Laurent Series and z-Transform
- Geometric Series
Double Pole Properties A

20190108 Tue

2 formulas of z

① $$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

z^{-1}

② $$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

$$f(z) = \begin{cases} f_1(z) \\ f_2(z^{-1}) \end{cases} \qquad g(z) = \begin{cases} g_1(z) \\ g_2(z^{-1}) \end{cases}$$

$$X(z) = \begin{cases} X_1(z) \\ X_2(z^{-1}) \end{cases} \qquad Y(z) = \begin{cases} Y_1(z) \\ Y_2(z^{-1}) \end{cases}$$

① $\frac{3}{2} \frac{-1}{(z-0.5)(z-2)}$

② $\frac{3}{2} \frac{-z^2}{(z-2)(z-0.5)}$

$+ \frac{1}{z-0.5} - \frac{1}{z-2}$

$- \frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}$

$- \frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad |z| < 0.5$

causal $f_1(z)$

anti-causal $X_1(z)$

$- \frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$ $\quad |z| > 2$

anti-causal $g_1(z)$

causal $Y_1(z)$

$+ \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $\quad |z| > 2$

anti-causal $f_2(z)$

causal $X_2(z)$

$+ \frac{z}{1-0.5z} - \frac{z}{1-2z}$ $\quad |z| < 0.5$

causal $g_2(z)$

anti-causal $Y_2(z)$

① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$ ② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

$+\frac{1}{z-0.5}-\frac{1}{z-2}$ $-\frac{2z}{(z-2)}+\frac{0.5z}{(z-0.5)}$

$\cdot 2$ $\cdot\frac{1}{2}$ $\cdot\frac{1}{z}$ $\cdot\frac{1}{z}$

$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$ $|z|<0.5$ $-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}$ $|z|>2$

$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$ $|z|>2$ $+\frac{z}{1-0.5z}-\frac{z}{1-2z}$ $|z|<0.5$

$\cdot\frac{1}{z}$ $\cdot\frac{1}{z}$ $\cdot\frac{1}{2}$ $\cdot 2$

$+\frac{1}{z-0.5}-\frac{1}{z-2}$ $-\frac{2z}{(z-2)}+\frac{0.5z}{(z-0.5)}$

① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$ ② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

$|2z|<1$ $|0.5z|<1$

$|z|<0.5$ $|z|<2$

$|2z^{-1}|<1$ $|0.5z^{-1}|<1$

$2<|z|$ $0.5<|z|$

$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z}$ $|z|<0.5$

$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}}$ $|z|>2$

$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}}$ $|z|>2$

$+\frac{z}{1-0.5z}-\frac{z}{1-2z}$ $|z|<0.5$

$|0.5z^{-1}|<1$ $|2z^{-1}|<1$

$0.5<|z|$ $2<|z|$

$|0.5z|<1$ $|2z|<1$

$|z|<2$ $|z|<0.5$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$\cdot\frac{1}{2z}$ $\qquad$ $\cdot\frac{z}{z}$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$\cdot\frac{z}{2}$ $\qquad$ $\cdot 2z$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$\cdot 2z$ $\qquad$ $\cdot\frac{z}{2}$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$\cdot\frac{2}{z}$ $\qquad$ $\cdot\frac{1}{2z}$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

Causal Sequence a_n & x_n

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

causal $f_1(z) =$

$-\left[2+2^2z^1+2^3z^2+\cdots\right]$ $\quad -2^{n+1}$

$+\left[\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2z^1+\left(\frac{1}{2}\right)^3z^2+\cdots\right]$ $\quad +\left(\frac{1}{2}\right)^{n+1}$

0 1 2

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

causal $Y_1(z) =$

$-\left[2^1z^0+2^2z^{-1}+2^3z^{-2}+\cdots\right]$ $\quad -2^{n+1}$

$+\left[\left(\frac{1}{2}\right)^1z^0+\left(\frac{1}{2}\right)^2z^{-1}+\left(\frac{1}{2}\right)^3z^{-2}+\cdots\right]$ $\quad +\left(\frac{1}{2}\right)^{n+1}$

0 1 2

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

causal $X_2(z)$

$+\left[\left(\frac{1}{2}\right)^0z^{-1}+\left(\frac{1}{2}\right)^1z^{-2}+\left(\frac{1}{2}\right)^2z^{-3}+\cdots\right]$ $\quad +\left(\frac{1}{2}\right)^{n-1}$

$-\left[2^0z^{-1}+2^1z^{-2}+2^2z^{-3}+\cdots\right]$ $\quad -2^{n-1}$

1 2 3

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

causal $g_2(z)$

$+\left[\left(\frac{1}{2}\right)^0z^1+\left(\frac{1}{2}\right)^1z^2+\left(\frac{1}{2}\right)^2z^3+\cdots\right]$ $\quad +\left(\frac{1}{2}\right)^{n-1}$

$-\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right]$ $\quad -2^{n-1}$

1 2 3

Anti-causal sequence a_n & x_n

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

anti-causal $X_1(z)$

$-\left[\left(\frac{1}{2}\right)^{-1}+\left(\frac{1}{2}\right)^{-2}z^{1}+\left(\frac{1}{2}\right)^{-3}z^{2}+\cdots\right] \quad -\left(\frac{1}{2}\right)^{n-1}$

$+\left[2^{-1}+2^{-2}z^{1}+2^{-3}z^{2}+\cdots\right] \quad +2^{n-1}$

0 -1 -2

$2=\left(\frac{1}{2}\right)^{-1}$

$\left(\frac{1}{2}\right)=2^{-1}$

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

anti-causal $g_1(z)$

$-\left[\left(\frac{1}{2}\right)^{-1}z^{0}+\left(\frac{1}{2}\right)^{-2}z^{-1}+\left(\frac{1}{2}\right)^{-3}z^{-2}+\cdots\right] \quad -\left(\frac{1}{2}\right)^{n-1}$

$+\left[2^{-1}z^{0}+2^{-2}z^{-1}+2^{-3}z^{-2}+\cdots\right] \quad +2^{n-1}$

0 -1 -2

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

anti-causal $f_2(z)$

$+\left[2^{0}z^{1}+2^{-1}z^{-2}+2^{-2}z^{-3}+\cdots\right] \quad +2^{n+1}$

$-\left[\left(\frac{1}{2}\right)^{0}z^{1}+\left(\frac{1}{2}\right)^{-1}z^{-2}+\left(\frac{1}{2}\right)^{-2}z^{-3}+\cdots\right] \quad -\left(\frac{1}{2}\right)^{n+1}$

-1 -2 -3

$2=\left(\frac{1}{2}\right)^{-1}$

$\left(\frac{1}{2}\right)=2^{-1}$

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

anti-causal $Y_2(z)$

$+\left[2^{0}z^{1}+2^{-1}z^{2}+2^{-2}z^{3}+\cdots\right] \quad +2^{n+1}$

$-\left[\left(\frac{1}{2}\right)^{0}z^{1}+\left(\frac{1}{2}\right)^{-1}z^{2}+\left(\frac{1}{2}\right)^{-2}z^{3}+\cdots\right] \quad -\left(\frac{1}{2}\right)^{n+1}$

