First Order Logic – Semantics (3A)

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9/17/17

Please send corrections (or suggestions) to youngwlim@hotmail.com.

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Based on

Contemporary Artificial Intelligence,
R.E. Neapolitan & X. Jiang

Logic and Its Applications,
Burkey & Foxley

Terms and Formulas

Terms

1. Variables
2. Functions

$x \quad y \quad f(x) \quad g(x, y)$

Formulas

Predicate symbols.

Equality.

Negation.

Binary connectives.

Quantifiers.

$P(x) \qquad Q(x, y)$

$x = f(y)$

$\neg Q(x, y)$

$P(x) \land \neg Q(x, y)$

$\forall x, y \; (P(x) \land \neg Q(x, y))$

https://en.wikipedia.org/wiki/First-order_logic#Formation_rules

Examples of Terms

no expression involving a predicate symbol is a **term**.

$x \quad y \quad f(x) \quad g(x, y)$

$father(x)$	A function returns neither True nor False	**term**
	The father of x	
$Father(x)$	A predicate returns always True or False	~~**term**~~
	Is x a father?	

∀x love(x,y) : free variable y
∀x tall(x) : no free variable

Bound variable x
Free variable y

https://en.wikipedia.org/wiki/First-order_logic#Formation_rules

Terms

1. **Variables**. Any variable is a term.
2. **Functions**. Any expression $f(t_1,...,t_n)$ of n arguments is a term
   where each argument t_i is a term and **f** is a function symbol of valence n
   In particular, symbols denoting individual constants are
   0-ary function symbols, and are thus terms.

Only expressions which can be obtained
by finitely many applications of rules 1 and 2 are terms.

no expression involving a predicate symbol is a term.

https://en.wikipedia.org/wiki/First-order_logic#Formation_rules

Formulas (1)

Formulas (wffs)

Predicate symbols.	$P(x) \qquad Q(x, y)$
Equality.	$x = f(y)$
Negation.	$\neg Q(x, y)$
Binary connectives.	$P(x) \land \neg Q(x, y)$
Quantifiers.	$\forall x, y \; (P(x) \land \neg Q(x, y))$

Only expressions which can be obtained
by finitely many applications of rules 1–5 are formulas.

The formulas obtained from the first two rules
are said to be **atomic formulas**.

https://en.wikipedia.org/wiki/First-order_logic#Formation_rules

Formulas (2)

Formulas (wffs)

Predicate symbols.

If **P** is an n-ary predicate symbol

and t_1, ..., t_n are terms

then **P**(t_1,...,t_n) is a formula.

Equality.

If the equality symbol is considered part of logic,

and t_1 and t_2 are terms,

then t_1 **=** t_2 is a formula.

$P(x)$ $Q(x, y)$

$x = f(y)$

https://en.wikipedia.org/wiki/First-order_logic#Formation_rules

Formulas (3)

Formulas (wffs)

Negation.

If φ is a formula,

then ¬φ is a formula.

Binary connectives.

If φ and ψ are formulas,

then (φ → ψ) is a formula.

Similar rules apply to other binary logical connectives.

Quantifiers.

If φ is a formula and x is a variable,

then ∀x φ (for all x, holds)

and ∃x φ (there exists x such that φ) are formulas.

$$\neg\, Q(x, y)$$

$$P(x) \wedge \neg Q(x, y)$$

$$\forall x, y \ \ (P(x) \wedge \neg Q(x, y))$$

https://en.wikipedia.org/wiki/First-order_logic#Formation_rules

Atoms and Compound Formulas

a formula that contains no logical connectives
a formula that has no strict subformulas

Atoms :

the simplest well-formed formulas of the logic. $P(x)$ $Q(x,y)$

Compound formulas :

formed by combining the atomic formulas using the logical connectives.

$$P(x) \wedge \neg Q(x,y)$$

$$\forall x, y \ (P(x) \wedge \neg Q(x,y))$$

https://en.wikipedia.org/wiki/Atomic_formula

Atomic Formula

for **propositional logic**
the atomic formulas are the propositional variables

p q

for **predicate logic**
the atoms are predicate symbols together with their arguments, each argument being a term.

$P(x)$ $Q(x, f(y))$

In **model theory**
atomic formula are merely strings of symbols with a given signature which may or may not be satisfiable with respect to a given model

https://en.wikipedia.org/wiki/Atomic_formula

Basic Entities in FOL

propositional logic assumes world contains **facts**

first-order logic assumes the world contains **objects**, **relations**, and **functions**

- **Objects**: people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries …

- **Relations**: red, round, bogus, prime, multistoried, is the brother of, is bigger than, is inside, is part of, has color, occurred after, owns, comes between, …

- **Functions**: father of, best friend, third inning of, one more than, end of

https://www.cs.umd.edu/~nau/cmsc421/chapter08.pdf

Types of Logic

Language	Ontological commitment* (what it talks about)	Epistemological commitment* (what it says about truth)
Prop. Logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief
Fuzzy logic	facts + degree of truth	known interval value

ontological commitment ≈ our assumptions about what things exist
epistemological commitment ≈ what we can know about those things

https://www.cs.umd.edu/~nau/cmsc421/chapter08.pdf

Model

A **model** is a pair **M**= (**D**,**I**),
- **D** is a domain and
- **I** is an interpretation

- **Objects**: people, houses, numbers, …
- **Relations**: red, round, bogus, prime, …
- **Functions**: father of, best friend, …

D contains
- more than 1 **objects** (domain elements)
- and **relations** among them

I specifies referents for
- constant symbols→**objects** in the domain
- predicate symbols→**relations** over objects in the domain
- function symbols→**functional relations** over objects in the domain

Truth Example

Suppose **M**= (**D**,**I**), where

D is the domain shown at right,

And **I** is an interpretation in which

Objects

Richard → Richard the Lionheart

John → the evil King John

Relations

Brother → the brotherhood relation

Predicate

Brother(Richard,John) is true in **M**

iff the pair consisting of Richard the Lionheart and the evil King John

https://www.cs.umd.edu/~nau/cmsc421/chapter08.pdf

Relation and Predicates

mathematically, a **relation** is a set of ordered n-tuples

An **atomic** sentence **predicate**(term$_1$,...,term$_n$) is ***true*** in **M**

