad -a_n \quad \overline{RNG(n)}\ \ n < 0}$

$$\boxed{f(z^{-1}) \quad ROC(z)\ \ |z| < p \quad \Longleftrightarrow \quad -a_{-n} \quad \overline{RNG(-n)}\ \ n \geq 1}$$

$$(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

$$(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$$

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$

Compare Ⅰ with Ⅳ

ROC(z)	f(z)	$\Longleftrightarrow$	a_n	RNG(n)
$\lvert z\rvert < p$				$n \geq 0$

Ⅰ $$(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

$f(z^{-1})$	ROC(z^{-1})	$\Longleftrightarrow$	a_{-n}	RNG($-n$)
	$\lvert z\rvert > \frac{1}{p}$			$n < 1$

Ⅳ $$(a_n, N) \Leftrightarrow (x_{-n}, -N)$$

$$(x_n, N) \Leftrightarrow (a_{-n}, -N)$$

$X(z)$	ROC(z)	$\Longleftrightarrow$	a_{-n}	RNG($-n$)
	$\lvert z\rvert < p$			$n < 1$

$-n$ $\quad$ $-n$

symmetrical

Ⅰ

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad X(z)$ $\quad a_n = \boxed{-\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n<1)$

$|z| > 2$ $\quad X(z)$ $\quad b_n = \boxed{+\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1}}$ $\quad (n \geq 1)$

$\{\, |z| < 0.5 \,\} \cap \{\, |z| > 2 \,\} = \phi \implies a_n + b_n = 0$

$a_n = -b_n$

ROC $|z| < a$

$$X(z) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$$

$-1\cdot$

ROC′ $|z| > a^{-1}$

$$X(z) = -\frac{z^{-1}}{1-a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad -a^{n+1} \qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^k$$

$$\boxed{\frac{a}{1-az}} = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad\qquad \boxed{\frac{z}{1-az}} = \sum_{n=1}^{\infty} a^{n-1} z^n$$

$$\boxed{-\frac{z^{-1}}{1-a^{-1}z^{-1}}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad\qquad \boxed{-\frac{a^{-1}}{1-a^{-1}z^{-1}}} = -\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^k \qquad\qquad = -\sum_{k=0}^{-\infty} a^{k-1} z^k$$

$$\frac{a}{1-az} \Rightarrow \sum_{n=0}^{\infty} a^{n+1} z^{n} \qquad\qquad \frac{z}{1-az} \Rightarrow \sum_{n=1}^{\infty} a^{n-1} z^{n}$$

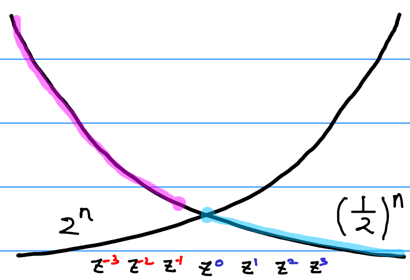
$$\| \qquad\qquad \|$$

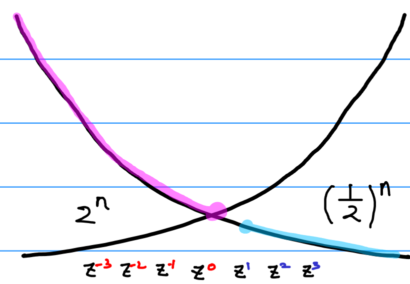
$$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad\qquad -\frac{a^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n-1} z^{-n}$$

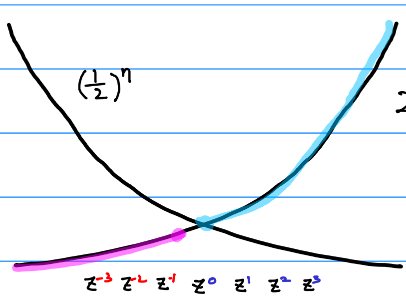
$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^{k} \qquad\qquad = -\sum_{k=0}^{-\infty} a^{k-1} z^{k}$$