-1 -2 -3

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

causal $f_1(z) =$

$-\left[2+2^2z^1+2^3z^2+\cdots\right] \quad -2^{n+1}$

$+\left[\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2z^1+\left(\frac{1}{2}\right)^3z^2+\cdots\right] \quad +\left(\frac{1}{2}\right)^{n+1}$

0 1 2

anti-causal $X_1(z)$

$-\left[\left(\frac{1}{2}\right)^{-1}+\left(\frac{1}{2}\right)^{-2}z^1+\left(\frac{1}{2}\right)^{-3}z^2+\cdots\right] \quad -\left(\frac{1}{2}\right)^{n-1}$

$+\left[2^{-1}+2^{-2}z^1+2^{-3}z^2+\cdots\right] \quad +2^{n-1}$

0 -1 -2

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

anti-causal $g_1(z)$

$-\left[\left(\frac{1}{2}\right)^{-1}z^0+\left(\frac{1}{2}\right)^{-2}z^{-1}+\left(\frac{1}{2}\right)^{-3}z^{-2}+\cdots\right] \quad -\left(\frac{1}{2}\right)^{n-1}$

$+\left[2^{-1}z^0+2^{-2}z^{-1}+2^{-3}z^{-2}+\cdots\right] \quad +2^{n-1}$

0 -1 -2

causal $Y_1(z) =$

$-\left[2^1z^0+2^2z^{-1}+2^3z^{-2}+\cdots\right] \quad -2^{n+1}$

$+\left[\left(\frac{1}{2}\right)^1z^0+\left(\frac{1}{2}\right)^2z^{-1}+\left(\frac{1}{2}\right)^3z^{-2}+\cdots\right] \quad +\left(\frac{1}{2}\right)^{n+1}$

0 1 2

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

anti-causal $f_2(z)$

$+\left[2^0z^1+2^{-1}z^{-2}+2^{-2}z^{-3}+\cdots\right] \quad +2^{n+1}$

$-\left[\left(\frac{1}{2}\right)^0z^{-1}+\left(\frac{1}{2}\right)^{-1}z^{-2}+\left(\frac{1}{2}\right)^{-2}z^{-3}+\cdots\right] \quad -\left(\frac{1}{2}\right)^{n+1}$

-1 -2 -3

causal $X_2(z)$

$+\left[\left(\frac{1}{2}\right)^0z^1+\left(\frac{1}{2}\right)^1z^2+\left(\frac{1}{2}\right)^2z^3+\cdots\right] \quad +\left(\frac{1}{2}\right)^{n-1}$

$-\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right] \quad -2^{n-1}$

1 2 3

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

causal $g_2(z)$

$+\left[\left(\frac{1}{2}\right)^0z^1+\left(\frac{1}{2}\right)^1z^2+\left(\frac{1}{2}\right)^2z^3+\cdots\right] \quad +\left(\frac{1}{2}\right)^{n-1}$

$-\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right] \quad -2^{n-1}$

1 2 3

anti-causal $Y_2(z)$

$+\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right] \quad +2^{n+1}$

$-\left[\left(\frac{1}{2}\right)^0z^1+\left(\frac{1}{2}\right)^{-1}z^2+\left(\frac{1}{2}\right)^{-2}z^3+\cdots\right] \quad -\left(\frac{1}{2}\right)^{n+1}$

-1 -2 -3

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$$f(z)=-\left[2+2^2z+2^3z^2+\cdots\right]+\left[\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2z+\left(\frac{1}{2}\right)^3z^2+\cdots\right]$$

$$a_n=-2^{n+1}+\left(\frac{1}{2}\right)^{n+1} \qquad (n\geq 0)$$

$$X(z)=-\left[\left(\frac{1}{2}\right)^{-1}+\left(\frac{1}{2}\right)^{-2}z^1+\left(\frac{1}{2}\right)^{-3}z^2+\cdots\right]+\left[2^{-1}+2^{-2}z^1+2^{-3}z^2+\cdots\right]$$

$$x_n=-\left(\frac{1}{2}\right)^{n-1}+2^{n-1} \qquad (n<1)$$

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$f(z)=+\left[2^0z^1+2^{-1}z^{-2}+2^{-2}z^{-3}+\cdots\right]-\left[\left(\frac{1}{2}\right)^0z^{-1}+\left(\frac{1}{2}\right)^{-1}z^{-2}+\left(\frac{1}{2}\right)^{-2}z^{-3}+\cdots\right]$$

$$a_n=+2^{n+1}-\left(\frac{1}{2}\right)^{n+1} \qquad (n<0)$$

$$X(z)=+\left[\left(\frac{1}{2}\right)^0z^1+\left(\frac{1}{2}\right)^1z^{-2}+\left(\frac{1}{2}\right)^2z^{-3}+\cdots\right]-\left[2^0z^{-1}+2^1z^{-2}+2^2z^{-3}+\cdots\right]$$

$$x_n=+\left(\frac{1}{2}\right)^{n-1}-2^{n-1} \qquad (n\geq 1)$$

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$$f(z)=-\left[\left(\frac{1}{2}\right)^{-1}z^0+\left(\frac{1}{2}\right)^{-2}z^{-1}+\left(\frac{1}{2}\right)^{-3}z^{-2}+\cdots\right]+\left[2^1z^0+2^2z^{-1}+2^3z^{-2}+\cdots\right]$$

$$a_n=-\left(\frac{1}{2}\right)^{n-1}+2^{n-1} \qquad (n<1)$$

$$X(z)=-\left[2^1z^0+2^2z^{-1}+2^3z^{-2}+\cdots\right]+\left[\left(\frac{1}{2}\right)^1z^0+\left(\frac{1}{2}\right)^2z^{-1}+\left(\frac{1}{2}\right)^3z^{-2}+\cdots\right]$$

$$x_n=-2^{n+1}+\left(\frac{1}{2}\right)^{n+1} \qquad (n\geq 0)$$

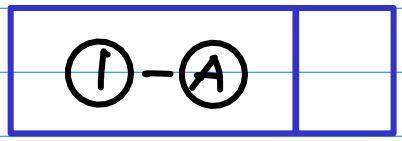
$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

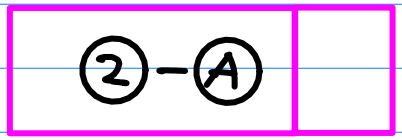
$$f(z)=+\left[\left(\frac{1}{2}\right)^0z^1+\left(\frac{1}{2}\right)^1z^2+\left(\frac{1}{2}\right)^2z^3+\cdots\right]-\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right]$$

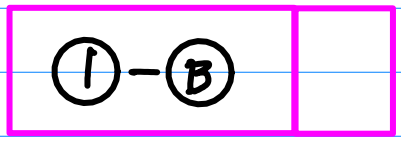
$$a_n=+\left(\frac{1}{2}\right)^{n-1}-2^{n-1} \qquad (n\geq 1)$$