iff the **objects** referred to by term$_1$,...,term$_n$

are in the **relation** referred to by **predicate**

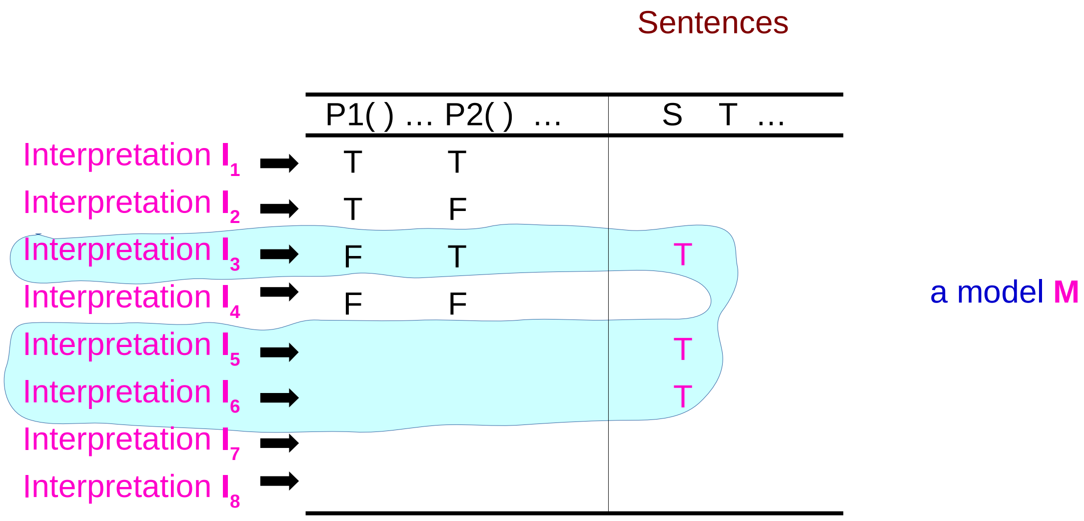
M is a **model** of a **sentence S**

If **S** is ***true*** in **M**

M= (**D**,**I**),

D is a domain and

I is an interpretation



A Signature

First specify a **signature**

Constant **Symbols**	$\{c_1, c_2, \ldots c_n\} = D$
Predicate **Symbols**	$\{\mathbf{P}_1, \mathbf{P}_2, \ldots \mathbf{P}_m\}$
Function **Symbols**	$\{\mathbf{f}_1, \mathbf{f}_2, \ldots \mathbf{f}_l\}$

A **model** is a pair **M**= (**D**,**I**),

D is a domain and

I is an interpretation

D contains

more than 1 **objects** (domain elements)

and **relations** among them

I specifies referents for

signature

constant symbols → **objects** in the domain

predicate symbols → **relations** over objects in the domain

function symbols → **functional** relations over objects in the domain

A Language

Determines the **language**

Given a language

A model is specified $M = (D, I)$

A **domain of discourse**

a set of entities $\{entity_1, entity_2, \dots entity_n\}$

An **interpretation**

constant assignments $\{c_1, c_2, \dots c_n\} = D$

function assignments $\mathbf{f}_1(\ ), \mathbf{f}_2(\ ), \dots \mathbf{f}_l(\ )$

truth value assignments $\mathbf{P}_1(\ ), \mathbf{P}_2(\ ), \dots \mathbf{P}_m(\ )$

Interpretation – assigning the signature

$M = (D, I)$

Constant assignments

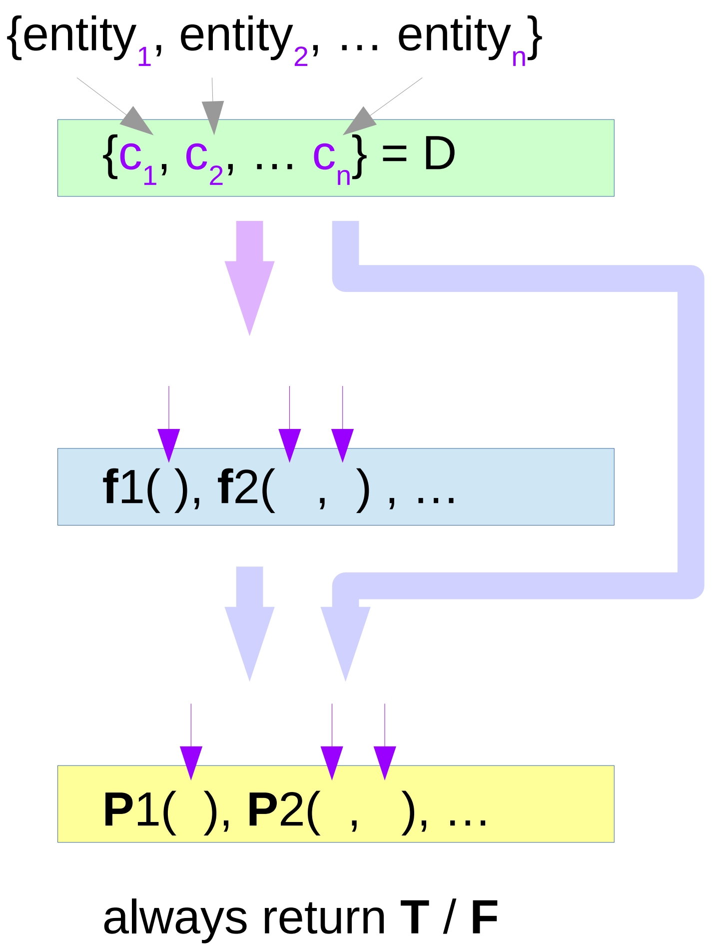
objects

Function assignments

functions

Truth value assignments

relations



Interpretation – assigning atoms

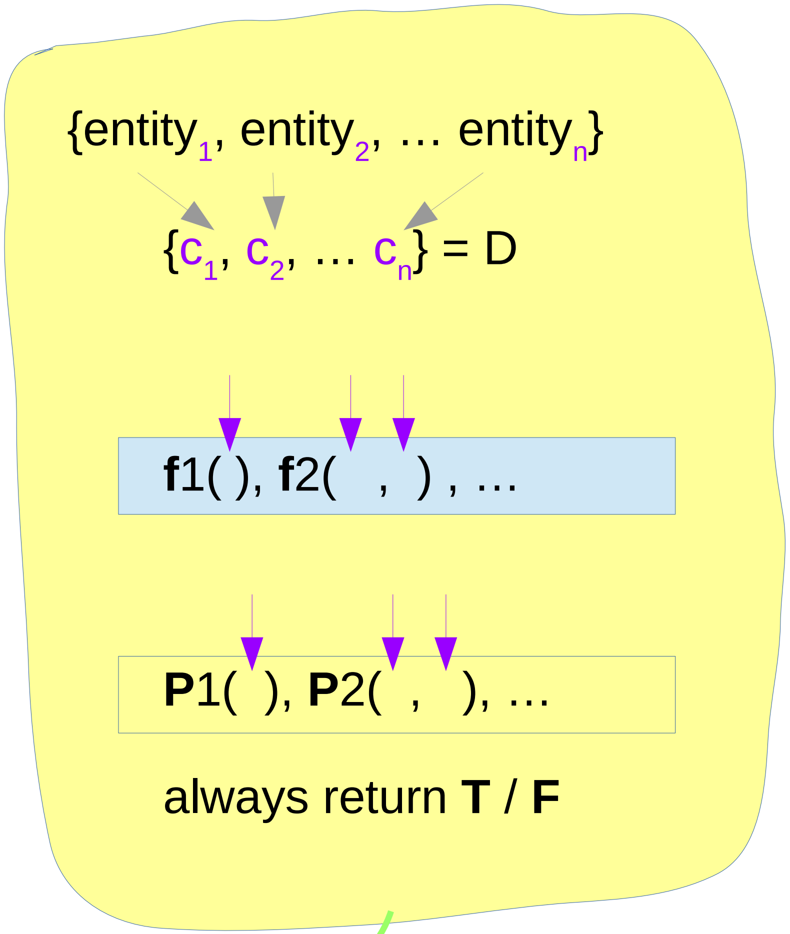
Propositional Logic

	A	B	
Interpretation I_1 ➡	T	T	
Interpretation I_2 ➡	T	F	
Interpretation I_3 ➡	F	T	
Interpretation I_4 ➡	F	F	

