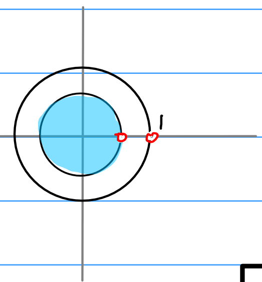
Laurent Series and z-Transform Examples case 2.A

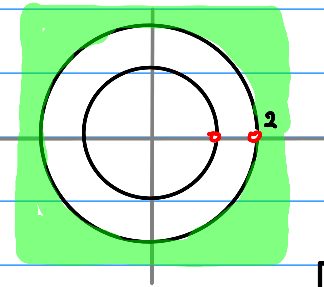
20171107

Ⅰ



$$a_n = \boxed{\begin{array}{ll} [1-2^{n-1}] & (n>0) \\ 0 & (n\le 0) \end{array}}$$

$$f(z) = \sum_{n=1}^{\infty} [1-2^{n-1}]\, z^{n}$$



$$x_n = \boxed{\begin{array}{ll} [1-2^{n-1}] & (n>0) \\ 0 & (n\le 0) \end{array}}$$

$$X(z) = \sum_{n=1}^{\infty} [1-2^{n-1}]\, z^{-n}$$

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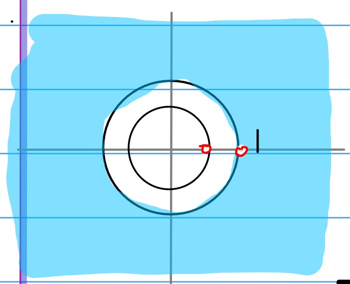
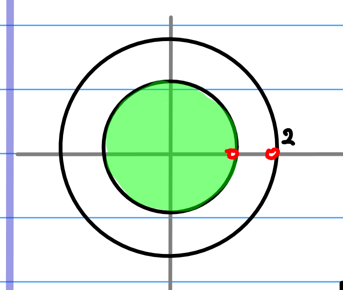


$$a_n = \boxed{\begin{array}{ll} 0 & (n>0) \\ [2^{n-1}-1] & (n\le 0) \end{array}}$$

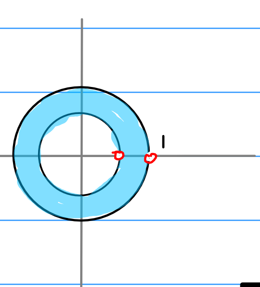
$$f(z) = \sum_{n=0}^{-\infty} [2^{n-1}-1]\, z^{n}$$



$$x_n = \boxed{\begin{array}{ll} 0 & (n>0) \\ [2^{n-1}-1] & (n\le 0) \end{array}}$$

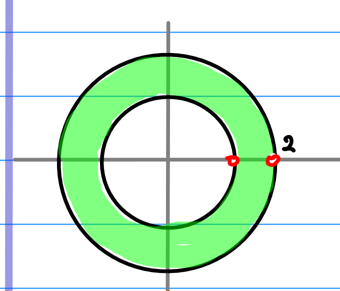
$$X(z) = \sum_{n=0}^{-\infty} [2^{n-1}-1]\, z^{-n}$$

Ⅲ



$$a_n = \boxed{\begin{array}{ll} 1 & (n>0) \\ 2^{n-1} & (n\le 0) \end{array}}$$

$$f(z) = \sum_{n=1}^{\infty} 1\cdot z^{n} + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^{n}$$

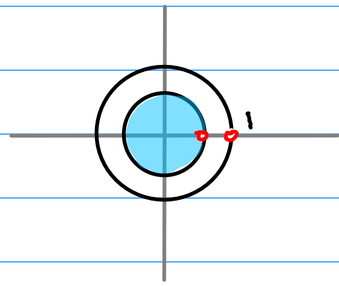


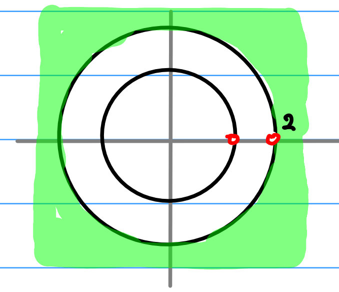
$$x_n = \boxed{\begin{array}{ll} 1 & (n>0) \\ 2^{n-1} & (n\le 0) \end{array}}$$

$$X(z) = \sum_{n=1}^{\infty} 1\cdot z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^{-n}$$

