Monad Background (3A)

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This document was produced by using OpenOffice.

Based on

Haskell in 5 steps
https://wiki.haskell.org/Haskell_in_5_steps

A List Comprehension Function

let removeLower x = [c | c **<-** x, c `elem` ['A'..'Z']]

a **list comprehension**

```
removeLower x
   [ c |
        c <- x,
        c `elem` ['A'..'Z']
   ]
```

“Hello”

```
[ c: ‘H’
  c: ‘e’
  c: ‘l’
  c: ‘l’
  c: ‘o’  ]
```

“H”

```
do { x1 <- action1
   ; x2 <- action2
   ; mk_action3 x1 x2 }
```

x1 : Return value of action1

x2: Return value of action2

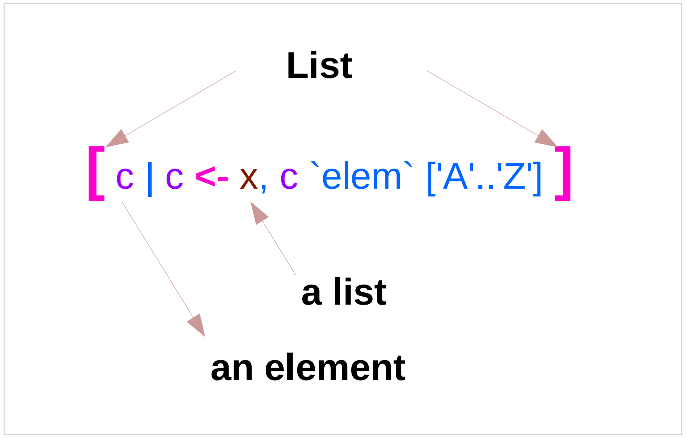
https://stackoverflow.com/questions/35198897/does-mean-assigning-a-variable-in-haskell

Pattern and Predicate

let removeLower x = [c | c **<-** x, c `elem` ['A'..'Z']]

a **list comprehension**

[c | c **<-** x, c `elem` ['A'..'Z']]



- c **<-** x is a **generator**
  - (x : argument of the function **removeLower**)
- c is a **pattern**
  - matching from the **elements** of the **list x**
  - successive binding of c to the **elements** of the **list x**
- c `elem` ['A'..'Z']
  - is a **predicate** which is applied to each successive binding of c
  - Only c which passes this predicate will appear in the output list

https://stackoverflow.com/questions/35198897/does-mean-assigning-a-variable-in-haskell

Assignment in Haskell

Assignment in Haskell : declaration with initialization:

- no uninitialized variables,
- must declare with an initial value
- no mutation
- a variable keeps its initial value throughout its scope.

https://stackoverflow.com/questions/35198897/does-mean-assigning-a-variable-in-haskell

Generator

```
[c| c <- x, c `elem` ['A'..'Z']]

filter (`elem` ['A' .. 'Z']) x
```

```
[ c | c <- x ]
```

c: an element
x: a list

```
do c <- x
   return c
```

```
x >>= ( \c -> return c )
```

```
x >>= return
```

c: an element
x: an element

or

c: a list
x: a list

```
action1 >>= (\ x1 ->
  action2 >>= (\ x2 ->
    mk_action3 x1 x2 ))
```

https://stackoverflow.com/questions/35198897/does-mean-assigning-a-variable-in-haskell

Anonymous Functions

```
(\x -> x + 1) 4
5 :: Integer
```

```
(\x y -> x + y) 3 5
8 :: Integer
```

```
inc1 = \x -> x + 1
```

```
incListA lst = map inc2 lst
    where inc2 x = x + 1
```

```
incListB lst = map (\x -> x + 1) lst
```

```
incListC = map (+1)
```