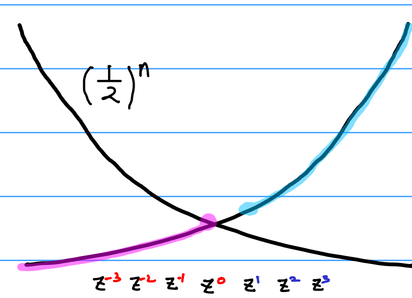
$$a + a^2 z^1 + a^3 z^2 + a^4 z^3 + \cdots \qquad\qquad z + a z^2 + a^2 z^3 + a^3 z^4 + \cdots$$

$$z^{-1} + a^{-1} z^{-2} + a^{-2} z^{-3} + a^{-3} z^{-4} + \cdots \qquad\qquad a^{-1} + a^{-2} z^{-1} + a^{-3} z^{-2} + a^{-4} z^{-3} + \cdots$$









(II)

ROC(z^{-1})	$f(z^{-1})$	a_{-n}	RNG($-n$)
$\lvert z\rvert > \frac{1}{p}$			$n < 1$

$|z| < 1$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -\left[1^1 + 1^2 z^1 + 1^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$\boxed{a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)}$$

$|z| > 1$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$f(z) = -\left[\left(\tfrac{1}{1}\right)^{-1} z^0 + \left(\tfrac{1}{1}\right)^{-2} z^{-1} + \left(\tfrac{1}{1}\right)^{-3} z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$\boxed{a_n = -1^{n-1} + 2^{n-1} \quad (n < 1)}$$

ROC

$|z| < a$

$$f(z) = \frac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n$$

a^{n+1}

$n \geq 0$ $n \geq 1$ $n < 0$ $n < 1$

z^{-1} $-n$

ROC'

$|z| > a^{-1}$

$$f(z^{-1}) = \frac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n} = \sum_{k=0}^{-\infty} a^{-k+1} z^k$$

$$a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$$

$n < 1$ $n < 0$ $n \geq 0$ $n \geq 0$

Ⅳ

ROC(z)	X(z)	⟺	a_{-n}	RNG(-n)
$\lvert z\rvert < p$				$n \le 0$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\qquad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\qquad \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $\quad (n \ge 0)$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad\qquad n=0 \quad n=1 \quad n=2$

||

$|z| < 0.5$ $\qquad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\qquad \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \le 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right)z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad\qquad n=0 \quad n=-1 \quad n=-2$

ROC $\qquad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n$ $\qquad a^{n+1}$ $\qquad n \ge 0 \quad n \ge 1 \quad n < 0 \quad n < 1$

|| $\qquad$ || $\qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$ $\qquad \Big\downarrow -n$

ROC $\qquad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$ $\qquad a^{-n+1}$ $\qquad n \le 0 \quad n \le 1 \quad n > 0 \quad n > 1$

$$= \left(\frac{1}{a}\right)^{n-1}$$

$z^{-1}X(z)$ Shifted Sequence

$$|z|>1 \qquad |z|>2$$

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_n = 1^n + 2^n \qquad (n \geq 0)$$

$$(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots) + (2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} + \cdots) \quad n \qquad n = 0, 1, 2, \cdots$$

$\cdot z^{-1}$ $\qquad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z^{-1}X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_{n-1} = 1^{n-1} + 2^{n-1} \qquad (n \geq 1)$$

$z f(z)$ Shifted Sequence

$$f(z) = (+1)\frac{1}{1-z} - \frac{1}{1-2z} \quad (|z|<0.5)$$

$$a_n = 1^n - 2^n \quad (n \geq 0)$$

$$(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots) - (2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots) \quad n \qquad n = 0, 1, 2, \cdots$$

$\cdot z$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z f(z) = (+1)\frac{z}{1-z} - \frac{z}{1-2z} \quad (|z|<0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$z^{-1} f(z^{-1})$ Shifted & Reflected Sequence

$$f(z) = \frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z| < 0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$\cdot z$$

$$(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots) - (2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots) \quad n \quad n = 0, 1, 2, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) - (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad n-1 \quad n = 1, 2, 3, \cdots$$

$$z f(z) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z| < 0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$$z$$

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n \quad n = 1, 2, 3, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$z^{-1}$$

$$(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots) + (2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots) \quad -n \quad n = -1, -2, -3, \cdots$$

$$z^{-1} f(z^{-1}) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z| > 2)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n < 0)$$

$$a_{-(n+1)}$$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5}-\frac{1}{z-2}\right)$$

$|z|<0.5$ $X(z)$ $a_n = \boxed{-(\tfrac{1}{2})^{n-1}+2^{n-1}}$ $(n<1)$

$|z|>2$ $X(z)$ $b_n = \boxed{+(\tfrac{1}{2})^{n-1}-2^{n-1}}$ $(n\geq 1)$

$\{|z|<0.5\}\cap\{|z|>2\} = \phi \Rightarrow a_n+b_n=0$

$a_n = -b_n$

$$|z|<a \quad X(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1} \qquad n\geq 0 \quad n\geq 1 \quad n<0 \quad n<1$$

$$= \sum_{n=0}^{\infty}\left(\frac{1}{a}\right)^{-n-1} z^n \qquad \downarrow -n$$

$$|z|>a \quad X(z) = \sum_{k=0}^{-\infty}\left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} \qquad n<0 \quad n<1 \quad n\geq 0 \quad n\geq 1$$

$$= \left(\frac{1}{a}\right)^{n-1}$$

$a^{n+1} z^n$

$a\,(az)^n$

$a\left(\frac{1}{az}\right)^{-n}$

$$\frac{a}{1-az} \qquad \sum_{n=0}^{\infty} a^{n+1} z^n \qquad \frac{z}{1-az} \qquad \sum_{n=1}^{\infty} a^{n-1} z^n$$

$$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \qquad -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} \qquad -\frac{a^{-1}}{1-a^{-1}z^{-1}} \qquad -\sum_{n=1}^{\infty} a^{-n-1} z^{-n}$$

$$\sum_{n=-1}^{-\infty} a^{n+1} z^n \qquad -\sum_{n=0}^{-\infty} a^{n-1} z^n$$

$zX(z)$ Shifted & Reflected Sequence

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$\cdot z^{-1}$

$$(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots) + (2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2}) \quad n \qquad n = 0, 1, 2, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \qquad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n-1 \qquad n = 1, 2, 3, \cdots$$

$$z^{-1}X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

z

$$(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots) + (2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots) \quad n \qquad n = 1, 2, 3, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \qquad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

z^{-1}

$$(1^0 z^{1} + 1^1 z^{2} + 1^2 z^{3} + \cdots) + (2^0 z^{1} + 2^1 z^{2} + 2^2 z^{3} + \cdots) \quad -n \qquad n = -1, -2, -3, \cdots$$

$$zX(z^{-1}) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z|<0.5)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n < 0)$$

$$a_{-(n+1)}$$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\qquad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\qquad \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $\qquad (n \geq 0)$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad\qquad n=0 \quad n=1 \quad n=2$

$|z| < 0.5$ $\qquad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\qquad \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\qquad (n \leq 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad\qquad n=0 \quad n=-1 \quad n=-2$

ROC $\qquad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n$ $\qquad a^{n+1}$ $\qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$||$ $\qquad ||$ $\qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$ $\qquad \Big\downarrow{\scriptstyle -n}$

ROC $\qquad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$ $\qquad a^{-n+1}$ $\qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$$= \left(\frac{1}{a}\right)^{n-1}$$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^{n} \quad a^{n+1} \quad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$z^{-1} \quad z^{-1} \quad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n} \quad -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} (a)^{k-1} z^{-k} \quad \left(\frac{1}{a}\right)^{-n+1} \quad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$

$= a^{n-1}$