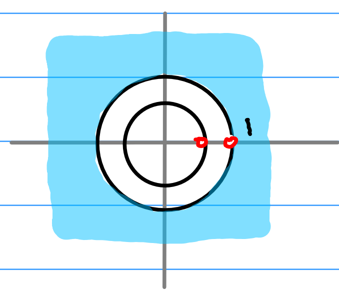
2.A

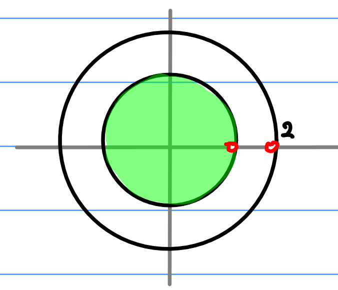
$$f(z) = \frac{-0.5 z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$



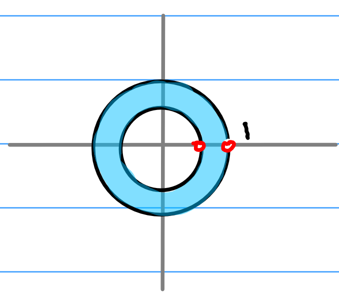


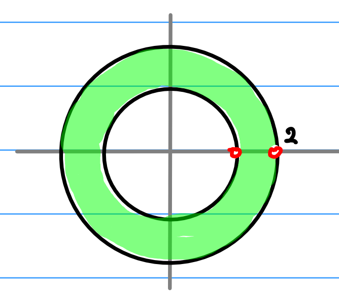
$$\sum_{n=1}^{\infty}\left[1-2^{n-1}\right] z^{n} \xrightarrow{z^{-1}} \sum_{n=1}^{\infty}\left[1-2^{n-1}\right] z^{-n}$$





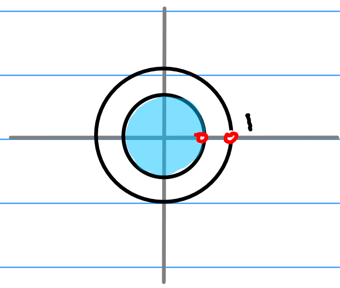
$$\sum_{n=0}^{-\infty}\left[2^{n-1}-1\right] z^{n} \xrightarrow{z^{-1}} \sum_{n=0}^{-\infty}\left[2^{n-1}-1\right] z^{-n}$$





$$\sum_{n=1}^{\infty} 1\cdot z^{n} + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^{n} \xrightarrow{z^{-1}} \sum_{n=1}^{\infty} 1\cdot z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^{-n}$$

$$f(z) = \frac{-0.5\, z^2}{(z-1)(z-0.5)} = \frac{-z}{z-1} + \frac{0.5z}{z-0.5}$$

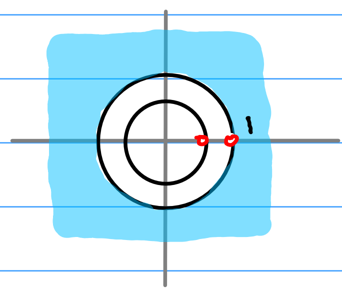


$$+\frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} - \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{2z}{1}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{z}{1}\right)^n - \sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{2z}{1}\right)^n$$

$$= +\sum_{n=0}^{\infty} z^{n+1} - \sum_{n=0}^{\infty} 2^n z^{n+1}$$

$$= \sum_{n=1}^{\infty}\left[1 - 2^{n-1}\right] z^n$$

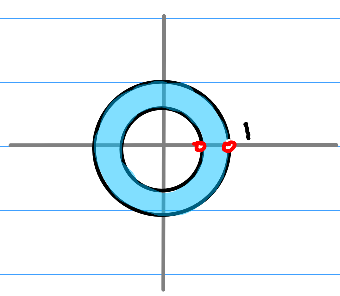


$$-\frac{(1)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$$

$$= -\sum_{n=0}^{\infty}(1)\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n$$

$$= \sum_{n=0}^{\infty}\left[2^{-n-1} - 1\right] z^{-n}$$

$$= \sum_{n=0}^{-\infty}\left[2^{n-1} - 1\right] z^n$$



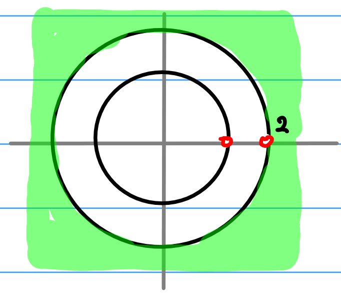
$$+\frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{z}{1}\right)\left(\frac{z}{1}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n$$

$$= +\sum_{n=0}^{\infty} z^{n+1} + \sum_{n=0}^{\infty} 2^{-n-1} z^{-n}$$

$$= +\sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n$$

$$X(z) = \frac{-1}{(z-1)(z-2)} = \frac{1}{(z-1)} - \frac{1}{(z-2)}$$



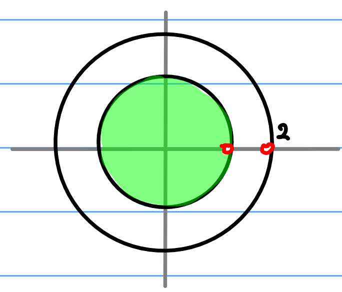
$$+\frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)} - \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{2}{z}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)\left(\frac{1}{z}\right)^n - \sum_{n=0}^{\infty}\left(\frac{1}{z}\right)\left(\frac{2}{z}\right)^n$$

$$= \sum_{n=0}^{\infty}\left[1-2^n\right] z^{-n-1}$$

$$= \sum_{n=-1}^{-\infty}\left[1-\left(\frac{1}{2}\right)^{n+1}\right] z^n$$

$$\boxed{\sum_{n=1}^{\infty}\left[1-2^{n-1}\right] z^{-n}}$$



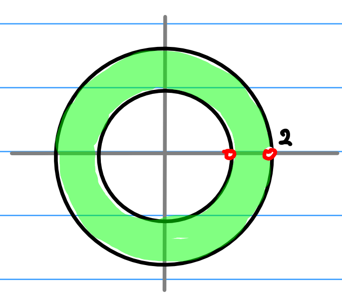
$$-\frac{\left(\frac{1}{1}\right)}{1-\left(\frac{z}{1}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)}$$

$$= -\sum_{n=0}^{\infty}(1)\left(\frac{z}{1}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{z}{2}\right)^n$$

$$= -\sum_{n=0}^{\infty} z^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^n$$

$$= \sum_{n=0}^{\infty}\left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^n$$

$$\boxed{\sum_{n=0}^{-\infty}\left[2^{n-1} - 1\right] z^{-n}}$$



$$+\frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)}$$

$$= +\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{z}{2}\right)^n$$

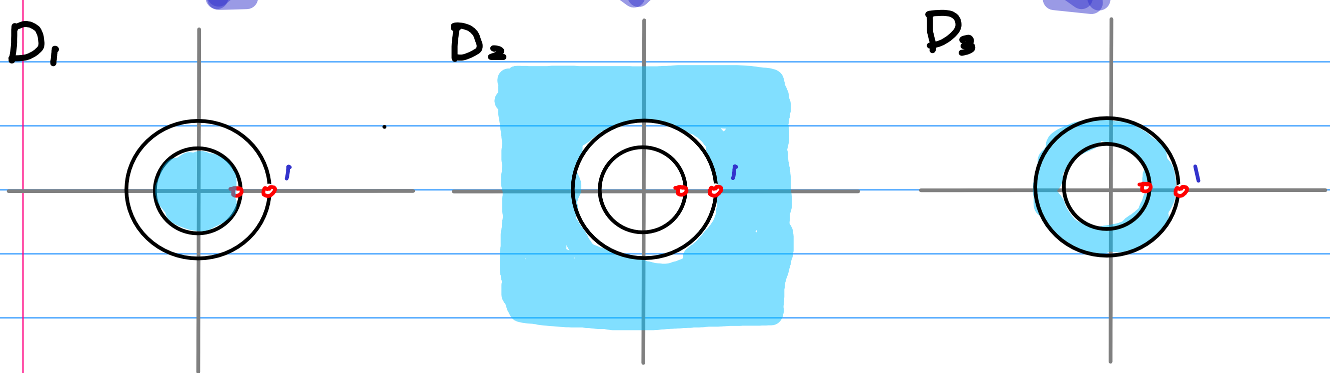
$$= +\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^n$$

$$= +\sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^n$$

$$\boxed{+\sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1} z^{-n}}$$

L.S. first

$$\boxed{f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}}$$



D_1	D_2	D_3
a_n ?	a_n ?	a_n ?
$\parallel$	$\parallel$	$\parallel$
x_n	x_n	x_n
$X(z)$?	$X(z)$?	$X(z)$?

