Undersampling (2A)

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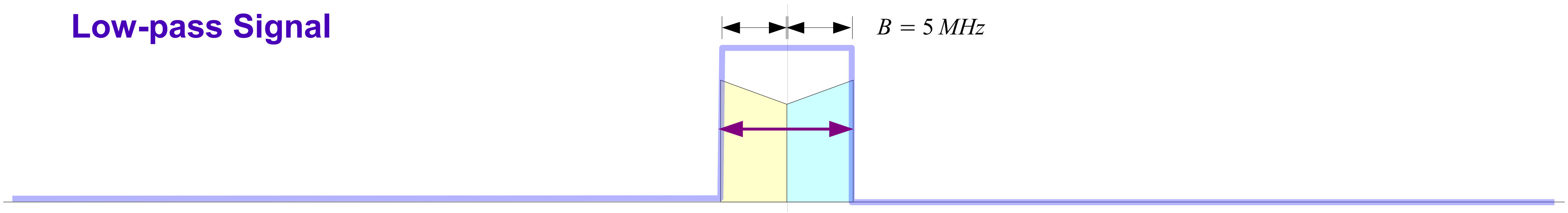
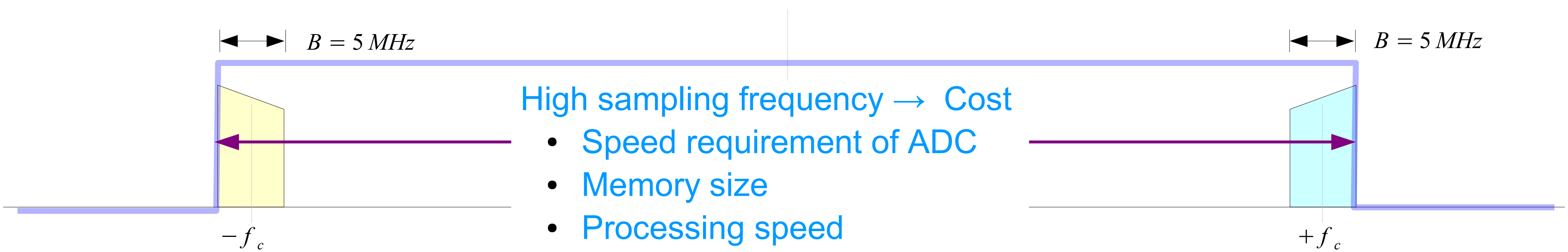
Low-pass Signal Sampling

Low-pass Signal



Nyquist Criterion	$f_s \geq 2B$

Band-pass Signal



Nyquist Criterion	$f_s \geq 2f_c + B$

Sub-Nyquist Rate ? → ***UnderSampling***

$$f_s < 2f_c + B$$

Sampling

Sampling in time domain

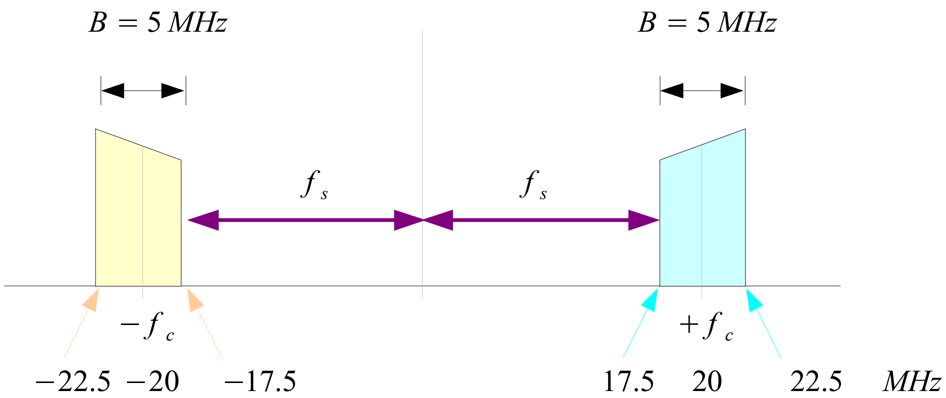
Multiplication by a comb of impulse functions

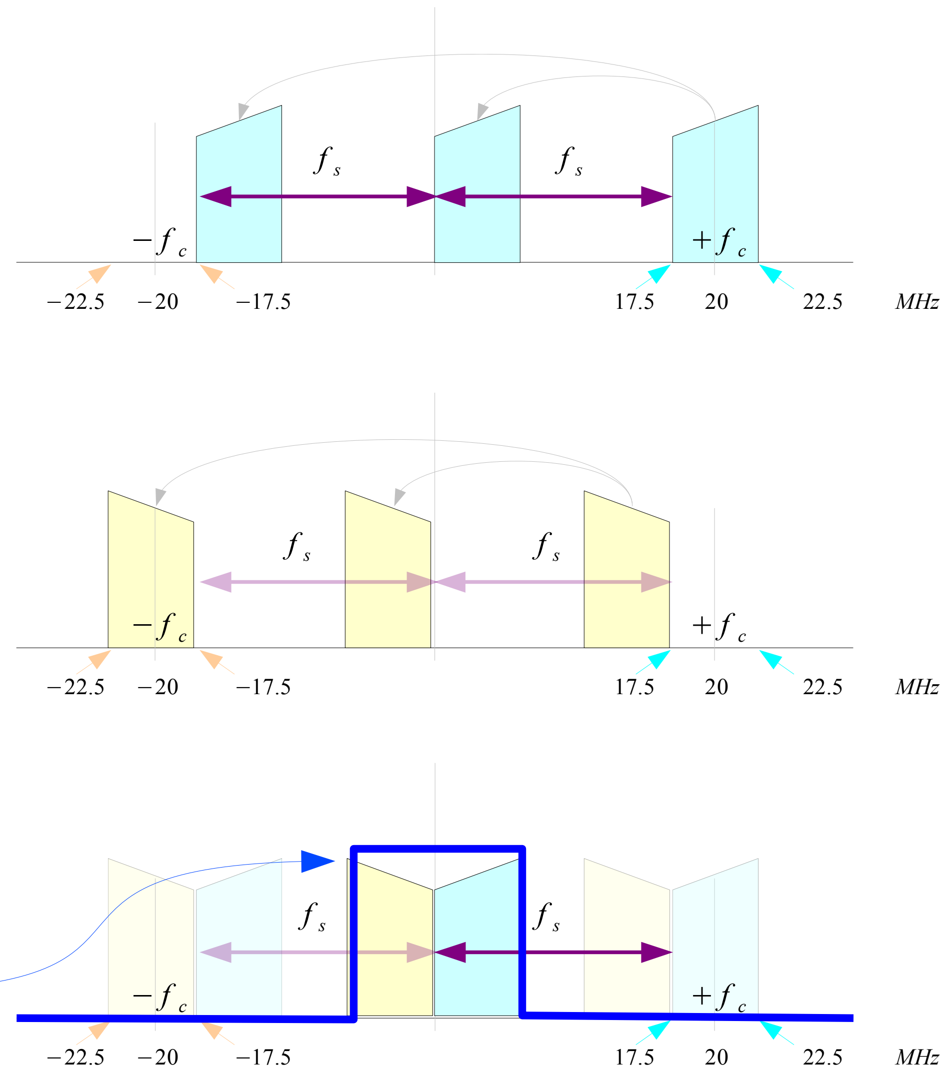
in frequency domain

convolution of the Fourier transformed impulse functions

REPLICATION

Band-pass Signal Sampling





- **Bandpass Sampling**
- **IF filtering**
- **Harmonic Sampling**
- **Sub-Nyquist Sampling**

Nyquist Criterion $2B \le f_s$

Sampling Frequency f_s (1)

Assume there are m multiples of f_s

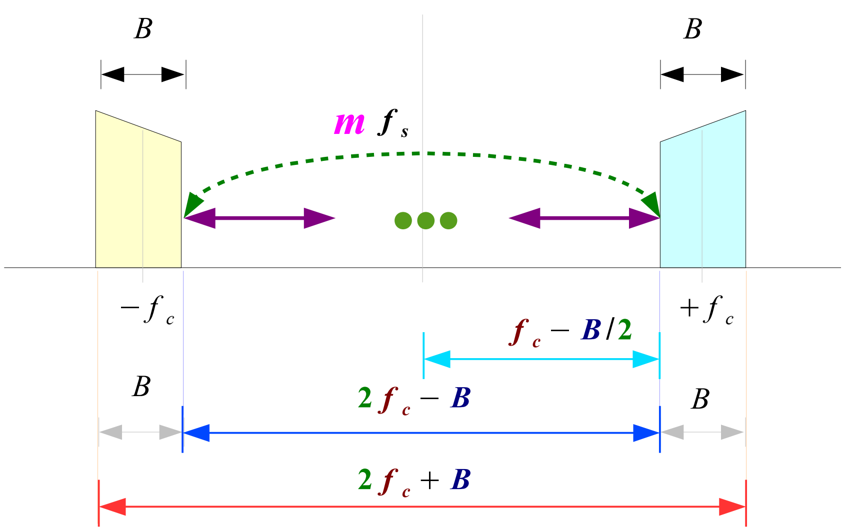
$$2f_c - B = m \cdot f_s$$

Given an integer m — **Max f_s condition**

f_s *can be decreased according to the following condition without introducing aliasing problems*

$$2f_c + B = (m+1) \cdot f_s$$

Min f_s condition



Given Band-pass Signal is characterized by

- *Bandwidth B*
- *Carrier Frequency f_c*

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

Sampling Frequency f_s (2)

$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

Given Band-pass Signal is characterized by

- ***Bandwidth B***
- ***Carrier Frequency f_c***

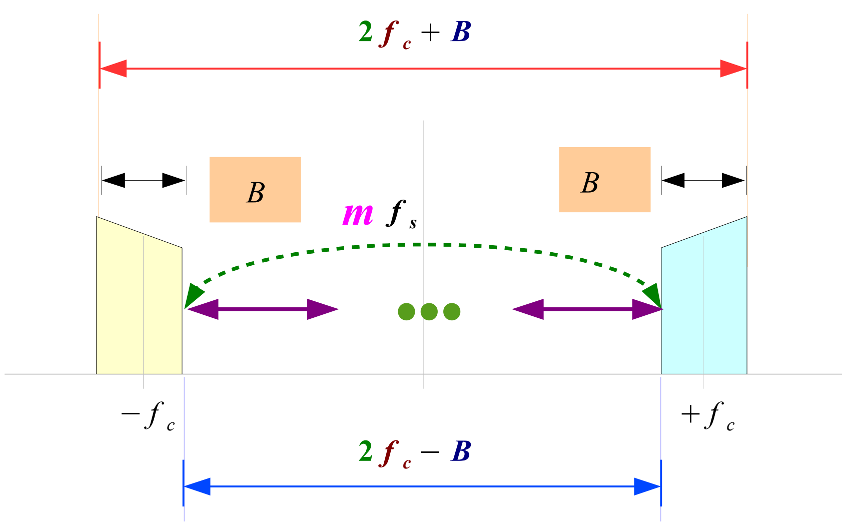
➡ ***Normalization by B***

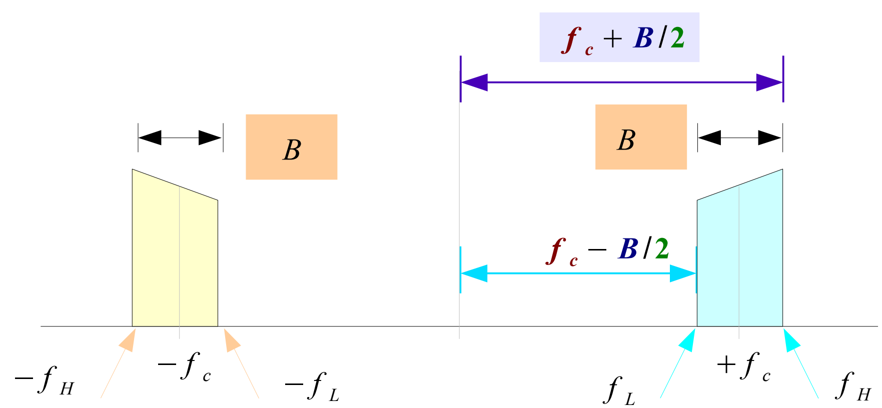
$$\frac{2f_c + B}{(m+1)B} \le \frac{f_s}{B} \le \frac{2f_c - B}{mB}$$

$$\frac{2f_H}{(m+1)B} \le \frac{f_s}{B} \le \frac{2f_L}{mB}$$

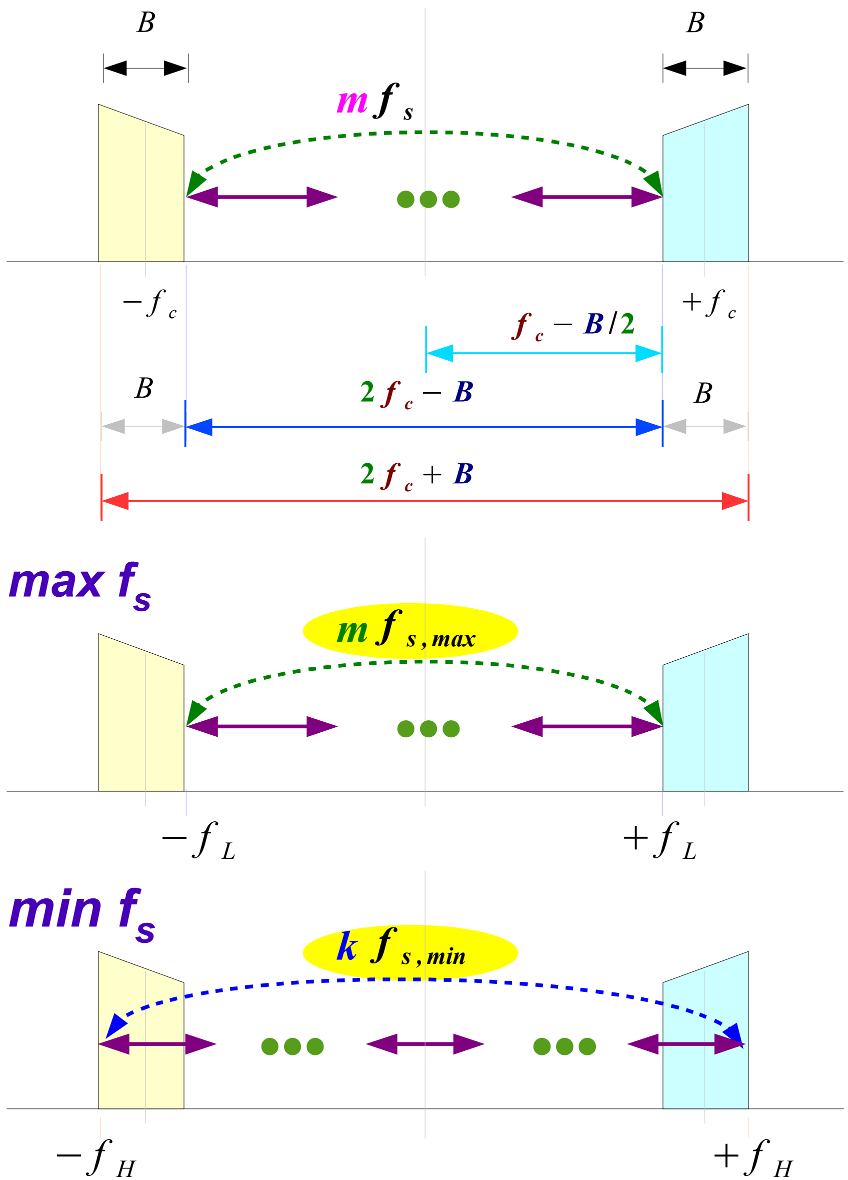
$f_H = f_c + B/2$ ***Highest frequency***

$f_L = f_c - B/2$ ***Lowest frequency***





Min, Max Condition on f_s (2)



min f_s		max f_s
$\frac{2f_H}{k}$	$\le f_s \le$	$\frac{2f_L}{m}$

$$k = m + 1$$

m represents how many f_s are in $2f_c - B$ in max f_s

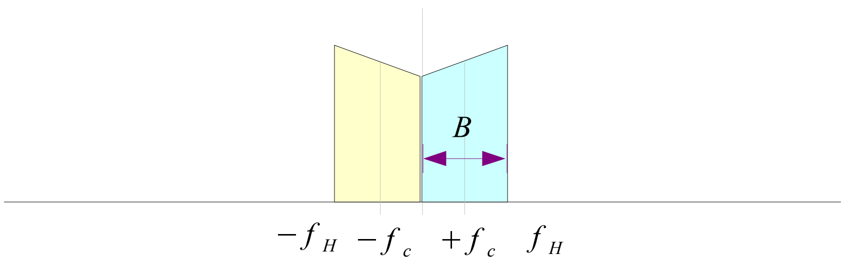
$$\text{max } f_s = \frac{2f_c - B}{m} = \frac{2f_L}{m}$$

k represents how many f_s are in $2f_c + B$ in min f_s

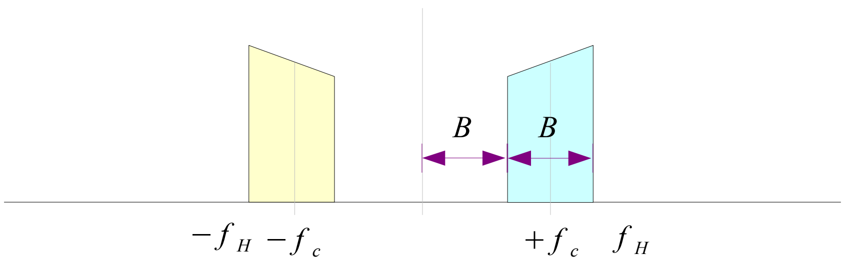
$$\text{min } f_s = \frac{2f_c + B}{k} = \frac{2f_H}{k}$$

Minimum f_s Plot (2)