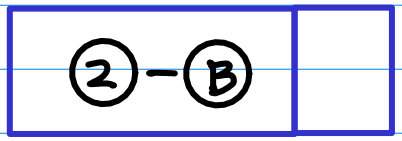
$$X(z)=+\left[2^0z^1+2^1z^2+2^2z^3+\cdots\right]-\left[\left(\frac{1}{2}\right)^0z^1+\left(\frac{1}{2}\right)^1z^2+\left(\frac{1}{2}\right)^2z^3+\cdots\right]$$

$$x_n=+2^{n+1}-\left(\frac{1}{2}\right)^{n+1} \qquad (n<0)$$


1-A


2-A


1-B


2-B

$f(z)$ $\quad |z| < 0.5$ causal $\quad |z| > 2$ anticausal

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

① - Ⓐ $\quad -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad |z| < 0.5$ $\quad -2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $\quad (n \geq 0)$

$$-\underset{n=0}{\left(2 + 2^1 z + 2^2 z^2 + \cdots\right)} + \underset{n=0}{\left(\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 z + \left(\frac{1}{2}\right)^3 z^2 + \cdots\right)}$$

① - Ⓑ $\quad \frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $\quad |z| > 2$ $\quad +2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $\quad (n < 0)$

$$\underset{n=-1}{\left(z^{-1} + \left(\frac{1}{2}\right)^1 z^{-2} + \left(\frac{1}{2}\right)^2 z^{-3} + \cdots\right)} - \underset{n=-1}{\left(z^{-1} + 2\, z^{-2} + 2^2 z^{-3} + \cdots\right)}$$

$$\underset{n=-1}{\left(2^0 z^{-1} + 2^{-1} z^{-2} + 2^{-2} z^{-3} + \cdots\right)} - \underset{n=-1}{\left(\left(\frac{1}{2}\right)^0 z^{-1} + \left(\frac{1}{2}\right)^{-1} z^{-2} + \left(\frac{1}{2}\right)^{-2} z^{-3} + \cdots\right)}$$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

② - Ⓐ $\quad -\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$ $\quad |z| > 2$ $\quad -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $\quad (n < 1)$

$$-\underset{n=0}{\left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right)} + \underset{n=0}{\left(\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2 z^{-1} + \left(\frac{1}{2}\right)^3 z^{-2} + \cdots\right)}$$

$$-\underset{n=0}{\left(\left(\frac{1}{2}\right)^{-1} + \left(\frac{1}{2}\right)^{-2} z^{-1} + \left(\frac{1}{2}\right)^{-3} z^{-2} + \cdots\right)} + \underset{n=0}{\left(2^{-1} + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right)}$$

② - Ⓑ $\quad +\frac{z}{1-0.5z} - \frac{z}{1-2z}$ $\quad |z| < 0.5$ $\quad +\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $\quad (n \geq 1)$

$$+\underset{n=1}{\left(z + \left(\frac{1}{2}\right) z^2 + \left(\frac{1}{2}\right)^2 z^3 + \cdots\right)} - \underset{n=1}{\left(z + 2z^2 + 2^2 z^3 + \cdots\right)}$$

$X(z)$ $|z| < 0.5$ $|z| > 2$

anticausal causal

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

①-A $-\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $|z| < 0.5$ $-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $(n < 1)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\tfrac{1}{2}\right) z^0 + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\tfrac{1}{2}\right)^{-1} z^0 + \left(\tfrac{1}{2}\right)^{-2} z^1 + \left(\tfrac{1}{2}\right)^{-3} z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0$ $n=-1$ $n=-2$ $\quad$ $n=0$ $n=-1$ $n=-2$

①-B $\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}}$ $|z| > 2$ $+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $(n \geq 1)$

$$\left(\left(\tfrac{1}{2}\right)^0 z^{-1} + \left(\tfrac{1}{2}\right)^1 z^{-2} + \left(\tfrac{1}{2}\right)^2 z^{-3} + \cdots\right) - \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)$$

$n=1$ $n=2$ $n=3$ $\quad$ $n=1$ $n=2$ $n=3$

$$\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)} = \left(\frac{0.5z}{(z-0.5)} - \frac{2z}{(z-2)}\right)$$

②-A $-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$ $|z| > 2$ $-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$

$$-\left(2 + 2^2 z^{-1} + 2^3 z^{-2} + \cdots\right) + \left(\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^{-1} + \left(\tfrac{1}{2}\right)^3 z^{-2} + \cdots\right)$$

$n=0$ $n=1$ $n=2$ $\quad$ $n=0$ $n=1$ $n=2$

②-B $+\frac{z}{1-0.5z} - \frac{z}{1-2z}$ $|z| < 0.5$ $+2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$

$$+\left(z + \left(\tfrac{1}{2}\right) z^2 + \left(\tfrac{1}{2}\right)^2 z^3 + \cdots\right) - \left(z + 2z^2 + 2^2 z^3 + \cdots\right)$$

$$+\left(2^0 z + 2^1 z^2 + 2^2 z^3 + \cdots\right) - \left(\left(\tfrac{1}{2}\right)^0 z + \left(\tfrac{1}{2}\right)^{-1} z^2 + \left(\tfrac{1}{2}\right)^{-2} z^3 + \cdots\right)$$

$n=-1$ $n=-2$ $n=-3$ $\quad$ $n=-1$ $n=-2$ $n=-3$

$f(z) \leftrightarrow a_n$

$X(z) \leftrightarrow x_n$

①-Ⓐ

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$a_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$

$x_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $(n < 1)$

②-Ⓐ

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$a_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $(n < 1)$

$x_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $(n \geq 0)$

①-Ⓑ

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$a_n = +2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$

$x_n = +\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $(n \geq 1)$

②-Ⓑ

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

$a_n = +\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $(n \geq 1)$

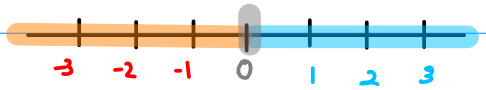
$x_n = +2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $(n < 0)$

$$x_n = a_{-n}$$

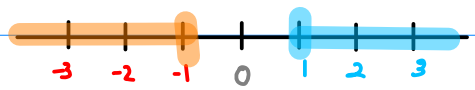
$$a_n = x_{-n}$$

$(n \geq 0) \leftrightarrow (n < 1)$

$(n \geq 1) \leftrightarrow (n < 0)$



$(n < 1)$ $(n \geq 0)$

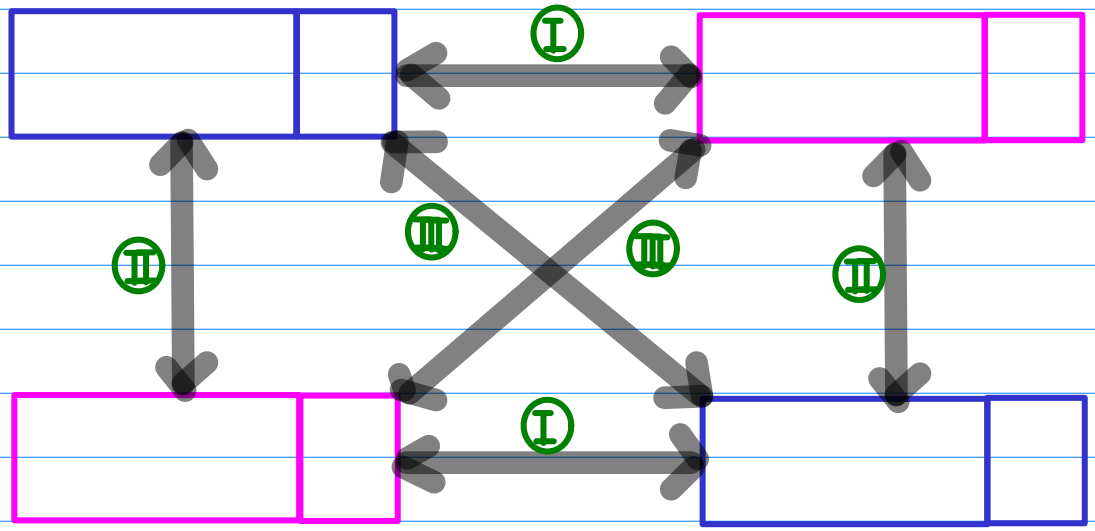


$(n < 0)$ $(n \geq 1)$

Ⅰ $(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

Ⅱ $(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$

Ⅲ $(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$



$(a_n, N) \Leftrightarrow (x_{-n}, -N)$

Ⅰ $(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$

①-Ⓐ

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$$a_n = -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \qquad (n \geq 0)$$

