RC Circuit

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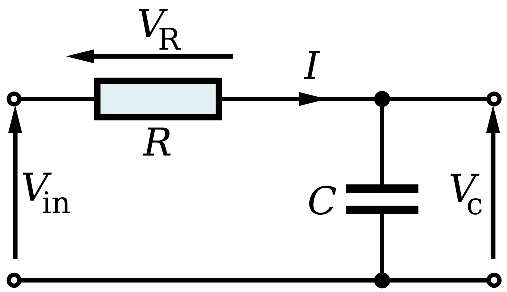
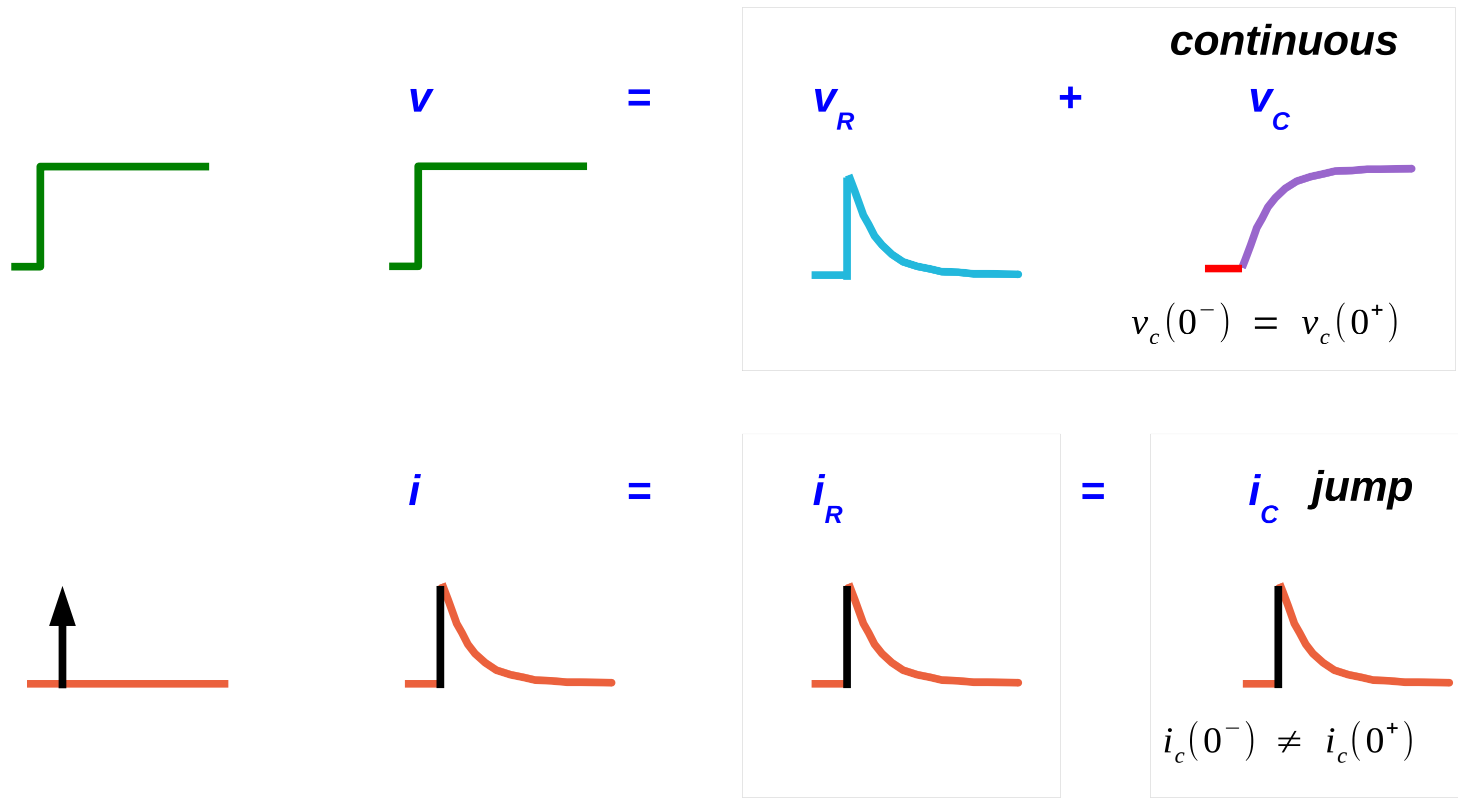
Charge



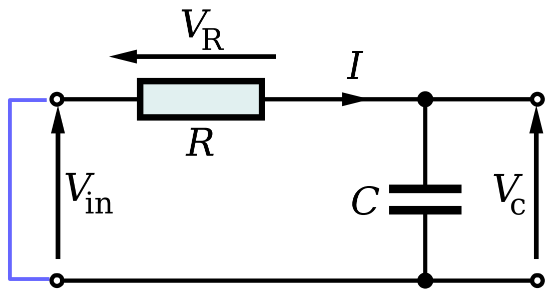
$$i_c = C \cdot \frac{d v_c}{d t}$$

unyielding voltage

current jump



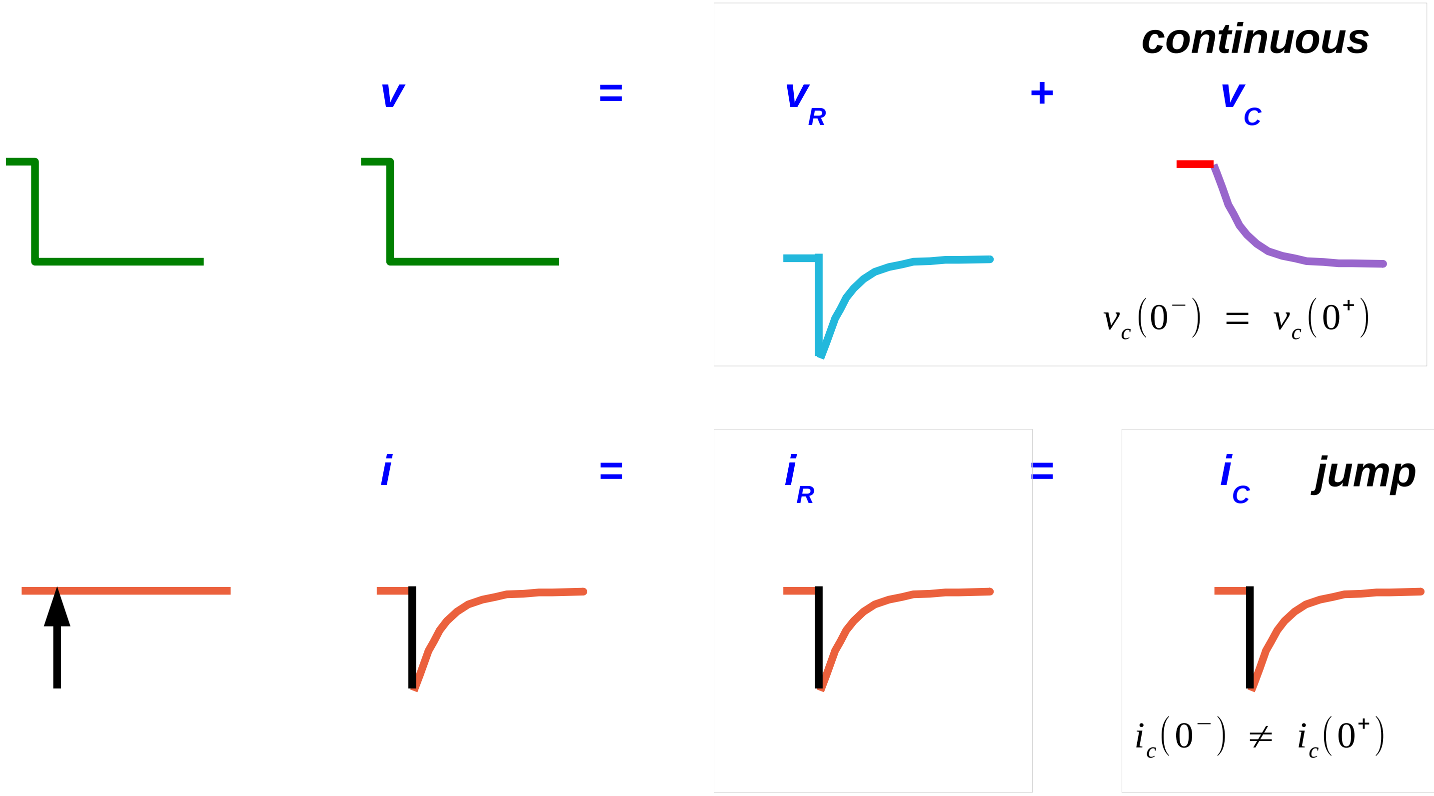
Discharge



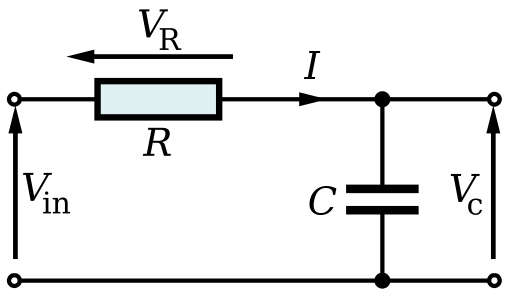
$$i_c = C \cdot \frac{d\,v_c}{d\,t}$$

unyielding voltage

current jump



Differential Equation (1)



$$RC \cdot \frac{dv_c}{dt} + v_c = V_s \qquad t \geq 0^+ \qquad v_c(0^+) = v_c(0^-) = V_0$$

$$\frac{dv_c}{dt} + \frac{1}{RC} v_c = \frac{1}{RC} V_s$$

$$\frac{dv_c}{dt} + \frac{1}{RC} v_c = 0 \qquad \frac{dv_c}{dt} = -\frac{1}{RC} v_c \qquad \frac{dv_c}{v_c} = -\frac{1}{RC} dt$$

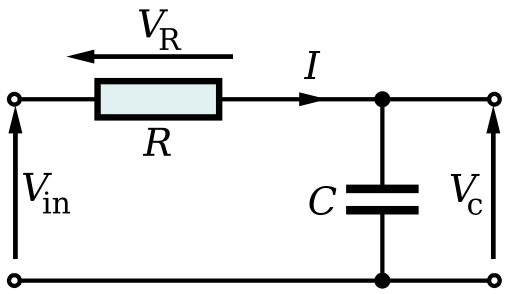
$$\ln v_c = -\int \frac{1}{RC} dt + C \qquad v_c = A e^{-\int \frac{1}{RC} dt} \qquad v_c = A e^{-\frac{t}{RC}}$$

$$\frac{dv_c}{dt} e^{\frac{t}{RC}} + \frac{1}{RC} v_c e^{\frac{t}{RC}} = \frac{V_s}{RC} e^{\frac{t}{RC}} \qquad \frac{d}{dt}\left(v_c e^{\frac{t}{RC}}\right) = \frac{V_s}{RC} e^{\frac{t}{RC}}$$

$$\left(v_c e^{\frac{t}{RC}}\right) = c + \int \frac{V_s}{RC} e^{\frac{t}{RC}} dt = c + V_s e^{\frac{t}{RC}} \qquad v_c(t) = c e^{-\frac{t}{RC}} + V_s$$

$$v_c(0) = c + V_s = V_0 \qquad v_c(t) = (V_0 - V_s) e^{-\frac{t}{RC}} + V_s$$

Differential Equation (2)



$$RC \cdot \frac{dv_c}{dt} + v_c = V_s \qquad t \geq 0^+ \qquad v_c(0^+) = v_c(0^-) = V_0$$

$$\frac{dv_c}{dt} + \frac{1}{RC} v_c = \frac{1}{RC} V_s$$

$$\frac{dv_c}{dt} + \frac{1}{RC} v_c = 0$$

assume $v_c = Ae^{st}$

$$\frac{d}{dt}(Ae^{st}) + \frac{1}{RC} Ae^{st} = 0$$

$$\left(s + \frac{1}{RC}\right) Ae^{st} = 0 \qquad s = -\frac{1}{RC} \qquad v_c = Ae^{-\frac{t}{RC}}$$

homogeneous solution

$$\frac{dv_c}{dt} + \frac{1}{RC} v_c = \frac{V_s}{RC}$$

assume $v_c = c$

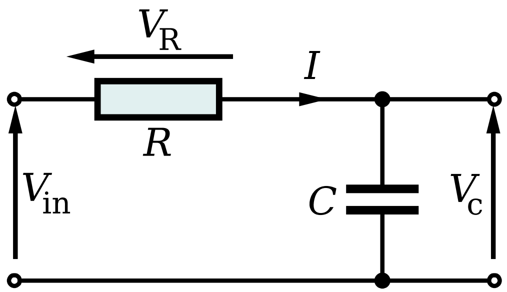
$$\frac{d}{dt}(c) + \frac{1}{RC} c = \frac{V_s}{RC} \qquad v_c = V_s$$

particular solution

$$v_c(t) = V_s + Ae^{-\frac{t}{RC}} \qquad v_c(0) = A + V_s = V_0 \qquad v_c(t) = (V_0 - V_s)e^{-\frac{t}{RC}} + V_s$$

complete solution

Differential Equation (3)



$$R i + \frac{1}{C}\int_{-\infty}^{t} i \, d\tau = V_s \qquad t \ge 0^+ \qquad v_c(0^+) = v_c(0^-) = V_0$$

$$i = C\frac{d v_c}{d t} \qquad RC \cdot \frac{d v_c}{d t} + v_c = V_s$$

$$(RC\,D + 1)v_c = V_s \qquad (RC\lambda + 1) = 0 \qquad \lambda = -\frac{1}{RC} \qquad v_{c0} = A e^{-\frac{t}{RC}}$$

$$v_{c0}(0) = A = V_0 \qquad v_{c0} = V_0 e^{-\frac{t}{RC}}$$

zero-input response

$$RC \cdot \dot{h}(t) + h(t) = \delta(t)$$

assume $v_c = c$

$$\frac{d}{dt}(c) + \frac{1}{RC}c = \frac{V_s}{RC} \qquad v_c = V_s$$

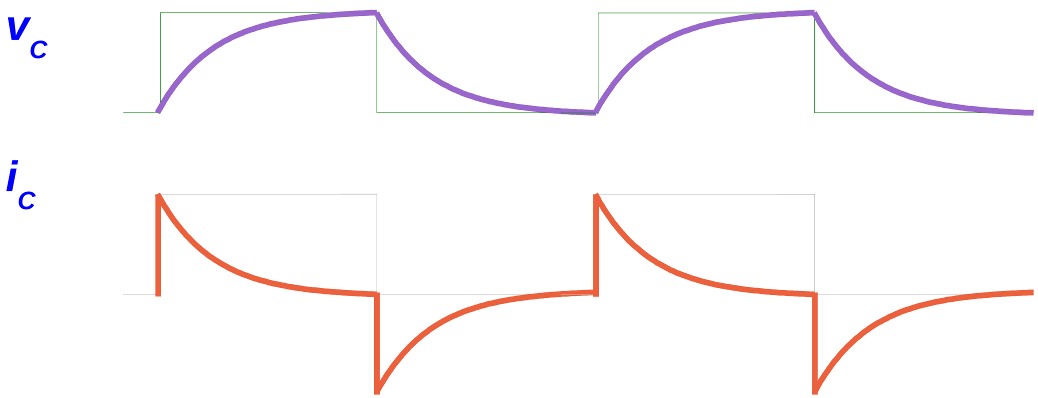
particular solution

$$\frac{d v_c}{d t} + \frac{1}{RC}v_c = \frac{V_s}{RC}$$

$$v_c(t) = V_s + A e^{-\frac{t}{RC}} \qquad v_c(t) = (V_0 - V_s)e^{-\frac{t}{RC}} + V_s$$

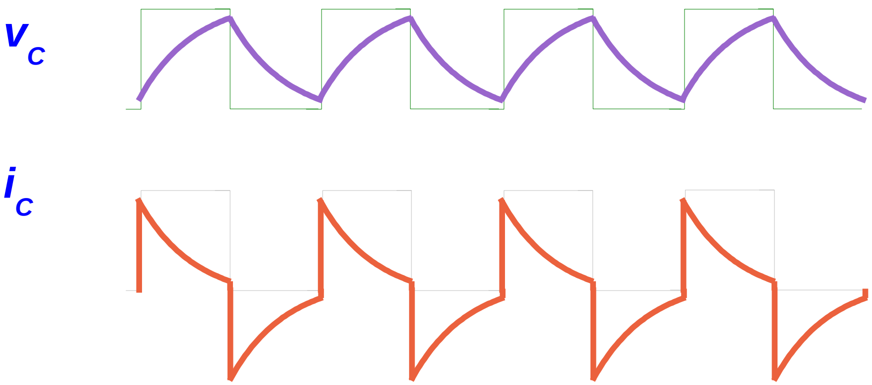
complete solution

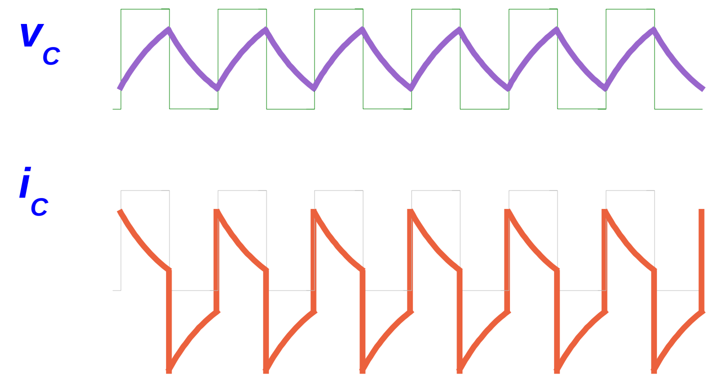
Pulse



$$i_C = C \frac{d v_C}{d t}$$

ω ↑ i_c ↑ X_c ↓





Phase Lags and Leads

References

[1] http://en.wikipedia.org/
[2] J.H. McClellan, et al., Signal Processing First, Pearson Prentice Hall, 2003