First Order Logic

Sentences

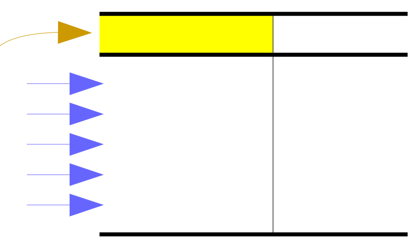
	P1( ) … P2( ) …		S1 S2
Interpretation I_1 ➡	T	T	
Interpretation I_2 ➡	T	F	
Interpretation I_3 ➡	F	T	
Interpretation I_4 ➡	F	F	



To assign truth values to predicates, constants and functions must be assigned

A **model** or a **possible world**:

Every **atomic proposition** is assigned a value **T** or **F**

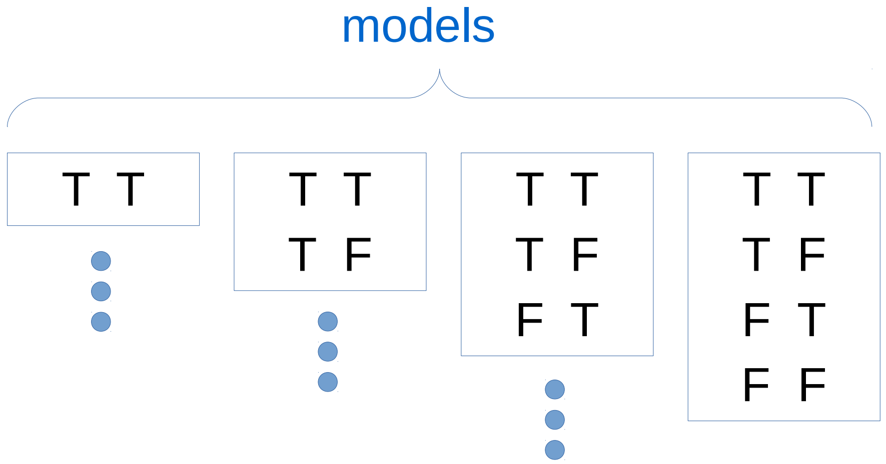


The set of **all** these assignments constitutes
A **model** or a **possible world**

All possible worlds (assignments) are permissible

A	B	A∧B	A∧B ⇒ A
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

Every atomic proposition : A, B



$$2^4 = 16$$

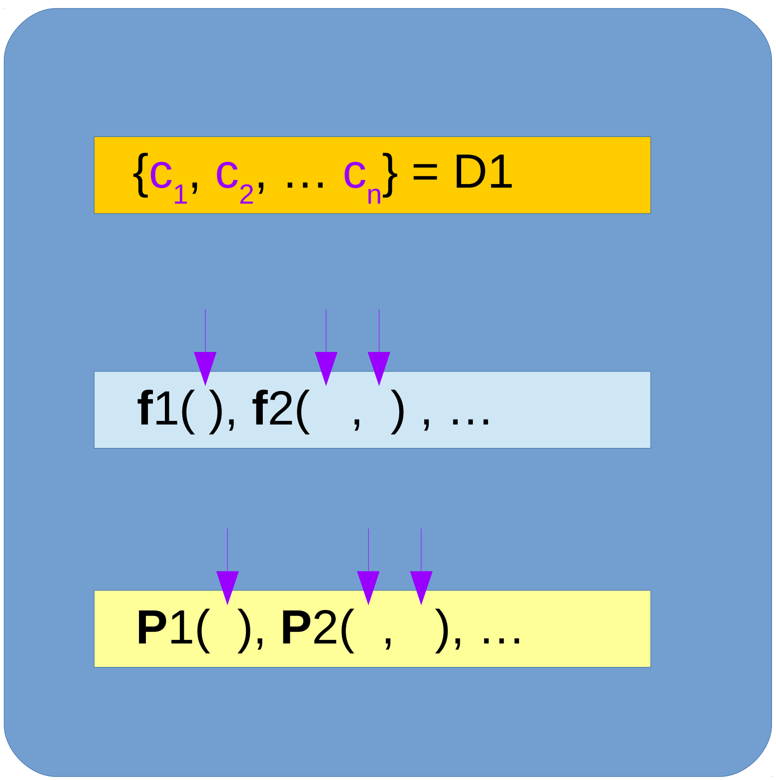
Models and Signatures

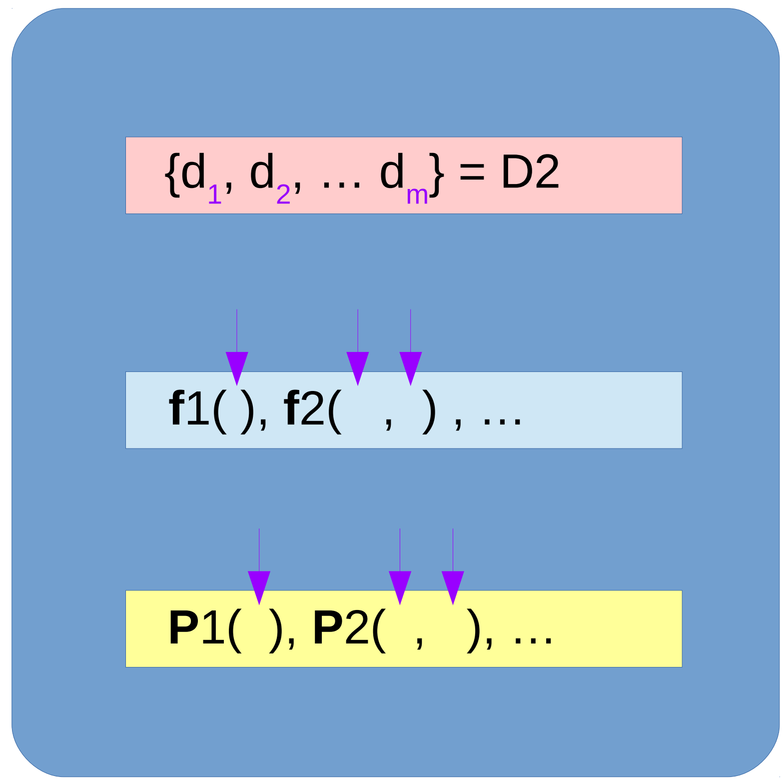
$\mathbf{M} = (\mathbf{D}, \mathbf{I})$

Different sets of constants (entities or objects)