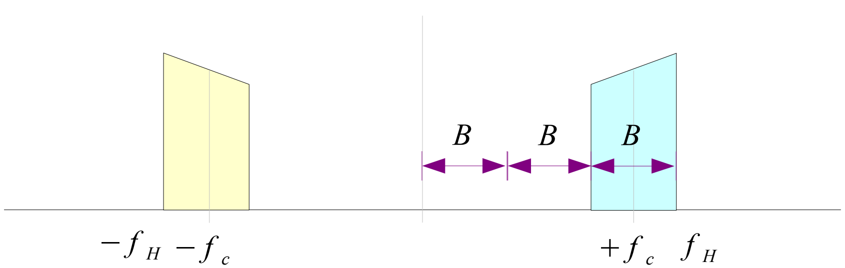
$f_H = f_c + B/2 = 1B$ $\quad$ $R = f_H / B = 1$

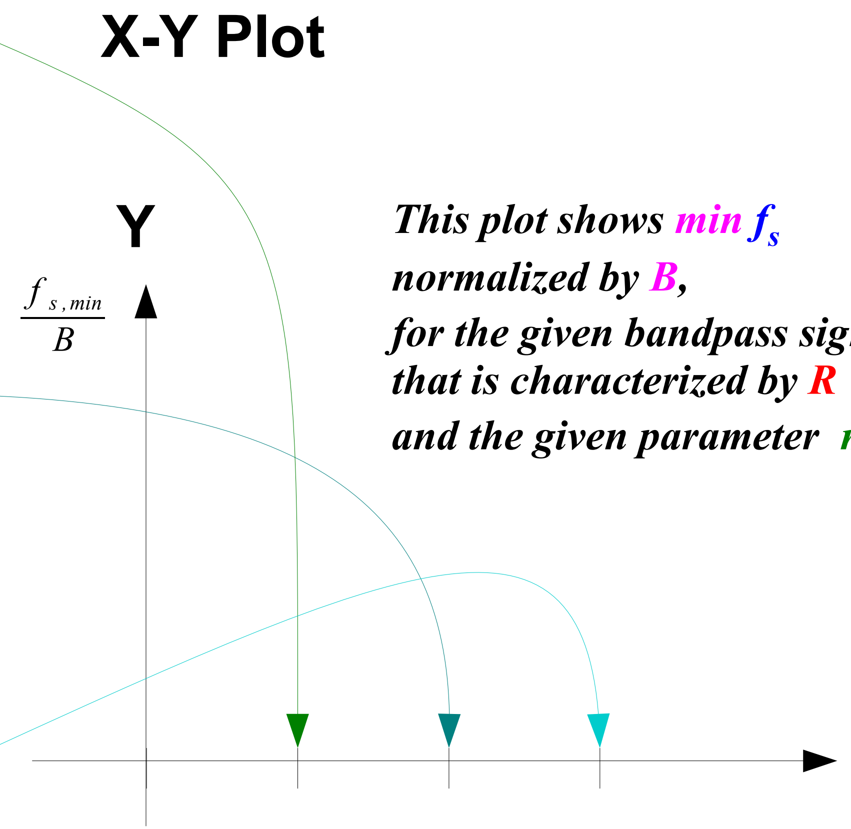


$f_H = f_c + B/2 = 2B$ $\quad$ $R = f_H / B = 2$



$f_H = f_c + B/2 = 3B$ $\quad$ $R = f_H / B = 3$





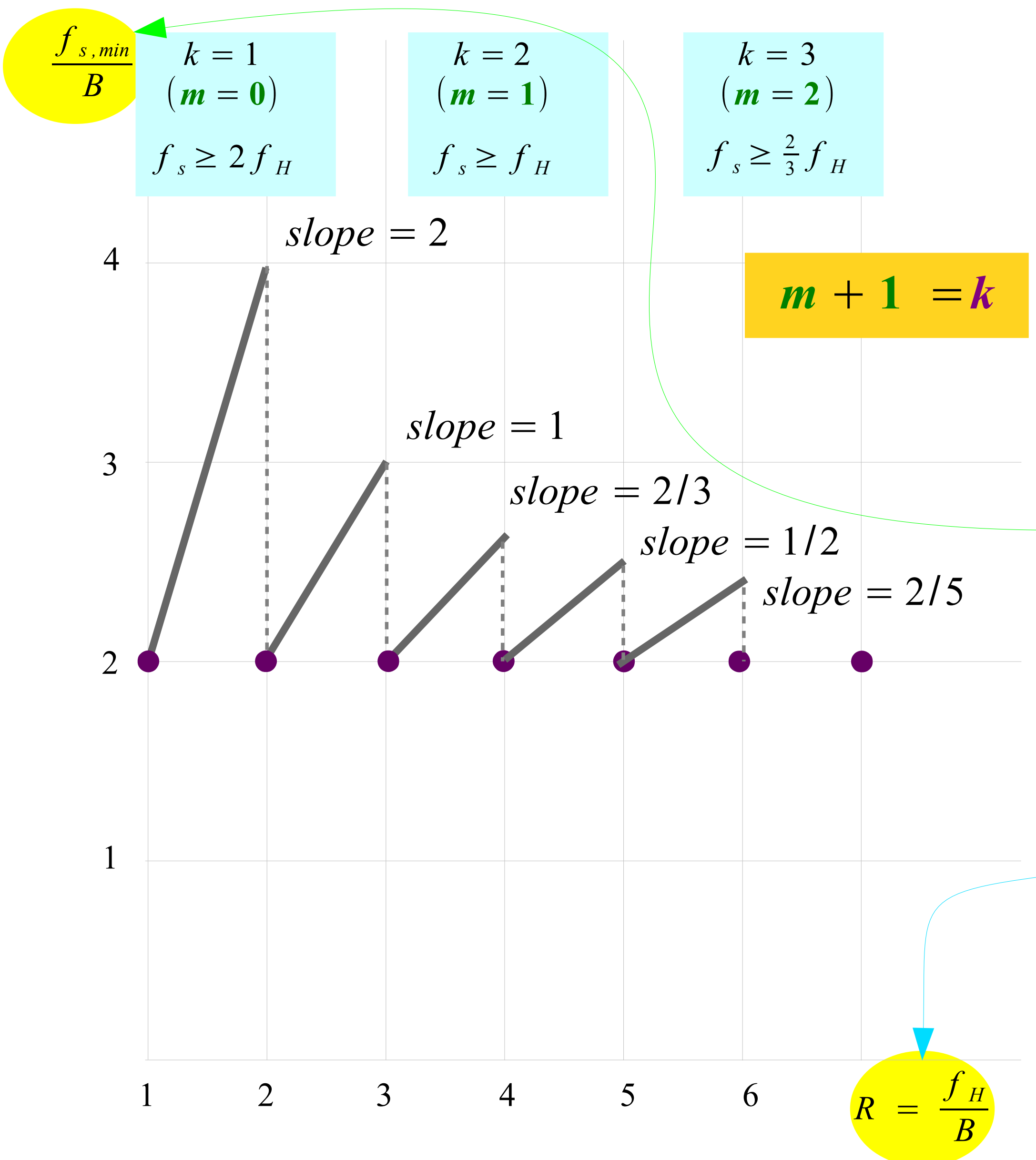
This plot shows min f_s normalized by B, for the given bandpass signal that is characterized by R and the given parameter m

Characterized by

- ***Bandwidth B***
- ***Carrier Frequency f_c***

$$R = \frac{f_H}{B} = \frac{f_c + B/2}{B}$$

Minimum f_s Plot (5)



$$\frac{2 f_c + B}{m + 1} \le f_s \le \frac{2 f_c - B}{m}$$

$$\frac{2 f_c + B}{k} \le f_s \le \frac{2 f_c - B}{k - 1}$$

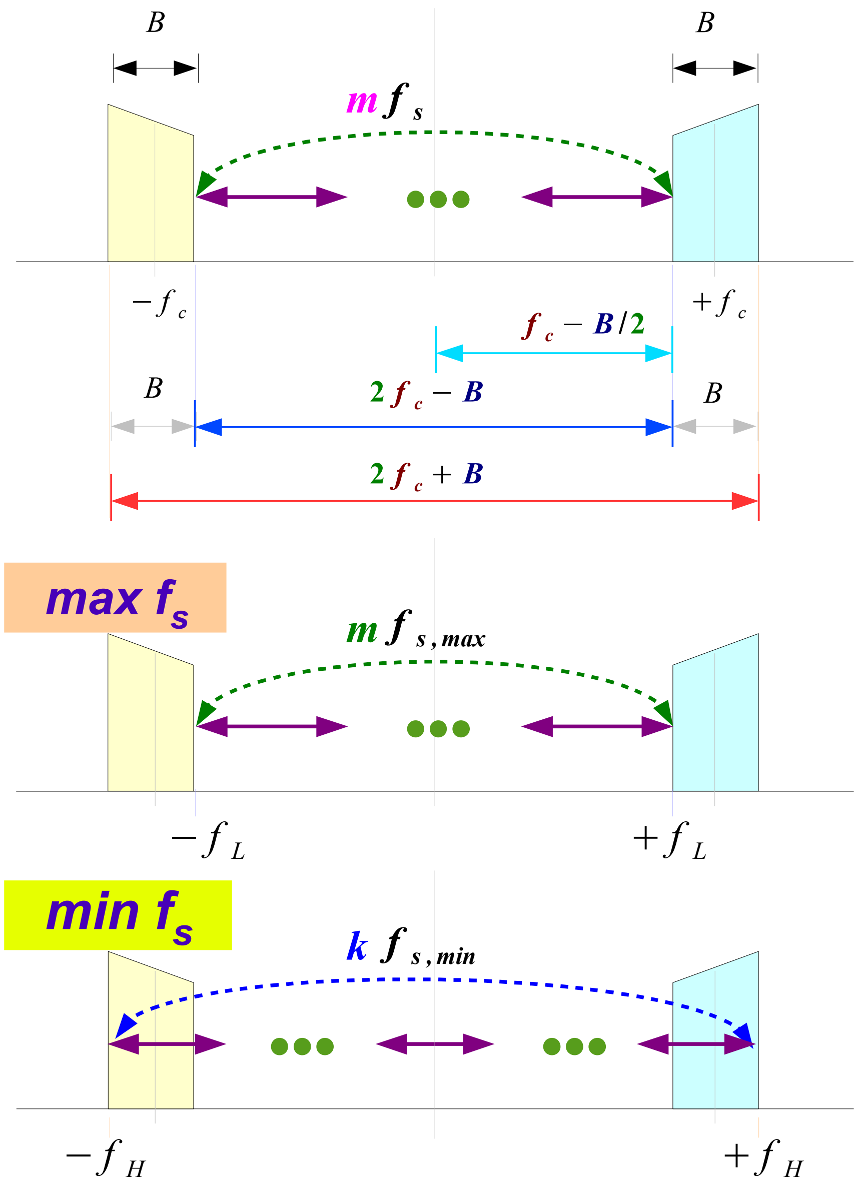
$$\frac{2 f_c + B}{(m + 1) B} = \frac{f_{s,min}}{B} = g(m, R)$$

$\Rightarrow$ $\dfrac{\text{minimum sampling rate}}{\text{bandwidth } B}$

$$\frac{f_c + B/2}{B} = R$$

$\Rightarrow$ $\dfrac{\text{highest signal frequency}}{\text{bandwidth } B}$

Min, Max Condition on f_s (2)



$$\frac{2f_c + B}{m+1} \le f_s \le \frac{2f_c - B}{m}$$

$$\frac{2f_c + B}{k} \le f_s \le \frac{2f_c - B}{k-1}$$

$$m + 1 = k$$

min f_s **max f_s**

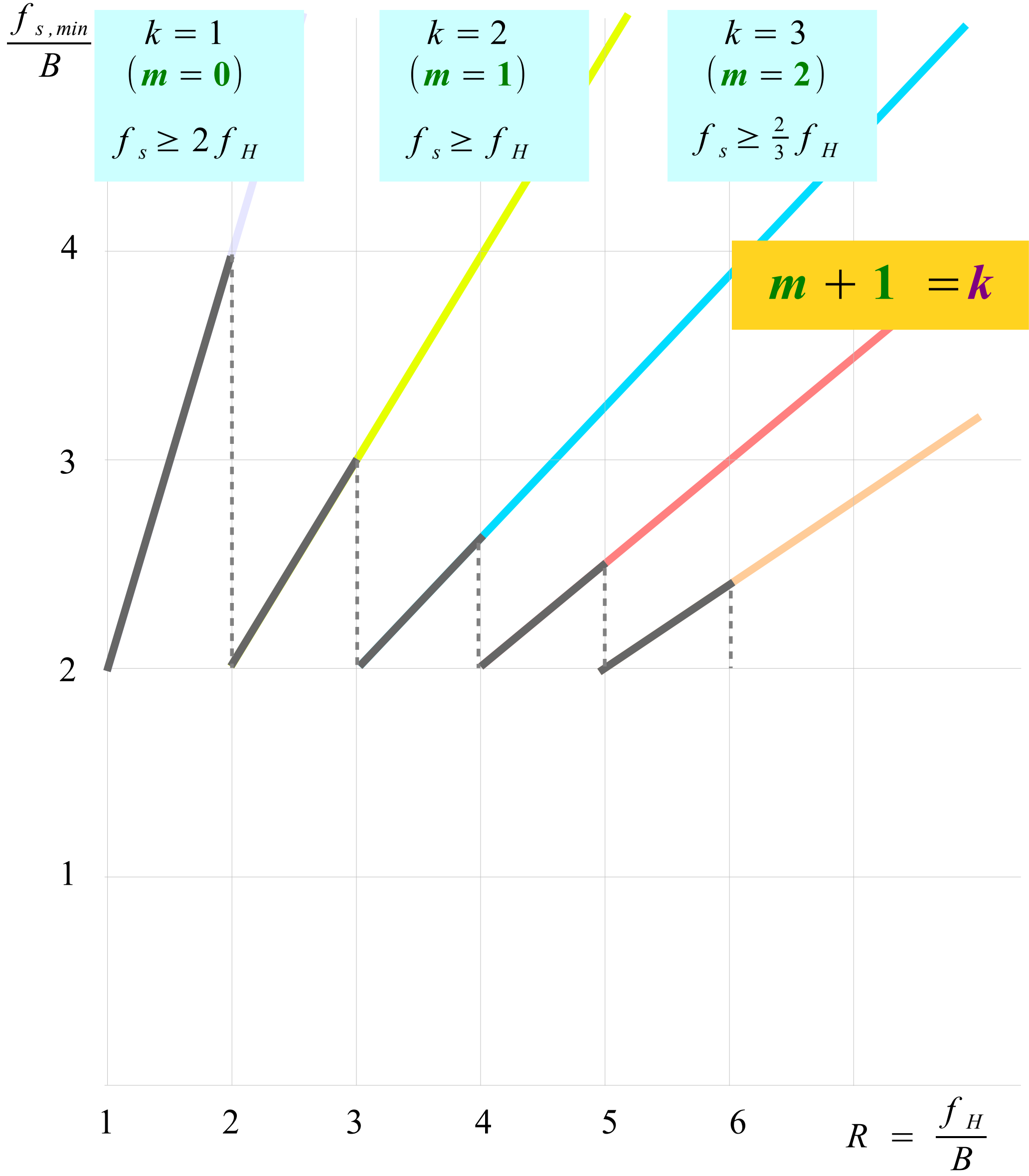
$$\frac{2f_H}{k} \le f_s \le \frac{2f_L}{m}$$

$$k = 2 \qquad f_H \le f_s \le 2f_L \qquad m = 1$$

$$k = 3 \qquad \frac{2}{3}f_H \le f_s \le f_L \qquad m = 2$$

$$k = 4 \qquad \frac{1}{2}f_H \le f_s \le \frac{2}{3}f_L \qquad m = 3$$

Min Max f_s Plot (1)

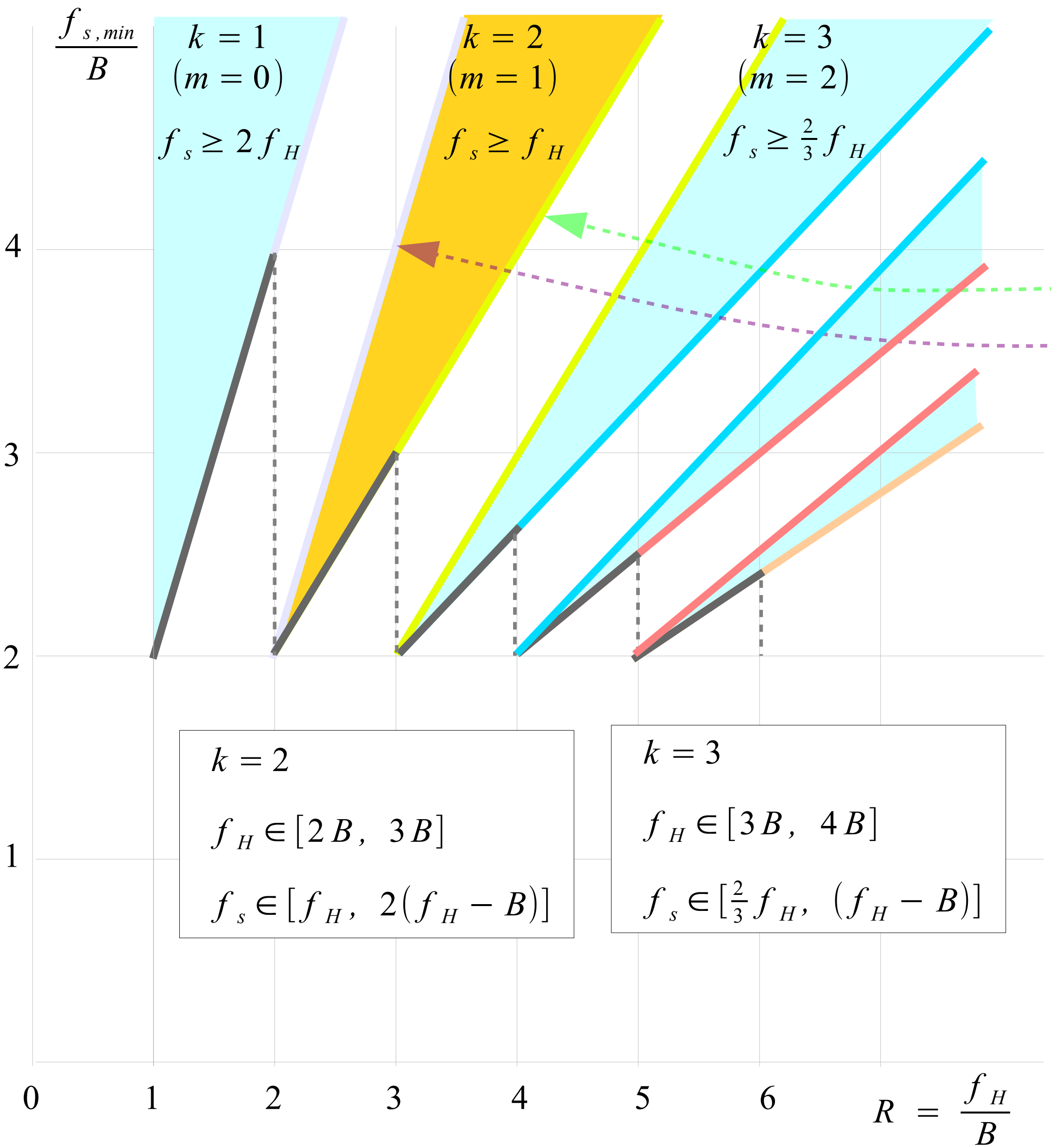


$$\frac{2 f_c + B}{m + 1} \leq f_s \leq \frac{2 f_c - B}{m}$$

$$\frac{2 f_c + B}{k} \leq f_s \leq \frac{2 f_c - B}{k - 1}$$

$$\frac{2 f_H}{k} \leq f_s \leq \frac{2 f_L}{m}$$

Min Max f_s Plot (2)



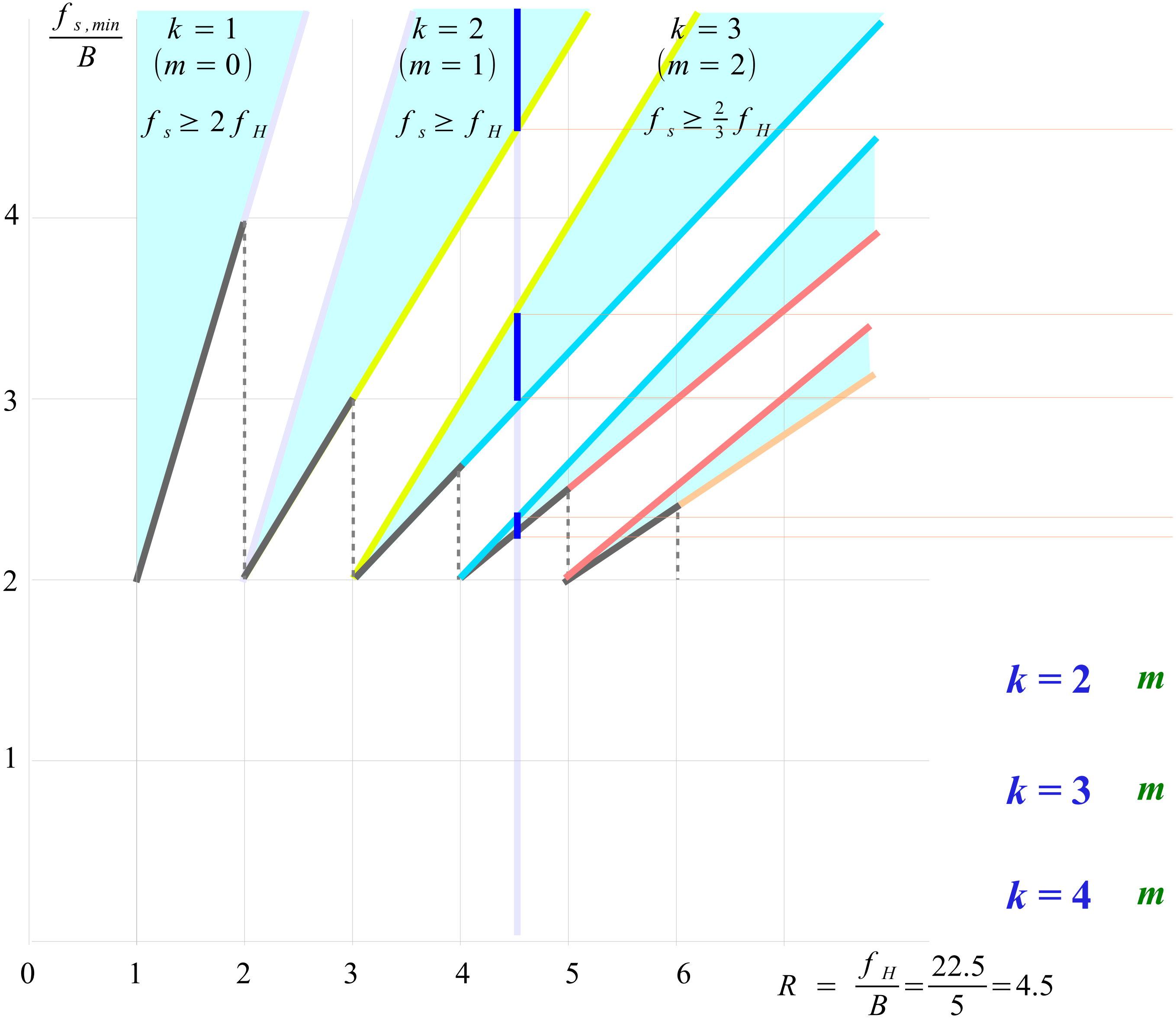
$$\frac{2 f_H}{k} \leq f_s \leq \frac{2 f_L}{m}$$

Min f_s **Max f_s**

$k = 2$	$f_H \leq f_s \leq 2 f_L$
$k = 3$	$\frac{2}{3} f_H \leq f_s \leq f_L$

Min f_s		**Max f_s**
$y = 1(x-2)+2$ $y = x$	$k = 2$	$y = 2(x-2)+2$ $y = 2x-2$
$y = \frac{2}{3}(x-3)+2$ $y = \frac{2}{3}x$	$k = 3$	$y = 1(x-3)+2$ $y = x-1$

Range of f_s when R=4.5, B=5 (7)