$$x_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1} \qquad (n < 1)$$

②-Ⓐ

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$$a_n = -\left(\tfrac{1}{2}\right)^{n-1} + 2^{n-1} \qquad (n < 1)$$

$$x_n = -2^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \qquad (n \geq 0)$$

①-Ⓑ

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$a_n = +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \qquad (n < 0)$$

$$x_n = +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

②-Ⓑ

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

$$a_n = +\left(\tfrac{1}{2}\right)^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$$x_n = +2^{n+1} - \left(\tfrac{1}{2}\right)^{n+1} \qquad (n < 0)$$

Ⅱ $(z,\ R^{-1}) \Leftrightarrow (-a_n,\ N^c)$

①-Ⓐ

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

①-Ⓑ

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

z : $\|$, R^{-1} : ↕

①-Ⓐ

$$a_n = -2^{n+1}+\left(\frac{1}{2}\right)^{n+1} \qquad (n\geq 0)$$

$-a_n$: ↕ , N^c : ↕

①-Ⓑ

$$a_n = +2^{n+1}-\left(\frac{1}{2}\right)^{n+1} \qquad (n<0)$$

①-Ⓐ

$$x_n = -\left(\frac{1}{2}\right)^{n-1}+2^{n-1} \qquad (n<1)$$

$-a_n$: ↕ , N^c : ↕

①-Ⓑ

$$x_n = +\left(\frac{1}{2}\right)^{n-1}-2^{n-1} \qquad (n\geq 1)$$

②-Ⓐ

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

②-Ⓑ

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

z : $\|$, R^{-1} : ↕

②-Ⓐ

$$a_n = -\left(\frac{1}{2}\right)^{n-1}+2^{n-1} \qquad (n<1)$$

$-a_n$: ↕ , N^c : ↕

②-Ⓑ

$$a_n = +\left(\frac{1}{2}\right)^{n-1}-2^{n-1} \qquad (n\geq 1)$$

②-Ⓐ

$$x_n = -2^{n+1}+\left(\frac{1}{2}\right)^{n+1} \qquad (n\geq 0)$$

$-a_n$: ↕ , N^c : ↕

②-Ⓑ

$$x_n = +2^{n+1}-\left(\frac{1}{2}\right)^{n+1} \qquad (n<0)$$

③

$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$

① $(z^{-1}, R^{-1}) \to (z, R^{-1})$ $\quad (a_{-n}, -N) \to (-a_n, N^c)$

①-A

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$a_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $\quad (n \geq 0)$

①-B

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$a_n = +2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $\quad (n < 0)$

②-A

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$a_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $\quad (n < 1)$

②-B

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

$a_n = +\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $\quad (n \geq 1)$

①-A

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$x_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}$ $\quad (n < 1)$

①-B

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$x_n = +\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}$ $\quad (n \geq 1)$

②-A

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$x_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1}$ $\quad (n \geq 0)$

②-B

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

$x_n = +2^{n+1} - \left(\frac{1}{2}\right)^{n+1}$ $\quad (n < 0)$

III

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c)$$

② $(z^{-1}, R^{-1}) \to (z, R^{-1})$ $\quad (a_{-n}, -N) \to (-a_n, N^c)$

① - Ⓐ

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad |z|<0.5$$

$$a_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

② - Ⓑ

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z} \quad |z|<0.5$$

$$a_n = +\left(\frac{1}{2}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)$$

① - Ⓑ

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z|>2$$

$$a_n = +2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

② - Ⓐ

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z|>2$$

$$a_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1} \quad (n < 1)$$

① - Ⓐ

$$-\frac{2}{1-2z} + \frac{0.5}{1-0.5z} \quad |z|<0.5$$

$$x_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1} \quad (n < 1)$$

② - Ⓑ

$$+\frac{z}{1-0.5z} - \frac{z}{1-2z} \quad |z|<0.5$$

$$x_n = +2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

① - Ⓑ

$$+\frac{z^{-1}}{1-0.5z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \quad |z|>2$$

$$x_n = +\left(\frac{1}{2}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)$$

② - Ⓐ

$$-\frac{2}{1-2z^{-1}} + \frac{0.5}{1-0.5z^{-1}} \quad |z|>2$$

$$x_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

Ⅳ $(a_n, N) \Leftrightarrow (x_{-n}, -N)$

①-Ⓐ

$$-\frac{2}{1-2z}+\frac{0.5}{1-0.5z} \qquad |z|<0.5$$

$$a_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

$a_n \leftrightarrow a_{-n}$, $N \leftrightarrow -N$

$$x_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1} \quad (n \leq 1)$$

②-Ⓐ

$$-\frac{2}{1-2z^{-1}}+\frac{0.5}{1-0.5z^{-1}} \qquad |z|>2$$

$$a_n = -\left(\frac{1}{2}\right)^{n-1} + 2^{n-1} \quad (n \leq 1)$$

$a_n \leftrightarrow a_{-n}$, $N \leftrightarrow -N$

$$x_n = -2^{n+1} + \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

①-Ⓑ

$$+\frac{z^{-1}}{1-0.5z^{-1}}-\frac{z^{-1}}{1-2z^{-1}} \qquad |z|>2$$

$$a_n = +2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n \leq 0)$$

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②-Ⓑ

$$+\frac{z}{1-0.5z}-\frac{z}{1-2z} \qquad |z|<0.5$$

$$a_n = +\left(\frac{1}{2}\right)^{n-1} - 2^{n-1} \quad (n \geq 1)$$

$a_n \leftrightarrow a_{-n}$, $N \leftrightarrow -N$

$$x_n = +2^{n+1} - \left(\frac{1}{2}\right)^{n+1} \quad (n \leq 0)$$

Some notations

$R1(z)$ $\quad R1(z^{-1})$

$R2(z)$ $\quad R2(z^{-1})$

R^{-1}

$-N$, $\quad N^{c}$

ROC's of interests

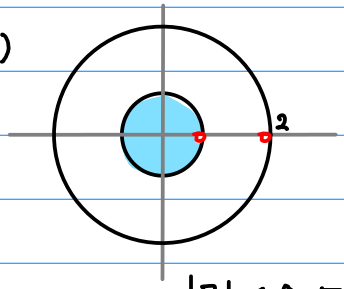
R1(z) R1(z^{-1})
R2(z) R2(z^{-1})