https://wiki.haskell.org/Anonymous_function

Then Operator (>>) and **do** Statements

a chain of actions

to *sequence* input / output operations

the (>>) (**then**) operator works almost identically in **do** notation

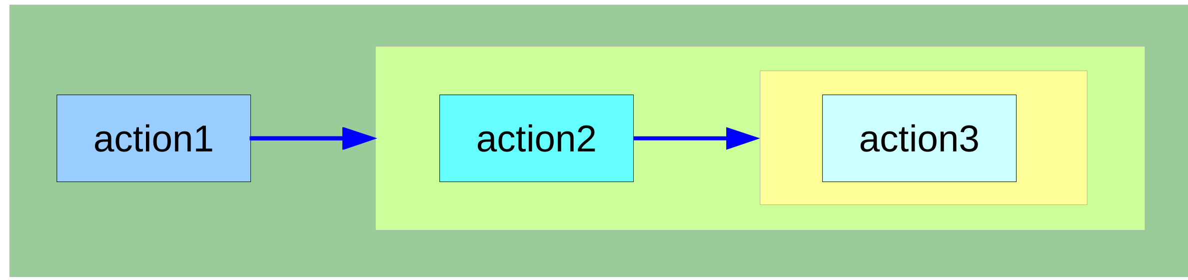
```
putStr "Hello"  >>
putStr " "      >>
putStr "world!" >>
putStr "\n"
```

```
do { putStr "Hello"
   ; putStr " "
   ; putStr "world!"
   ; putStr "\n" }
```

https://en.wikibooks.org/wiki/Haskell/do_notation

Chaining in **do** and >> notations

```
do { action1
   ; action2
   ; action3 }
```




```
do { action1
   ; do { action2
        ; action3 } }
```

```
action1 >>
do { action2
   ; action3 }
```

can **chain** any actions
all of which are in **the same monad**

```
do { action1
   ; do { action2
        ; do { action3 } } }
```

```
action1 >>
  action2 >>
    action3
```

https://en.wikibooks.org/wiki/Haskell/do_notation

Bind Operator (>==) and **do** statements

The bind operator (>>=)
passes a value ->
(the result of an action or function),
downstream in the binding sequence.

do notation assigns a variable name
to the passed value using the <-

```
action1 >>= (\ x1 ->
  action2 >>= (\ x2 ->
    mk_action3 x1 x2 ))
```

```
do { x1 <- action1
   ; x2 <- action2
   ; mk_action3 x1 x2 }
```

anonymous function
(lambda expression)
is used

https://en.wikibooks.org/wiki/Haskell/do_notation

Chaining >>= and do notations

->

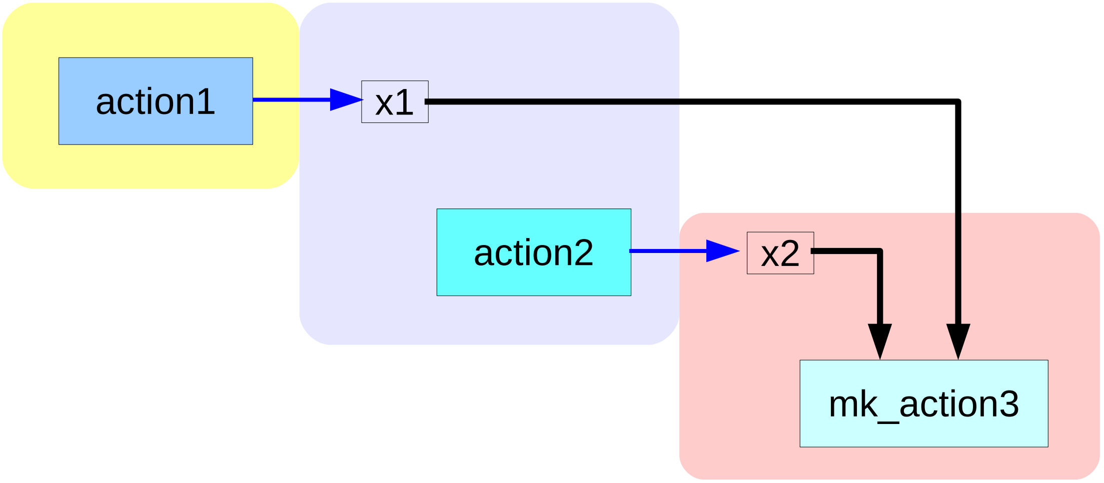
```
action1 >>= (\ x1 -> action2 >>= (\ x2 -> mk_action3 x1 x2 ))
```

```
action1
  >>=
    (\ x1 -> action2
      >>=
        (\ x2 -> mk_action3 x1 x2 ))
```

```
action1 >>= (\ x1 ->
  action2 >>= (\ x2 ->
    mk_action3 x1 x2 ))
```

<-

```
do { x1 <- action1
   ; x2 <- action2
   ; mk_action3 x1 x2 }
```



https://en.wikibooks.org/wiki/Haskell/do_notation

fail method

```
do { Just x1 <- action1
   ;      x2 <- action2
   ; mk_action3 x1 x2   }
```

```
do { x1 <- action1
   ; x2 <- action2
   ; mk_action3 x1 x2 }
```