$$\boxed{X(z) = ?}$$

$$f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$\frac{z}{(z-1)(z-0.5)} = \frac{2}{z-1} - \frac{1}{z-0.5}$$

$$= \frac{2z-1-z+1}{(z-1)(z-0.5)}$$

$$\frac{-0.5\ z^2}{(z-1)(z-0.5)} = -0.5\left(\frac{2z}{z-1} - \frac{z}{z-0.5}\right) = \frac{-z}{z-1} + \frac{0.5z}{z-0.5}$$

$$\frac{-z}{z-1} = \begin{cases} \dfrac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} = \displaystyle\sum_{n=0}^{\infty} z \cdot z^n = \sum_{n=0}^{\infty} 1 \cdot z^{n+1} & \left|\frac{z}{1}\right| < 1 \\ \dfrac{-1}{1-\left(\frac{1}{z}\right)} = -\displaystyle\sum_{n=0}^{\infty} 1 \cdot z^{-n} = -\sum_{n=0}^{\infty} 1 \cdot z^{-n} & \left|\frac{1}{z}\right| < 1 \end{cases}$$

$$\frac{0.5z}{z-0.5} = \begin{cases} \dfrac{-\left(\frac{z}{1}\right)}{1-\left(\frac{2z}{1}\right)} = -\displaystyle\sum_{n=0}^{\infty} \left(\frac{z}{1}\right)\left(\frac{2z}{1}\right)^n = -\sum_{n=0}^{\infty} 2^n z^{n+1} & \left|\frac{2z}{1}\right| < 1 \\ \dfrac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)} = \displaystyle\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n} & \left|\frac{1}{2z}\right| < 1 \end{cases}$$

$$f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)} \qquad X(z) = \frac{-1}{(z-1)(z-2)}$$

L.S. first

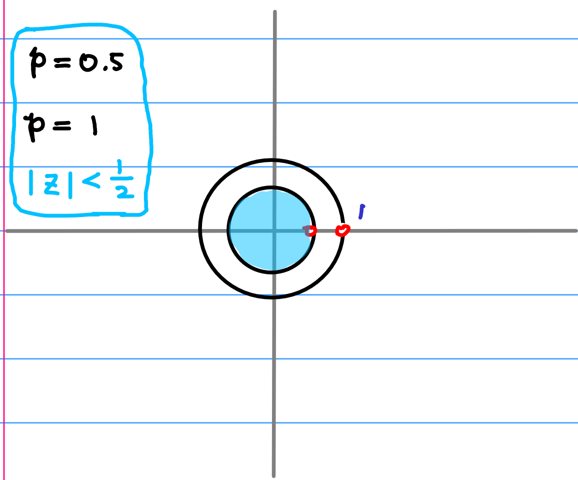
Ⓘ $-$ 1

Ⓘ D_1 $\quad |z| < 0.5$	$\left[\left|\frac{z}{1}\right| < 1 , \quad \left|\frac{2z}{1}\right| < 1 \right]$

$p = 0.5$

$p = 1$

$|z| < \frac{1}{2}$



$$f(z) = \frac{-z}{z-1} + \frac{0.5z}{z-0.5}$$

$$= \frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)} + \frac{-\left(\frac{z}{1}\right)}{1-\left(\frac{2z}{1}\right)}$$

$$= \sum_{n=0}^{\infty} 1 \cdot z^{n+1} - \sum_{n=0}^{\infty} 2^n z^{n+1}$$

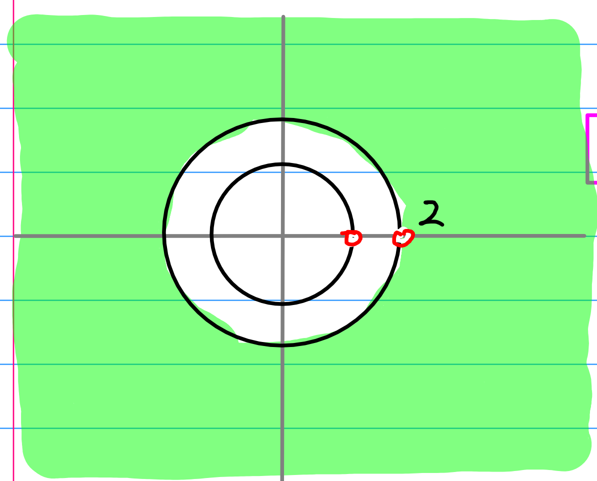
$$= \sum_{n=0}^{\infty} \left[1 - 2^n\right] z^{n+1}$$

$$= \sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^n \qquad |z| < \frac{1}{2}$$

$p =$

$p =$

$|z| > 1$



$$\boxed{X(z) = f(z^{-1})} = \sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^{-n} \qquad |z| > 2$$

$$= \sum_{n=1}^{\infty} 1 \cdot z^{-n} - \sum_{n=1}^{\infty} 2^{n-1} \cdot z^{-n}$$

$$= \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)} - \frac{\left(\frac{1}{z}\right)}{1-\left(\frac{2}{z}\right)}$$

$$= \frac{1}{z-1} - \frac{1}{z-2}$$

$$= \frac{z-2-z+1}{(z-1)(z-2)}$$

$$= \frac{-1}{(z-1)(z-2)}$$

$$f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$

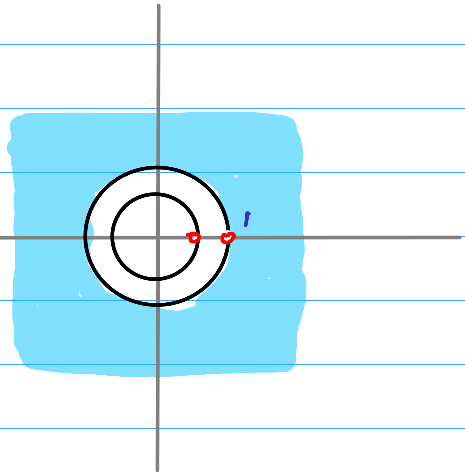
$$f(z^{-1}) = \frac{-0.5z^{-2}}{(z^{-1}-1)(z^{-1}-0.5)} = \frac{-0.5}{(1-z)(1-0.5z)} = \frac{-1}{(z-1)(z-2)} = X(z)$$

② D_2 $\quad 1<|z| \quad \left[\left|\frac{1}{z}\right|<1, \quad \left|\frac{1}{2z}\right|<1 \right]$

$p=0.5$

$p=1$

$|z|>\frac{1}{2}$



$$f(z) = \frac{-z}{z-1} + \frac{0.5z}{z-0.5}$$

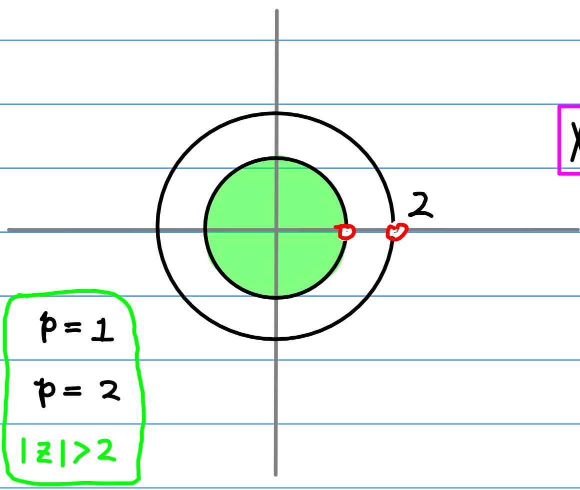
$$= \frac{-1}{1-\left(\frac{1}{z}\right)} + \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$$

$$= -\sum_{n=0}^{\infty} 1\cdot\left(\frac{1}{z}\right)^n + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n$$

$$= \sum_{n=0}^{\infty}\left[2^{-n-1}-1\right] z^{-n}$$

$$= \sum_{n=0}^{-\infty}\left[2^{n-1}-1\right] z^{n} \qquad |z|>1$$

$$\boxed{X(z) = f(z^{-1})} = \sum_{n=0}^{-\infty}\left[2^{n-1}-1\right] z^{-n} \qquad |z|>2$$



$p=1$

$p=2$

$|z|>2$

$$= \sum_{n=0}^{\infty} 2^{-n-1}\cdot z^n - \sum_{n=0}^{\infty} 1\cdot z^n$$

$$= \frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)} - \frac{1}{1-\left(\frac{z}{1}\right)}$$

$$= \frac{-1}{z-2} + \frac{1}{z-1}$$

$$= \frac{-z+1+z-2}{(z-1)(z-2)}$$

$$= \frac{-1}{(z-1)(z-2)}$$

$$f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$

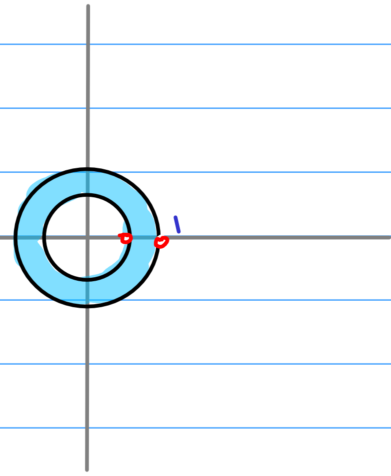
$$f(z^{-1}) = \frac{-0.5z^{-2}}{(z^{-1}-1)(z^{-1}-0.5)} = \frac{-0.5}{(1-z)(1-0.5z)} = \frac{-1}{(z-1)(z-2)} = X(z)$$

$$\text{③}\quad D_3 \quad 0.5<|z|<1 \qquad \left[\ \left|\frac{z}{1}\right|<1,\quad \left|\frac{1}{2z}\right|<1\ \right]$$

$p=1$

$p=0.5$

$\frac{1}{2}<|z|<1$



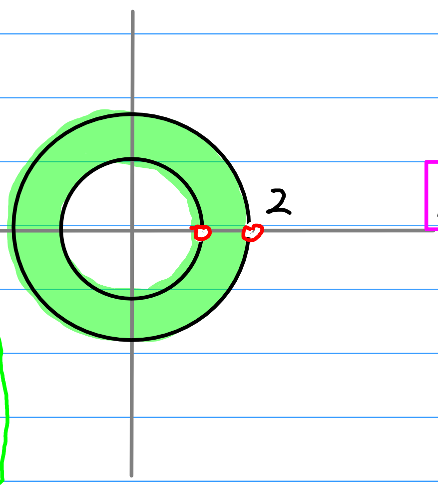
$$f(z)=\frac{-z}{z-1}+\frac{0.5z}{z-0.5}$$

$$=\frac{\left(\frac{z}{1}\right)}{1-\left(\frac{z}{1}\right)}+\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2z}\right)}$$

$$=\sum_{n=0}^{\infty} z\cdot z^n+\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)\left(\frac{1}{2z}\right)^n$$

$$=\sum_{n=0}^{\infty}1\cdot z^{n+1}+\sum_{n=0}^{\infty}2^{-n-1}\,z^{-n}$$

$$=\sum_{n=1}^{\infty}1\cdot z^{n}+\sum_{n=0}^{-\infty}2^{n-1}\cdot z^{n} \qquad |z|<\frac{1}{2}$$



$$\boxed{X(z)=f(z^{-1})}=\sum_{n=1}^{\infty}1\cdot z^{-n}+\sum_{n=0}^{-\infty}2^{n-1}\cdot z^{-n} \qquad |z|>2$$

$$=\sum_{n=1}^{\infty}1\cdot z^{-n}+\sum_{n=0}^{\infty}2^{-n-1}\cdot z^{n}$$

$$=\frac{\left(\frac{1}{z}\right)}{1-\left(\frac{1}{z}\right)}+\frac{\left(\frac{1}{2}\right)}{1-\left(\frac{z}{2}\right)}$$