① $\frac{3}{2}\frac{-1}{(z-0.5)(z-2)}$ $\xleftrightarrow{z^{-1}}$ ② $\frac{3}{2}\frac{-z^2}{(z-2)(z-0.5)}$

$+\frac{1}{z-0.5} - \frac{1}{z-2}$ $\xleftrightarrow{z^{-1}}$ $-\frac{2z}{(z-2)} + \frac{0.5z}{(z-0.5)}$

$p_1 = 0.5$ $\xleftrightarrow{z^{-1}}$ $p_1^{-1} = 2$
$p_2 = 2$ $p_2^{-1} = 0.5$

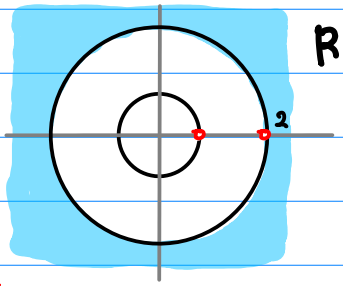
R1(z)



$|z| < 0.5$

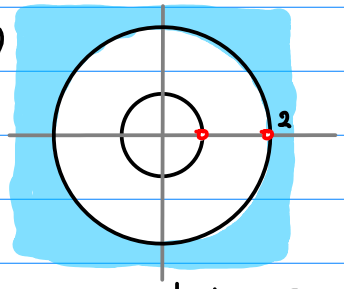
$\xleftrightarrow{z^{-1}}$

R1(z^{-1})



$|z^{-1}| < 0.5$ $|z| > 2$

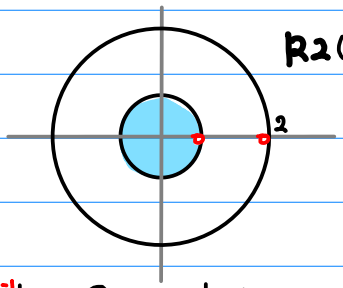
R2(z)



$|z| > 2$

$\xleftrightarrow{z^{-1}}$

R2(z^{-1})

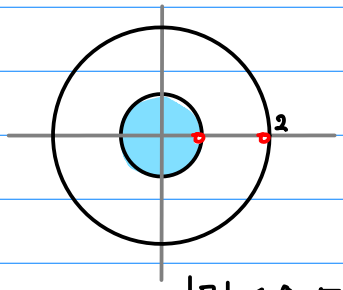


$|z^{-1}| > 2$ $|z| < 0.5$

$R1(z^{-1}) = R2(z)$
$R2(z^{-1}) = R1(z)$

R^{-1}

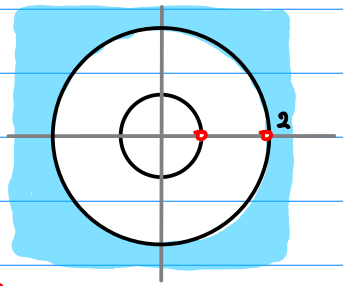
$R1(z) \triangleq R1$



$|z| < 0.5$

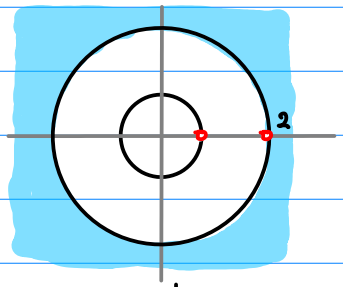
z^{-1}

$R1(z^{-1}) \triangleq R1^{-1}$



$|z^{-1}| < 0.5 \quad |z| > 2$

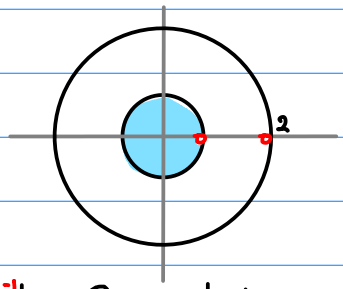
$R2(z)$



$|z| > 2$

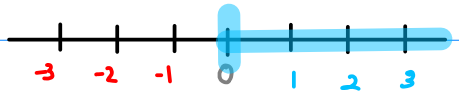
z^{-1}

$R2(z^{-1}) \triangleq R2^{-1}$



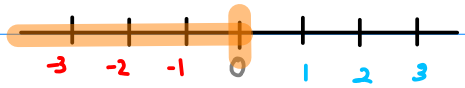
$|z^{-1}| > 2 \quad |z| < 0.5$

$-N$, N^c

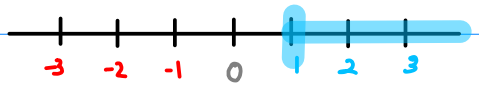


$(n \geq 0)$

$-N$

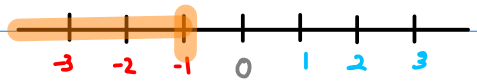


$(n < 1)$

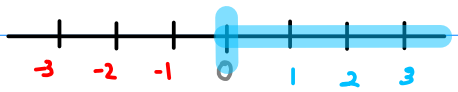


$(n \geq 1)$

$-N$

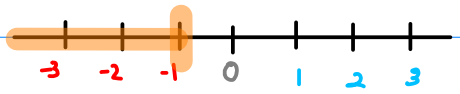


$(n < 0)$

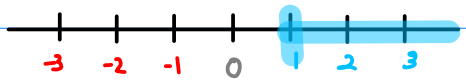


$(n \geq 0)$

$N^c = \overline{N}$

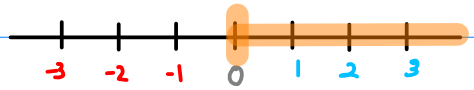


$(n < 0)$



$(n \geq 1)$

$N^c = \overline{N}$



$(n < 1)$

$-$: -1

$-N \triangleq \mathrm{RNG}(-n)$

—— : complement

$N^c \triangleq \overline{\mathrm{RNG}(n)}$

$\ll \quad \gg$

$(-N)^c = -(N^c)$

$$(z,\ R) \Leftrightarrow (a_n,\ N)$$

$f(z)$	$ROC(z)$	$\Leftrightarrow$	a_n	$RNG(n)$
	$\lvert z\rvert < p$			$n \geq 0$

Ⅰ $$(z^{-1},\ R^{-1}) \Leftrightarrow (a_{-n},\ -N)$$

$f(z^{-1})$	$ROC(z^{-1})$	$\Leftrightarrow$	a_{-n}	$RNG(-n)$
	$\lvert z\rvert > \frac{1}{p}$			$n < 1$

Ⅱ $$(z,\ R^{-1}) \Leftrightarrow (-a_n,\ N^c)$$

$f(z)$	$ROC(z^{-1})$	$\Leftrightarrow$	$-a_n$	$\overline{RNG}(n)$
	$\lvert z\rvert > \frac{1}{p}$			$n < 0$

Ⅲ $$(z^{-1},\ R) \Leftrightarrow (-a_{-n},\ (-N)^c) = (-a_{-n},\ -(N^c))$$

$f(z^{-1})$	$ROC(z)$	$\Leftrightarrow$	$-a_{-n}$	$\ll RNG(n) \gg$
	$\lvert z\rvert < p$			$n \geq 1$