O.K. when action1 returns **Just** x1

when action1 returns **Nothing**
crash with an non-exhaustive patterns error

Handling failure with **fail** method

```
action1 >>= f where
   f (Just x1) = do { x2 <- action2
                    ; mk_action3 x1 x2 }
   f _         = fail "..."
```

-- *A compiler-generated message.*

https://en.wikibooks.org/wiki/Haskell/do_notation

Example

```
nameDo :: IO ()
nameDo = do { putStr "What is your first name? "
            ; first <- getLine
            ; putStr "And your last name? "
            ; last <- getLine
            ; let full = first ++ " " ++ last
            ; putStrLn ("Pleased to meet you, " ++ full ++ "!") }
```

```
do { x1 <- action1
   ; x2 <- action2
   ; mk_action3 x1 x2 }
```

using the **do** statement

A possible translation into vanilla monadic code:

```
nameLambda :: IO ()
nameLambda = putStr "What is your first name? " >>
             getLine >>= \ first ->
             putStr "And your last name? " >>
             getLine >>= \ last ->
             let full = first ++ " " ++ last
             in putStrLn ("Pleased to meet you, " ++ full ++ "!")
```

using **then (>>)** and **Bind (>>=)** operators

https://en.wikibooks.org/wiki/Haskell/do_notation

return method

```
nameReturn :: IO String
nameReturn = do putStr "What is your first name? "
                first <- getLine
                putStr "And your last name? "
                last <- getLine
                let full = first ++ " " ++ last
                putStrLn ("Pleased to meet you, " ++ full ++ "!")
                return full
```

```
greetAndSeeYou :: IO ()
greetAndSeeYou = do name <- nameReturn
                    putStrLn ("See you, " ++ name ++ "!")
```

https://en.wikibooks.org/wiki/Haskell/do_notation

Without a **return** method

```
nameReturn :: IO String
nameReturn = do putStr "What is your first name? "
                first <- getLine
                putStr "And your last name? "
                last <- getLine
                let full = first ++ " " ++ last
                putStrLn ("Pleased to meet you, " ++ full ++ "!")
                return full
```

explicit return statement

returns **IO String** monad

```
nameDo :: IO ()
nameDo = do { putStr "What is your first name? "
            ; first <- getLine
            ; putStr "And your last name? "
            ; last <- getLine
            ; let full = first ++ " " ++ last
            ; putStrLn ("Pleased to meet you, " ++ full ++ "!") }
```

no return statement

returns **empty IO** monad

https://en.wikibooks.org/wiki/Haskell/do_notation

return method – not a final statement

```
nameReturnAndCarryOn :: IO ()
nameReturnAndCarryOn = do putStr "What is your first name? "
                          first <- getLine
                          putStr "And your last name? "
                          last <- getLine
                          let full = first++" "++last
                          putStrLn ("Pleased to meet you, "++full++"!")
                          return full
                          putStrLn "I am not finished yet!"
```

the return statement does not interrupt the flow

the last statements of the sequence returns a value

https://en.wikibooks.org/wiki/Haskell/do_notation

Data Constructor

data **Color** = **Red | Green | Blue**

Color is a type

Red is a *constructor* that contains a *value* of type **Color**.
Green is a *constructor* that contains a *value* of type **Color**.
Blue is a *constructor* that contains a *value* of type **Color**.

https://stackoverflow.com/questions/18204308/haskell-type-vs-data-constructor

Data Constructor with Parameters

data **Color** = **RGB** Int Int Int

Color is a type

RGB is not a value but a *function* taking three Int's and *returning a value*

RGB :: Int -> Int -> Int -> Color

RGB is a **data constructor** that is a *function*
taking three Int values as its arguments,
and then uses them to construct a new value.

https://stackoverflow.com/questions/18204308/haskell-type-vs-data-constructor

Type Constructor

Consider a binary tree to store Strings

```
data SBTree = Leaf String | Branch String SBTree SBTree
```

a type

SBTree	is a **type**
Leaf	is a **data constructor** (a function)
Branch	is a **data constructor** (a function)

```
Leaf :: String -> SBTree
Branch :: String -> SBTree -> SBTree -> SBTree
```

https://stackoverflow.com/questions/18204308/haskell-type-vs-data-constructor

Similar Type Constructors

Consider a binary tree to store Strings

```
data SBTree = Leaf String | Branch String SBTree SBTree
```

Consider a binary tree to store Bool

```
data BBTree = Leaf Bool | Branch Bool BBTree BBTree
```

Consider a binary tree to store a parameter type

```
data BTree a = Leaf a | Branch a (BTree a) (BTree a)
```

https://stackoverflow.com/questions/18204308/haskell-type-vs-data-constructor

Type Constructor with a Parameter

Type constructors

Both **SBTree** and **BBTree** are type constructors

```
data SBTree = Leaf String  |  Branch String SBTree SBTree
data BBTree = Leaf Bool   |  Branch Bool BBTree BBTree
```

```
data BTree a = Leaf a  |  Branch a (BTree a) (BTree a)
```

Now we introduce a type variable a as a parameter to the type constructor.