{John, Baker, …, Paul} = D1

{Mary, Jane, …, Elizabeth} = D2

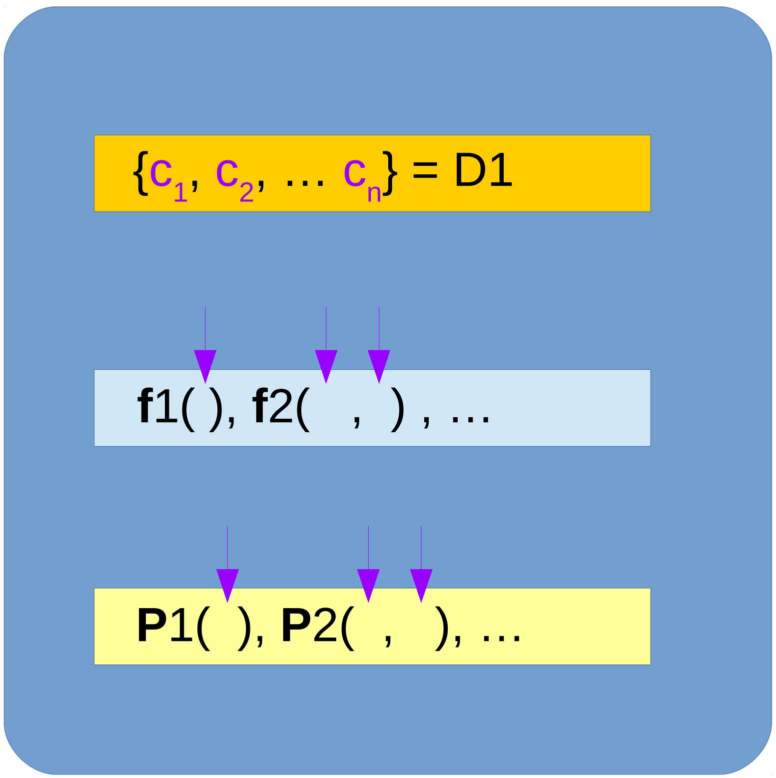




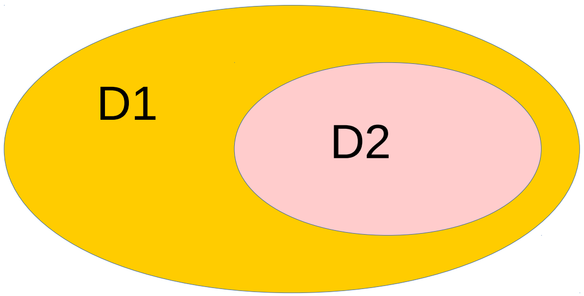
Models and Signatures

$\mathbf{M} = (\mathbf{D}, \mathbf{I})$

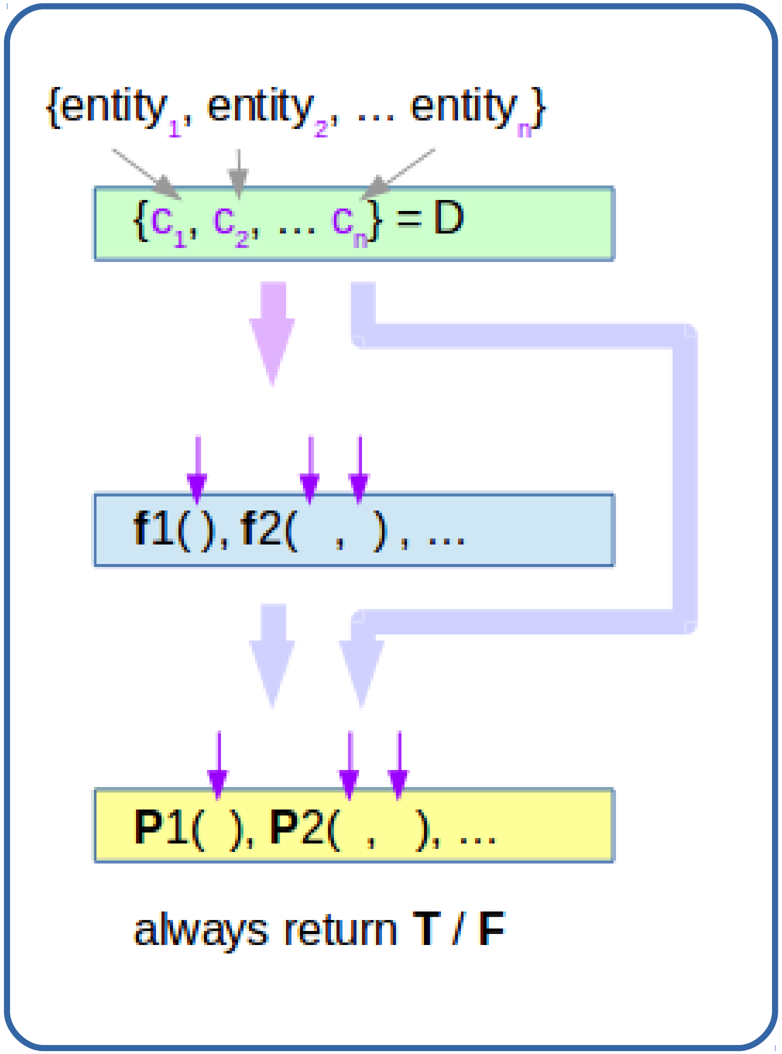
a subset of constants



$\{d_1, d_2, \ldots\ d_m\} = D2$



Truth values of sentences



terms	$x \quad y \quad f(x) \quad g(x,y)$
atomic formulas	$P(x) \qquad Q(x, f(y))$
formulas / sentences	$\forall x, y \ (P(x) \land \neg Q(x,y))$