$$=\frac{1}{z-1}-\frac{1}{z-2}$$

$$=\frac{z-2-z+1}{(z-1)(z-2)}$$

$$=\frac{-1}{(z-1)(z-2)}$$

$p=1$

$p=2$

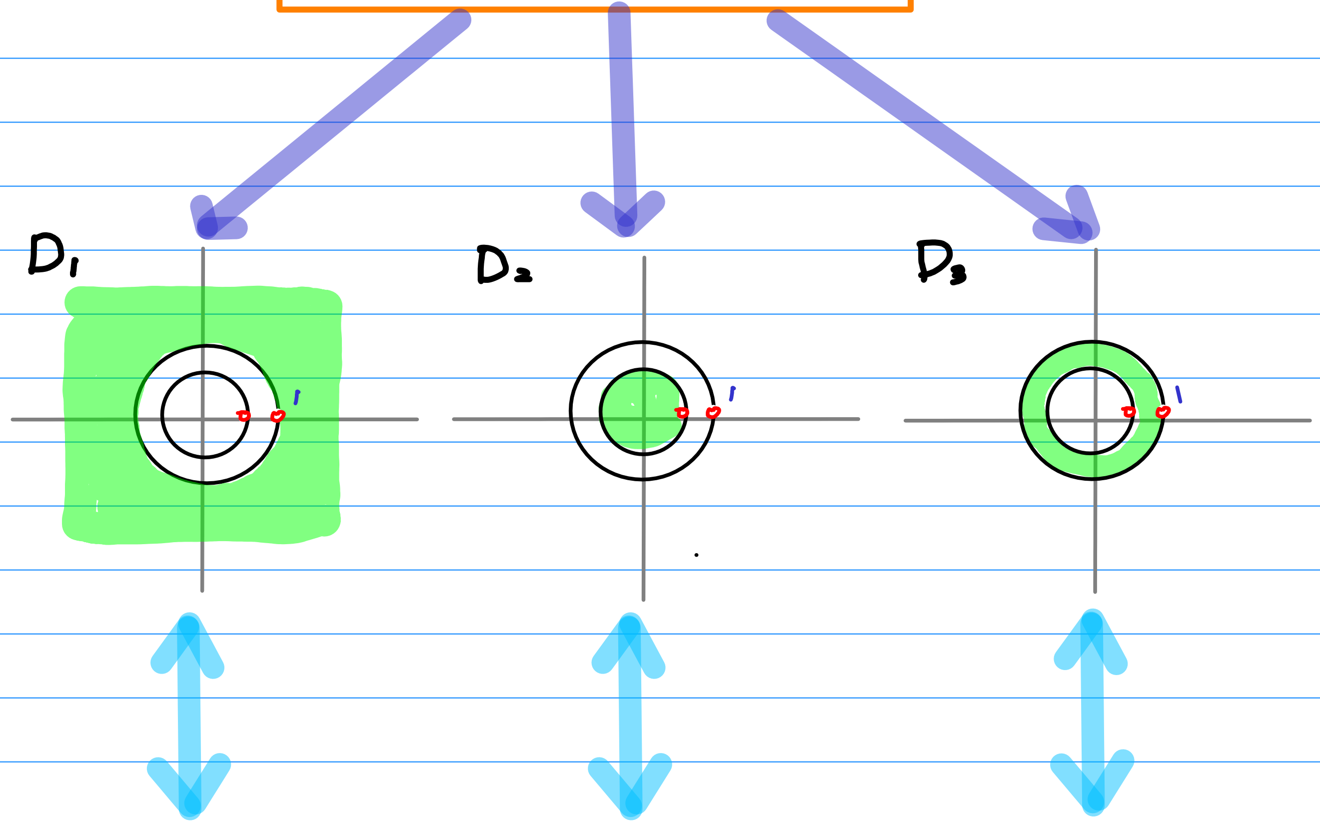
$1<|z|<2$

$$f(z)=\frac{-0.5z^2}{(z-1)(z-0.5)}$$

$$f(z^{-1})=\frac{-0.5z^{-2}}{(z^{-1}-1)(z^{-1}-0.5)}=\frac{-0.5}{(1-z)(1-0.5z)}=\frac{-1}{(z-1)(z-2)}=X(z)$$

Z.T. first

$$\boxed{X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}}$$



D_1	D_2	D_3
x_n?	x_n?	x_n?
$\parallel$	$\parallel$	$\parallel$
a_n	a_n	a_n
$\updownarrow$	$\updownarrow$	$\updownarrow$
$f(z)$?	$f(z)$?	$f(z)$?