BTree has become a function.
It takes a type as its argument and it returns a new type.

https://stackoverflow.com/questions/18204308/haskell-type-vs-data-constructor

Type Constructors and Data Constructors

A **type constructor**

- a "function" that takes 0 or more types
- gives you back a new **type**.

Type constructors with parameters
allows slight variations in types

type **SBTree** = **BTree** String
type **BBTree** = **BTree** Bool

A **data constructor**

- a "function" that takes 0 or more values
- gives you back a new **value**.

Data constructors with parameters
allows slight variations in values

RGB 12 92 27
#0c5c1b
RGB 255 0 0
RGB 0 255 0
RGB 0 0 255

https://stackoverflow.com/questions/18204308/haskell-type-vs-data-constructor

( )

( ) is both a **type** and a **value**.

( ) is a special **type**, pronounced “unit”,
has one **value** **( )**, sometimes pronounced “void”

the **unit type** has only one **value** which is called **unit**.

( ) :: ( ) **Type :: Expression**

It is the same as the void type **void** in Java or C/C++.

https://stackoverflow.com/questions/20380465/what-do-parentheses-used-on-their-own-mean

Unit Type

a **unit type** is a type that allows *only one value* (and thus can hold *no information*).

It is the same as the void type **void** in Java or C/C++.

```
:t
Expression :: Type
```

```
data Unit = Unit

Prelude> :t Unit
Unit :: Unit
```

```
Prelude> :t ()
() :: ()
```

https://stackoverflow.com/questions/20380465/what-do-parentheses-used-on-their-own-mean

Type Language and Expression Language

data **T**const **T**var … **T**var = **V**const type … type | …
Vconst type … type

A new datatype declaration

Tconst (Type Constructor) is added to *the type language*

Vconst (Value Constructor) is added to *the expression language* and *its pattern sublanguage*

must not appear in types

Argument types in **V**const type … type

are the types given to the arguments (**T**const **T**var … **T**var)

are used in expressions

https://stackoverflow.com/questions/16892570/what-is-in-haskell-exactly

Datatype Declaration Examples

data Tree a = **Leaf** | **Node** (Tree a) (Tree a)

Tree (Type Constructor)
Leaf or **Node** (Value Constructor)

data Type = **Value**

data ( ) = **( )**

( ) (Type Constructor)
( ) (Value Constructor)

the type (), often pronounced "Unit"
the value (), sometimes pronounced "void"

the type () containing only one value ()

https://stackoverflow.com/questions/16892570/what-is-in-haskell-exactly

Monadic Effect

```
class Monad m where
    return :: a -> m a
    (>>=) :: m a -> (a -> m b) -> m b
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/IO
https://stackoverflow.com/questions/2488646/why-are-side-effects-modeled-as-monads-in-haskell
https://stackoverflow.com/questions/7840126/why-monads-how-does-it-resolve-side-effects
https://stackoverflow.com/questions/2488646/why-are-side-effects-modeled-as-monads-in-haskell

https://www.cs.hmc.edu/~adavidso/monads.pdf

IO ( )

Monadic operations tend to have types which look like

val-in-type-1 -> ... -> val-in-type-n -> **effect-monad** val-out-type

where the **return type** is a type application:
the function tells you which **effects** are possible
and the argument tells you what sort of value
is produced by the operation

https://stackoverflow.com/questions/16892570/what-is-in-haskell-exactly

IO ( )

put :: s -> State s ( )

put :: s -> (State s) ( )

one value input type **s**

the **effect-monad State s**

the value output type **( )**

the operation is used *only for its effect*;

the *value* delivered is *uninteresting*

putStr :: String -> IO ()

delivers a string to stdout but does not return anything exciting.

https://stackoverflow.com/questions/16892570/what-is-in-haskell-exactly

Variable definition in a file

Var1.hs