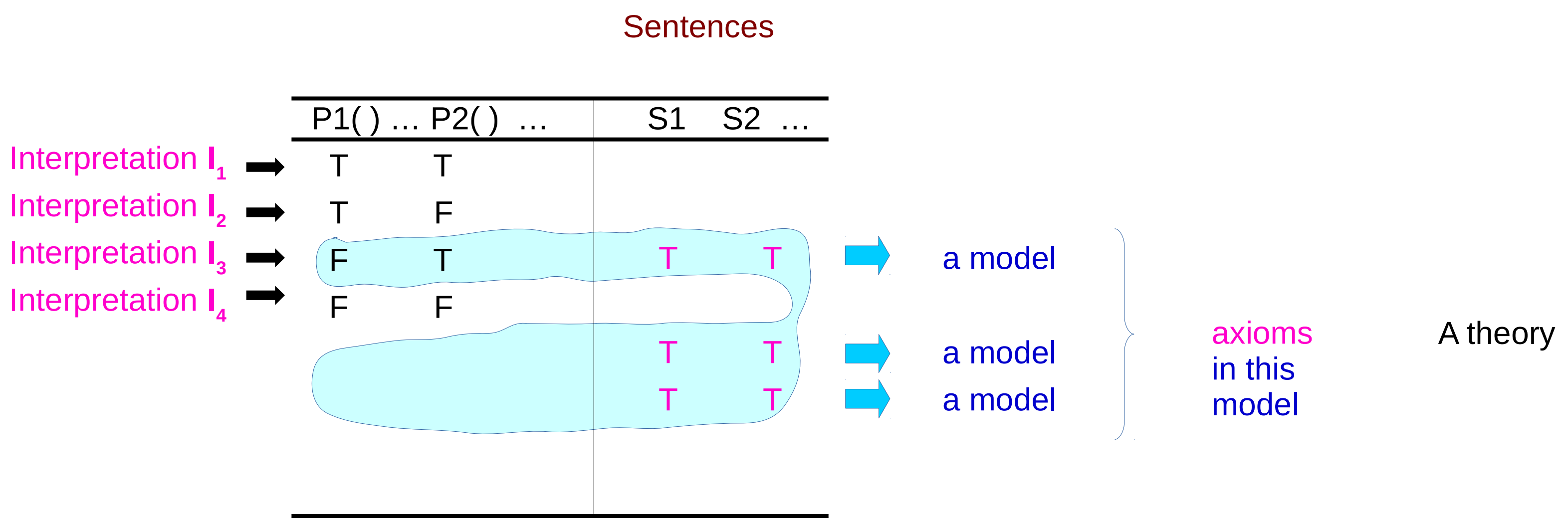
First Order Logic

Sentences

	P1( )	P2( )	…	S1	S2
Interpretation $\mathbf{I}_1$ ➡	T	T			
Interpretation $\mathbf{I}_2$ ➡	T	F			
Interpretation $\mathbf{I}_3$ ➡	F	T			
Interpretation $\mathbf{I}_4$ ➡	F	F			

Model Theory (1)

A first-order theory of a particular signature is
a set of axioms,
which are sentences
consisting of symbols from that signature.

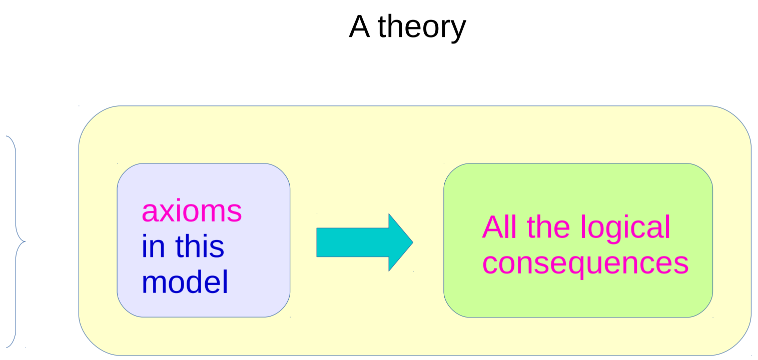


https://en.wikipedia.org/wiki/First-order_logic#First-order_theories.2C_models.2C_and_elementary_classes

Model Theory (2)

The set of axioms is
often **finite** or **recursively enumerable**,
in which case the theory is called **effective**.

Sometimes theories often include
all logical consequences of the axioms.



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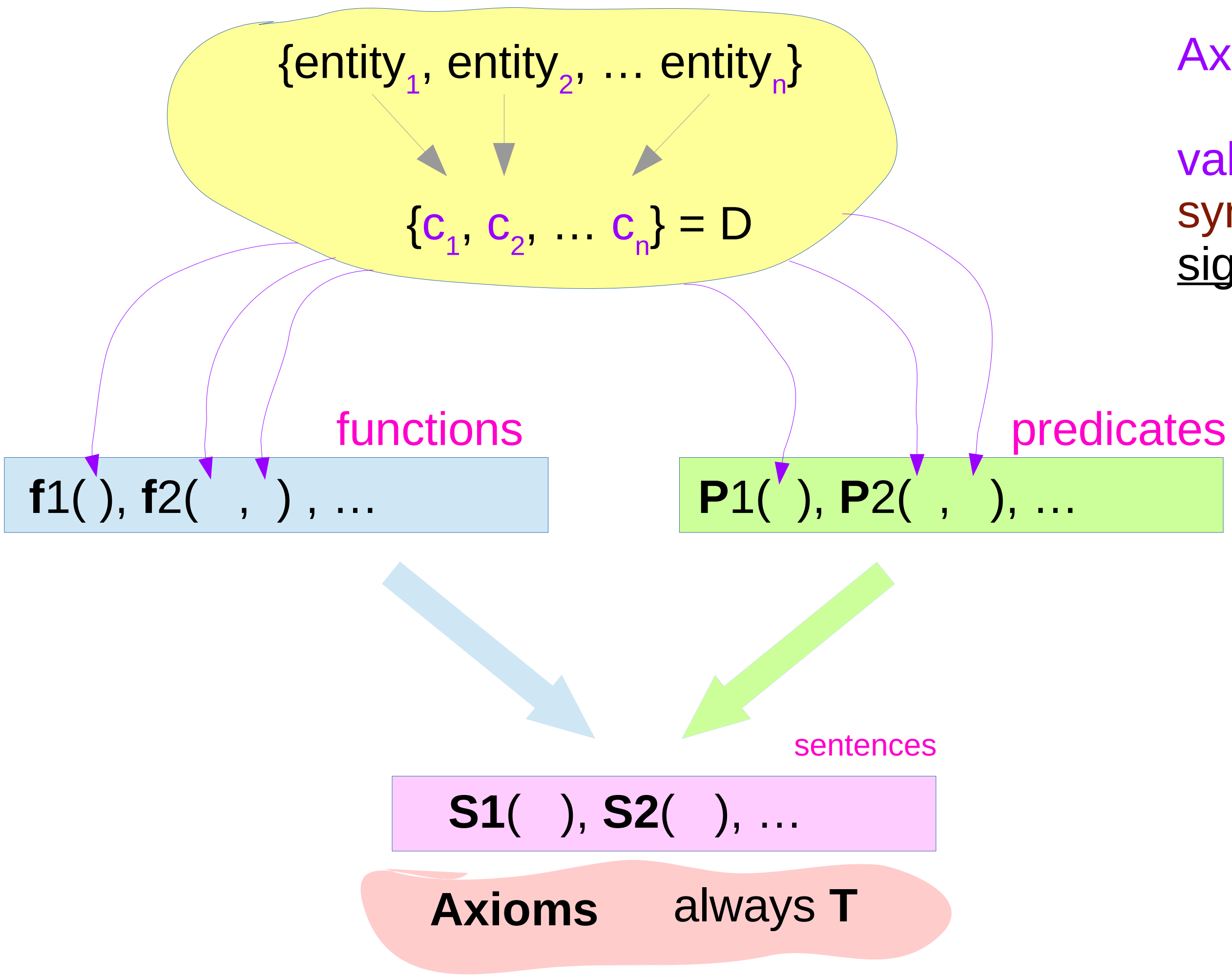
Axioms of a model theory



Axioms

valid sentences consisting of symbols from a particular signature.

Models

Propositional Logic

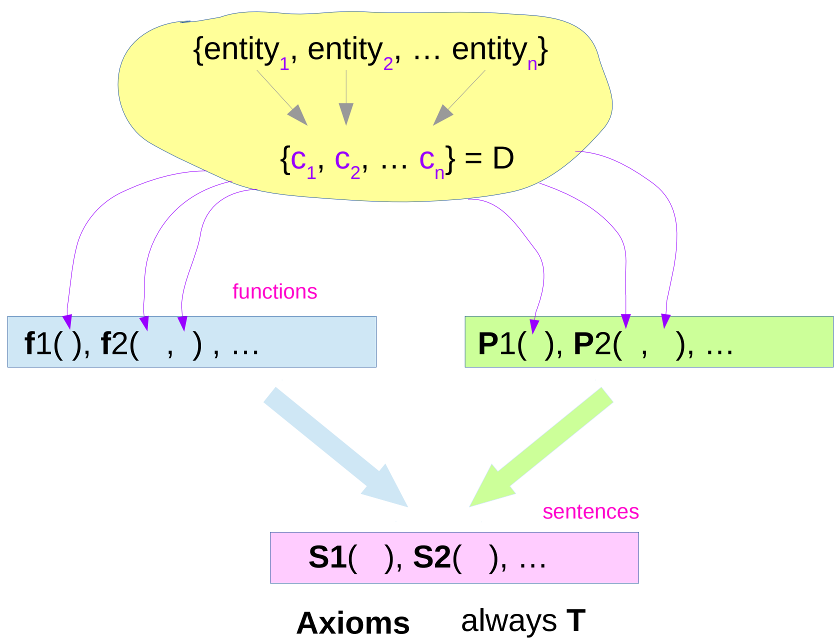
	A	B					
Interpretation I_1 ➡	T	T					
Interpretation I_2 ➡	T	F	T	T	T	➡	a model
Interpretation I_3 ➡	F	T	T	T	T	➡	a model
Interpretation I_4 ➡	F	F					

First Order Logic

Sentences

	P1( ) …	P2( ) …	S1	S2 …		
Interpretation I_1 ➡	T	T				
Interpretation I_2 ➡	T	F				
Interpretation I_3 ➡	F	T	T	T	➡	a model
Interpretation I_4 ➡	F	F				
			T	T	➡	a model
			T	T	➡	a model

Signature



Axioms

Logical Axioms	- axioms
Non-logical Axioms	- postulate – deductive system

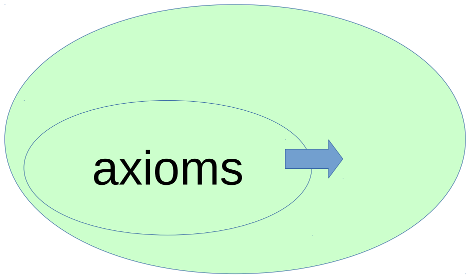
https://en.wikipedia.org/wiki/Axiom

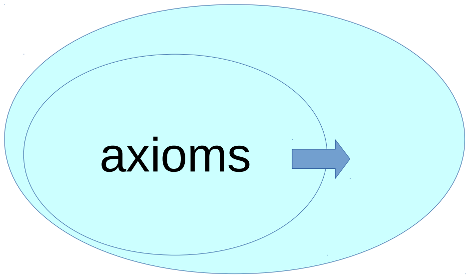
Logical Axioms

- formulas in a formal language that are universally valid
- formulas that are satisfied by every assignment of values (interpretations)

usually one takes as **logical axioms**
at least some minimal set of tautologies
that is sufficient for proving all tautologies in the language

in the case of predicate logic *more* **logical axioms** than that are required,
in order to prove logical truths that are not tautologies in the strict sense.





https://en.wikipedia.org/wiki/Axiom

Non-logical Axioms

formulas that play the role of theory-specific assumptions

reasoning about two different structures,
for example the natural numbers and the integers,
may involve the same **logical axioms**;

the purpose is to find out
what is special about *a particular structure*
(or set of structures, such as groups).

Thus non-logical axioms are not tautologies.

https://en.wikipedia.org/wiki/Axiom

Mathematical Discourse

Also called

- **postulate**
- **axioms in mathematical discourse**

this does not mean that it is claimed
that they are true in some absolute sense

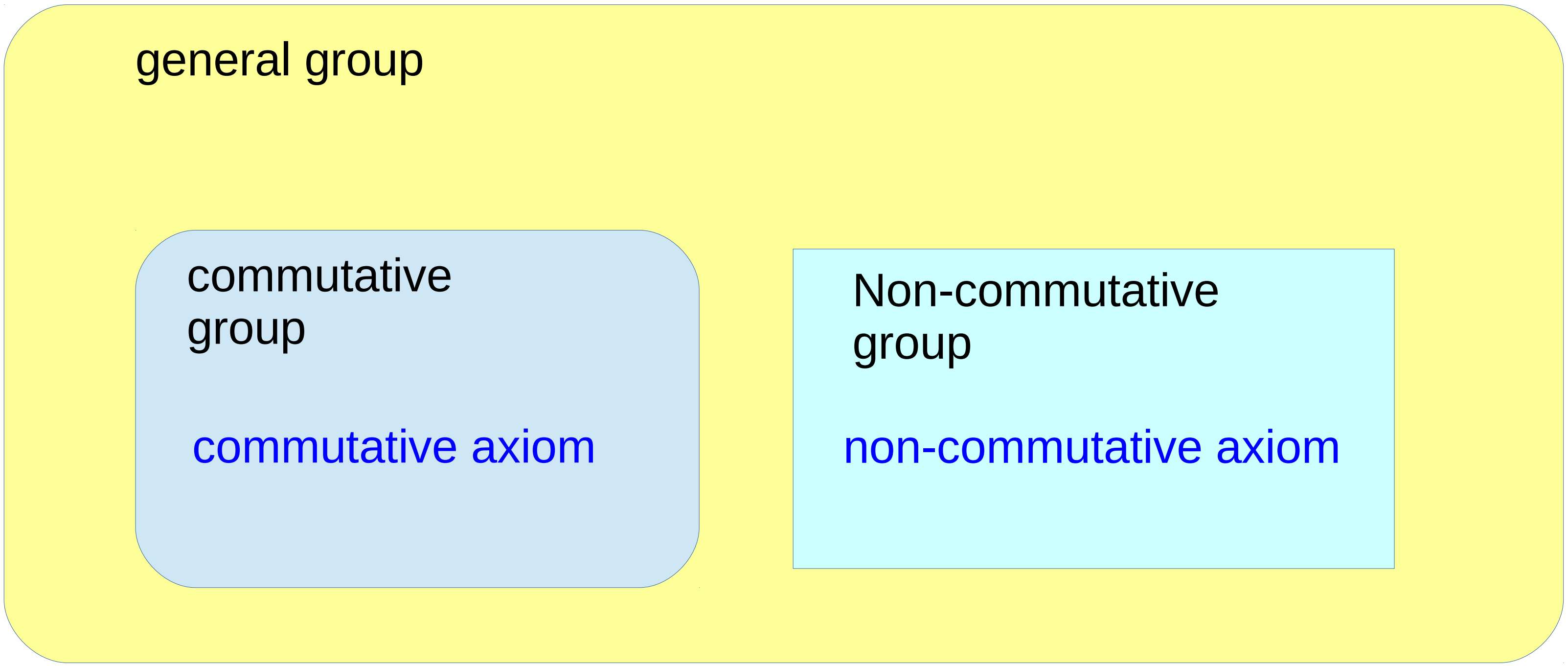
an elementary basis for a formal logic system

A deductive system

- **axioms** (non-logical)
- **rules of inference**

https://en.wikipedia.org/wiki/Axiom

Need not be tautologies



this does not mean that it is claimed

that they are true in some absolute sense

- Commutative axiom
- Non-commutative axiom

https://en.wikipedia.org/wiki/Axiom

Model Theory

The axioms are considered
to *hold* within the theory and

From axioms,
other sentences that *hold* within the theory
can be derived.