$f_s \geq 22.5$ $\quad \frac{f_s}{B} = 4.5$

$f_s \leq 17.5$ $\quad \frac{f_s}{B} = 3.5$

$f_s \geq 15.0$ $\quad \frac{f_s}{B} = 3.0$

$f_s \leq 11.67$ $\quad \frac{f_s}{B} = 2.33$

$f_s \geq 11.2$ $\quad \frac{f_s}{B} = 2.24$

$\mathbf{k = 2}$ $\quad \mathbf{m = 1}$ ➡ $22.5 \leq f_s \leq 35$

$\mathbf{k = 3}$ $\quad \mathbf{m = 2}$ ➡ $15.0 \leq f_s \leq 17.5$

$\mathbf{k = 4}$ $\quad \mathbf{m = 3}$ ➡ $11.2 \leq f_s \leq 11.67$

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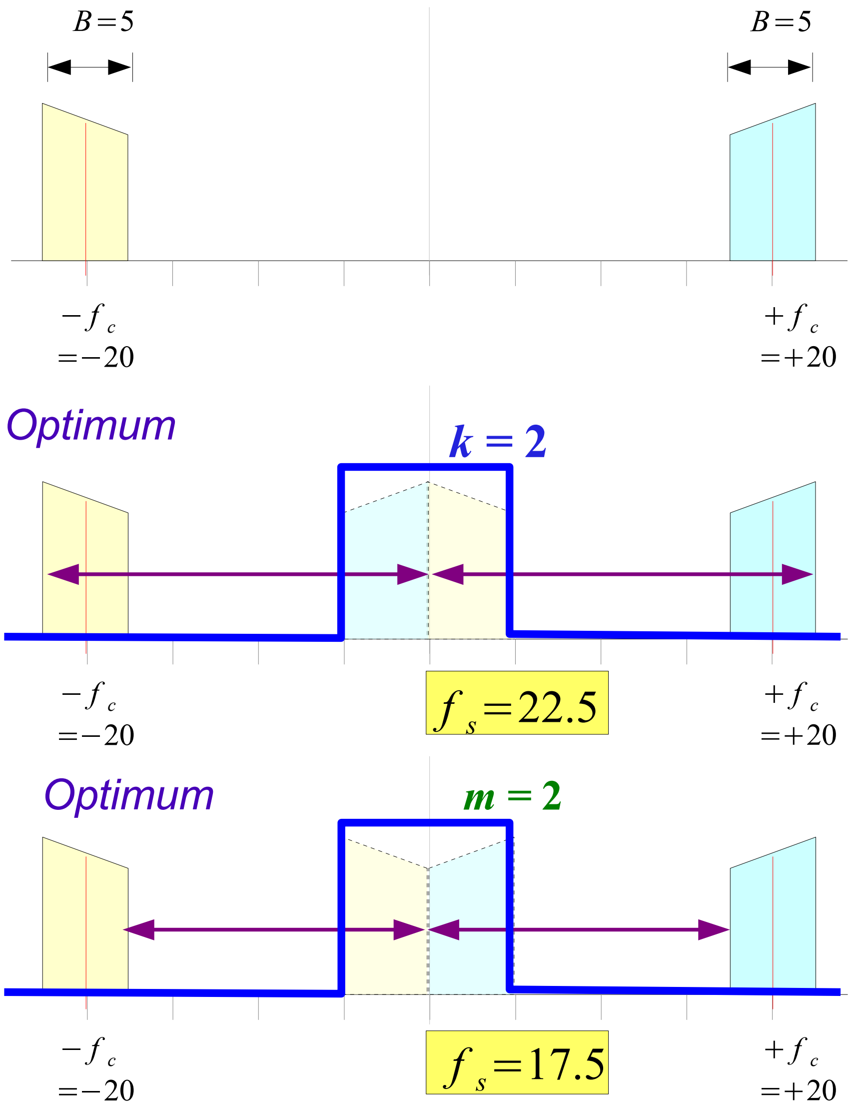
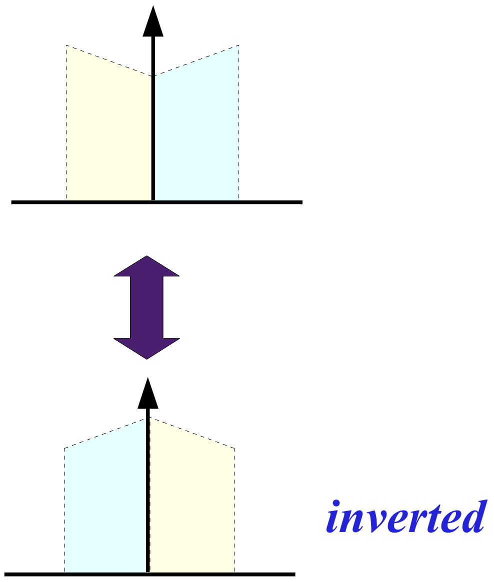
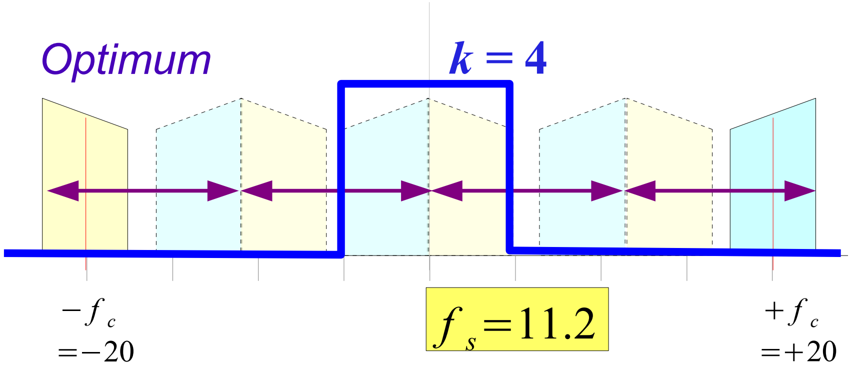
Spectral Inversion







References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003
[3] A "graphical interpretation" of the DFT and FFT, by Steve Mann
[4] R. G. Lyons, Understanding Digital Signal Processing, 1997