Ⅰ + Ⅱ

Ⅳ $$(a_n,\ N) \Leftrightarrow (x_{-n},\ -N)$$

$X(z)$	$ROC(z)$	$\Leftrightarrow$	a_{-n}	$RNG(-n)$
	$\lvert z\rvert < p$			$n < 1$

Ⅲ = Ⅰ + Ⅱ

Ⅲ | $f(z^{-1})$ | $ROC(z)$ $|z| < p$ | $\Longleftrightarrow$ | $-a_{-n}$ | $\ll RNG(n) \gg$ $n \geq 1$ | Ⅰ + Ⅱ

$f(z)$ | $ROC(z)$ $|z| < p$ | $\Longleftrightarrow$ | a_n | $RNG(n)$ $n \geq 0$

Ⅰ | $f(z^{-1})$ | $ROC(z^{-1})$ $|z| > \frac{1}{p}$ | $\Longleftrightarrow$ | a_{-n} | $RNG(-n)$ $n < 1$

Ⅱ | $f(z)$ | $ROC(z^{-1})$ $|z| > \frac{1}{p}$ | $\Longleftrightarrow$ | $-a_n$ | $\overline{RNG(n)}$ $n < 0$

$f(z^{-1})$ | $ROC(z)$ $|z| < p$ | $\Longleftrightarrow$ | $-a_{-n}$ | $\overline{RNG(-n)}$ $n \geq 1$

$$(z^{-1}, R^{-1}) \Leftrightarrow (a_{-n}, -N)$$

$$(z, R^{-1}) \Leftrightarrow (-a_n, N^c)$$

$$(z^{-1}, R) \Leftrightarrow (-a_{-n}, (-N)^c) = (-a_{-n}, -(N^c))$$

Compare Ⓘ with Ⓘⅴ

ROC(z)	f(z)		a_n	RNG(n)
$\lvert z\rvert < p$				$n \geq 0$

Ⓘ $\quad (z^{-1},\ R^{-1}) \Leftrightarrow (a_{-n},\ -N)$

$f(z^{-1})$	$ROC(z^{-1})$		a_{-n}	RNG(−n)
	$\lvert z\rvert > \frac{1}{p}$			$n < 1$

Ⓘⅴ $\quad (a_n,\ N) \Leftrightarrow (x_{-n},\ -N)$

$(x_n,\ N) \Leftrightarrow (a_{-n},\ -N)$

X(z)	ROC(z)		a_{-n}	RNG(−n)
	$\lvert z\rvert < p$			$n < 1$

−n −n

Symmetrical

(I)

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad X(z)$ $\quad a_n = \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \le 1)$

$|z| > 2$ $\quad X(z)$ $\quad b_n = \boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}}$ $\quad (n \ge 1)$

$\{|z| < 0.5\} \cap \{|z| > 2\} = \phi \;\Rightarrow\; a_n + b_n = 0$

$a_n = -b_n$

ROC

$|z| < a$ $\quad X(z) = \dfrac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n$ $\quad a^{n+1}$ $\quad n \ge 0$ $\quad n \ge 1$ $\quad n < 0$ $\quad n < 1$

$\times(-1)$ ↓

ROC′

$|z| > a^{-1}$ $\quad X(z) = -\dfrac{z^{-1}}{1-a^{-1}z^{-1}} = -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$ $\quad -a^{n+1}$ $\quad n < 0$ $\quad n < 1$ $\quad n \ge 0$ $\quad n \ge 1$

$$= -\sum_{k=-1}^{-\infty} a^{k+1} z^k$$

$\dfrac{a}{1-az}$	$= \sum_{n=0}^{\infty} a^{n+1} z^n$	$\dfrac{z}{1-az}$	$= \sum_{n=1}^{\infty} a^{n-1} z^n$
$-\dfrac{z^{-1}}{1-a^{-1}z^{-1}}$	$= -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} = -\sum_{k=-1}^{-\infty} a^{k+1} z^k$	$-\dfrac{a^{-1}}{1-a^{-1}z^{-1}}$	$= -\sum_{n=0}^{\infty} a^{-n-1} z^{-n} = -\sum_{k=0}^{-\infty} a^{k-1} z^k$

$$\frac{a}{1-az} \Rightarrow \sum_{n=0}^{\infty} a^{n+1} z^{n}$$

$$\frac{z}{1-az} \Rightarrow \sum_{n=1}^{\infty} a^{n-1} z^{n}$$

$$\|$$

$$-\frac{z^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n} z^{-n-1} = -\sum_{k=-1}^{-\infty} a^{k+1} z^{k}$$

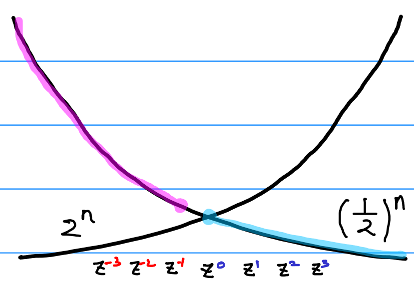
$$-\frac{a^{-1}}{1-a^{-1}z^{-1}} \Rightarrow -\sum_{n=0}^{\infty} a^{-n-1} z^{-n} = -\sum_{k=0}^{-\infty} a^{k-1} z^{k}$$

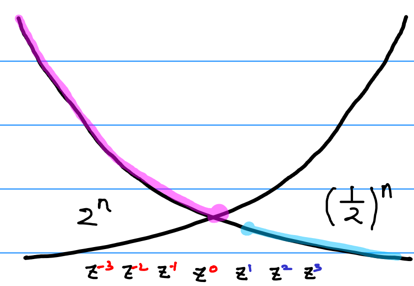
$$a + a^{2}z^{1} + a^{3}z^{2} + a^{4}z^{3} + \cdots$$

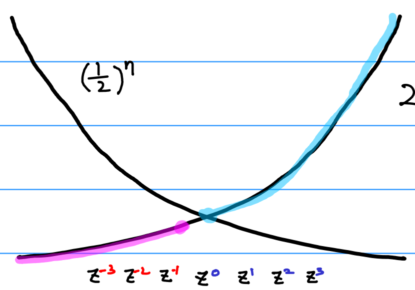
$$z^{-1} + a^{-1}z^{-2} + a^{-2}z^{-3} + a^{-3}z^{-4} + \cdots$$

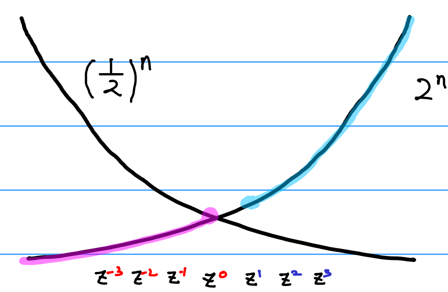
$$z + az^{2} + a^{2}z^{3} + a^{3}z^{4} + \cdots$$

$$a^{-1} + a^{-2}z^{-1} + a^{-3}z^{-2} + a^{-4}z^{-3} + \cdots$$









(II)

ROC(z^{-1})	$f(z^{-1})$	a_{-n}	RNG($-n$)
$\lvert z\rvert > \frac{1}{p}$			$n < 1$

$|z| < 1$

$$-\frac{1}{1-z} + \frac{0.5}{1-0.5z}$$

$$f(z) = -\left[1^1 + 1^2 z^1 + 1^3 z^2 + \cdots\right] + \left[\left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right)^2 z^1 + \left(\tfrac{1}{2}\right)^3 z^2 + \cdots\right]$$

$$a_n = -1^{n+1} + \left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

$|z| > 1$

$$-\frac{1}{1-z^{-1}} + \frac{0.5}{1-0.5z^{-1}}$$

$$f(z) = -\left[\left(\tfrac{1}{1}\right)^1 z^0 + \left(\tfrac{1}{1}\right)^2 z^{-1} + \left(\tfrac{1}{1}\right)^3 z^{-2} + \cdots\right] + \left[2^{-1} z^0 + 2^{-2} z^{-1} + 2^{-3} z^{-2} + \cdots\right]$$

$$a_n = -1^{n-1} + 2^{n-1} \quad (n < 1)$$

ROC

$|z| < a$ $\quad f(z) = \dfrac{a}{1-az} = \sum_{n=0}^{\infty} a^{n+1} z^n$ $\quad a^{n+1}$ $\quad n \geq 0$ $\quad n \geq 1$ $\quad n < 0$ $\quad n < 1$

z^{-1} $\quad -n$

ROC′

$|z| > a^{-1}$ $\quad f(z^{-1}) = \dfrac{a}{1-az^{-1}} = \sum_{n=0}^{\infty} a^{n+1} z^{-n} = \sum_{k=0}^{-\infty} a^{-k+1} z^{k}$ $\quad a^{-n+1} = \left(\tfrac{1}{a}\right)^{n-1}$ $\quad n < 1$ $\quad n < 0$ $\quad n \geq 0$ $\quad n \geq 0$

Ⅳ

ROC(z)	X(z)	⟷	a_{-n}	RNG(-n)
$\lvert z\rvert < p$				$n \le 0$

$$\frac{3}{2}\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}}$ $\quad (n \ge 0)$

$$-\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\frac{1}{2}\right)z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \cdots\right)$$

$n=0 \quad n=1 \quad n=2 \qquad\qquad n=0 \quad n=1 \quad n=2$

$|z| < 0.5$ $\quad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\quad \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \le 0)$

$$-\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right) + \left(\left(\frac{1}{2}\right)z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \cdots\right)$$

$$-\left(\left(\frac{1}{2}\right)^1 z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

$n=0 \quad n=-1 \quad n=-2 \qquad\qquad n=0 \quad n=-1 \quad n=-2$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n$ $\qquad a^{n+1}$ $\qquad n \ge 0 \quad n \ge 1 \quad n < 0 \quad n < 1$

$\qquad\qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$ $\qquad -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$ $\qquad a^{-n+1}$ $\qquad n \le 0 \quad n \le 1 \quad n > 0 \quad n > 1$

$\qquad\qquad = \left(\frac{1}{a}\right)^{n-1}$

$z^{-1}X(z)$ Shifted Sequence

$$X(z) = \underbrace{\frac{1}{1-z^{-1}}}_{|z|>1} - \underbrace{\frac{1}{1-2z^{-1}}}_{|z|>2} \qquad (|z|>2)$$

$$a_n = 1^n + 2^n \qquad (n \geq 0)$$

$\bullet z^{-1}$

$$\underbrace{\left(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots\right) + \left(2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} + \cdots\right)}_{n} \qquad n = 0, 1, 2, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$\underbrace{\left(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right) + \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right)}_{n-1} \qquad n = 1, 2, 3, \cdots$$

$$z^{-1}X(z) = \underbrace{\frac{z^{-1}}{1-z^{-1}}}_{|z|>1} - \underbrace{\frac{z^{-1}}{1-2z^{-1}}}_{|z|>2} \qquad (|z|>2)$$

$$a_{n-1} = 1^{n-1} + 2^{n-1} \qquad (n \geq 1)$$

$z\,f(z)$ Shifted Sequence

$$f(z) = (+1)\frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z|<0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$\begin{array}{l} \left(1^0 z^0 + 1^1 z^1 + 1^2 z^2 + \cdots\right) - \left(2^0 z^0 + 2^1 z^1 + 2^2 z^2 + \cdots\right) \quad (n) \quad n = 0, 1, 2, \cdots \\ \bullet z \downarrow \qquad 1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots \\ \left(1^0 z^1 + 1^1 z^2 + 1^2 z^3 + \cdots\right) - \left(2^0 z^1 + 2^1 z^2 + 2^2 z^3 + \cdots\right) \quad (n-1) \quad n = 1, 2, 3, \cdots \end{array}$$

$$z\,f(z) = (+1)\frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z|<0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

$z^{-1}f(z^{-1})$ Shifted & Reflected Sequence

$$f(z) = \frac{1}{1-z} - \frac{1}{1-2z} \qquad (|z|<0.5)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$\cdot z$

$$\left(1^0z^0 + 1^1z^1 + 1^2z^2 + \cdots\right) - \left(2^0z^0 + 2^1z^1 + 2^2z^2 + \cdots\right) \quad (n) \quad n = 0, 1, 2, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$\left(1^0z^1 + 1^1z^2 + 1^2z^3 + \cdots\right) - \left(2^0z^1 + 2^1z^2 + 2^2z^3 + \cdots\right) \quad (n-1) \quad n = 1, 2, 3, \cdots$$

$$zf(z) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z|<0.5)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

z

$$\left(1^0z^{-1} + 1^1z^{-2} + 1^2z^{-3} + \cdots\right) + \left(2^0z^{-1} + 2^1z^{-2} + 2^2z^{-3} + \cdots\right) \quad (n) \quad n = 1, 2, 3, \cdots$$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

z^{-1}

$$\left(1^0z^1 + 1^1z^2 + 1^2z^3 + \cdots\right) + \left(2^0z^1 + 2^1z^2 + 2^2z^3 + \cdots\right) \quad (-n) \quad n = -1, -2, -3, \cdots$$

$$z^{-1}f(z^{-1}) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n<0)$$

$$a_{-(n+1)}$$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5$ $\quad X(z)$ $\quad a_n = \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}}$ $\quad (n \leq 1)$

$|z| > 2$ $\quad X(z)$ $\quad b_n = \boxed{+\left(\frac{1}{2}\right)^{n-1} - 2^{n-1}}$ $\quad (n \geqslant 1)$

$\{|z| < 0.5\} \cap \{|z| > 2\} = \phi \longrightarrow a_n + b_n = 0$

$a_n = -b_n$

$|z| < a$ $\quad X(z) = \sum_{n=0}^{\infty} a^{n+1} z^n$

$= \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n$

$a^{n+1} \xrightarrow{-n} a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$

$n \geqslant 0$ $\quad n \geqslant 1$ $\quad n < 0$ $\quad n < 1$

$|z| > a$ $\quad X(z) = -\sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k}$