A first-order structure
that satisfies **all** sentences in a given theory
is said to be a **model** of the theory.

An elementary class is the set of **all** structures satisfying a particular theory.

These classes are a main subject of study in model theory.

https://en.wikipedia.org/wiki/First-order_logic#First-order_theories.2C_models.2C_and_elementary_classes

Truth values of sentences

Entailment in propositional logic can be computed
By **enumerating** the possible worlds (i.e. model checking)

How to **enumerate** possible worlds in FOL?

For each number of domain number n from 1 to infinity
For each k-ary predicate P_k in the vocabulary
For each possible k-ary relation on n objects
For each constant symbol C in the vocabulary
For each choice of referent for C from n objects. ..

Computing entailment in this way is not easy.

https://www.cs.umd.edu/~nau/cmsc421/chapter08.pdf

Truth values of sentences

domain number **n** [1, ∞)

k-ary predicate $\mathbf{P}_k$

k-ary relation $\mathbf{f}_k$ on n objects

constant symbol **C**

referent for C from n objects. ..

https://www.cs.umd.edu/~nau/cmsc421/chapter08.pdf

Model – domain of discourse

1. a nonempty set D of **entities** called a **domain of discourse**
   - this domain is a set
   - each element in the set : entity
   - each constant symbol : one entity in the domain

If we considering all individuals in a class,
The constant symbols might be
‘Mary’, - an entity
‘Fred’, - an entity
‘John’, - an entity
‘Tom’ - an entity

Model – interpretation

2. an **interpretation**

   (a) an entity in D is assigned to each of the constant symbols.

      Normally, every entity is assigned to a constant symbol.

   (b) for each **function**,

      an entity is assigned to each possible input of entities to the **function**

   (c) the predicate '**True**' is always assigned the value T

      The predicate '**False**' is always assigned the value F

   (d) for every other **predicate**,

      the value T or F is assigned

      to each possible input of entities to the **predicate**

Each possible input of entities

$\{entity_1, entity_2, \ldots\ entity_n\}$

$\{c_1, c_2, \ldots\ c_n\} = D$

Arity one: C(n, 1)
Arity two: C(n, 2)
Arity three: C(n, 3)
…

f1( ), **f**2(,) , …

Arity one functions & predicates: C(n, 1)
Arity two: C(n, 2)
Arity three: C(n, 3)
…

P1( ), **P**2(,), …

always return **T** / **F**

Interpretation

Constant assignments

(a) an entity → the constant symbols.

Function assignments

(b) an entity → each possible input of entities to the **function**

Truth value assignments

(c) the value T →the predicate '**True**'
the value F → the predicate '**False**'

(d) for every other **predicate**,
the value T or F is assigned → every other predicate
to each possible input of entities to the **predicate**

Signature Model Examples A – (1)

Signature

1. constant symbols = { Mary, Fred, Sam }
2. predicate symbols = { married, young }
   married(x, y) : arity two
   young(x) : arity one

Model

1. domain of discourse D : the set of three particular *individuals*
   - this domain is a set
   - each element in the set : entity (= individuals)
   - each constant symbol : one entity in the domain (= one individual)
2. interpretation
   (a) a different individual is assigned to each of the constant symbols

   (a) an entity in D is assigned to each of the constant symbols.
   Normally, every entity is assigned to a constant symbol.

Signature Model Examples A – (2)

(b) for each **function**,
an entity is assigned to each possible input of entities to the **function**

(c) the predicate '**True**' is always assigned the value T
The predicate '**False**' is always assigned the value F

(d) the truth value assignments for every predicate

young(Mary) = F, young(Fred) = F, young(Sam) = T

married(Mary, Mary) = F, married(Mary, Fred) = T, married(Mary, Sam) = F
married(Fred, Mary) = T, married(Fred, Fred) = F, married(Fred, Sam) = F
married(Sam, Mary) = F, married(Sam, Fred) = F, married(Sam, Sam) = F

(d) for every other **predicate**,
the value T or F is assigned
to each possible input of entities to the **predicate**

(Mary, Mary), (Mary, Fred), (Mary, Sam)
(Fred, Mary), (Fred, Fred), (Fred, Sam)
(Sam, Mary), (Sam, Fred), (Sam, Sam)

Signature Model Examples B – (1)

Signature

1. constant symbols = { Fred, Mary, Sam }
2. predicate symbols = { love } love(x, y) : arity two
3. function symbols = { mother } mother(x) : arity one

Model

1. domain of discourse D : the set of three particular individuals
2. interpretation
   (a) a different individual is assigned to each of the constant symbols

   (b) the truth value assignments for every predicate
   love(Fred, Fred) = F, love(Fred, Mary) = F, love(Fred, Ann) = F
   love(Mary, Fred) = T, love(Mary, Mary) = F, love(Mary, Ann) = T
   love(Ann, Fred) = T, love(Ann, Mary) = T, love(Ann, Ann) = F

   (c) the function assignments
   mother(Fred) = Mary, mother(Mary) = Ann, mother(Ann) = - (no assignment)

Signature Model Examples B – (2)

2. interpretation

(a) a different individual is assigned to each of the constant symbols

(a) an entity in D is assigned to each of the constant symbols.
Normally, every entity is assigned to a constant symbol.

(b) the truth value assignments

(b) for each **function**,
an entity is assigned to each possible input of entities to the **function**

love(Fred, Fred) = F, love(Fred, Mary) = F, love(Fred, Ann) = F
love(Mary, Fred) = T, love(Mary, Mary) = F, love(Mary, Ann) = T
love(Ann, Fred) = T, love(Ann, Mary) = T, love(Ann, Ann) = F

(c) the function assignments

(d) for every other **predicate**,
the value T or F is assigned
to each possible input of entities to the **predicate**

mother(Fred) = Mary, mother(Mary) = Ann, mother(Ann) = - (no assignment)

The truth value of sentences

The truth values of **all sentences** are assigned :

1. the truth values for **sentences** developed with the symbols ¬, ∧, ∨, ⇒, ⇔ are assigned as in propositional logic.

