Monad Overview (3B)

Based on

Haskell in 5 steps
https://wiki.haskell.org/Haskell_in_5_steps

Please send corrections (or suggestions) to youngwlim@hotmail.com.

This document was produced by using LibreOffice.

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Haskell in 5 steps
https://wiki.haskell.org/Haskell_in_5_steps

A Type **Monad**

Haskell does not have states

But it's type system is powerful enough
to construct the stateful program flow

defining a **Monad** type in Haskell
- similar to defining a **class** in an object oriented language (C++, Java)
- a **Monad** can do much more than a class:

A **Monad** is a **type** that can be used for

- **exception handling**
- **parallel program workflow**
- **a parser generator**

http://www.idryman.org/blog/2014/01/23/yet-another-monad-tutorial/

Types: rules and data

types are the ***rules*** associated with the **data**,
not the actual **data** itself.

Object-Oriented Programming enable us
to use classes / interfaces
to define **types**,
the **rules** (**methods**) that interacts with the actual **data**.

to use templates(c++) or generics(java)
to define more **abstracted rules** that are more reusable

Monad is pretty much like **generic class**.

http://www.idryman.org/blog/2014/01/23/yet-another-monad-tutorial/

Monad Rules

A **type** is just a set of rules, or methods in Object-Oriented terms

A **Monad** is just yet another type, and
the definition of this type is defined by **four rules**:

1) **bind (>>=)**
2) **then (>>)**
3) **return**
4) **fail**

http://www.idryman.org/blog/2014/01/23/yet-another-monad-tutorial/

Monad Applications

1. **Exception Handling**

2. **Accumulate States**

3. **IO Monad**

http://www.idryman.org/blog/2014/01/23/yet-another-monad-tutorial/

A value of type **M a**

a **value** of type **M a** is interpreted

as a **statement** in an imperative language **M**
that returns a value of type **a** as its result;

and the **semantics** of this language
are determined by the **monad M**.

Maybe a
IO a
ST a
State a

the type **M a** ⇕ an imperative language **M**

a value ⇕ a statement

https://en.wikibooks.org/wiki/Haskell/Understanding_monads#cite_note-3

A value of type **M a**

a **value** of type **M** **a** is interpreted

as a **statement** in an imperative language **M**
that returns a value of type **a** as its result;

and the **semantics** of this language
are determined by the **monad M**.

computations that result in values

an immediate abort
a valueless return in the middle of a computation.

types are the ***rules*** associated with the **data**,
not the actual **data** itself. (*classes in C++)*

Maybe a
IO a
ST a
State a

https://en.wikibooks.org/wiki/Haskell/Understanding_monads#cite_note-3

Semantics of a language M

Semantics : what the language allows us to say.

In the case of **Maybe**,

the **semantics** allow us to express failures

when a statement fails to produce a result,

allow statements that are following to be *skipped*

https://en.wikibooks.org/wiki/Haskell/Understanding_monads#cite_note-3

Semicolon and Assignment

the **then** operator **(>>)**

an implementation of the **semicolon**

The **bind** operator **(>>=)**

an implementation of the **semicolon** and

assignment (**binding**) of the result

of a previous computational step.

https://en.wikibooks.org/wiki/Haskell/Understanding_monads#cite_note-3

A function application and the bind operator

a **let** expression as a **function** application,

let x = foo in (x + 3)

foo & (\x -> id (x + 3)) -- v & f = f v

& and **id** are trivial;

id is the identity function
just returns its parameter unmodified

an **assignment** and **semicolon** as the **bind** operator:

x <- foo; return (x + 3)

foo >>= (\x -> return (x + 3))

>>= and **return** are substantial.

https://en.wikibooks.org/wiki/Haskell/Understanding_monads#cite_note-3

& and id

a **let** expression as a **function** application,

let x = foo in (x + 3)

foo & (\x -> id (x + 3))   -- v & f = f v

The **&** operator combines together two *pure calculations*,
foo and **id (x + 3)**

while creating a new binding for the variable **x** to hold **foo**'s value,

making **x** available to the second computational step: **id (x + 3)**.

https://en.wikibooks.org/wiki/Haskell/Understanding_monads#cite_note-3

>>= and return

an **assignment** and **semicolon** as the **bind** operator:

```
x <- foo; return (x + 3)

foo >>= (\x -> return (x + 3))
```

The bind operator **>>=** combines together two computational steps,
foo and **return (x + 3)**,
in a manner particular to the **monad M**,

while creating a new binding for the variable **x** to hold **foo**'s result,

making **x** available to the next computational step, **return (x + 3)**.

In the particular case of **Maybe**,
if **foo** fails to produce a result,
the second step will be skipped and
the whole combined computation will also fail immediately.

https://en.wikibooks.org/wiki/Haskell/Understanding_monads#cite_note-3

Monad Class Function >>= & >>

>>= and **>>** : functions from the Monad *class*

Monad Sequencing Operator with value passing

>>= **passes** the result of the expression on the left
as an argument to the expression on the right,
while preserving the context the argument and function use

Monad Sequencing Operator

>> is used to **order** the **evaluation** of expressions within some context;
it makes evaluation of the right depend on the evaluation of the left

https://www.quora.com/What-do-the-symbols-and-mean-in-haskell

Monad Definition

A **monad** is defined by

a **type constructor m**;
a function **return**;
an operator (**>>=**) "**bind**"

The function and operator are methods of the Monad type class and have types

return :: a -> **m** a

(>>=) :: **m** a -> (a -> **m** b) -> **m** b

are required to obey three laws

https://en.wikibooks.org/wiki/Haskell/Understanding_monads

Monad Laws

every instance of the Monad type class must obey the following three laws:

```
m >>= return     =  m                      -- right unit
return x >>= f   =  f x                    -- left unit
(m >>= f) >>= g  =  m >>= (\x -> f x >>= g)  -- associativity
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads

Monadic Effect

```
class Monad m where
    return :: a -> m a
    (>>=) :: m a -> (a -> m b) -> m b
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/IO
https://stackoverflow.com/questions/2488646/why-are-side-effects-modeled-as-monads-in-haskell
https://stackoverflow.com/questions/7840126/why-monads-how-does-it-resolve-side-effects
https://stackoverflow.com/questions/2488646/why-are-side-effects-modeled-as-monads-in-haskell

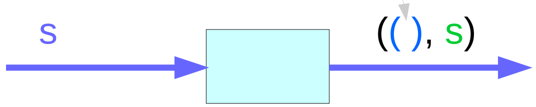
https://www.cs.hmc.edu/~adavidso/monads.pdf

Monadic Operations

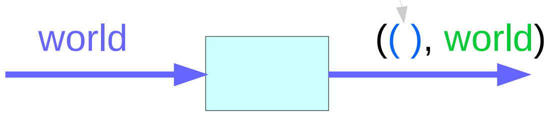
Monadic operations tend to have types which look like

val-in-type-1 -> ... -> val-in-type-n -> **effect-monad** val-out-type

put :: s -> (State s) ( )



putStr :: String -> IO ( )



returning a function as a value
executable function
executing an action (**effect-monad**)
produce a result val-out-type

https://stackoverflow.com/questions/16892570/what-is-in-haskell-exactly

Monadic Operations

val-in-type-1 -> ... -> val-in-type-n -> **effect-monad** val-out-type

where the **return type** is a type application:

a type with a parameter type

effect-monad

an executable function

giving information about which **effects** are possible

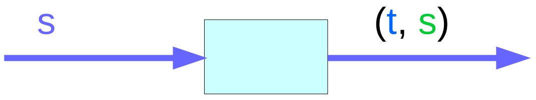
val-out-type

the argument of the executable function

the type of the **result** produced by the function

(the result of executing the function)

returning a function as a value



put :: s -> (State s) ( )

putStr :: String -> IO ( )

https://stackoverflow.com/questions/16892570/what-is-in-haskell-exactly

Monadic Operations – **put**, **putStr**

put :: s -> State s ( )

put :: s -> (State s) ( )

one value input type **s**

the **effect-monad** **State s**

the value output type **( )**

the operation is used *only for its effect*;

the *value* delivered is *uninteresting*

putStr :: String -> IO ( )

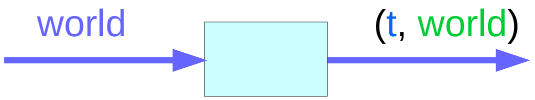
delivers a string to stdout but does not return anything meaningful

https://stackoverflow.com/questions/16892570/what-is-in-haskell-exactly

IO t and State s a types

```
type    IO t   =   World   ->   (t, World)
```

type synonym

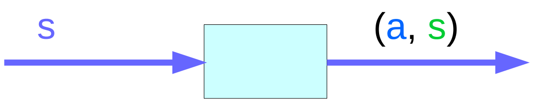



```
newtype State s a = State { runState :: s -> (a, s) }
```

s : the type of the state,

a : the type of the produced result

s -> (a, s) : function type



Monad Definition

```
class Monad m where
    return :: a -> m a
    (>>=) :: m a -> (a -> m b) -> m b
    (>>) :: m a -> m b -> m b
    fail :: String -> m a
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads

Maybe Monad Instance

```
instance Monad Maybe where
        return x = Just x
        Nothing >>= f = Nothing
        Just x >>= f  = f x
        fail _ = Nothing
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads

IO Monad Instance

```
instance  Monad IO  where
    m >> k    = m >>= \ _ -> k
    return    = returnIO
    (>>=)     = bindIO
    fail s    = failIO s

returnIO :: a -> IO a
returnIO x = IO $ \ s -> (# s, x #)

bindIO :: IO a -> (a -> IO b) -> IO b
bindIO (IO m) k
   = IO $ \ s ->  case m s of (# new_s, a #)
                   -> unIO (k a) new_s
```

https://stackoverflow.com/questions/9244538/what-are-the-definitions-for-and-return-for-the-io-monad

State Monad Instance

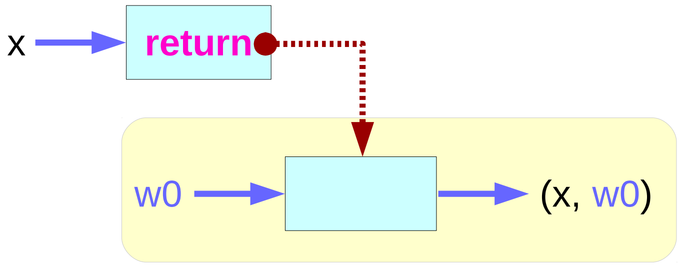
```
instance Monad (State s) where

return :: a -> State s a
return x = state ( \ s -> (x, s) )

(>>=) :: State s a -> (a -> State s b) -> State s b
p >>= k = q where
    p' = runState p        -- p' :: s -> (a, s)
    k' = runState . k      -- k' :: a -> s -> (b, s)
    q' s0 = (y, s2) where  -- q' :: s -> (b, s)
        (x, s1) = p' s0    -- (x, s1) :: (a, s)
        (y, s2) = k' x s1  -- (y, s2) :: (b, s)
    q = State q'
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads/State

IO Monad - return method

The **return** function takes x
and gives back a function
 that takes a w0 :: World
 and returns x along with the updated World,

 but not modifying the w0 :: World it was given



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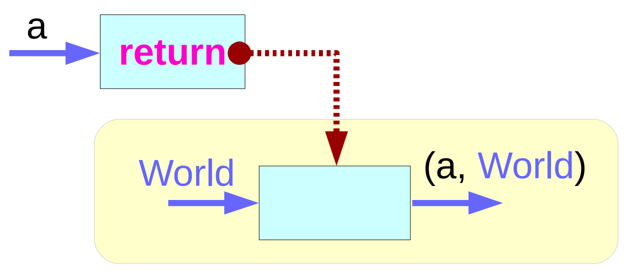
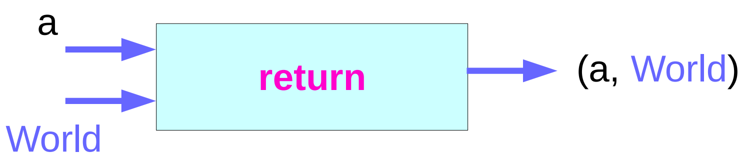
https://www.cs.hmc.edu/~adavidso/monads.pdf

IO Monad - return method type

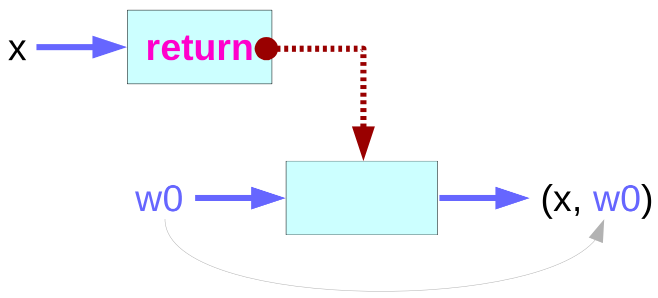
return a :: a -> IO a



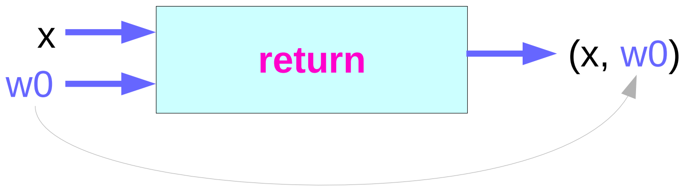
return a World :: (a, World)



let (x, w0) = return x w0



let (x, w0) = return x w0



https://www.cs.hmc.edu/~adavidso/monads.pdf

IO Monad - >>= operator

the expression (**ioX >>= f**) has

type IO **a -> (a -> IO b) -> IO b**

IoX :: IO **a** has a function type of World **-> (a, World)**

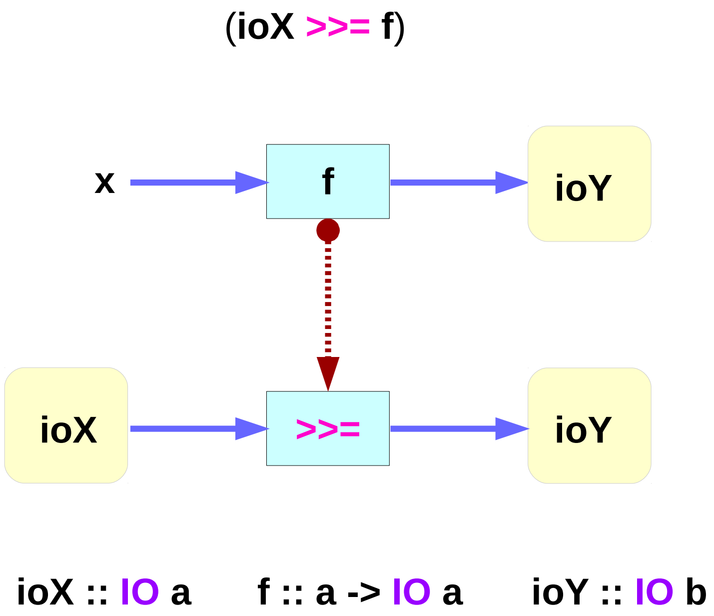
a function that takes **w0 ::** World,

returns **x :: a** and the new, updated **w1 ::** World

x and **w1** get passed to **f**, resulting in another **IO** monad,

which again is a function that takes **w1 ::** World

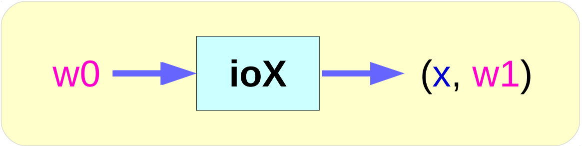
and returns **y** computed from **x** and the same **w1 ::** World



https://www.cs.hmc.edu/~adavidso/monads.pdf

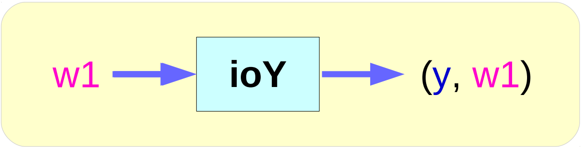
IO Monad - **>>=** operator binding

We give the **IOx** the **w0**	**w0 :: World**
we got back the updated **w1**	**w1 :: World**
and **x** out of its monad	**x :: a**
the **f** is given with	
the **x** with	**x :: a**
the updated **w1**	**w1 :: World**
.	
The final **IO** Monad	
takes **w1**	**w1 :: World**
returns **w1**	**w1 :: World**
and **y** out of its monad	**y :: a**



let (x, w1) = ioX w0

bind variables



let (y, w1) = ioY w0

https://www.cs.hmc.edu/~adavidso/monads.pdf

IO Monad - >>= operator implementation

(t,World)		(t,World)		(t,World)
(–,w0)	→	(x,w1)	→	(y,w1)

first, execute the **action**	then, compute the new **result**
State updated	**No State transition**
Result extracted	**Result computed**

the implementation of bind

https://www.cs.hmc.edu/~adavidso/monads.pdf

IO Monad Instance

```
instance Monad IO where
    return x w0 = (x, w0)

    (ioX >>= f) w0  =
        let  (x, w1) = ioX w0
        in   f x w1            -- has type (t, World)
```

```
type   IO t   =   World   ->   (t, World)
```
type synonym

https://www.cs.hmc.edu/~adavidso/monads.pdf

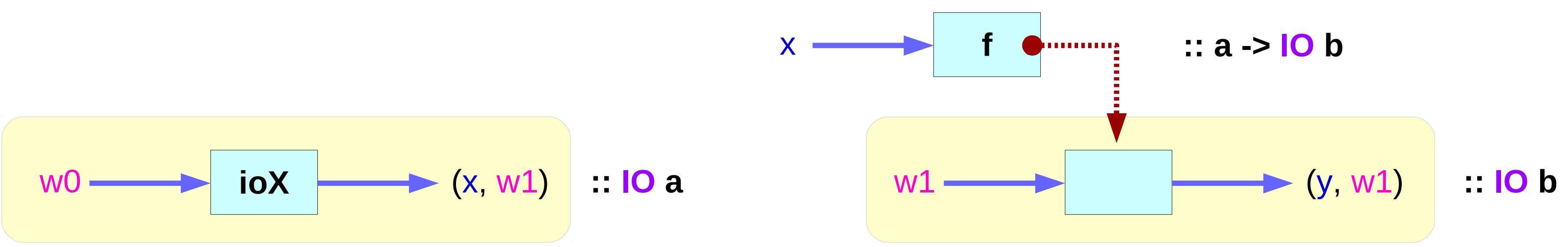
IO Monad – IO a, IO b

```
instance Monad IO where
   return x w0 = (x, w0)

   (ioX >>= f) w0  =
      let  (x, w1) = ioX w0
      in   f x w1          -- has type (t, World)
```

ioX >>= f :: IO a -> (a -> IO b) -> IO b

type   IO t =   World   ->   (t, World)   type synonym



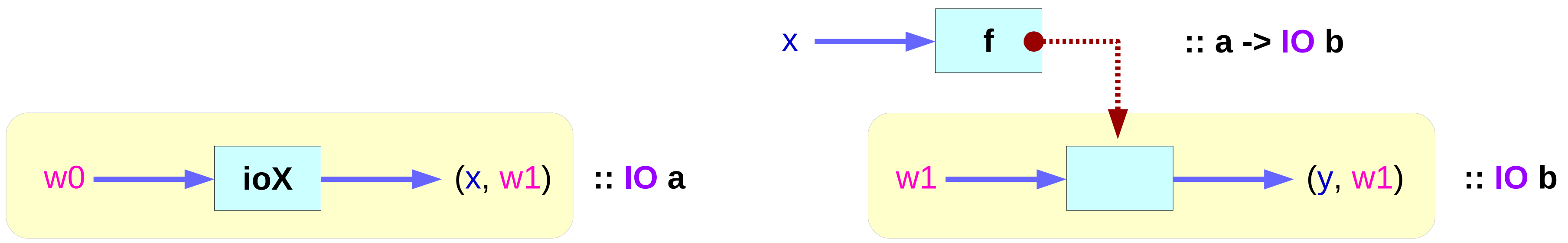
https://www.cs.hmc.edu/~adavidso/monads.pdf

IO Monad – (a -> IO b) type

ioX >>= f :: IO a -> (a -> IO b) -> IO b

ioX :: IO a w0 :: World x :: a
f :: a -> IO b w1 :: World

ioX w0 :: IO a World ⇒ (x, w1)
f x :: IO b
f x w1 :: IO b World ⇒ (y, w1)



https://www.cs.hmc.edu/~adavidso/monads.pdf

IO Monad – binding variables

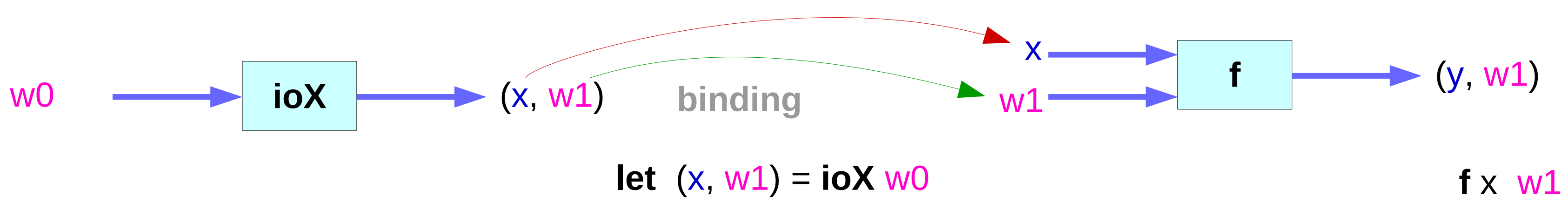
ioX >>= f :: IO a -> (a -> IO b) -> IO b

ioX :: IO a
f :: a -> IO b w0 :: World

x :: a
w1 :: World

internal variables

ioX w0 **:: IO a** World → (x, w1)
f x :: a -> a -> IO b
f x w1 **:: IO b** World → (y, w1)

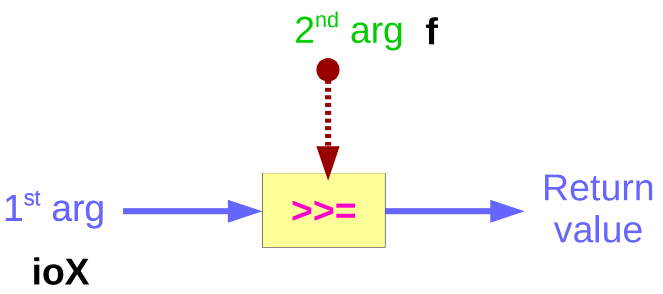


https://www.cs.hmc.edu/~adavidso/monads.pdf

IO Monad - (>>=) operator type

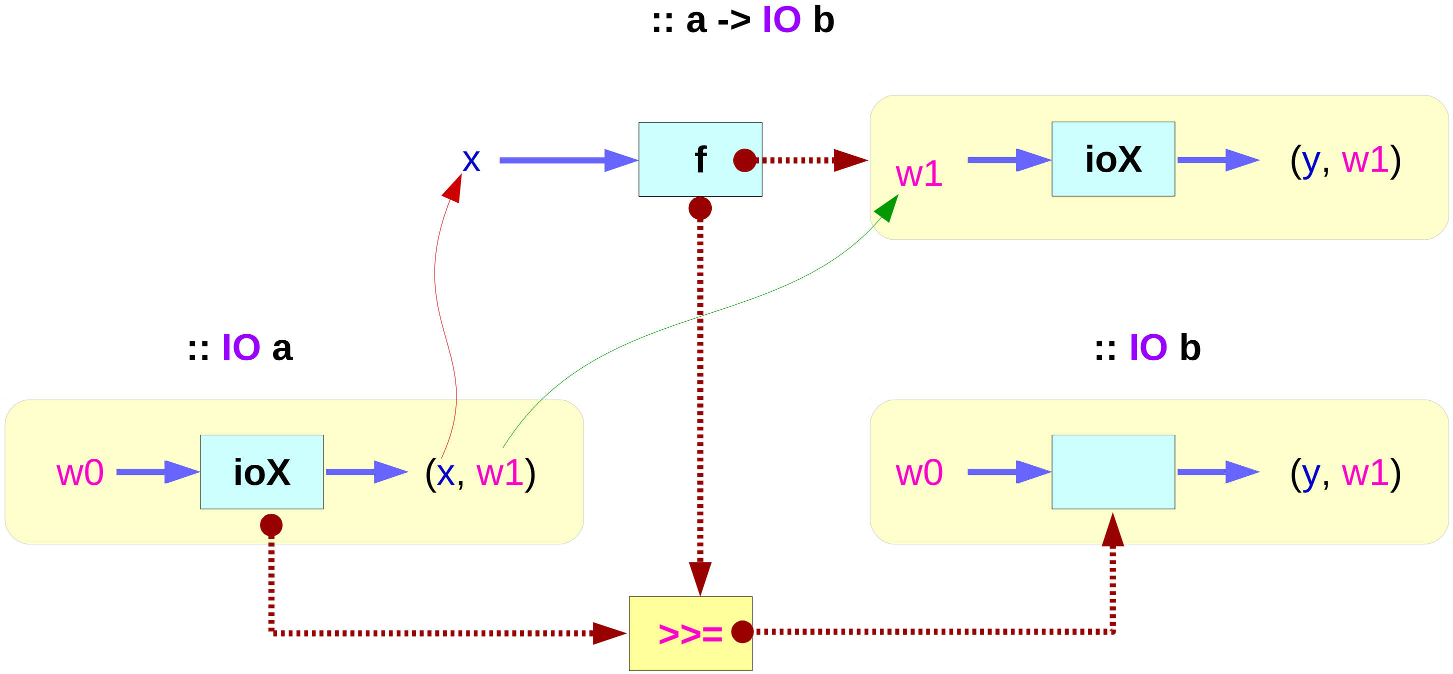
Monad	Function	Monad
1st arg	2nd arg	return

(>>=) :: IO a -> (a -> IO b) -> IO b

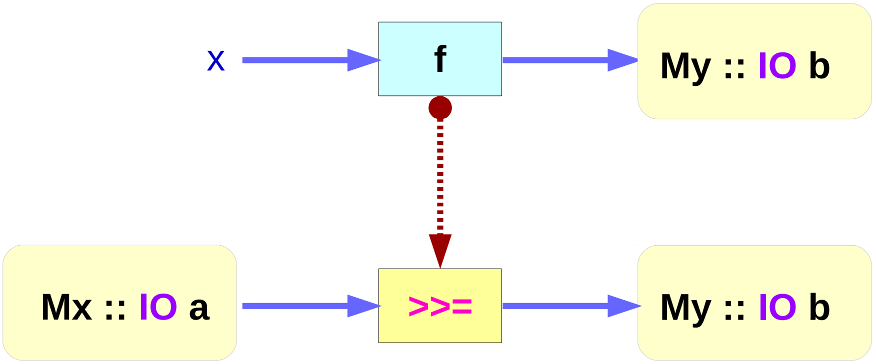



```
(ioX >>= f) w0 =
    let (x, w1) = ioX w0
    in  f x w1          -- has type (t, World)
```

IO Monad - (>>=) operator type



IO Monad - (>>=) operator type



IO Monad and ST Monad

```
instance Monad IO where
    return x w0 = (x, w0)

    (ioX >>= f) w0  =
        let    (x, w1) = ioX w0
        in     f x w1                -- has type (t, World)
```

```
instance Monad ST where
    -- return :: a -> ST a
    return x  =  \s -> (x,s)

    -- (>>=)  :: ST a -> (a -> ST b) -> ST b
    st >>= f  =  \s -> let (x,s') = st s
                        in f x s'
```

```
type    IO t   =    World    ->    (t, World)
```

type synonym

https://www.cs.hmc.edu/~adavidso/monads.pdf

State Transformers **ST Monad**

```
instance Monad ST where
  -- return :: a -> ST a
  return x  =  \s -> (x,s)

  -- (>>=)  :: ST a -> (a -> ST b) -> ST b
  st >>= f  =  \s -> let (x,s') = st s in f x s'
```

>>= provides a means of sequencing state transformers:
st >>= f applies the state transformer **st** to an initial state s,
then applies the function **f** to the resulting value x
to give a second state transformer (**f** x),
which is then applied to the modified state s' to give the final result:

```
st >>= f  =  \s -> f x s'
         where (x,s') = st s

st >>= f  =  \s -> (y,s')
         where (x,s') = st s
               (y,s') = f x s'
```

(x,s') = **st** s

f x s'

https://cseweb.ucsd.edu/classes/wi13/cse230-a/lectures/monads2.html

Another Monad Definition

```
fmap   :: (a -> b) -> M a -> M b  -- functor

return :: a -> M a
join   :: M (M a) -> M a
```

the functors-as-containers metaphor
a functor M can be thought of as container
so that M a "contains" values of type a,
with a corresponding mapping function, i.e. fmap,
that allows functions to be applied to values inside it.

Under this interpretation, the functions behave as follows:

fmap applies a given function to every element in a container
return <u>packages</u> an element into a container,
join takes a container of containers and <u>flattens</u> it into a single container.

https://en.wikibooks.org/wiki/Haskell/Understanding_monads#cite_note-3

fmap and **join**

fmap applies a given function to every element in a container
return packages an element into a container,
join takes a container of containers and flattens it into a single container.

the bind combinator can be defined as follows:

m >>= g = join (fmap g m)

Likewise, we could give a definition of **fmap** and **join** in terms of (**>>=**) and **return**:

fmap f x = x >>= (return . f)
join x   = x >>= id

https://en.wikibooks.org/wiki/Haskell/Understanding_monads#cite_note-3

liftM Function

a **Monad** is just a special **Functor** with extra features

Monads

map types to new types

that represent "computations that result in values"

can lift regular functions into **Monad** types

via a **liftM** function (like a **fmap** function)

liftM transform a regular function

into a "computations that results in the value

obtained by evaluating the function."

https://stackoverflow.com/questions/18808258/what-does-the-just-syntax-mean-in-haskell

liftM Function

Control.Monad defines **liftM**,
a function with a strangely familiar type signature...

```
liftM :: (Monad m) => (a -> b) -> m a -> m b
```

liftM is merely
fmap implemented with (**>>=**) and **return**

liftM and **fmap** are therefore interchangeable.

Another Control.Monad function with an uncanny type is **ap**:

```
ap :: Monad m => m (a -> b) -> m a -> m b
```

Analogously to the other cases, **ap** is a monad-only version of (**<*>**).

https://en.wikibooks.org/wiki/Haskell/Understanding_monads#cite_note-3

liftM vs fmap and ap vs <*>

```
liftM :: Monad m =>  (a -> b) -> m a -> m b
fmap ::  Functor f =>   (a -> b) -> f a -> f b


ap :: Monad m => m (a -> b) -> m a -> m b
(<*>) :: Applicative f => f (a -> b) -> f a -> f b


(>>=) ::  Monad m => m a  -> (a -> m b)  -> m b
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads#cite_note-3

Lift

The function **return** lifts a plain *value* **a** to **M a**

The *statements* in the imperative language **M** when executed, will result in the value **a** without any additional effects particular to **M**.

This is ensured by **Monad Laws**,

foo >>= return === foo

```
foo >>= return
foo
```

return x >>= k === k x;

```
return x >>= k
k x;
```

https://en.wikibooks.org/wiki/Haskell/Understanding_monads#cite_note-3

References

[1] ftp://ftp.geoinfo.tuwien.ac.at/navratil/HaskellTutorial.pdf
[2] https://www.umiacs.umd.edu/~hal/docs/daume02yaht.pdf