$n < 0$ $\quad n \leq 1$ $\quad n \geqslant 0$ $\quad n \geqslant$

$a^{n+1} z^n$

$a\,(az)^n$

$a\left(\frac{1}{az}\right)^{-n}$

$\frac{a}{1-az}$ $\quad \sum_{n=0}^{\infty} a^{n+1} z^n$ $\qquad \frac{z}{1-az}$ $\quad \sum_{n=1}^{\infty} a^{n-1} z^n$

$-\frac{z^{-1}}{1-a^{-1}z^{-1}}$ $\quad -\sum_{n=0}^{\infty} a^{-n} z^{-n-1}$ $\qquad -\frac{a^{-1}}{1-a^{-1}z^{-1}}$ $\quad -\sum_{n=1}^{\infty} a^{-n-1} z^{-n}$

$\sum_{n=-1}^{-\infty} a^{n+1} z^n$ $\qquad -\sum_{n=0}^{-\infty} a^{n-1} z^n$

$zX(z)$ Shifted & Reflected Sequence

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_n = 1^n - 2^n \qquad (n \geq 0)$$

$$\left(1^0 z^0 + 1^1 z^{-1} + 1^2 z^{-2} + \cdots\right) + \left(2^0 z^0 + 2^1 z^{-1} + 2^2 z^{-2} \right) \quad (n) \quad n = 0, 1, 2, \cdots$$

$\bullet z^{-1}$

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$\left(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right) + \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right) \quad (n-1) \quad n = 1, 2, 3, \cdots$$

$$z^{-1}X(z) = \frac{z^{-1}}{1-z^{-1}} - \frac{z^{-1}}{1-2z^{-1}} \qquad (|z|>2)$$

$$a_{n-1} = 1^{n-1} - 2^{n-1} \qquad (n \geq 1)$$

z

$$\left(1^0 z^{-1} + 1^1 z^{-2} + 1^2 z^{-3} + \cdots\right) + \left(2^0 z^{-1} + 2^1 z^{-2} + 2^2 z^{-3} + \cdots\right) \quad (n) \quad n = 1, 2, 3, \cdots$$

z^{-1}

$$1^0 \quad 1^1 \quad 1^2 \quad \cdots \quad 2^0 \quad 2^1 \quad 2^2 \quad \cdots$$

$$\left(1^0 z^{1} + 1^1 z^{2} + 1^2 z^{3} + \cdots\right) + \left(2^0 z^{1} + 2^1 z^{2} + 2^2 z^{3} + \cdots\right) \quad (-n) \quad n = -1, -2, -3, \cdots$$

$$zX(z^{-1}) = \frac{z}{1-z} - \frac{z}{1-2z} \qquad (|z|<0.5)$$

$$a_{-n-1} = 1^{n+1} - \left(\frac{1}{2}\right)^{n+1} \qquad (n<0)$$

$$a_{-(n+1)}$$

$$\frac{3}{2}\,\frac{-1}{(z-0.5)(z-2)} = \left(\frac{1}{z-0.5} - \frac{1}{z-2}\right)$$

$|z| < 0.5 \qquad f(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\qquad \boxed{-2^{n+1} + \left(\frac{1}{2}\right)^{n+1}} \qquad (n \geq 0)$

$$-\underbrace{\left(2z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right)}_{n=0 \quad n=1 \quad n=2} + \underbrace{\left(\left(\frac{1}{2}\right)z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \cdots\right)}_{n=0 \quad n=1 \quad n=2}$$

$|z| < 0.5 \qquad X(z) = -\frac{2}{1-2z} + \frac{0.5}{1-0.5z}$ $\qquad \boxed{-\left(\frac{1}{2}\right)^{n-1} + 2^{n-1}} \qquad (n \leq 0)$

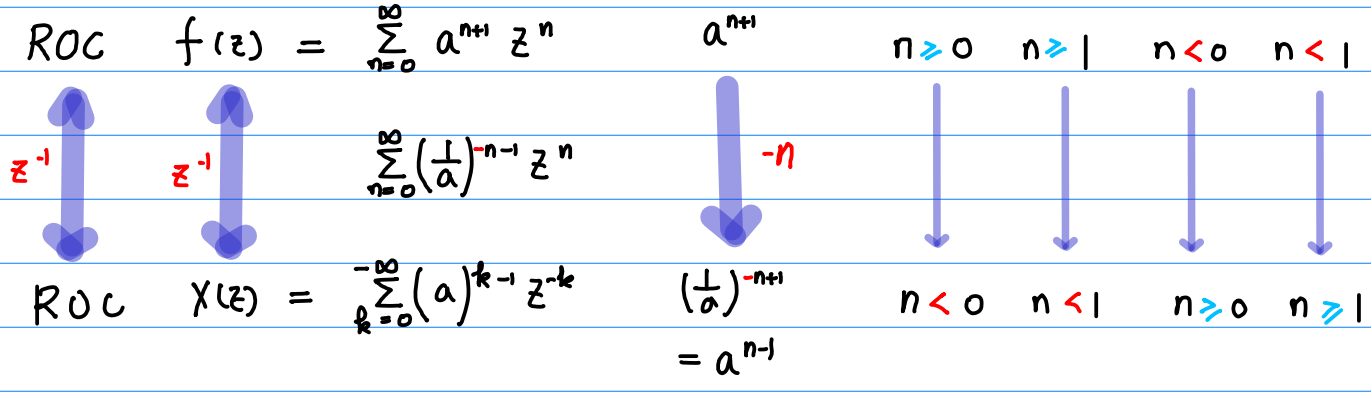
$$-\underbrace{\left(2^1 z^0 + 2^2 z^1 + 2^3 z^2 + \cdots\right)}_{n=0 \quad n=-1 \quad n=-2} + \underbrace{\left(\left(\frac{1}{2}\right)z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \cdots\right)}_{n=0 \quad n=-1 \quad n=-2}$$

$$-\left(\left(\frac{1}{2}\right)^1 z^0 + \left(\frac{1}{2}\right)^2 z^1 + \left(\frac{1}{2}\right)^3 z^2 + \cdots\right) + \left(2^{-1} z^0 + 2^{-2} z^1 + 2^{-3} z^2 + \cdots\right)$$

ROC $\quad f(z) = \sum_{n=0}^{\infty} a^{n+1} z^n \qquad a^{n+1}$ $\qquad n \geq 0 \quad n \geq 1 \quad n < 0 \quad n < 1$

$\qquad\qquad \sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^n \qquad -n$

ROC $\quad X(z) = \sum_{k=0}^{-\infty} \left(\frac{1}{a}\right)^{k-1} z^{-k} \qquad a^{-n+1} = \left(\frac{1}{a}\right)^{n-1}$ $\qquad n < 0 \quad n < 1 \quad n \geq 0 \quad n \geq 1$


ROC
f(z) = $\sum_{n=0}^{\infty} a^{n+1} z^{n}$
a^{n+1}
$n \geq 0$
$n \geq 1$
$n < 0$
$n < 1$
z^{-1}
z^{-1}
$\sum_{n=0}^{\infty} \left(\frac{1}{a}\right)^{-n-1} z^{n}$
$-n$
ROC
X(z) = $\sum_{k=0}^{-\infty} (a)^{k-1} z^{-k}$
$\left(\frac{1}{a}\right)^{-n+1}$
$= a^{n-1}$
$n < 0$
$n < 1$
$n \geq 0$
$n \geq 1$