```
r = 5
```

Var2.hs

```
r = 55
```

definition with initialization

```
young@Sys ~ $ ghci
GHCi, version 7.10.3: http://www.haskell.org/ghc/  :? for help
Prelude> :load Var1.hs
[1 of 1] Compiling Main             ( var.hs, interpreted )
Ok, modules loaded: Main.
*Main> r
5
*Main> :t r
r :: Integer
*Main>
*Main> :load Var2.hs
[1 of 1] Compiling Main             ( var2.hs, interpreted )
Ok, modules loaded: Main.
*Main> r
55
```

https://en.wikibooks.org/wiki/Haskell/Variables_and_functions

No Mutation

Var1.hs

```
r = 5
```

Var2.hs

```
r = 55
```

No mutation

```
*Main> r = 33
<interactive>:12:3: parse error on input '='



young@Sys ~ $ ghci
GHCi, version 7.10.3: http://www.haskell.org/ghc/  :? for help
Prelude> r = 333
<interactive>:2:3: parse error on input '='
Prelude>



let r = 33
```

https://en.wikibooks.org/wiki/Haskell/Variables_and_functions

Functional & Imperative Languages

Imperative programming:

- variables as changeable locations in a computer's memory
- imperative programs explicitly commands the computer what to do

functional programming

- a way to think in higher-level mathematical terms
- defining how variables relate to one another
- leaving the **compiler** to translate these

  to the step-by-step instructions that the computer can process.

https://en.wikibooks.org/wiki/Haskell/Variables_and_functions

Redefinition : not allowed

```
r = 5

r = 2
```

imperative programming:

after setting r = 5 and then changing it to r = 2.

Hakell programming:

an error: "multiple declarations of r".

Within a given scope, a variable in Haskell

gets defined only once and cannot change.

like variables in mathematics.

Immutable: They vary only based on the data we enter into a program.

We can't define r two ways in the same code,

but we could change the value by changing the file

https://en.wikibooks.org/wiki/Haskell/Variables_and_functions

Recursion

r = r + 1

imperative programming:

incrementing the variable r

(updating the value in memory)

Hakell programming:

a recursive definition of r

(defining it in terms of itself)

if r had been defined with any value beforehand,

then r = r + 1 in Haskell would bring an error message.

https://en.wikibooks.org/wiki/Haskell/Variables_and_functions

Data Dependence

y = x * 2	x = 3
x = 3	y = x * 3

Hakell programming:

because their values of variables do not change within a program

variables can be defined in any order

there is no notion of "x being declared before y" or the other way around.

https://en.wikibooks.org/wiki/Haskell/Variables_and_functions

Evaluation

```
   area 5
=>    { replace the LHS  area r = ...  by the RHS  ... = pi * r^2 }
   pi * 5 ^ 2
=>    { replace  pi  by its numerical value }
   3.141592653589793 * 5 ^ 2
=>    { apply exponentiation (^) }
   3.141592653589793 * 25
=>    { apply multiplication (*) }
   78.53981633974483
```

```
area r = pi * r^2
```

replace each function with its definition

calculate the results until a single value remains.

to apply or call a function means

to replace the LHS of its definition by its RHS.

https://en.wikibooks.org/wiki/Haskell/Variables_and_functions

Type Synonyms

```
type String = [Char]

phoneBook :: [(String,String)]
```

```
type PhoneBook = [(String,String)]

phoneBook :: PhoneBook
```

```
type PhoneNumber = String
type Name = String
type PhoneBook = [(Name,PhoneNumber)]

phoneBook :: PhoneBook
```

```
phoneBook =
    [("betty","555-2938")
    ,("bonnie","452-2928")
    ,("patsy","493-2928")
    ,("lucille","205-2928")
    ,("wendy","939-8282")
    ,("penny","853-2492")
    ]
```

http://learnyouahaskell.com/making-our-own-types-and-typeclasses

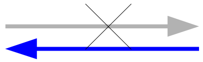
Record Syntax (named field)

```
data Configuration = Configuration
    { username          :: String
    , localHost         :: String
    , currentDir        :: String
    , homeDir           :: String
    , timeConnected     :: Integer
    }
```

```
username :: Configuration -> String                        -- accessor function  (automatic)
localHost :: Configuration -> String
-- etc.