2. the truth values for two terms connected by the = symbol is **T** if both terms refer to the same entity; otherwise it is **F**

3. the truth values for ∀x p(x) has value **T** if p(x) has value **T** for **every assignment** to x of an entity in the domain D; otherwise it has value **F**

4. the truth values for ∃x p(x) has value **T** if p(x) has value **T** for **at least one assignment** to x of an entity in the domain D; otherwise it has value **F**

5. the operator **precedence** is as follows ¬, =, ∧, ∨, ⇒, ⇔

6. the **quantifiers** have precedence over the operators

7. **parentheses** change the order of the precedence

Formulas and Sentences

An **formula**
- A **atomic formula**
- The operator ¬ followed by a **formula**
- Two formulas separated by ∧, ∨, ⇒, ⇔
- A quantifier following by a variable followed by a formula

A **sentence**
- A **formula** with no free variables

∀x love(x,y)	: free variable y	: not a sentence
∀x tall(x)	: no free variable	: a sentence

Finding the truth value

Find the truth values of **all sentences**

1. ¬, ∧, ∨, ⇒, ⇔

2. = symbol

3. ∀x p(x)

4. ∃x p(x)

5. the **operator precedence** is as follows ¬, =, ∧, ∨, ⇒, ⇔

6. the **quantifiers** (∀, ∃) have precedence over the **operators**

7. **parentheses** change the order of the precedence

Sentence Examples (1)

Signature

Constant Symbols = {Socrates, Plato, Zeus, Fido}
Predicate Symbols = {human, mortal, legs} all arity one

Model
D: the set of these four particular individuals

Interpretation

(a) a different individual is assigned to each of the constant symbols

(b) the truth value assignment

human(Socrates)=T, human(Plato)=T, human(Zeus)=F, human(Fido)=F
mortal(Socrates)=T, mortal(Plato)=T, mortal(Zeus)=F, mortal(Fido)=T
legs(Socrates)=T, legs(Plato)=T, legs(Zeus)=T, legs(Fido)=T

Sentence Examples (2)

Sentence 1: $human(Zeus) \land human(Fido) \lor human(Socrates) = T$

$F \land F \lor T$

Sentence 2: $human(Zeus) \land (human(Fido) \lor human(Socrates)) = F$

$F \land (F \lor T)$

Sentence 3: $\forall x\ human(x) = F$

$human(Zeus)=F$, $human(Fido)=F$

Sentence 4: $\forall x\ mortal(x) = F$

$mortal(Zeus)=F$

Sentence 5: $\forall x\ legs(x) = T$

$legs(Socrates)=T$, $legs(Plato)=T$, $legs(Zeus)=T$, $legs(Fido)=T$

Sentence 6: $\exists x\ human(x) = T$

$human(Socrates)=T$, $human(Plato)=T$

Sentence 7: $\forall x\ (human(x) \Rightarrow mortal(x)) = T$

Sentence Examples (3)

Sentence 7: $\forall x\ (human(x) \Rightarrow mortal(x)) = T$

human(Socrates)=T,	mortal(Socrates)=T,	$T \Rightarrow T$: T
human(Plato)=T,	mortal(Plato)=T,	$T \Rightarrow T$: T
human(Zeus)=F,	mortal(Zeus)=F,	$F \Rightarrow F$: T
human(Fido)=F	mortal(Fido)=T	$F \Rightarrow T$: T

A **vocabulary τ** is a set consisting of
relation symbols, function symbols and constant symbols.

relation symbols such as P , Q, R, ≤, . . . , (arity ≥ 1)
function symbols such as f, g, h, ·, +, . . . , (arity ≥ 1)
constant symbols such as c, d, 0, 1, ...

http://www.math.uni-hamburg.de/home/geschke/teaching/ModelTheory.pdf

Model Theory (1)

Given a **vocabulary** τ .

A **structure** $\mathcal{A}$ for τ (a τ-structure) is a

nonempty set $\mathcal{A}$ together with

(S1) relations $R^{\mathcal{A}} \subseteq \mathcal{A}^n$ for every *n-ary relation* symbol $R \in \tau$,

(S2) functions $f^{\mathcal{A}} : \mathcal{A}^m \rightarrow \mathcal{A}$ for every *m-ary function* symbol $f \in \tau$ and

(S3) constants $c^{\mathcal{A}} \in \mathcal{A}$ for every *constant* symbol $c \in \tau$

.

http://www.math.uni-hamburg.de/home/geschke/teaching/ModelTheory.pdf

Model Theory (2)

If $\varphi(x,x_1,...,x_n)$ is a formula,

then $\exists x\ \varphi(a_1,...,a_n)$ holds in $\mathcal{A}$

iff there is $a \in \mathcal{A}$ such that $\varphi(a,a_1,...,a_n)$ holds in $\mathcal{A}$.

If $\varphi(a_1,...,a_n)$ holds in $\mathcal{A}$, we write $\mathcal{A} \vDash \varphi(a_1,...,a_n)$.

We extend the model relation $\vDash$ to (possibly infinite) sets of formulas.

http://www.math.uni-hamburg.de/home/geschke/teaching/ModelTheory.pdf

Model Theory (3)

Let a: Var→*A* be any function,

an assignment of elements of *A* to each of the variables.

Also, let Φ be a set of formulas over τ.

Then Φ holds in A under the assignment a (or with respect toa)

iff for every formula $\varphi(x_1,...,x_n) \in \Phi$ we have $A \vDash \varphi(a(x_1),...,a(x_n))$.

In this case we write $A \vDash \Phi[a]$ and say that (A,a) is a model of Φ.

If Φ holds in A with respect to every assignment,

then we write $A \vDash \Phi$ and say that A is a model of Φ.

http://www.math.uni-hamburg.de/home/geschke/teaching/ModelTheory.pdf

Model Theory (4)

Basically, when we define a proof system, we want that it is sound and complete. In the "most general" form, we expect that :

$$\Gamma \vdash \varphi \text{ iff } \Gamma \vDash \varphi.$$

About soundenss: no problem, this is the easy task, while regarding completeness, we may have some "imperfection".

For example, in Mendelson's proof system we have generalization and the (standard) definition of derivation allows us to have :

$$P(x) \vdash \forall x\, P(x) \quad .$$

Of course, M's proof system is sound; due to the restrictions on the Deduction Theorem, we cannot derive the (invalid) : $\vdash P(x) \rightarrow \forall x P(x)$

https://math.stackexchange.com/questions/131706/what-is-the-common-definition-of-model-in-first-order-logic

Model Theory (5)

Why ? Because the semantics give us : B
logically implies A iff $B \rightarrow A$ is valid, and we know that $P(x) \rightarrow \forall x P(x)$

is not valid !

In conclusion, Mendelson is not licensed to state that, in general :

if $\Gamma \vdash \varphi$

, then $\Gamma \vDash \varphi$

and he does not state it ...

https://math.stackexchange.com/questions/131706/what-is-the-common-definition-of-model-in-first-order-logic

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