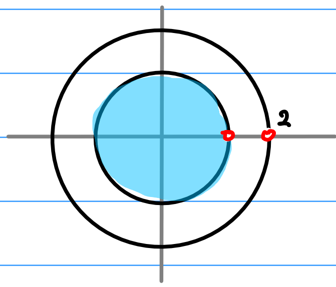
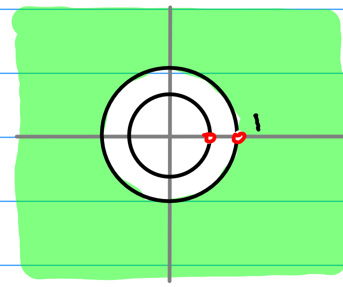
Laurent Series and z-Transform Examples case 0.A

20171125

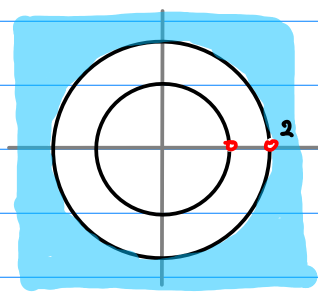
1. A

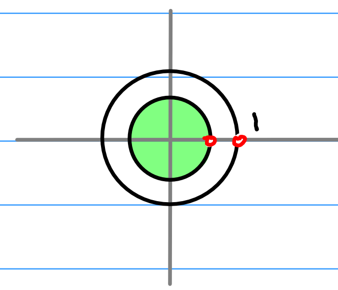
$$f(z) = \frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$



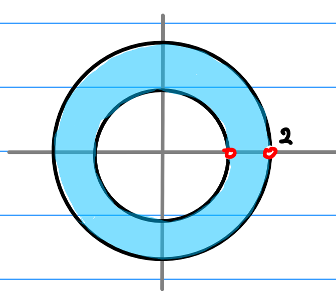


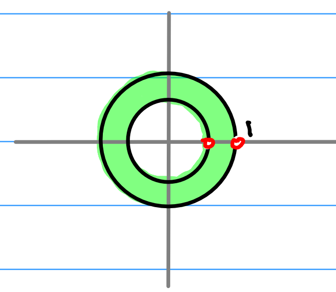
$$\sum_{n=0}^{\infty}\left[\left(\frac{1}{2}\right)^{n+1}-1\right]z^{n} \xrightarrow{z^{-1}} \sum_{n=0}^{\infty}\left[\left(\frac{1}{2}\right)^{n+1}-1\right]z^{-n}$$





$$\sum_{n=-1}^{-\infty}\left[1-\left(\frac{1}{2}\right)^{n+1}\right]z^{n} \xrightarrow{z^{-1}} \sum_{n=-1}^{-\infty}\left[1-\left(\frac{1}{2}\right)^{n+1}\right]z^{-n}$$

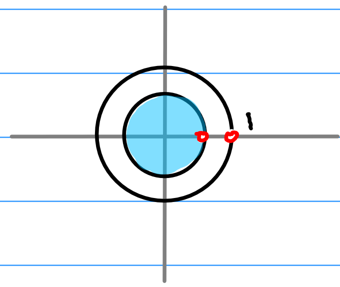


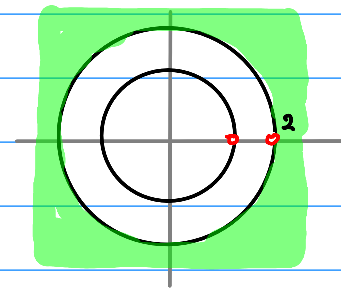


$$\sum_{n=-1}^{-\infty} 1\cdot z^{n} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^{n} \xrightarrow{z^{-1}} \sum_{n=-1}^{-\infty} 1\cdot z^{-n} + \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{n+1} z^{-n}$$

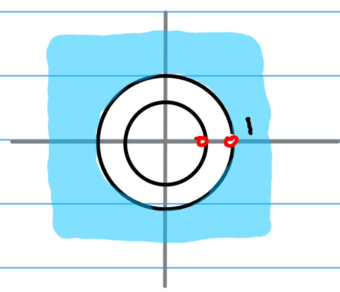
2.A

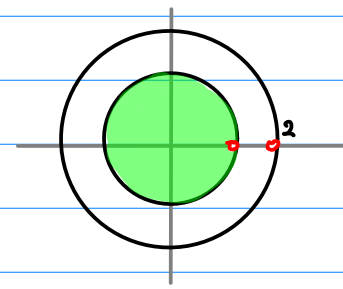
$$f(z) = \frac{-0.5\,z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$



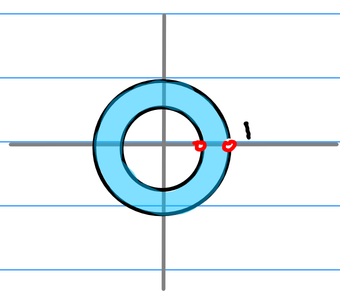


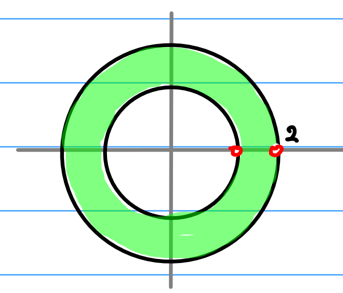
$$\sum_{n=1}^{\infty}\left[1-2^{n-1}\right] z^{n} \xrightarrow{z^{-1}} \sum_{n=1}^{\infty}\left[1-2^{n-1}\right] z^{-n}$$





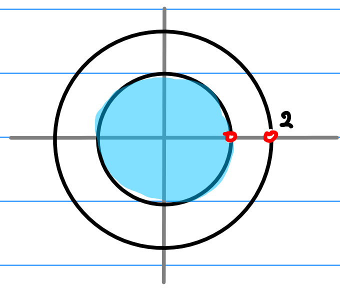
$$\sum_{n=0}^{-\infty}\left[2^{n-1}-1\right] z^{n} \xrightarrow{z^{-1}} \sum_{n=0}^{-\infty}\left[2^{n-1}-1\right] z^{-n}$$

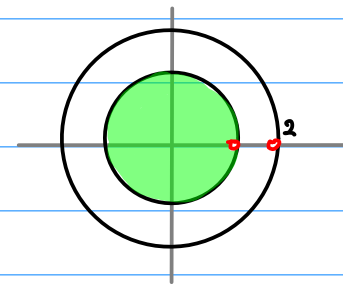




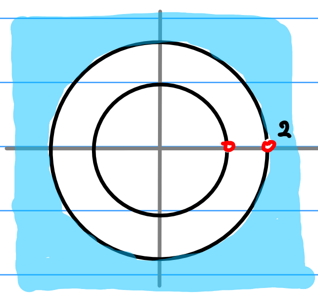
$$\sum_{n=1}^{\infty} 1\cdot z^{n} + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^{n} \xrightarrow{z^{-1}} \sum_{n=1}^{\infty} 1\cdot z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^{-n}$$

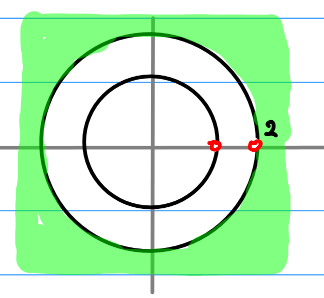
3 . A $f(z) = \frac{-1}{(z-1)(z-2)}$ $=$ $X(z) = \frac{-1}{(z-1)(z-2)}$



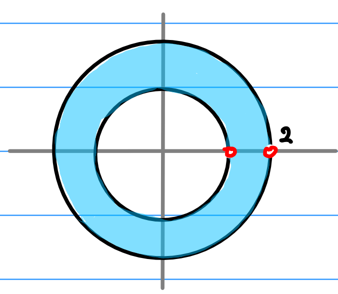


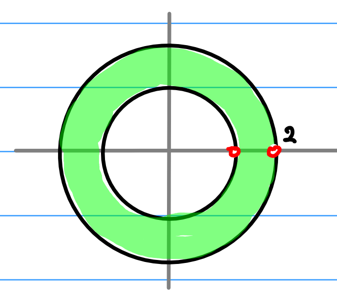
$$\sum_{n=0}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1 \right] z^{n} \xrightarrow{-n} \sum_{n=0}^{-\infty} \left[2^{n-1} - 1 \right] z^{-n}$$





$$\sum_{n=-1}^{-\infty} \left[1 - \left(\tfrac{1}{2}\right)^{n+1} \right] z^{n} \xrightarrow{-n} \sum_{n=1}^{\infty} \left[1 - 2^{n-1} \right] z^{-n}$$

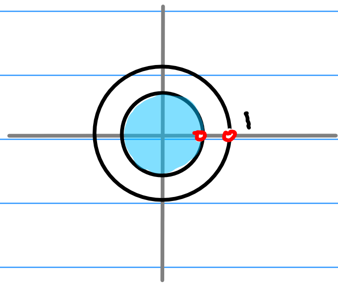


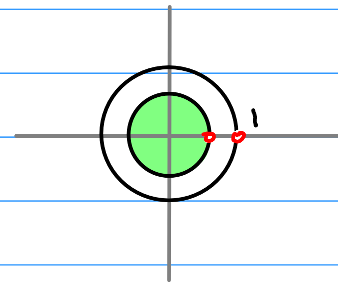


$$\sum_{n=-1}^{-\infty} z^{n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{n} \xrightarrow{-n} + \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

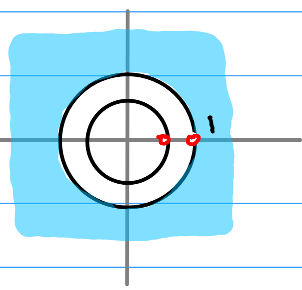
4.A

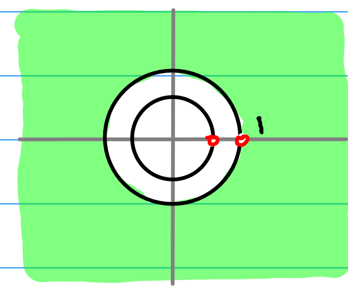
$$f(z) = \frac{-0.5\, z^2}{(z-1)(z-0.5)} = X(z) = \frac{-0.5\, z^2}{(z-1)(z-0.5)}$$



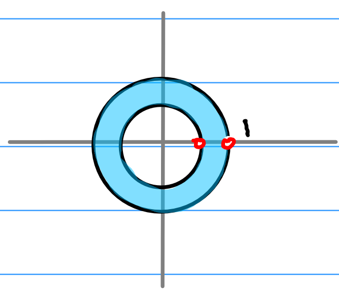


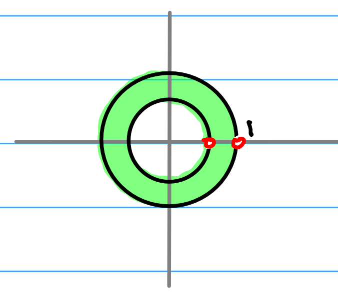
$$\sum_{n=1}^{\infty} \left[1 - 2^{n-1}\right] z^n \xrightarrow{-n} \sum_{n=-1}^{\infty} \left[1 - \left(\tfrac{1}{2}\right)^{n+1}\right] z^{-n}$$





$$\sum_{n=-1}^{-\infty} \left[2^{n-1} - 1\right] z^n \xrightarrow{-n} \sum_{n=1}^{\infty} \left[\left(\tfrac{1}{2}\right)^{n+1} - 1\right] z^{-n}$$





$$+\sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1} z^n \xrightarrow{-n} +\sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\tfrac{1}{2}\right)^{n+1} z^{-n}$$

1. A

$$f(z) = \frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$

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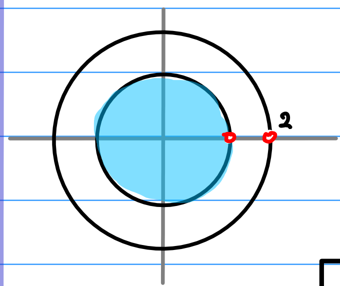

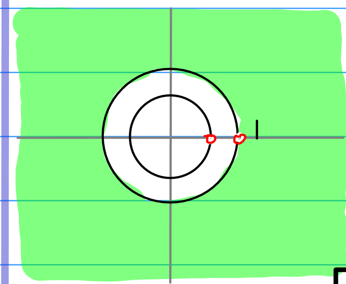

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases} \quad = \quad x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^n \qquad X(z) = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^{n+1} - 1\right] z^{-n}$$

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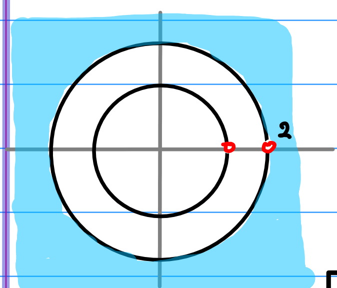

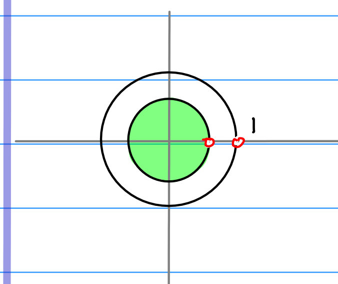

$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases} \quad = \quad x_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) z^n \qquad X(z) = \sum_{n=-1}^{-\infty} \left(1 - \left(\frac{1}{2}\right)^{n+1}\right) z^{-n}$$

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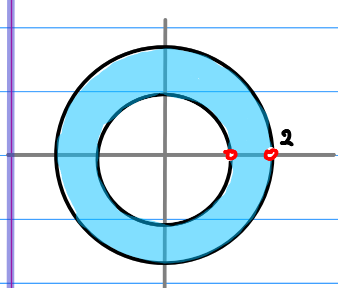

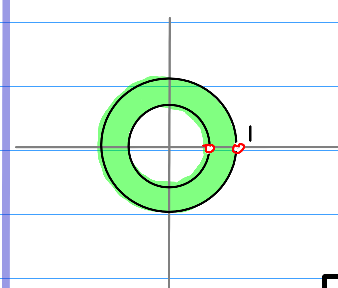

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases} \quad = \quad x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$$f(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^n \qquad X(z) = \sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n}$$

2.A

$$f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} X(z) = \frac{-1}{(z-1)(z-2)}$$

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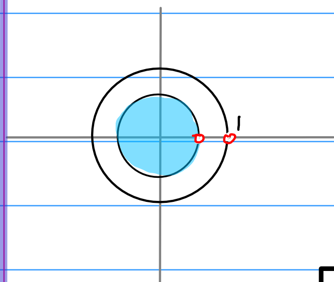

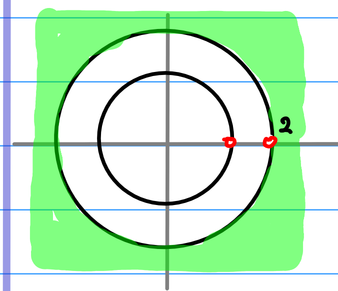

$$a_n = \boxed{\begin{matrix} [1-2^{n-1}] & (n>0) \\ 0 & (n\leq 0) \end{matrix}} \quad = \quad x_n = \boxed{\begin{matrix} [1-2^{n-1}] & (n>0) \\ 0 & (n\leq 0) \end{matrix}}$$

$$f(z) = \sum_{n=1}^{\infty} [1-2^{n-1}]\, z^n \qquad X(z) = \sum_{n=1}^{\infty} [1-2^{n-1}]\, z^{-n}$$

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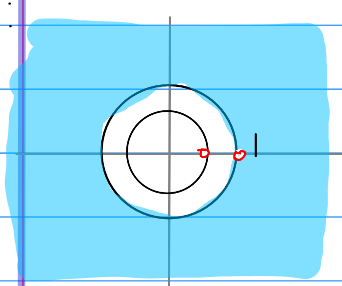

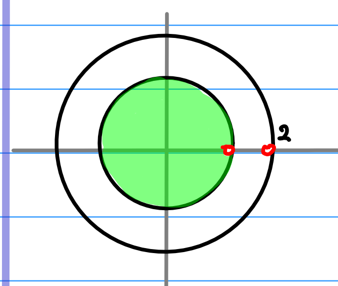

$$a_n = \boxed{\begin{matrix} 0 & (n>0) \\ [2^{n-1}-1] & (n\leq 0) \end{matrix}} \quad = \quad x_n = \boxed{\begin{matrix} 0 & (n>0) \\ [2^{n-1}-1] & (n\leq 0) \end{matrix}}$$

$$f(z) = \sum_{n=0}^{-\infty} [2^{n-1}-1]\, z^n \qquad X(z) = \sum_{n=0}^{-\infty} [2^{n-1}-1]\, z^{-n}$$

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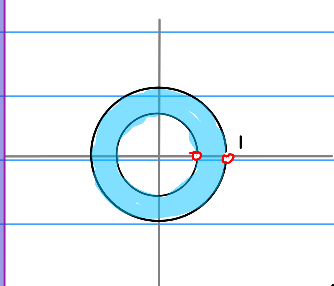

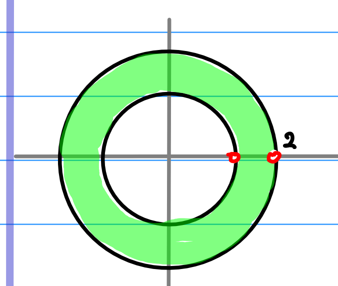

$$a_n = \boxed{\begin{matrix} 1 & (n>0) \\ 2^{n-1} & (n\leq 0) \end{matrix}} \quad = \quad x_n = \boxed{\begin{matrix} 1 & (n>0) \\ 2^{n-1} & (n\leq 0) \end{matrix}}$$

$$f(z) = \sum_{n=1}^{\infty} 1\cdot z^n + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^n \qquad X(z) = \sum_{n=1}^{\infty} 1\cdot z^{-n} + \sum_{n=0}^{-\infty} 2^{n-1}\cdot z^{-n}$$

3 . A $\quad f(z) = \dfrac{-1}{(z-1)(z-2)} \quad = \quad X(z) = \dfrac{-1}{(z-1)(z-2)}$

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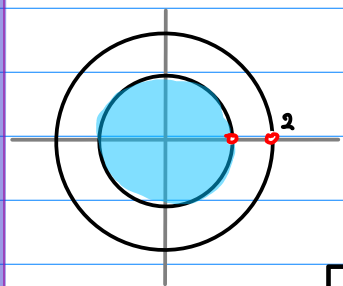

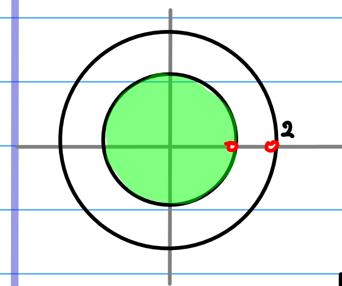

$$a_n = \boxed{\begin{matrix} (\frac{1}{2})^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{matrix}} \xrightarrow{-n} x_n = \boxed{\begin{matrix} 0 & (n > 0) \\ [2^{n-1} - 1] & (n \leq 0) \end{matrix}}$$

$$f(z) = \sum_{n=0}^{\infty} [(\tfrac{1}{2})^{n+1} - 1]\, z^n \qquad X(z) = \sum_{n=0}^{-\infty} [2^{n-1} - 1]\, z^{-n}$$

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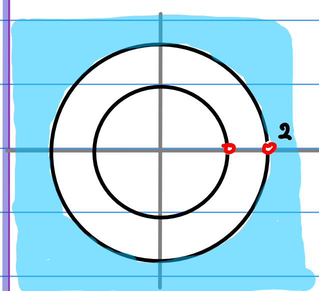

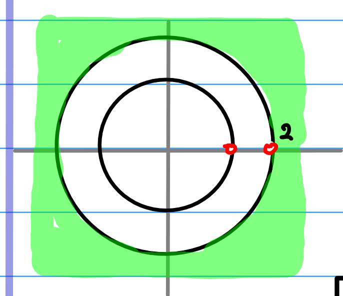

$$a_n = \boxed{\begin{matrix} 0 & (n \geq 0) \\ 1 - (\frac{1}{2})^{n+1} & (n < 0) \end{matrix}} \xrightarrow{-n} x_n = \boxed{\begin{matrix} [1 - 2^{n-1}] & (n > 0) \\ 0 & (n \leq 0) \end{matrix}}$$

$$f(z) = \sum_{n=-1}^{-\infty} [1 - (\tfrac{1}{2})^{n+1}]\, z^n \qquad X(z) = \sum_{n=1}^{\infty} [1 - 2^{n-1}]\, z^{-n}$$

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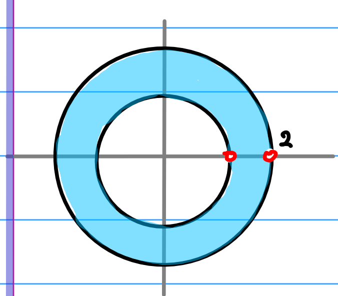

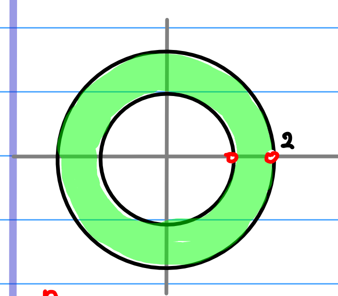

$$a_n = \boxed{\begin{matrix} (\frac{1}{2})^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{matrix}} \xrightarrow{-n} x_n = \boxed{\begin{matrix} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{matrix}}$$

$$f(z) = \sum_{n=-1}^{-\infty} z^n + \sum_{n=0}^{\infty} (\tfrac{1}{2})^{n+1} z^n \qquad X(z) = + \sum_{n=1}^{\infty} z^{-n} + \sum_{n=0}^{\infty} 2^{n-1} z^{-n}$$

4.A

$$f(z) = \frac{-0.5z^2}{(z-1)(z-0.5)} \quad = \quad X(z) = \frac{-0.5z^2}{(z-1)(z-0.5)}$$

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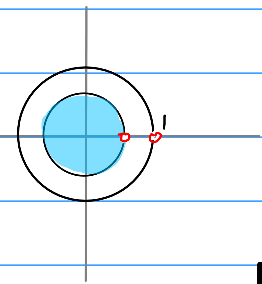

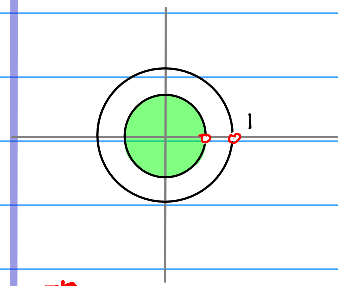

$$a_n = \boxed{\begin{matrix} [1-2^{n-1}] & (n>0) \\ 0 & (n\leq 0) \end{matrix}} \xrightarrow{-n} x_n = \boxed{\begin{matrix} 0 & (n\geq 0) \\ 1-(\frac{1}{2})^{n+1} & (n<0) \end{matrix}}$$

$$f(z) = \sum_{n=1}^{\infty} [1-2^{n-1}]z^n \qquad X(z) = \sum_{n=-1}^{\infty} [1-(\tfrac{1}{2})^{n+1}]z^{-n}$$

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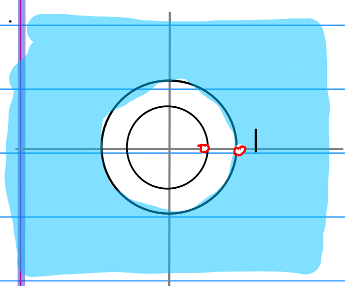

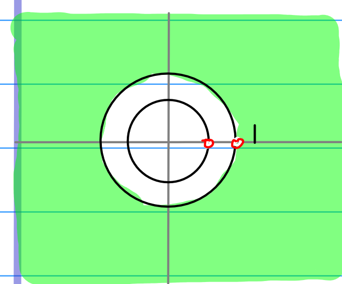

$$a_n = \boxed{\begin{matrix} 0 & (n>0) \\ [2^{n-1}-1] & (n\leq 0) \end{matrix}} \xrightarrow{-n} x_n = \boxed{\begin{matrix} (\frac{1}{2})^{n+1}-1 & (n\geq 0) \\ 0 & (n<0) \end{matrix}}$$

$$f(z) = \sum_{n=-1}^{-\infty} [2^{n-1}-1]z^n \qquad X(z) = \sum_{n=1}^{\infty} [(\tfrac{1}{2})^{n+1}-1]z^{-n}$$

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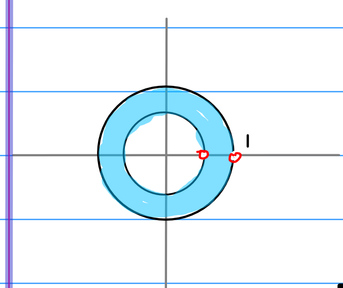
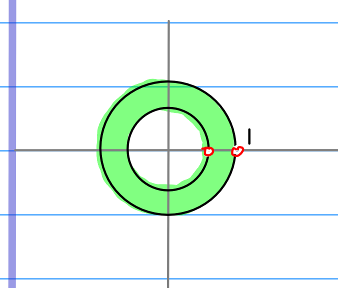
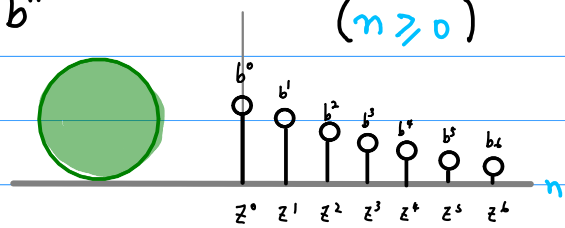




$$a_n = \boxed{\begin{matrix} 1 & (n>0) \\ 2^{n-1} & (n\leq 0) \end{matrix}} \xrightarrow{-n} x_n = \boxed{\begin{matrix} (\frac{1}{2})^{n+1} & (n\geq 0) \\ 1 & (n<0) \end{matrix}}$$

$$f(z) = +\sum_{n=1}^{\infty} z^n + \sum_{n=0}^{-\infty} 2^{n-1}z^n \qquad X(z) = +\sum_{n=-1}^{-\infty} z^{-n} + \sum_{n=0}^{\infty} (\tfrac{1}{2})^{n+1}z^{-n}$$

b^n $(n \geq 0)$



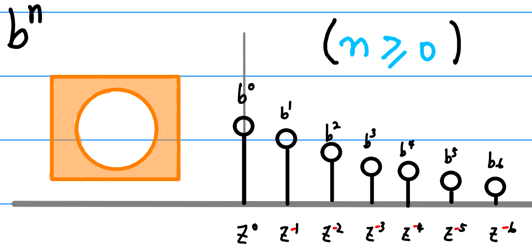
$X(z^{-1}) = \frac{z^{-1}}{z^{-1} - 0.5}$ $|z| < 2$

$f(z) = \frac{2}{2-z} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^n$

$a_n = \left(\frac{1}{2}\right)^n$

$= p^{-n}$ $\boxed{p = 2}$

b^n $(n \geq 0)$



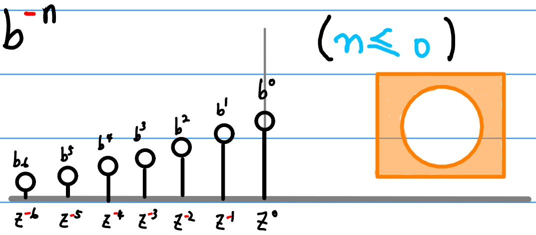
$|z| > \frac{1}{2}$

$X(z) = \frac{z}{z - 0.5} = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$

$x_n = \left(\frac{1}{2}\right)^n$

$= p^n$ $\boxed{p = \frac{1}{2}}$

b^{-n} $(n \leq 0)$



$X(z^{-1}) = \frac{2}{2 - z^{-1}}$ $|z| > \frac{1}{2}$

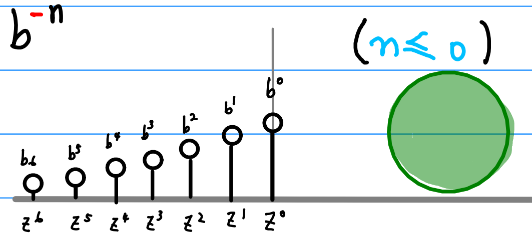
$f(z) = \frac{z}{z - 0.5} = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-n} z^n$

$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n}$

$a_n = \left(\frac{1}{2}\right)^{-n}$

$= p^{-n}$ $\boxed{p = \frac{1}{2}}$

b^{-n} $(n \leq 0)$



$|z| < 2$

$X(z) = \frac{2}{2-z} = \sum_{n=-\infty}^{0} \left(\frac{1}{2}\right)^{-n} z^{-n}$

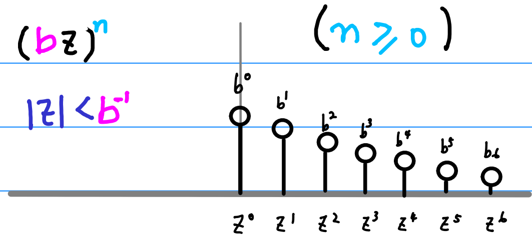
$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^n$

$x_n = \left(\frac{1}{2}\right)^{-n}$

$= p^n$ $\boxed{p = 2}$

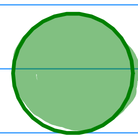
$(bz)^n$ $\quad (n \geq 0)$

$|z| < b^{-1}$



$$f(z) = \frac{1}{1 - bz} = \frac{b^{-1}}{b^{-1} - z}$$

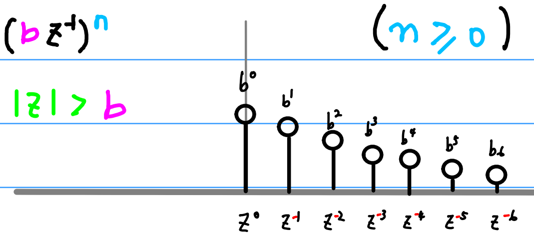
$$a_n = b^n$$
$$= p^{-n}$$



$\boxed{p = b^{-1}}$

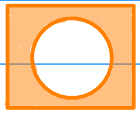
$(bz^{-1})^n$ $\quad (n \geq 0)$

$|z| > b$



$$X(z) = \frac{1}{1 - b/z} = \frac{z}{z - b}$$

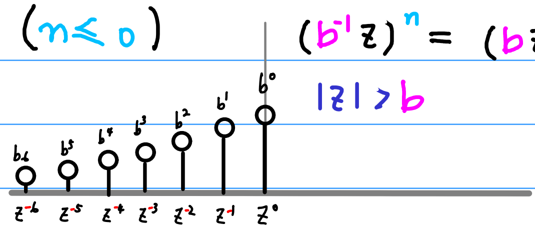
$$x_n = b^n$$
$$= p^n$$



$\boxed{p = b}$

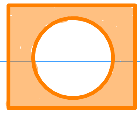
$(n \leq 0)$ $\quad (b^{-1}z)^n = (bz^{-1})^{-n}$

$|z| > b$



$$f(z) = \frac{1}{1 - (bz^{-1})} = \frac{z}{z - b}$$

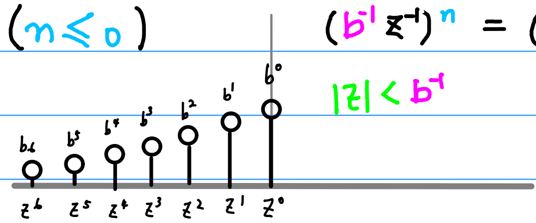
$$a_n = b^{-n}$$
$$= p^{-n}$$



$\boxed{p = b}$

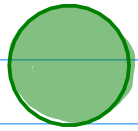
$(n \leq 0)$ $\quad (b^{-1}z^{-1})^n = (bz)^{-n}$

$|z| < b^{-1}$



$$X(z) = \frac{1}{1 - (bz)} = \frac{b^{-1}}{b^{-1} - z}$$

$$x_n = b^{-n}$$
$$= p^n$$

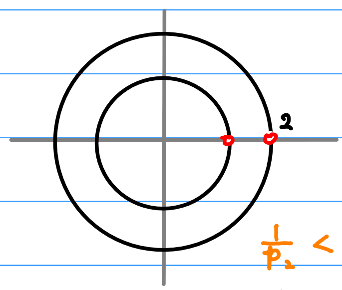


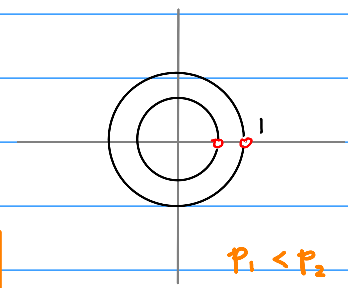
$\boxed{p = b^{-1}}$

$$f(z) \xrightarrow{z^{-1}} X(z)$$

1.A

$$\frac{-1}{(z-1)(z-2)} \xrightarrow{z^{-1}} \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$





$\frac{1}{p_2} < \frac{1}{p_1}$

$\frac{1}{2} \quad \frac{1}{1}$

$\boxed{p_1 = 1,\ p_2 = 2}$

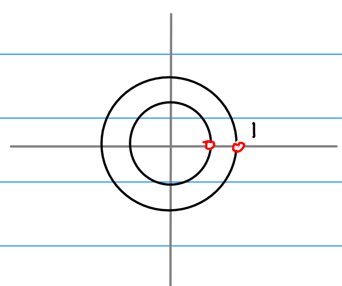
$p_1 < p_2$

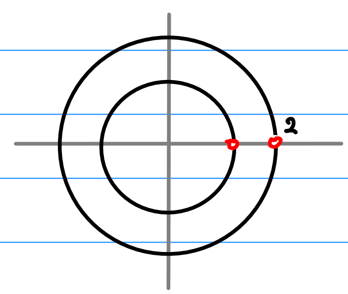
$0.5 \quad 1.0$

$\boxed{p_1 = 0.5,\ p_2 = 1}$

2.A

$$\frac{-0.5\,z^2}{(z-1)(z-0.5)} \xrightarrow{z^{-1}} \frac{-1}{(z-1)(z-2)}$$





$\frac{1}{p_2} < \frac{1}{p_1}$

$\frac{1}{1} \quad \frac{1}{1/2}$

$\boxed{p_1 = 0.5,\ p_2 = 1}$

$p_1 < p_2$

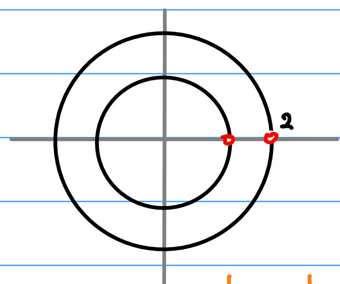
$1 \quad 2$

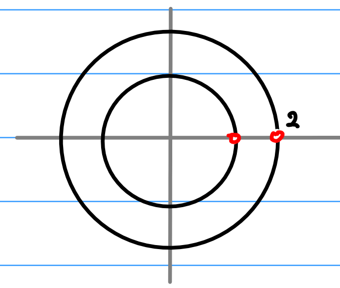
$\boxed{p_1 = 1,\ p_2 = 2}$

$$f(z) = X(z)$$

3.A

$$\frac{-1}{(z-1)(z-2)} = \frac{-1}{(z-1)(z-2)}$$





$\frac{1}{p_2} < \frac{1}{p_1}$

$\frac{1}{2} \quad \frac{1}{1}$

$\boxed{p_1 = 1,\ p_2 = 2}$

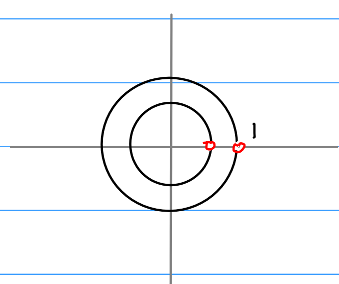
$p_1 < p_2$

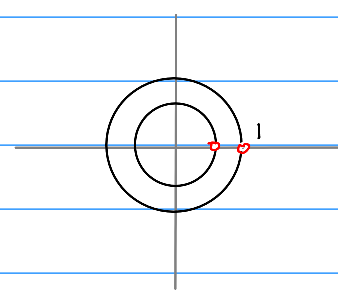
$1 \quad 2$

$\boxed{p_1 = 1,\ p_2 = 2}$

4.A

$$\frac{-0.5\,z^2}{(z-1)(z-0.5)} = \frac{-0.5\,z^2}{(z-1)(z-0.5)}$$





$\frac{1}{p_2} < \frac{1}{p_1}$

$\frac{1}{1} \quad \frac{1}{1/2}$

$\boxed{p_1 = 0.5,\ p_2 = 1}$

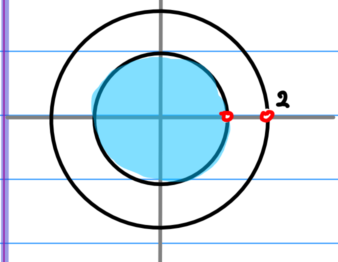
$p_1 < p_2$

$1 \quad 2$

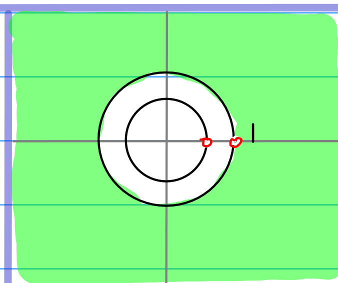
$\boxed{p_1 = 0.5,\ p_2 = 1}$

$$f(z) \xleftrightarrow{Z^{-1}} X(z) \qquad\qquad a_n = x_n$$

Ⅰ

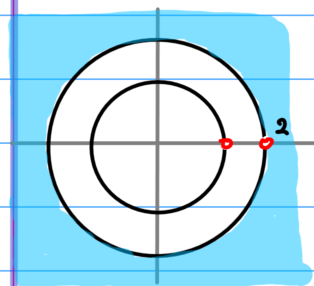


$$\left(\tfrac{1}{2}\right)^{n+1} - 1 \quad (n \geq 0)$$
$$0 \quad (n < 0)$$

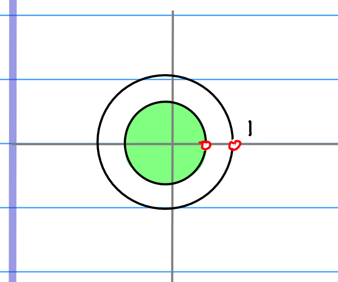


$$\left(\tfrac{1}{2}\right)^{n+1} - 1 \quad (n \geq 0)$$
$$0 \quad (n < 0)$$

Ⅱ

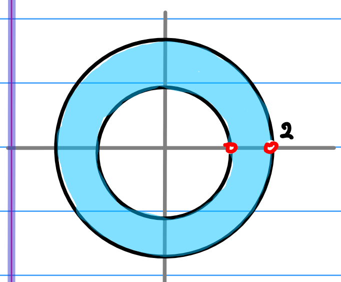


$$0 \quad (n \geq 0)$$
$$1 - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)$$

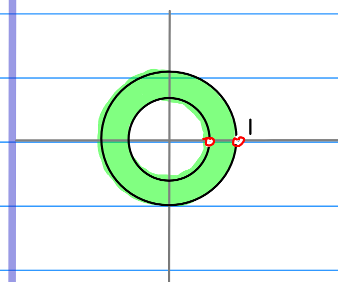


$$0 \quad (n \geq 0)$$
$$1 - \left(\tfrac{1}{2}\right)^{n+1} \quad (n < 0)$$

Ⅲ

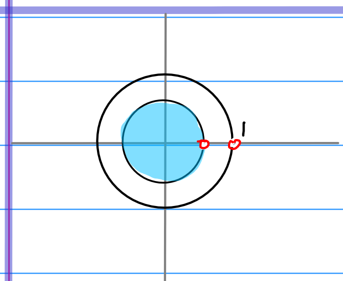


$$\left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)$$
$$1 \quad (n < 0)$$

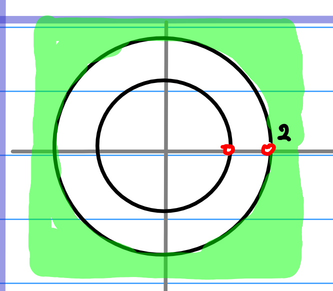


$$\left(\tfrac{1}{2}\right)^{n+1} \quad (n \geq 0)$$
$$1 \quad (n < 0)$$

Ⅰ

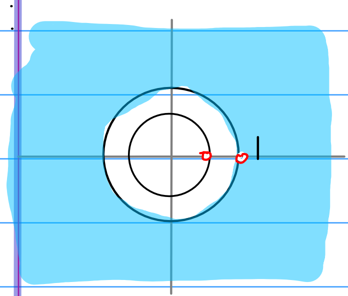


$$1 - 2^{n-1} \quad (n > 0)$$
$$0 \quad (n \leq 0)$$

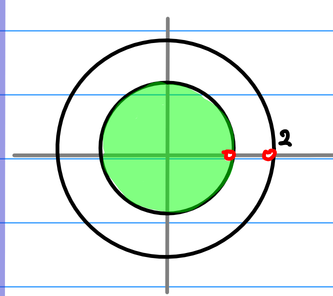


$$1 - 2^{n-1} \quad (n > 0)$$
$$0 \quad (n \leq 0)$$

Ⅱ

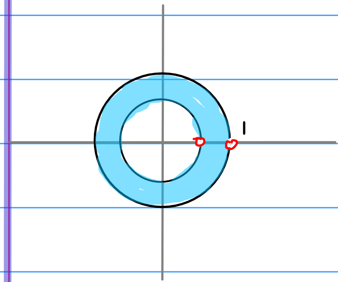


$$0 \quad (n > 0)$$
$$2^{n-1} - 1 \quad (n \leq 0)$$

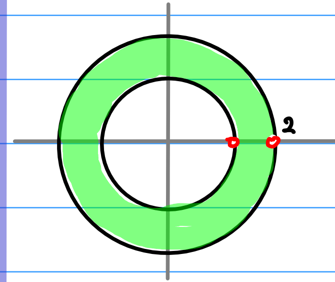


$$0 \quad (n > 0)$$
$$2^{n-1} - 1 \quad (n \leq 0)$$

Ⅲ



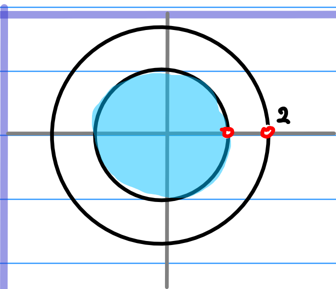
$$1 \quad (n > 0)$$
$$2^{n-1} \quad (n \leq 0)$$



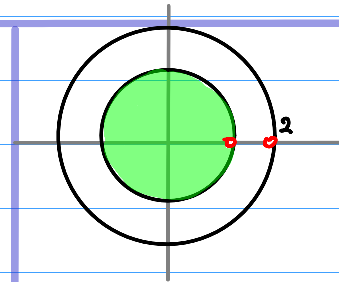
$$1 \quad (n > 0)$$
$$2^{n-1} \quad (n \leq 0)$$

$f(z) = X(z)$ $\qquad$ $a_n \xleftrightarrow{-n} x_n$

Ⅰ

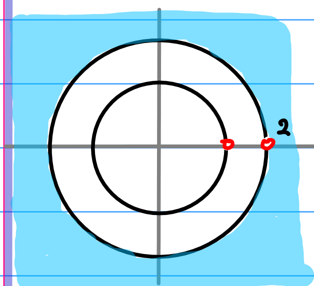


$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$

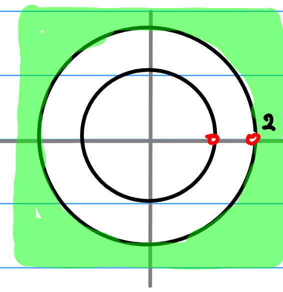


$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$

Ⅱ

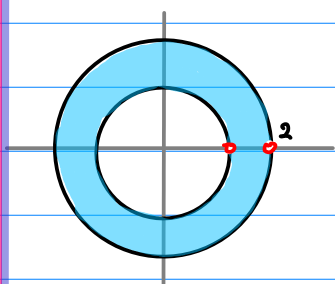


$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$

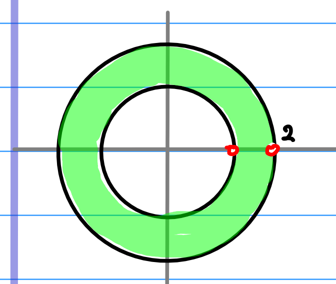


$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$

Ⅲ

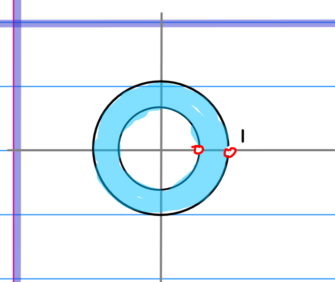


$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$

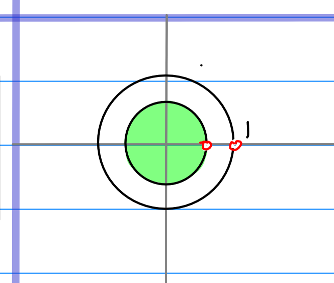


$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$

Ⅰ

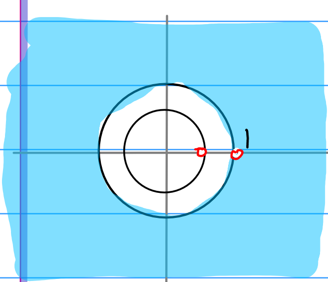


$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$

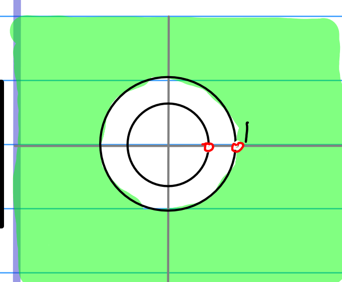


$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$

Ⅱ

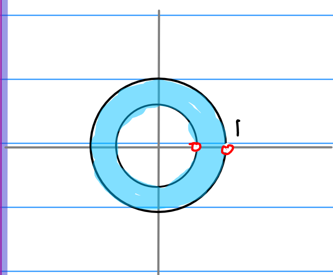


$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$

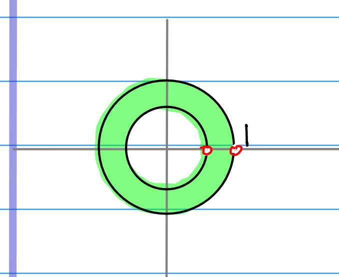


$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$

Ⅲ



$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$



$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$

$f(z) \xleftrightarrow{z^{-1}} X(z)$ $\qquad$ $a_n = x_n$

Ⅰ

$p_1 = 1$, $p_2 = 2$

$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$p_1 = 0.5$, $p_2 = 1$

$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$* -1$ $\qquad$ $* -1$

Ⅱ

$p_1 = 1$, $p_2 = 2$

$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$p_1 = 0.5$, $p_2 = 1$

$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

Ⅲ

$p_1 = 1$, $p_2 = 2$

$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$p_1 = 0.5$, $p_2 = 1$

$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

Ⅰ

$p_1 = 0.5$, $p_2 = 1$

$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$p_1 = 1$, $p_2 = 2$

$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$* -1$ $\qquad$ $* -1$

Ⅱ

$p_1 = 0.5$, $p_2 = 1$

$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$p_1 = 1$, $p_2 = 2$

$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

Ⅲ

$p_1 = 0.5$, $p_2 = 1$

$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$p_1 = 1$, $p_2 = 2$

$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$f(z) = X(z)$ $\qquad$ $a_n \xleftrightarrow{-n} x_n$

I

$p_1 = 1$, $p_2 = 2$

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases} \qquad x_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

$p_1 = 1$, $p_2 = 2$

$* -1$ $\qquad$ $* -1$

II

$p_1 = 1$, $p_2 = 2$

$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases} \qquad x_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

$p_1 = 1$, $p_2 = 2$

III

$p_1 = 1$, $p_2 = 2$

$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases} \qquad x_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

$p_1 = 1$, $p_2 = 2$

I

$p_1 = 0.5$, $p_2 = 1$

$$a_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases} \qquad x_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

$p_1 = 0.5$, $p_2 = 1$

$* -1$ $\qquad$ $* -1$

II

$p_1 = 0.5$, $p_2 = 1$

$$a_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases} \qquad x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

$p_1 = 0.5$, $p_2 = 1$

III

$p_1 = 0.5$, $p_2 = 1$

$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases} \qquad x_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

$p_1 = 0.5$, $p_2 = 1$

$$\boxed{\begin{array}{ll} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{array}} \longrightarrow \boxed{\begin{array}{ll} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{array}}$$

n	-3	-2	-1	0	1	2	3	4
$\left(\frac{1}{2}\right)^{n+1}$	$\left(\frac{1}{2}\right)^{-3+1}$	$\left(\frac{1}{2}\right)^{-2+1}$	$\left(\frac{1}{2}\right)^{-1+1}$	$\left(\frac{1}{2}\right)^{0+1}$	$\left(\frac{1}{2}\right)^{1+1}$	$\left(\frac{1}{2}\right)^{2+1}$	$\left(\frac{1}{2}\right)^{3+1}$	$\left(\frac{1}{2}\right)^{4+1}$
	$\left(\frac{1}{2}\right)^{-2}$	$\left(\frac{1}{2}\right)^{-1}$	$\left(\frac{1}{2}\right)^{0}$	$\left(\frac{1}{2}\right)^{1}$	$\left(\frac{1}{2}\right)^{2}$	$\left(\frac{1}{2}\right)^{3}$	$\left(\frac{1}{2}\right)^{4}$	$\left(\frac{1}{2}\right)^{5}$
2^{-n-1}	2^{2}	2^{1}	2^{0}	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
$-n$	$+3$	$+2$	$+1$	0	-1	-2	-3	-4
2^{-n-1}	2^{3-1}	2^{2-1}	2^{1-1}	2^{0-1}	2^{-1-1}	2^{-2-1}	2^{-3-1}	

n'	m	-2	-1	0	$+1$	$+2$	$+3$
$2^{n'-1}$	2^{m-1}	2^{-2-1}	2^{-1-1}	2^{0-1}	2^{1-1}	2^{2-1}	2^{3-1}

n	-3	-2	-1	0	1	2	3
2^n	2^{-3}	2^{-2}	2^{-1}	2^0	2^1	2^2	2^3
$-n$	3	2	1	0	-1	-2	-3
2^{-n}	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}

rearrange

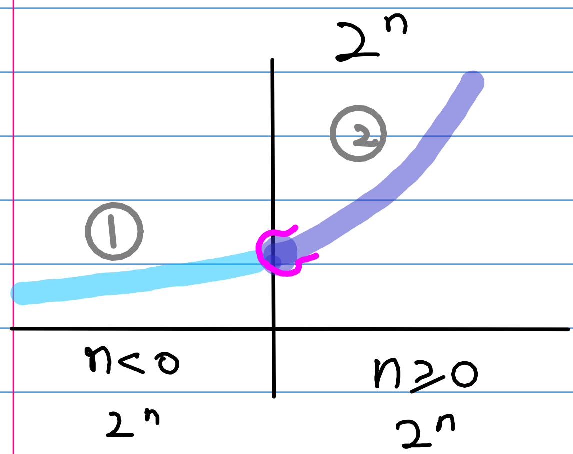
n'	m	$-n$	-3	-2	-1	0	1	2	3
$2^{n'}$	2^m	2^{-n}	2^{-3}	2^{-2}	2^{-1}	2^0	2^1	2^2	2^3

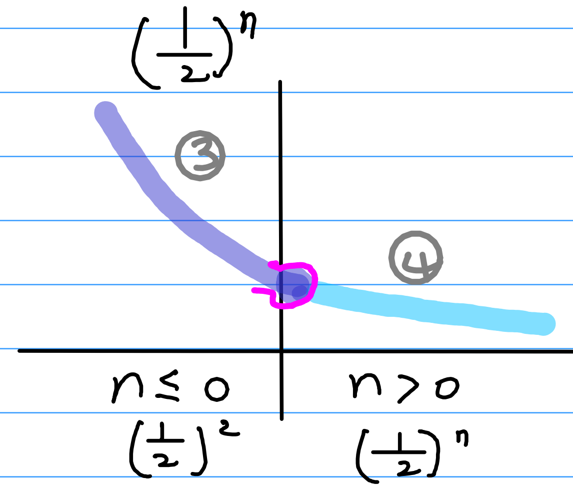
$$\begin{array}{ccc}
 & \vdots & \\
2 & \longrightarrow & 2^2 \\
1 & \longrightarrow & 2^1 \\
0 & \longrightarrow & 2^0 \\
-1 & \longrightarrow & 2^{-1} \\
-2 & \longrightarrow & 2^{-2} \\
 & \vdots &
\end{array}
\qquad\qquad
\begin{array}{ccc}
 & \vdots & \\
-2 & \longrightarrow & 2^{-2} \\
-1 & \longrightarrow & 2^{-1} \\
0 & \longrightarrow & 2^0 \\
1 & \longrightarrow & 2^1 \\
2 & \longrightarrow & 2^2 \\
 & \vdots &
\end{array}$$

the same function

n	-3	-2	-1	0	1	2	3
2^n	2^{-3}	2^{-2}	2^{-1}	2^0	2^1	2^2	2^3
n	-3	-2	-1	0	1	2	3
2^{-n}	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}

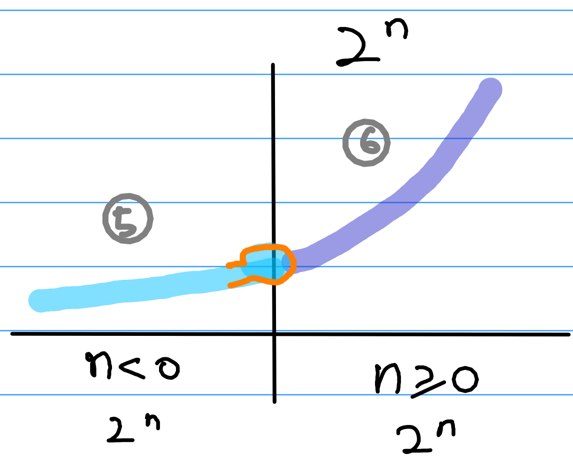
different functions

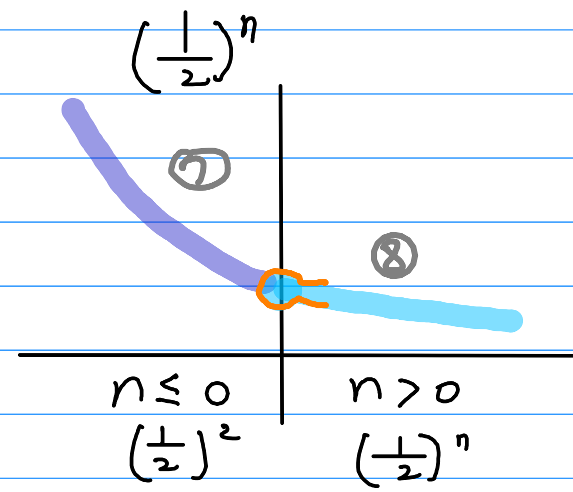




① 2^n $(n<0)$ ③ $(\frac{1}{2})^n$ $(n\leq 0)$

② 2^n $(n\geq 0)$ ④ $(\frac{1}{2})^n$ $(n>0)$





⑤ 2^n $(n\leq 0)$ ⑦ $(\frac{1}{2})^n$ $(n<0)$

⑥ 2^n $(n>0)$ ⑧ $(\frac{1}{2})^n$ $(n\geq 0)$

$$2^{n-1} \xleftrightarrow{-n} 2^{-n-1} = \left(\frac{1}{2}\right)^{n+1}$$

$$2^{n-1} \quad (n < 0) \qquad -n \qquad \left(\frac{1}{2}\right)^{n+1} \quad (n \geq 0)$$

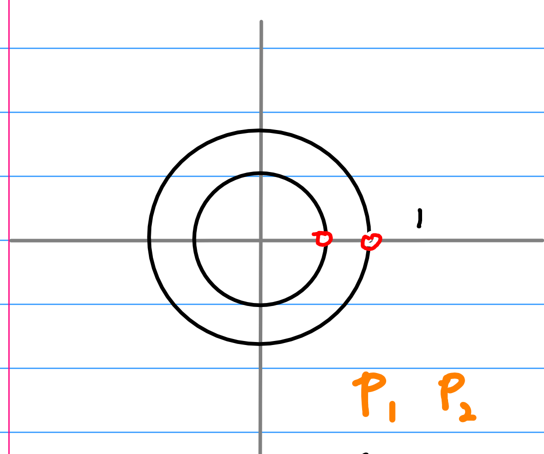
$$2^{n-1} \quad (n \geq 0) \qquad -n \qquad \left(\frac{1}{2}\right)^{n+1} \quad (n < 0)$$

$$2^{n-1} \quad (n \leq 0) \qquad -n \qquad \left(\frac{1}{2}\right)^{n+1} \quad (n > 0)$$

$$2^{n-1} \quad (n > 0) \qquad -n \qquad \left(\frac{1}{2}\right)^{n+1} \quad (n \leq 0)$$

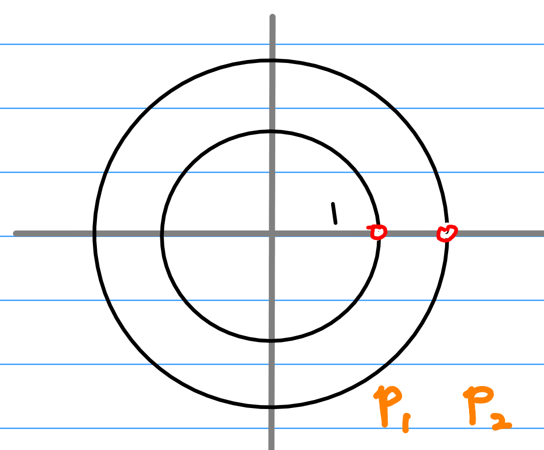
$f(z) \qquad p_1 < p_2$

$p_1 \leq p_2 \leq 1$		$1 \leq p_1 \leq p_2$		
$\left(\frac{1}{p_2}\right)^{n-1} - \left(\frac{1}{p_1}\right)^{n-1}$	$(n > 0)$	$\left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1}$	$(n \geq 0)$	$\lvert z \rvert < p_1$
0	$(n \leq 0)$	0	$(n < 0)$	
0	$(n > 0)$	0	$(n \geq 0)$	$\lvert z \rvert > p_2$
$\left(\frac{1}{p_1}\right)^{n-1} - \left(\frac{1}{p_2}\right)^{n-1}$	$(n \leq 0)$	$\left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1}$	$(n < 0)$	
$\left(\frac{1}{p_2}\right)^{n-1}$	$(n > 0)$	$\left(\frac{1}{p_2}\right)^{n+1}$	$(n \geq 0)$	$p_1 < \lvert z \rvert < p_2$
$\left(\frac{1}{p_1}\right)^{n-1}$	$(n \leq 0)$	$\left(\frac{1}{p_1}\right)^{n+1}$	$(n < 0)$	



p_1	p_2
0.5	1

$\frac{1}{p_1}$	$\frac{1}{p_2}$
2	1



p_1	p_2
1	2

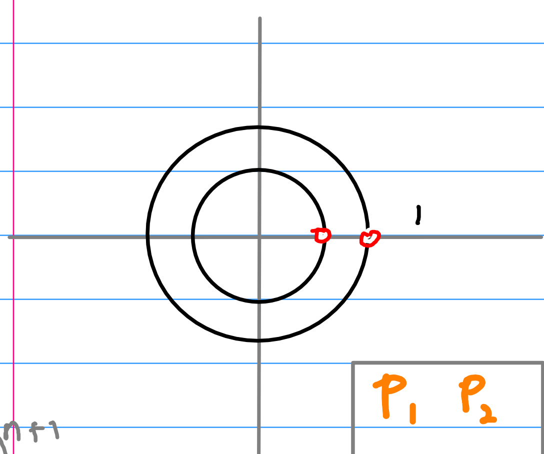
$\frac{1}{p_1}$	$\frac{1}{p_2}$
1	0.5

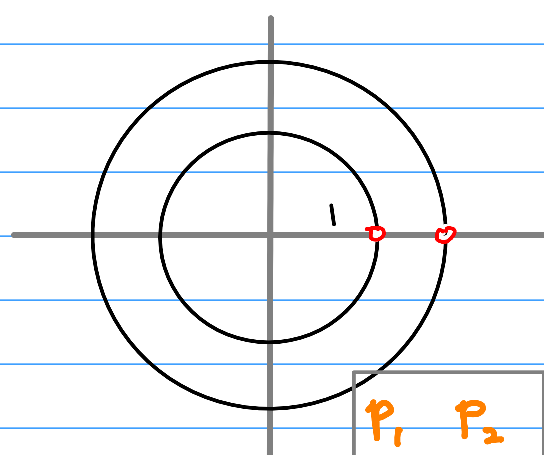
2^n

$\left(\frac{1}{2}\right)^{n+1}$

$X(z)$ $\quad p_1 < p_2$

$p_1 \leq p_2 \leq 1$	$1 \leq p_1 \leq p_2$	
$0 \quad (n \geq 0)$ $(p_1)^{n+1} - (p_2)^{n+1} \quad (n < 0)$	$0 \quad (n > 0)$ $(p_1)^{n-1} - (p_2)^{n-1} \quad (n \leq 0)$	$\lvert z \rvert < p_1$
$(p_2)^{n+1} - (p_1)^{n+1} \quad (n \geq 0)$ $0 \quad (n < 0)$	$(p_2)^{n-1} - (p_1)^{n-1} \quad (n > 0)$ $0 \quad (n \leq 0)$	$\lvert z \rvert > p_2$
$(p_2)^{n+1} \quad (n \geq 0)$ $(p_1)^{n+1} \quad (n < 0)$	$(p_2)^{n-1} \quad (n > 0)$ $(p_1)^{n-1} \quad (n \leq 0)$	$p_1 < \lvert z \rvert < p_2$





$\left(\frac{1}{2}\right)^{n+1}$

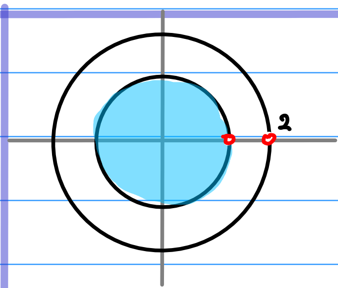
p_1	p_2
0.5	1.0
$\frac{1}{p_1}$	$\frac{1}{p_2}$
2	1

p_1	p_2
1	2
$\frac{1}{p_1}$	$\frac{1}{p_2}$
1	0.5

2^n

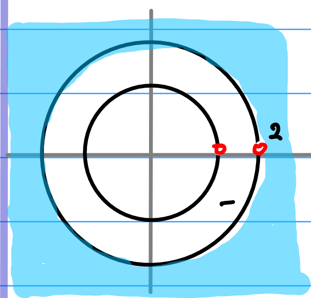
$f(z) \xleftrightarrow{Z^{-1}} X(z)$ $\qquad a_n = x_n$

Ⅰ



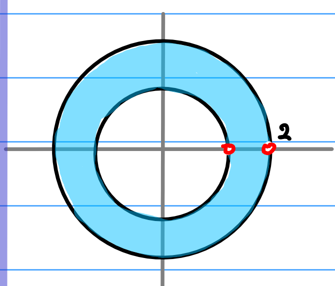
$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases} \qquad \boxed{\begin{matrix} p_1 = 1 \\ p_2 = 2 \end{matrix}} \qquad x_n = \begin{cases} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

Ⅱ



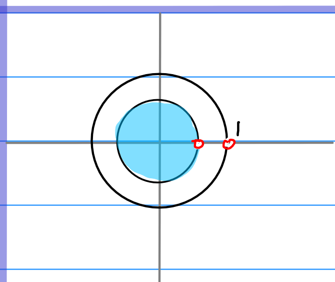
$$a_n = \begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases} \qquad \boxed{\begin{matrix} p_1 = 1 \\ p_2 = 2 \end{matrix}} \qquad x_n = \begin{cases} 0 & (n \geq 0) \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} & (n < 0) \end{cases}$$

Ⅲ



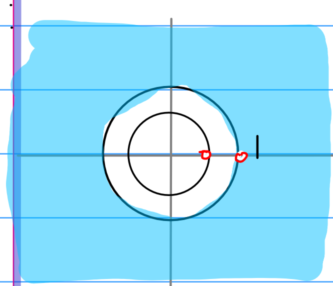
$$a_n = \begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases} \qquad \boxed{\begin{matrix} p_1 = 1 \\ p_2 = 2 \end{matrix}} \qquad x_n = \begin{cases} \left(\frac{1}{p_2}\right)^{n+1} & (n \geq 0) \\ \left(\frac{1}{p_1}\right)^{n+1} & (n < 0) \end{cases}$$

Ⅰ



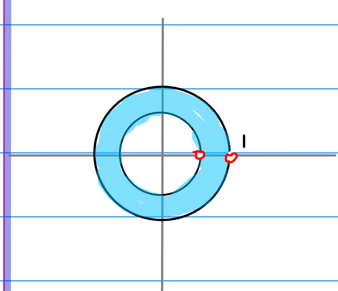
$$a_n = \begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases} \qquad \boxed{\begin{matrix} p_1 = 0.5 \\ p_2 = 1 \end{matrix}} \qquad x_n = \begin{cases} \left(\frac{1}{p_2}\right)^{n-1} - \left(\frac{1}{p_1}\right)^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

Ⅱ



$$a_n = \begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases} \qquad \boxed{\begin{matrix} p_1 = 0.5 \\ p_2 = 1 \end{matrix}} \qquad x_n = \begin{cases} 0 & (n > 0) \\ \left(\frac{1}{p_1}\right)^{n-1} - \left(\frac{1}{p_2}\right)^{n-1} & (n \leq 0) \end{cases}$$

Ⅲ



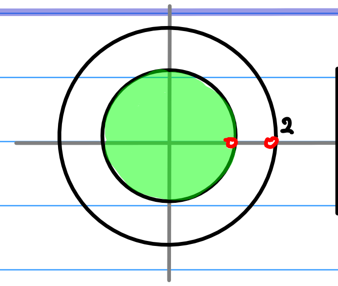
$$a_n = \begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases} \qquad \boxed{\begin{matrix} p_1 = 0.5 \\ p_2 = 1 \end{matrix}} \qquad x_n = \begin{cases} \left(\frac{1}{p_2}\right)^{n-1} & (n > 0) \\ \left(\frac{1}{p_1}\right)^{n-1} & (n \leq 0) \end{cases}$$

$f(z) \xleftrightarrow{z^{-1}} X(z)$ $\qquad a_n = x_n$

Ⅰ

$p_1 = 1$
$p_2 = 2$

$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1} \\ 0 \end{cases}$$

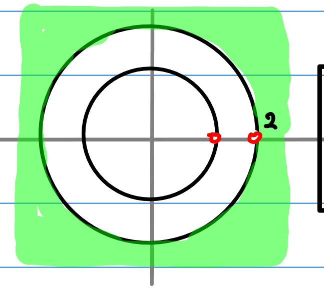


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} - 1 & (n \geq 0) \\ 0 & (n < 0) \end{cases}$$

Ⅱ

$p_1 = 1$
$p_2 = 2$

$$\begin{cases} 0 \\ \left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1} \end{cases}$$

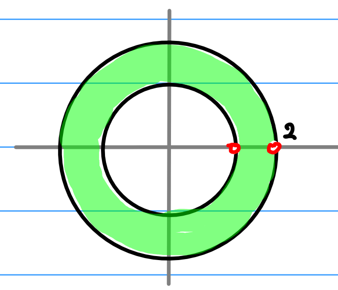


$$\begin{cases} 0 & (n \geq 0) \\ 1 - \left(\frac{1}{2}\right)^{n+1} & (n < 0) \end{cases}$$

Ⅲ

$p_1 = 1$
$p_2 = 2$

$$\begin{cases} \left(\frac{1}{p_2}\right)^{n+1} \\ \left(\frac{1}{p_1}\right)^{n+1} \end{cases}$$

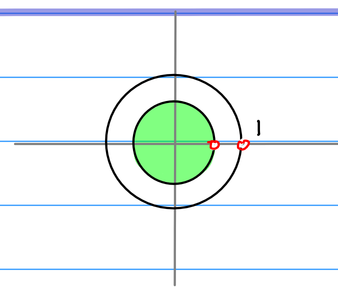


$$\begin{cases} \left(\frac{1}{2}\right)^{n+1} & (n \geq 0) \\ 1 & (n < 0) \end{cases}$$

Ⅰ

$p_1 = 0.5$
$p_2 = 1$

$$\begin{cases} \left(\frac{1}{p_2}\right)^{n-1} - \left(\frac{1}{p_1}\right)^{n-1} \\ 0 \end{cases}$$

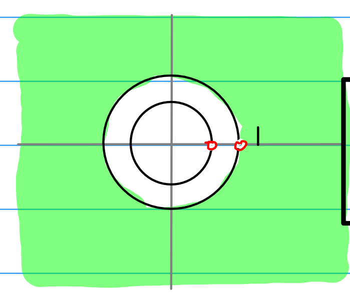


$$\begin{cases} 1 - 2^{n-1} & (n > 0) \\ 0 & (n \leq 0) \end{cases}$$

Ⅱ

$p_1 = 0.5$
$p_2 = 1$

$$\begin{cases} 0 \\ \left(\frac{1}{p_1}\right)^{n-1} - \left(\frac{1}{p_2}\right)^{n-1} \end{cases}$$

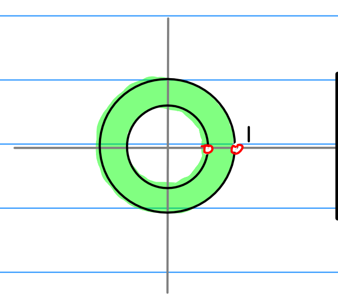


$$\begin{cases} 0 & (n > 0) \\ 2^{n-1} - 1 & (n \leq 0) \end{cases}$$

Ⅲ

$p_1 = 0.5$
$p_2 = 1$

$$\begin{cases} \left(\frac{1}{p_2}\right)^{n-1} \\ \left(\frac{1}{p_1}\right)^{n-1} \end{cases}$$



$$\begin{cases} 1 & (n > 0) \\ 2^{n-1} & (n \leq 0) \end{cases}$$

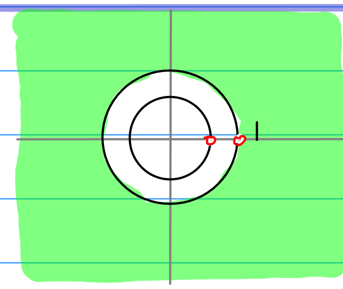
$f(z) = X(z)$ $\qquad$ $a_n \xleftrightarrow{-n} x_n$

I

$p_1 = 0.5$, $p_2 = 1$

0

$(p_1)^{n+1} - (p_2)^{n+1}$



$0 \quad (n > 0)$

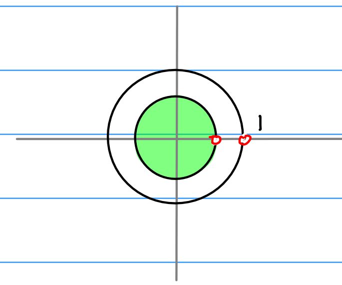
$\left(\frac{1}{2}\right)^{n+1} - 1 \quad (n \leq 0)$

II

$p_1 = 0.5$, $p_2 = 1$

$(p_2)^{n+1} - (p_1)^{n+1}$

0



$1 - \left(\frac{1}{2}\right)^{n+1} \quad (n > 0)$

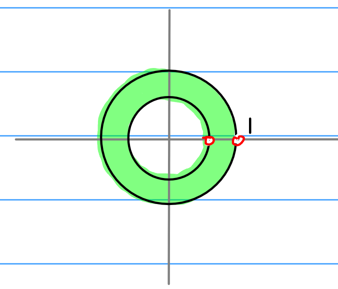
$0 \quad (n \leq 0)$

III

$p_1 = 0.5$, $p_2 = 1$

$(p_2)^{n+1}$

$(p_1)^{n+1}$



$1 \quad (n > 0)$

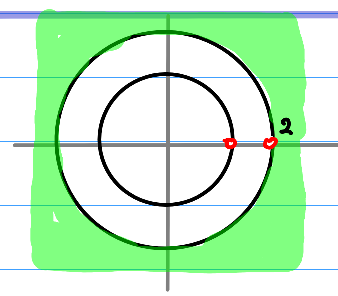
$\left(\frac{1}{2}\right)^{n+1} \quad (n \leq 0)$

I

$p_1 = 1$, $p_2 = 2$

0

$(p_1)^{n-1} - (p_2)^{n-1}$



$0 \quad (n \geq 0)$

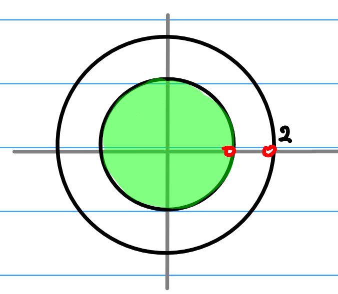
$1 - 2^{n-1} \quad (n < 0)$

II

$p_1 = 1$, $p_2 = 2$

$(p_2)^{n-1} - (p_1)^{n-1}$

0



$2^{n-1} - 1 \quad (n \geq 0)$

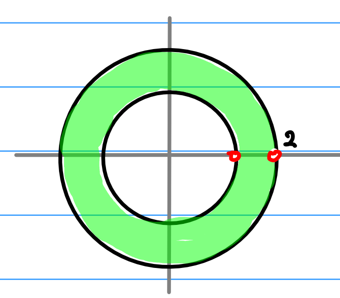
$0 \quad (n < 0)$

III

$p_1 = 1$, $p_2 = 2$

$(p_2)^{n-1}$

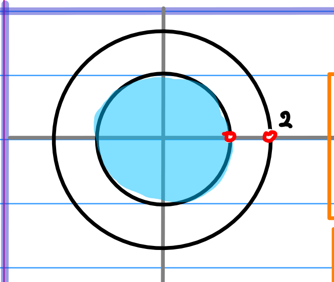
$(p_1)^{n-1}$



$2^{n-1} \quad (n \geq 0)$

$1 \quad (n < 0)$

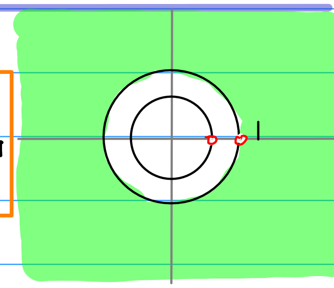
Ⅰ



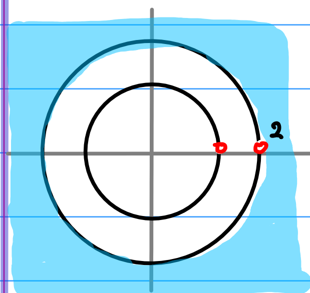
$\left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1}$ $(n \geq 0)$ $(n > 0)$ 0

0 $(n < 0)$ $(n \leq 0)$ $(p_1)^{n+1} - (p_2)^{n+1}$

$p_1 = 1$, $p_2 = 2$ $p_1 = 0.5$, $p_2 = 1$



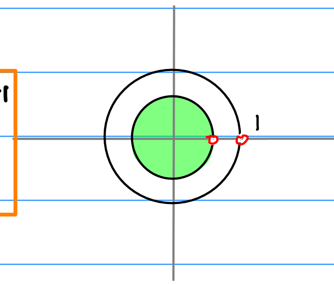
Ⅱ



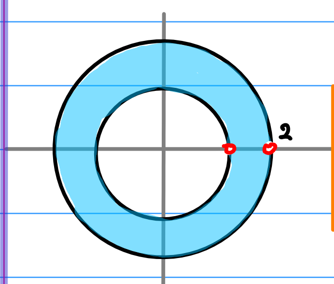
0 $(n \geq 0)$ $(n > 0)$ $(p_2)^{n+1} - (p_1)^{n+1}$

$\left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1}$ $(n < 0)$ $(n \leq 0)$ 0

$p_1 = 1$, $p_2 = 2$ $p_1 = 0.5$, $p_2 = 1$



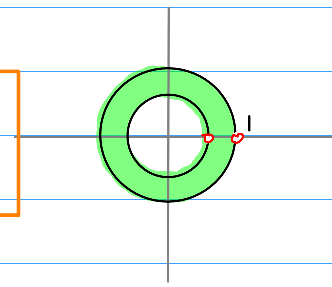
Ⅲ



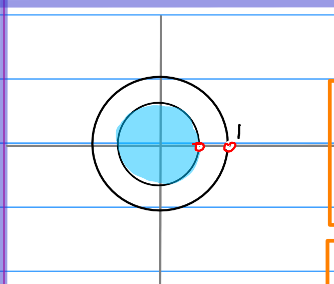
$\left(\frac{1}{p_2}\right)^{n+1}$ $(n \geq 0)$ $(n > 0)$ $(p_1)^{n+1}$

$\left(\frac{1}{p_1}\right)^{n+1}$ $(n < 0)$ $(n \leq 0)$ $(p_2)^{n+1}$

$p_1 = 1$, $p_2 = 2$ $p_1 = 0.5$, $p_2 = 1$



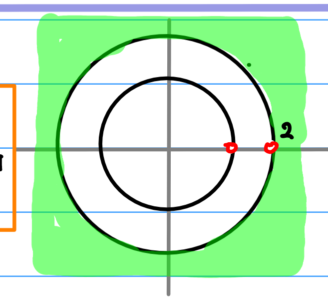
Ⅰ



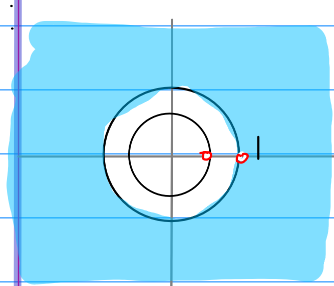
$\left(\frac{1}{p_2}\right)^{n-1} - \left(\frac{1}{p_1}\right)^{n-1}$ $(n > 0)$ $(n \geq 0)$ 0

0 $(n \leq 0)$ $(n < 0)$ $(p_1)^{n-1} - (p_2)^{n-1}$

$p_1 = 0.5$, $p_2 = 1$ $p_1 = 1$, $p_2 = 2$



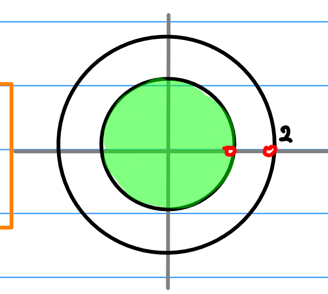
Ⅱ



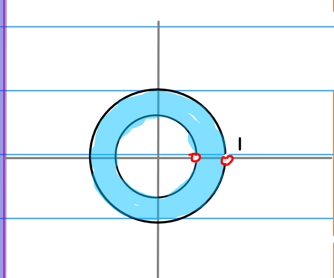
0 $(n > 0)$ $(n \geq 0)$ $(p_2)^{n-1} - (p_1)^{n-1}$

$\left(\frac{1}{p_1}\right)^{n-1} - \left(\frac{1}{p_2}\right)^{n-1}$ $(n \leq 0)$ $(n < 0)$ 0

$p_1 = 0.5$, $p_2 = 1$ $p_1 = 1$, $p_2 = 2$



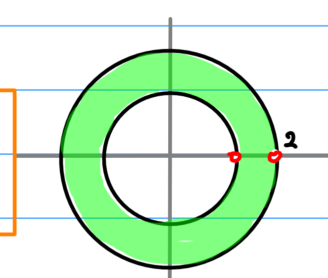
Ⅲ

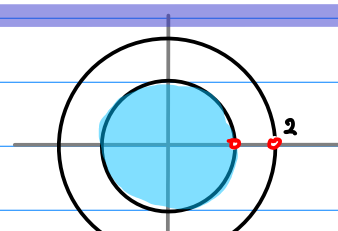
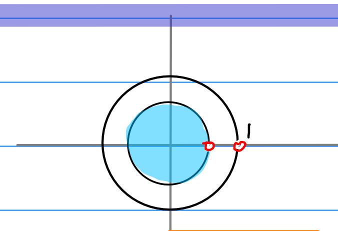
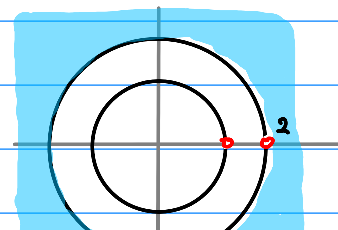
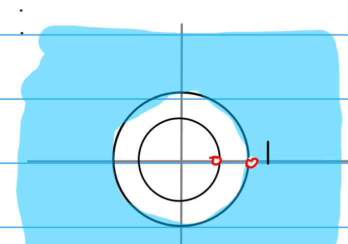
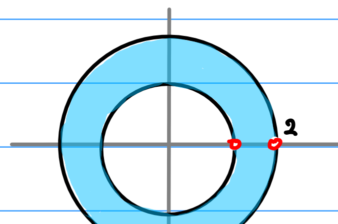
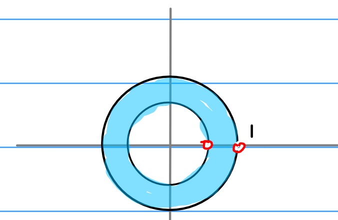
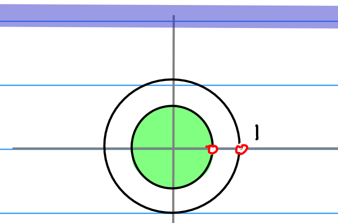
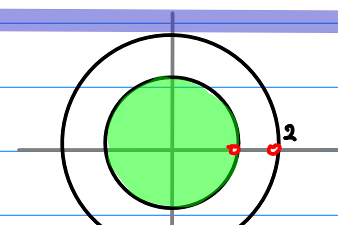
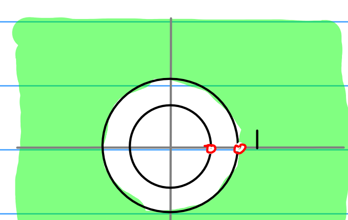
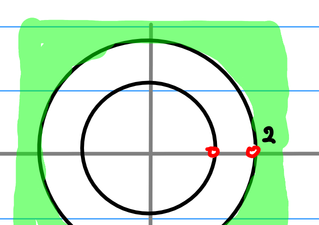
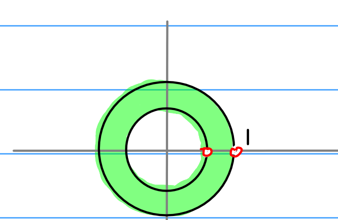
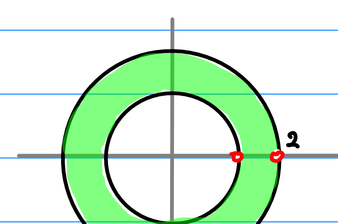


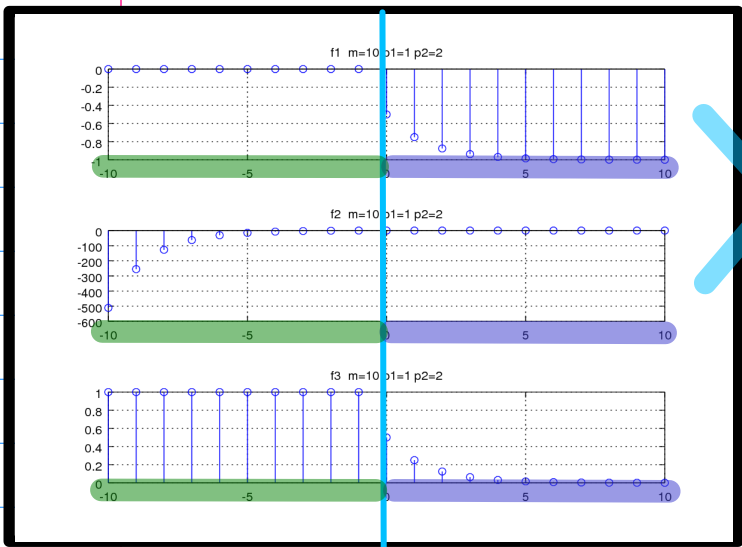
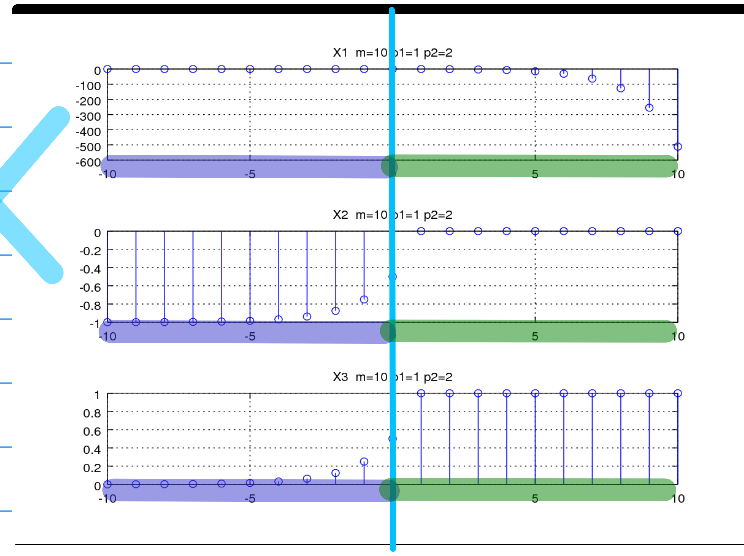
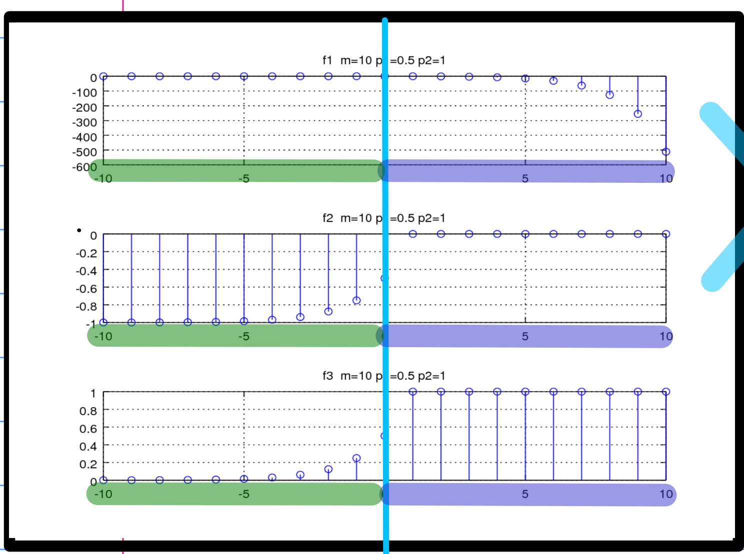
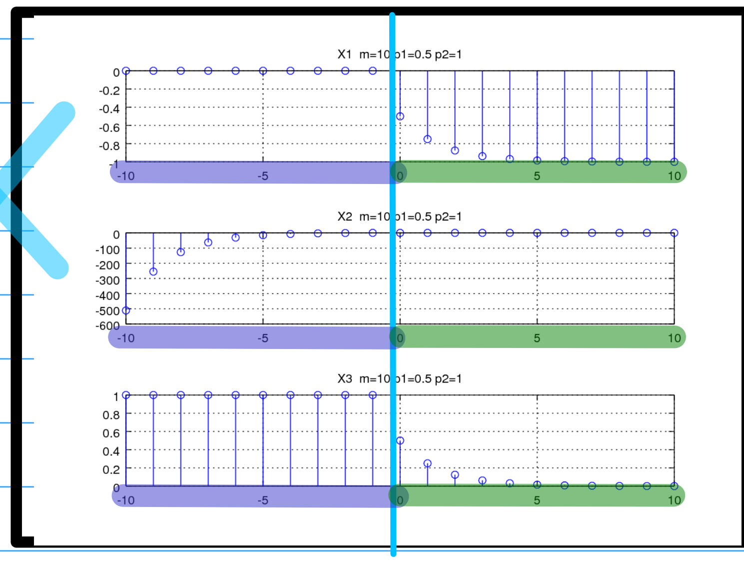
$\left(\frac{1}{p_2}\right)^{n-1}$ $(n > 0)$ $(n \geq 0)$ $(p_2)^{n-1}$

$\left(\frac{1}{p_1}\right)^{n-1}$ $(n \leq 0)$ $(n < 0)$ $(p_1)^{n-1}$

$p_1 = 0.5$, $p_2 = 1$ $p_1 = 1$, $p_2 = 2$



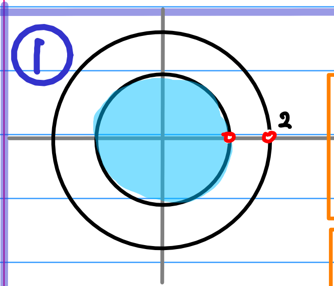
	$n \geq 0$ $n < 0$	$n > 0$ $n \leq 0$
$\left(\frac{1}{p_2}\right)^{n+1} - \left(\frac{1}{p_1}\right)^{n+1}$ 0	 1, 2	 0.5, 1.0
0 $\left(\frac{1}{p_1}\right)^{n+1} - \left(\frac{1}{p_2}\right)^{n+1}$	 1, 2	 0.5, 1.0
$\left(\frac{1}{p_2}\right)^{n+1}$ $\left(\frac{1}{p_1}\right)^{n+1}$	 1, 2	 0.5, 1.0
$(p_2)^{n+1} - (p_1)^{n+1}$ 0	 0.5, 1.0	 1, 2
0 $(p_1)^{n+1} - (p_2)^{n+1}$	 0.5, 1.0	 1, 2
$(p_2)^{n+1}$ $(p_1)^{n+1}$	 0.5, 1.0	 1, 2


f1 m=10 p1=1 p2=2
f2 m=10 p1=1 p2=2
f3 m=10 p1=1 p2=2

X1 m=10 p1=1 p2=2
X2 m=10 p1=1 p2=2
X3 m=10 p1=1 p2=2

f1 m=10 p1=0.5 p2=1
f2 m=10 p1=0.5 p2=1
f3 m=10 p1=0.5 p2=1

X1 m=10 p1=0.5 p2=1
X2 m=10 p1=0.5 p2=1
X3 m=10 p1=0.5 p2=1

```
% Laurent Series and sequences
function plotseq1(m=1, p1=2, p2=2.1)


if (p1 >=1  && p2 >= 1 && p1 < p2)
  t1n = -m: -1;
  t1p =  0: m;
  t1 = [t1n, t1p];
  f1 = [zeros(1,m), ((1/p2).^(t1p+1) - (1/p1).^(t1p+1))];
  f2 = [((1/p1).^(t1n+1) -(1/p2).^(t1n+1)), zeros(1,m+1)];
  f3 = [(1/p1).^(t1n+1), (1/p2).^(t1p+1)];
endif

if (p1 <= 1 && p2 <= 1 && p1 < p2)
  t1n = -m: 0;
  t1p = +1: m;
  t1 = [t1n, t1p];
  f1 = [zeros(1,m+1), ((1/p2).^(t1p-1) - (1/p1).^(t1p-1))];
  f2 = [((1/p1).^(t1n-1) -(1/p2).^(t1n-1)), zeros(1,m)];
  f3 = [(1/p1).^(t1n-1), (1/p2).^(t1p-1)];
endif

subplot(3, 1, 1);
stem(t1, f1);
grid on
%axis([0, m])
title(sprintf("f1  m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 2);
stem(t1, f2);
grid on
%axis([0, m])
title(sprintf("f2  m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 3);
stem(t1, f3);
grid on
%axis([0, m])
title(sprintf("f3  m=%d p1=%g p2=%g", m, p1, p2))

endfunction
```

```
% z-Transform and sequences
function plotseq2(m=1, p1=2, p2=2.1)

cla;

if (p1 <= 1 && p2 <= 1 && p1 < p2)
  t1n = -m: -1;
  t1p =  0: m;
  t1 = [t1n, t1p];
  X1 = [zeros(1,m), ((p1).^(t1p+1) - (p2).^(t1p+1))];
  X2 = [((p2).^(t1n+1) -(p1).^(t1n+1)), zeros(1,m+1)];
  X3 = [(p2).^(t1n+1), (p1).^(t1p+1)];
endif

if (p1 >= 1 && p2 >= 1 && p1 < p2)
  t1n = -m: 0;
  t1p = +1: m;
  t1 = [t1n, t1p];
  X1 = [zeros(1,m+1), ((p1).^(t1p-1) - (p2).^(t1p-1))];
  X2 = [((p2).^(t1n-1) -(p1).^(t1n-1)), zeros(1,m)];
  X3 = [(p2).^(t1n-1), (p1).^(t1p-1)];
endif


subplot(3, 1, 1);
stem(t1, X1);
grid on
%axis([0, m])
title(sprintf("X1  m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 2);
stem(t1, X2);
grid on
%axis([0, m])
title(sprintf("X2  m=%d p1=%g p2=%g", m, p1, p2))

subplot(3, 1, 3);
stem(t1, X3);
grid on
%axis([0, m])
title(sprintf("X3  m=%d p1=%g p2=%g", m, p1, p2))

endfunction
```

Ⅰ ①



$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$

0

$(n \geq 0)$ $(n > 0)$

$(n < 0)$ $(n \leq 0)$

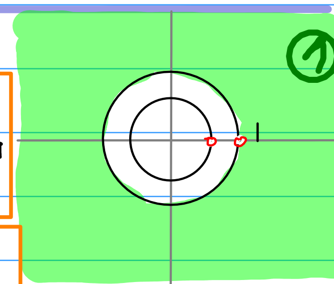
0

$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$

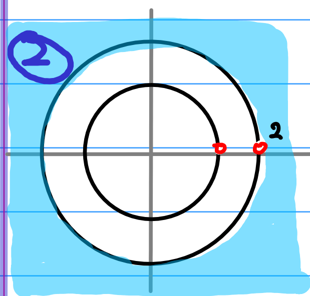
$p_1 = 1 \quad = p$
$p_2 = 2 \quad = q$

$p_1 = 0.5 \quad = 1/q$
$p_2 = 1 \quad = 1/p$

⑦



Ⅱ ②



0

$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$

$(n \geq 0)$ $(n > 0)$

$(n < 0)$ $(n \leq 0)$

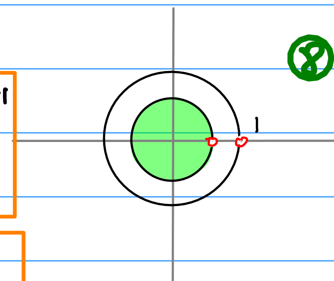
$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$

0

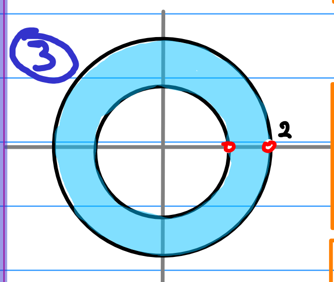
$p_1 = 1 \quad = p$
$p_2 = 2 \quad = q$

$p_1 = 0.5 \quad = 1/q$
$p_2 = 1 \quad = 1/p$

⑧



Ⅲ ③



$\left(\frac{1}{q}\right)^{n+1}$

$\left(\frac{1}{p}\right)^{n+1}$

$(n \geq 0)$ $(n > 0)$

$(n < 0)$ $(n \leq 0)$

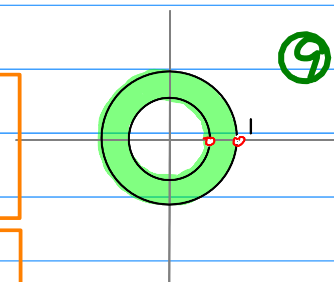
$\left(\frac{1}{q}\right)^{n+1}$

$\left(\frac{1}{p}\right)^{n+1}$

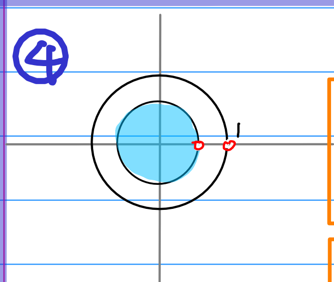
$p_1 = 1 \quad = p$
$p_2 = 2 \quad = q$

$p_1 = 0.5 \quad = 1/q$
$p_2 = 1 \quad = 1/p$

⑨



Ⅰ ④



$(p)^{n-1} - (q)^{n-1}$

0

$(n > 0)$ $(n \geq 0)$

$(n \leq 0)$ $(n < 0)$

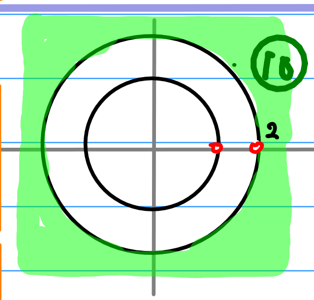
0

$(p)^{n-1} - (q)^{n-1}$

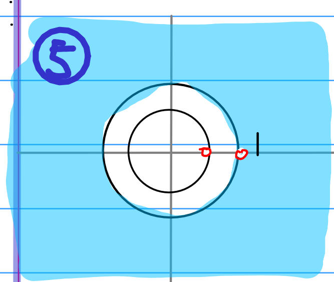
$p_1 = 0.5 \quad = 1/q$
$p_2 = 1 \quad = 1/p$

$p_1 = 1 \quad = p$
$p_2 = 2 \quad = q$

⑩



Ⅱ ⑤



0

$(q)^{n-1} - (p)^{n-1}$

$(n > 0)$ $(n \geq 0)$

$(n \leq 0)$ $(n < 0)$

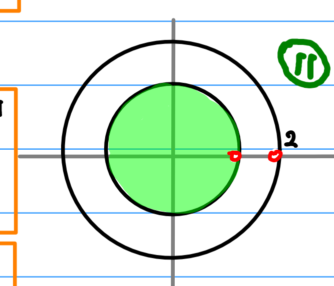
$(q)^{n-1} - (p)^{n-1}$

0

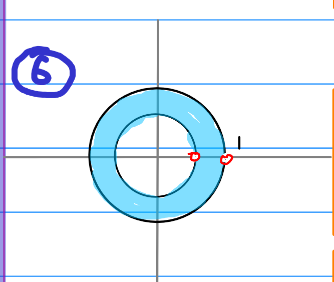
$p_1 = 0.5 \quad = 1/q$
$p_2 = 1 \quad = 1/p$

$p_1 = 1 \quad = p$
$p_2 = 2 \quad = q$

⑪



Ⅲ ⑥



$(p)^{n-1}$

$(q)^{n-1}$

$(n > 0)$ $(n \geq 0)$

$(n \leq 0)$ $(n < 0)$

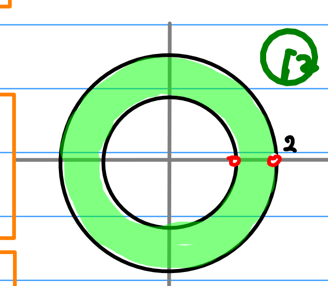
$(p)^{n-1}$

$(q)^{n-1}$

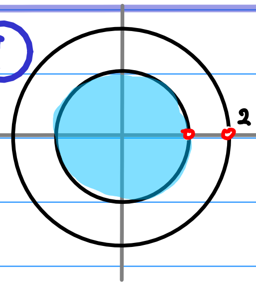
$p_1 = 0.5 \quad = 1/q$
$p_2 = 1 \quad = 1/p$

$p_1 = 1 \quad = p$
$p_2 = 2 \quad = q$

⑫



I

① 

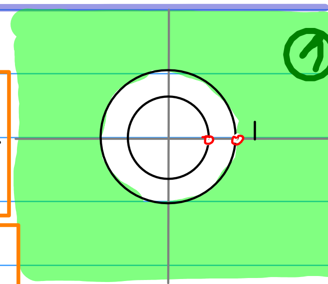
$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$ / 0

$(n \geq 0)$ $(n > 0)$ / $(n < 0)$ $(n \leq 0)$

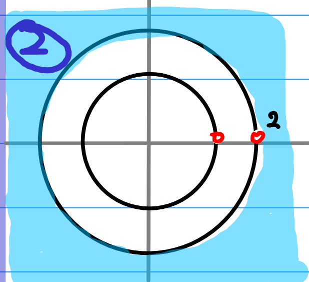
0 / $\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$

$p_1 = 1 = p$
$p_2 = 2 = q$

$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

⑦ 

II

② 

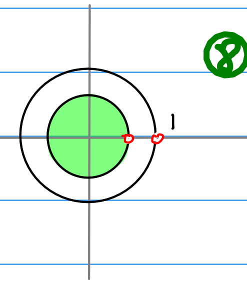
0 / $\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$

$(n \geq 0)$ $(n > 0)$ / $(n < 0)$ $(n \leq 0)$

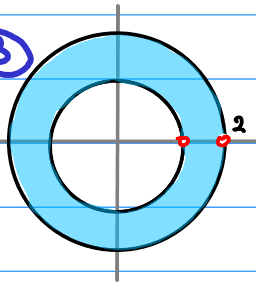
$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$ / 0

$p_1 = 1 = p$
$p_2 = 2 = q$

$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

⑧ 

III

③ 

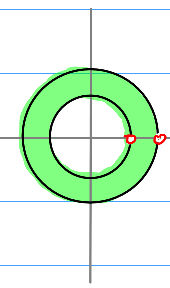
$\left(\frac{1}{q}\right)^{n+1}$ / $\left(\frac{1}{p}\right)^{n+1}$

$(n \geq 0)$ $(n > 0)$ / $(n < 0)$ $(n \leq 0)$

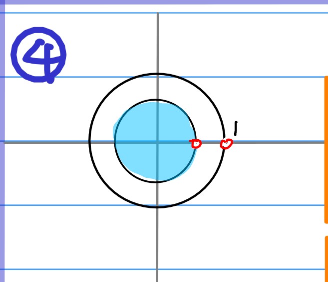
$\left(\frac{1}{q}\right)^{n+1}$ / $\left(\frac{1}{p}\right)^{n+1}$

$p_1 = 1 = p$
$p_2 = 2 = q$

$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

⑨ 

I

④ 

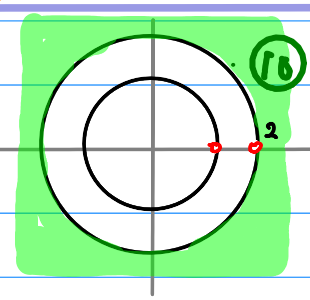
$(p)^{n-1} - (q)^{n-1}$ / 0

$(n > 0)$ $(n \geq 0)$ / $(n \leq 0)$ $(n < 0)$

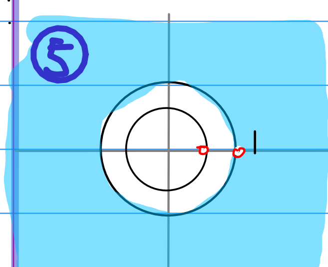
0 / $(p)^{n-1} - (q)^{n-1}$

$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

$p_1 = 1 = p$
$p_2 = 2 = q$

⑩ 

II

⑤ 

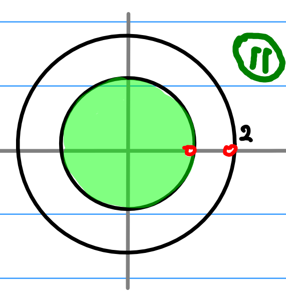
0 / $(q)^{n-1} - (p)^{n-1}$

$(n > 0)$ $(n \geq 0)$ / $(n \leq 0)$ $(n < 0)$

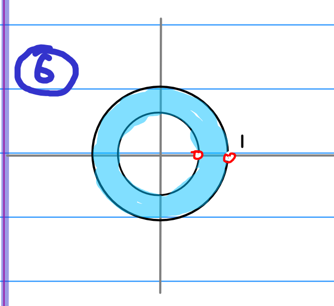
$(q)^{n-1} - (p)^{n-1}$ / 0

$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

$p_1 = 1 = p$
$p_2 = 2 = q$

⑪ 

III

⑥ 

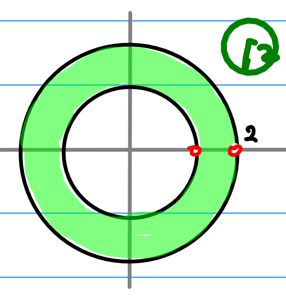
$(p)^{n-1}$ / $(q)^{n-1}$

$(n > 0)$ $(n \geq 0)$ / $(n \leq 0)$ $(n < 0)$

$(p)^{n-1}$ / $(q)^{n-1}$

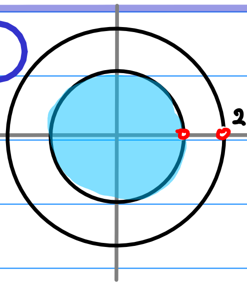
$p_1 = 0.5 = 1/q$
$p_2 = 1 = 1/p$

$p_1 = 1 = p$
$p_2 = 2 = q$

⑫ 

$f(z) \xleftrightarrow{z^{-1}} X(z)$ $\qquad a_n = x_n$

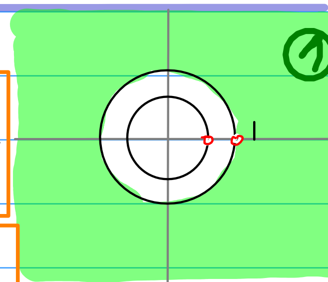
Ⅰ ①



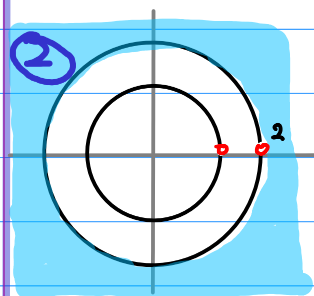
$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$	$(n \geq 0)$	$(n > 0)$	0
0	$(n < 0)$	$(n \leq 0)$	$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$

$p_1 = 1 = p$, $p_2 = 2 = q$ $\qquad$ $p_1 = 0.5 = 1/q$, $p_2 = 1 = 1/p$

⑦



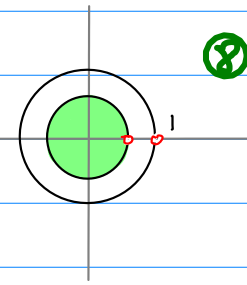
Ⅱ ②



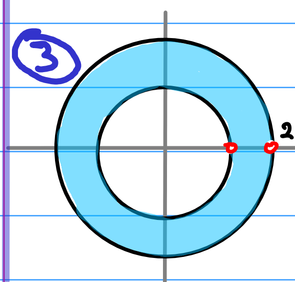
0	$(n \geq 0)$	$(n > 0)$	$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$
$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$	$(n < 0)$	$(n \leq 0)$	0

$p_1 = 1 = p$, $p_2 = 2 = q$ $\qquad$ $p_1 = 0.5 = 1/q$, $p_2 = 1 = 1/p$

⑧



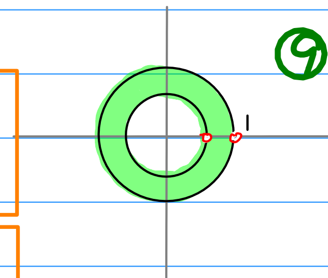
Ⅲ ③



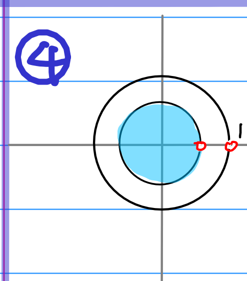
$\left(\frac{1}{q}\right)^{n+1}$	$(n \geq 0)$	$(n > 0)$	$\left(\frac{1}{q}\right)^{n+1}$
$\left(\frac{1}{p}\right)^{n+1}$	$(n < 0)$	$(n \leq 0)$	$\left(\frac{1}{p}\right)^{n+1}$

$p_1 = 1 = p$, $p_2 = 2 = q$ $\qquad$ $p_1 = 0.5 = 1/q$, $p_2 = 1 = 1/p$

⑨



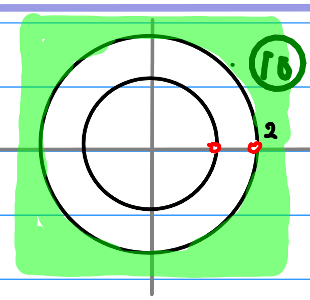
Ⅰ ④



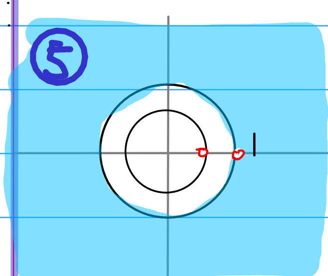
$(p)^{n-1} - (q)^{n-1}$	$(n > 0)$	$(n \geq 0)$	0
0	$(n \leq 0)$	$(n < 0)$	$(p)^{n-1} - (q)^{n-1}$

$p_1 = 0.5 = 1/q$, $p_2 = 1 = 1/p$ $\qquad$ $p_1 = 1 = p$, $p_2 = 2 = q$

⑩



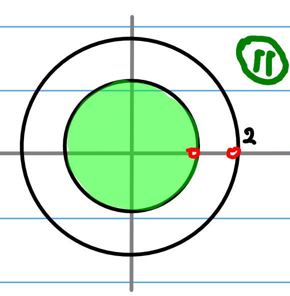
Ⅱ ⑤



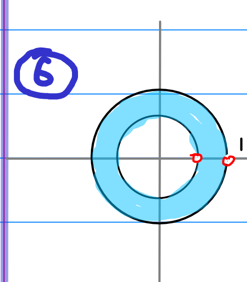
0	$(n > 0)$	$(n \geq 0)$	$(q)^{n-1} - (p)^{n-1}$
$(q)^{n-1} - (p)^{n-1}$	$(n \leq 0)$	$(n < 0)$	0

$p_1 = 0.5 = 1/q$, $p_2 = 1 = 1/p$ $\qquad$ $p_1 = 1 = p$, $p_2 = 2 = q$

⑪



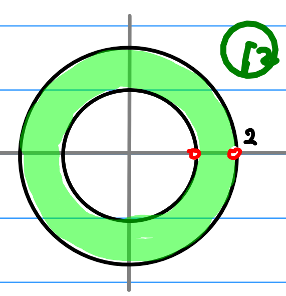
Ⅲ ⑥



$(p)^{n-1}$	$(n > 0)$	$(n \geq 0)$	$(p)^{n-1}$
$(q)^{n-1}$	$(n \leq 0)$	$(n < 0)$	$(q)^{n-1}$

$p_1 = 0.5 = 1/q$, $p_2 = 1 = 1/p$ $\qquad$ $p_1 = 1 = p$, $p_2 = 2 = q$

⑫

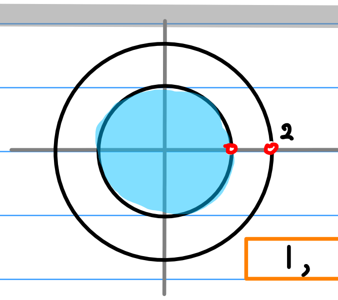


$n \geq 0$ | $n > 0$

$n < 0$ | $n \leq 0$

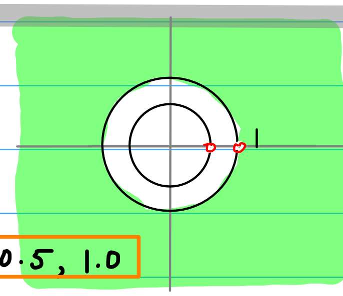
$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$

0



1, 2

0.5, 1.0

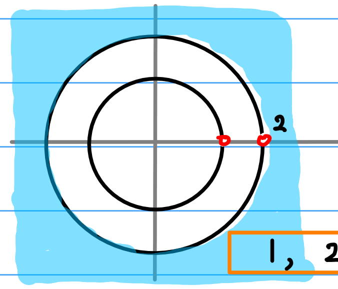


0

$\left(\frac{1}{q}\right)^{n+1} - \left(\frac{1}{p}\right)^{n+1}$

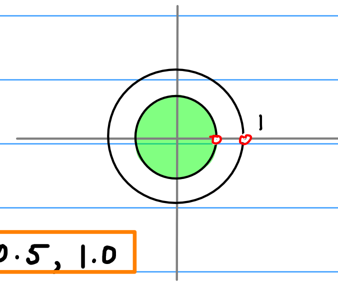
0

$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$



1, 2

0.5, 1.0

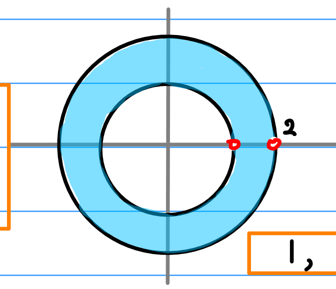


$\left(\frac{1}{p}\right)^{n+1} - \left(\frac{1}{q}\right)^{n+1}$

0

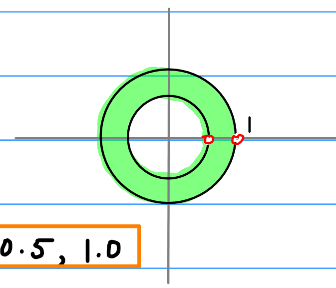
$\left(\frac{1}{q}\right)^{n+1}$

$\left(\frac{1}{p}\right)^{n+1}$



1, 2

0.5, 1.0

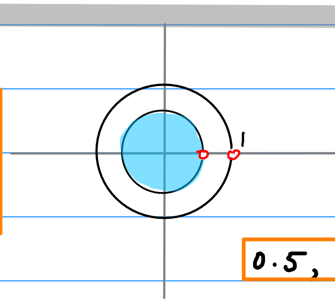


$\left(\frac{1}{p}\right)^{n+1}$

$\left(\frac{1}{q}\right)^{n+1}$

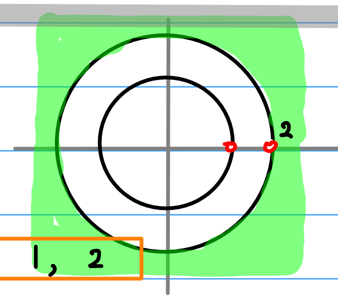
$(p)^{n-1} - (q)^{n-1}$

0



0.5, 1.0

1, 2

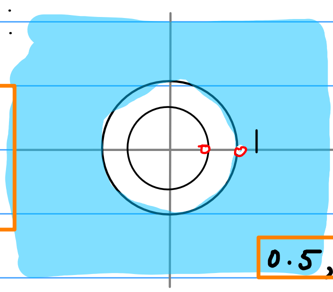


0

$(p)^{n-1} - (q)^{n-1}$

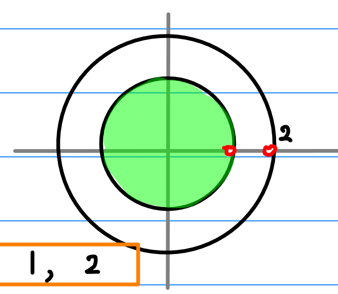
0

$(q)^{n-1} - (p)^{n-1}$



0.5, 1.0

1, 2

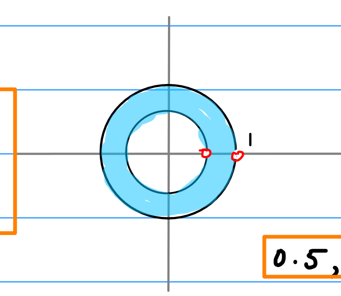


$(q)^{n-1} - (p)^{n-1}$

0

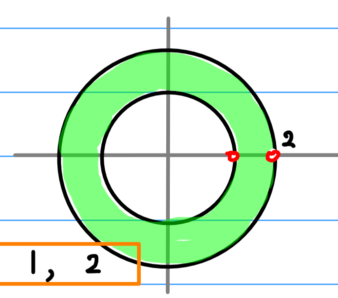
$(p)^{n-1}$

$(q)^{n-1}$



0.5, 1.0

1, 2



$(q)^{n-1}$

$(p)^{n-1}$