changeDir :: Configuration -> String -> Configuration      -- update function
changeDir cfg newDir =
    if directoryExists newDir               -- make sure the directory exists
        then cfg { currentDir = newDir }
        else error "Directory does not exist"
```

https://en.wikibooks.org/wiki/Haskell/More_on_datatypes

newtype and **data**

data  **newtype**

Data can only be replaced with newtype
if the type has exactly *one constructor* with exactly *one field* inside it.

It ensures that the trivial **wrapping** and **unwrapping**
of the single field is eliminated by the **compiler**.

simple wrapper types such as **State** are usually defined with **newtype**.

type : used for type synonyms

```
newtype State s a = State { runState :: s -> (s, a) }
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

newtype examples

```
newtype Fd = Fd CInt
-- data Fd = Fd CInt would also be valid

-- newtypes can have deriving clauses just like normal types
newtype Identity a = Identity a
  deriving (Eq, Ord, Read, Show)

-- record syntax is still allowed, but only for one field
newtype State s a = State { runState :: s -> (s, a) }

-- this is *not* allowed:
-- newtype Pair a b = Pair { pairFst :: a, pairSnd :: b }
-- but this is:
data Pair a b = Pair { pairFst :: a, pairSnd :: b }
-- and so is this:
newtype NPair a b = NPair (a, b)
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

Side Effects in Haskell

Generally, a monad <u>cannot</u> perform side effects in Haskell.
there is one exception: **IO monad**

Suppose there is a type called World,
which contains all the <u>state</u> of the external universe

A way of thinking what IO monad does

```
type   IO t   =   World   ->   (t, World)
```

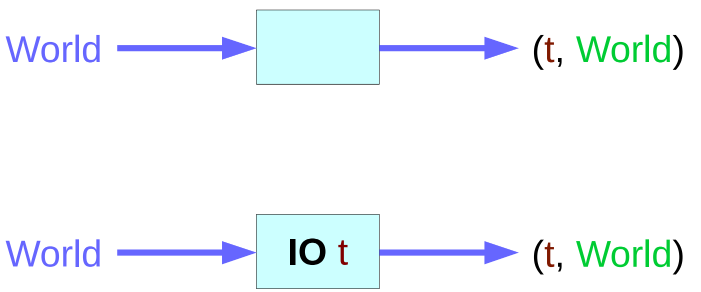
type synonym

IO t is a <u>function</u>

input : a World

output: the t and a new, updated World
obtained by modifying the given World
in the process of computing the t.

World -> (t, World)



IO x world0 (x, world1)

https://www.cs.hmc.edu/~adavidso/monads.pdf

Side Effects in Haskell

IO t is a function

input : a World

output: the t and a new, updated World
obtained by modifying the given World
in the process of computing the t.

It is impossible to store the extra copies of the contents of your hard drive that each of the Worlds contains

World → World

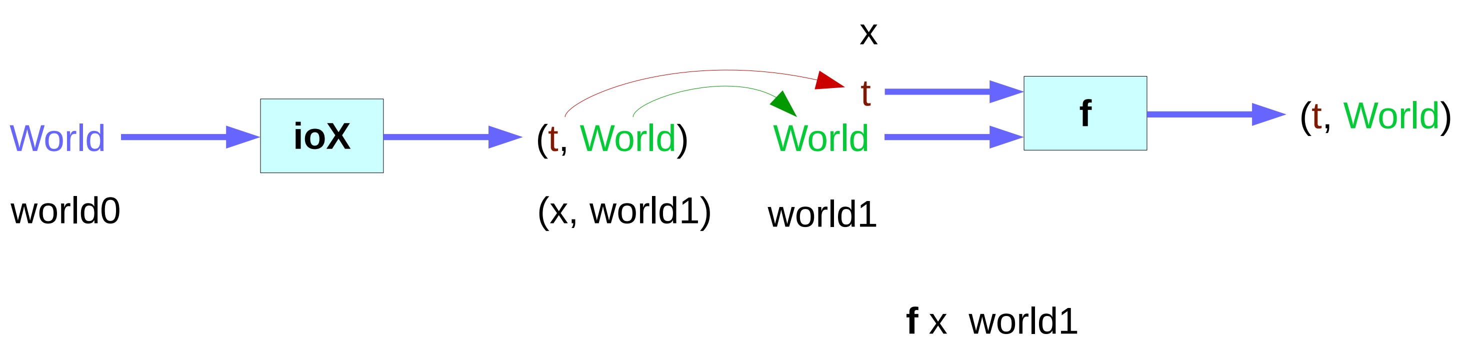
https://www.cs.hmc.edu/~adavidso/monads.pdf

Side Effects in Haskell

```
instance Monad IO where
    return x world = (x, world)
    (ioX >>= f) world0  =
        let     (x, world1) = ioX world0
        in      f x world1                           -- Has type (t, World)
```

```
type    IO t    =    World    ->    (t, World)
```

type synonym



https://www.cs.hmc.edu/~adavidso/monads.pdf

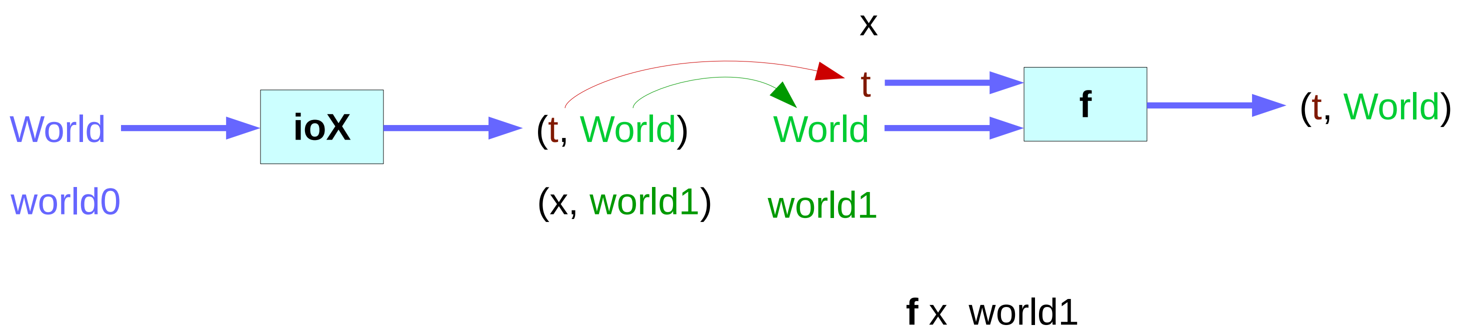
Side Effects in Haskell

```
instance Monad IO where
    return x world = (x, world)

    (ioX >>= f) world0  =
        let    (x, world1) = ioX world0
        in     f x world1           -- has type (t, World)
```

```
instance Monad ST where
    -- return :: a -> ST a
    return x  =  \s -> (x,s)

    -- (>>=)  :: ST a -> (a -> ST b) -> ST b
    st >>= f  =  \s -> let (x,s') = st s
                           in f x s'
```

```
type    IO t    =    World    ->    (t, World)
```

type synonym



https://www.cs.hmc.edu/~adavidso/monads.pdf

State Transformers ST

```
instance Monad ST where
  -- return :: a -> ST a
  return x  =  \s -> (x,s)

  -- (>>=)  :: ST a -> (a -> ST b) -> ST b
  st >>= f  =  \s -> let (x,s') = st s in f x s'
```

>>= provides a means of sequencing state transformers:
st >>= f applies the state transformer **st** to an initial state s,
then applies the function **f** to the resulting value x
to give a second state transformer (**f** x),
which is then applied to the modified state s' to give the final result:

```
st >>= f  =  \s -> f x s'
        where (x,s') = st s

st >>= f  =  \s -> (y,s')
        where (x,s') = st s
              (y,s') = f x s'
```

(x,s') = **st** s

f x s'

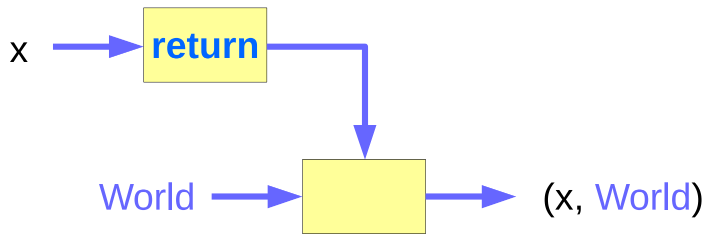
https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Side Effects in Haskell

The return function takes x
and gives back a function
  that takes a World
  and returns x along with the "new, updated" World
  formed by not modifying the World it was given

.

return x world = (x, world)



https://www.cs.hmc.edu/~adavidso/monads.pdf

Side Effects in Haskell

the expression (**ioX** **>>=** **f**) has

type World -> (t, World)

a function **ioX** that takes world0 of the type World,

which is used to extract x from its **IO** monad.

x gets passed to **f**, resulting in another **IO** monad,

which again is a function that takes a World

and returns a t and a new, updated World.

We give it the World we got back from getting x out of its monad,

and the thing it gives back to us is the t with a final version of the World

.

the implementation of bind

https://www.cs.hmc.edu/~adavidso/monads.pdf

Side Effects in Haskell

Which World was given initially?

Which World was updated?

In **GHC**, a **main** must be defined somewhere with type **IO ()**

a program execution starts from the **main**

the initial World is contained in the **main** to start everything off

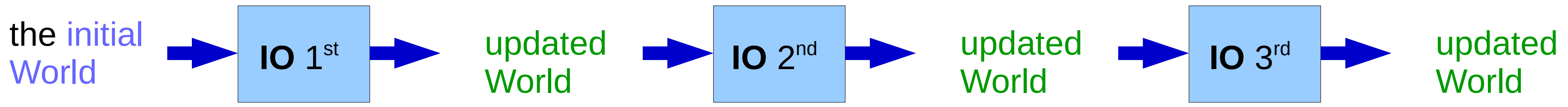
the **main** passes the updated World from each **IO**

to the next **IO** as its initial World

an **IO** that is not reachable from **main** will never be executed

an initial / updated World is not passed to such an **IO**

The modification of the World



https://www.cs.hmc.edu/~adavidso/monads.pdf

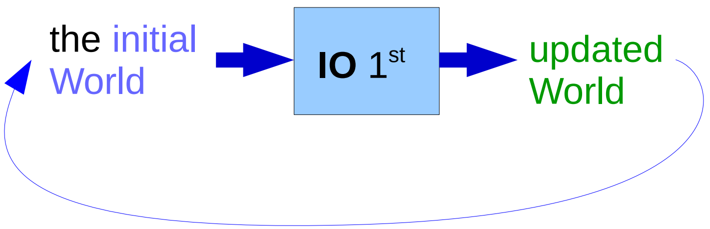
Side Effects in Haskell

when using **GHCI**,
everything is wrapped in **an implicit IO**,
since the results get printed out to the screen.

Every time you give it a new command, it passes in the current World,
gets the result of your command back, calls print on it
(which updates the World by modifying the contents of the screen or
the list of defined variables or the list of loaded modules or whatever),
and then saves the new World to give to the next command.

The neat thing about this is that there's only 1 World in existence
at any given moment.
Each IO takes that one and only World, consumes it,
and gives back a single new World.
Consequently, there's no way to accidentally run out of Worlds,
or have multiple ones running around.

the implementation of bind



https://www.cs.hmc.edu/~adavidso/monads.pdf

Side Effects in Haskell

the expression (**ioX >>= f**) has type World -> (t, World)

a function that takes a World, called world0,

which is used to extract x from its **IO** monad.

This gets passed to **f**, resulting in another **IO** monad,

which again is a function that takes a World

and returns a x and a new, updated World.

We give it the World we got back from getting x out of its monad,

and the thing it gives back to us is the t with a final version of the World

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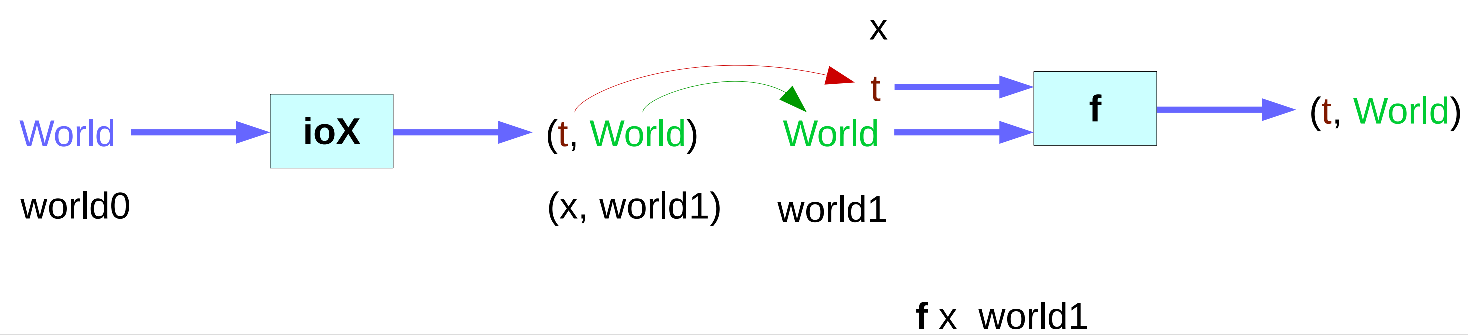
the implementation of bind



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References

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[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf